Concepts of Experiments at Future Colliders II

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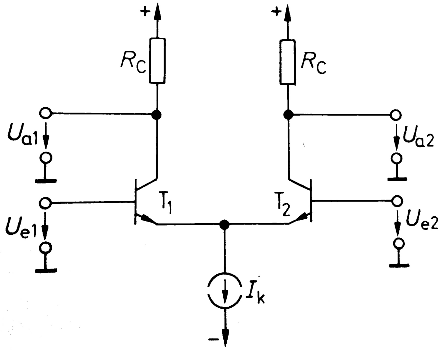
23.05.2025

Operation of a differential amplifier



- Constant current source at the emitter. $\Rightarrow dI_k = 0$.
- Internal resistance of the constant current source: r_k.

- $I_k = I_{C1} + I_{C2} \Rightarrow dI_{C1} = -dI_{C2}$.
- So $dU_{a1} = -dU_{a2}$.
- Also
  $dU_{e1} = dU_{BE1} = -dU_{BE2} = -dU_{e2}$.
- $U_D := U_{e1} - U_{e2}$.
  $dU_{e1} = d(U_{e1} - U_{e2} + U_{e2})$
  $= dU_D + dU_{e2} = dU_D - dU_{e1}$,
  thus $dU_D = \frac{1}{2} dU_{e1}$.

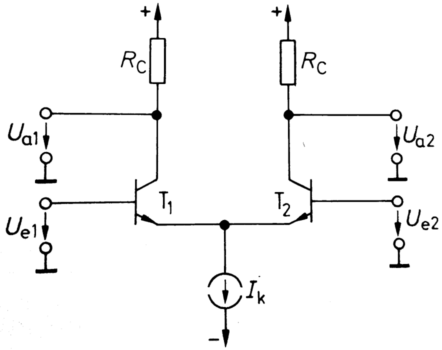
$\Rightarrow$ Differential amplification $A_D = \frac{dU_{a1}}{dU_D}$

$A_D = \frac{dU_{a1}}{2dU_{BE1}} = -\frac{1}{2} S(R_C || r_{CE})$.

Since S is large, A_D is also large.

Recapitulation of the previous lecture

Operation of a differential amplifier



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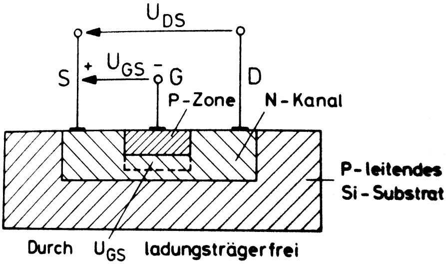
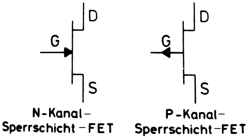
Besides the differential amplification, there is also a much smaller common-mode amplification $A_{CM} := \frac{dU_{a1}}{d(U_{e1}+U_{e2})/2} = -\frac{1}{2}\frac{R_C}{r_k}$, which immediately follows from the formula for the amplification of the emitter circuit with current feedback.

Alternative to bip. transistors: field-effect transistors
Construction of an n-channel junction field-effect transistor

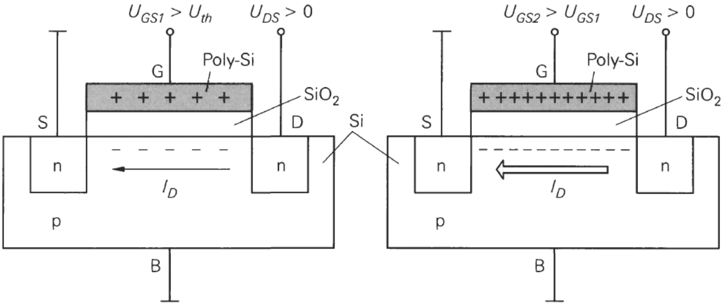


S: Source.
G: Gate.
D: Drain.



- Control of the size of the charge carrier-free zone via the value of the voltage U_{GS}.
- Thickness of the charge carrier-free zone determines the resistance between drain and source.
- Advantage of field-effect transistors over bipolar transistors: Lower power consumption, as the control is done via the applied electric field and not via a current.

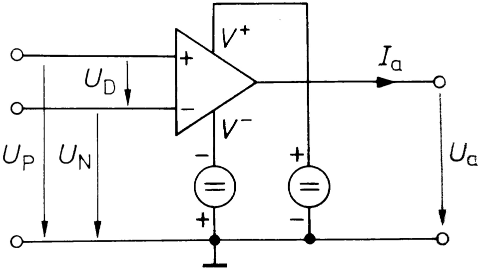
Metal-oxid-semiconductor field-effect transistor



- Structure forms a capacitor from gate terminal, dielectric, and bulk terminal.
- Application of positive voltage between gate and bulk charges the capacitor.
- Electric field causes migration of minority carriers (electrons in p-silicon) to the junction and recombination with majority carriers (defect electrons in p-silicon), known as depletion.
- Space charge region forms at the junction with negative space charge.
- At threshold voltage U_{th}, displacement of majority carriers becomes significant, limiting recombination.
- Accumulation of minority carriers results in near-inversion of p-doped substrate close to the oxide, known as strong inversion
- Increased gate voltage induces band bending of conduction and valence bands at the junction in band model.
- Fermi level shifts closer to the conduction band than the valence band, inverting the semiconductor material.
- Formed thin n-type conducting channel connects source and drain n-regions, allowing charge carriers to flow (almost) unimpeded from source to drain.

Operational amplifiers

- Operational amplifiers are broadband differential amplifiers with high gain and high input impedance.
- Operational amplifiers are available as integrated circuits made of bipolar and field-effect transistors.



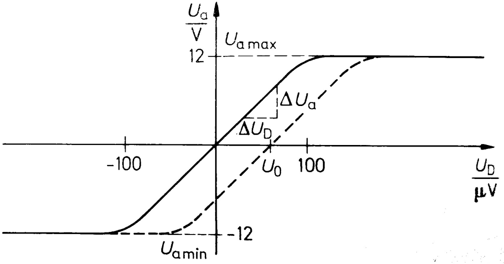
- Input stage designed as a differential amplifier, hence two inputs (+ and -).
- Positive and negative supply voltage required to drive the inputs and outputs positively and negatively.

- Open-loop gain:

$$A_D := \frac{dU_a}{dU_D}.$$

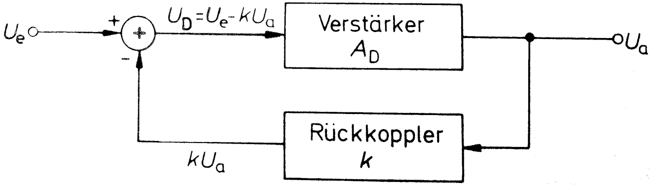
Recapitulation of the previous lecture

Characteristic of an operational amplifier



- Offset voltage U_0 adjustable in most operational amplifiers.
- Linear dependency of U_a on U_D in a small range of U_D around U_0.
- Constant output voltage outside of this range (amplifier saturation).

Principle of negative feedback

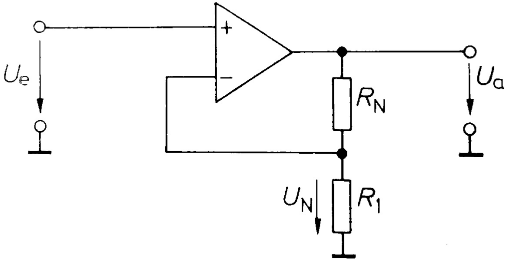


- $U_a = A_D(U_e - kU_a) \Leftrightarrow U_a = \frac{A_D}{1+kA_D} U_e \underset{A_D \to \infty}{\approx} \frac{1}{k} U_e$.
- $U_P = U_e$, $U_N = kU_a$, $|U_a|$¡const. Thus,

$$|U_P - U_N| = \frac{U_a}{A_D} \underset{A_D \to \infty}{\to} 0,$$

i.e., $\boxed{U_P = U_N}$.

Non-inverting amplifier

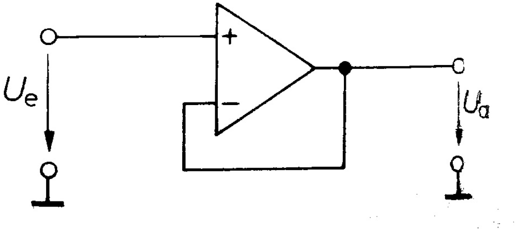


$$U_e = U_P = U_N = \frac{R_1}{R_1 + R_N} U_a$$

$$\Leftrightarrow \quad U_a = \left(1 + \frac{R_N}{R_1}\right) U_e .$$

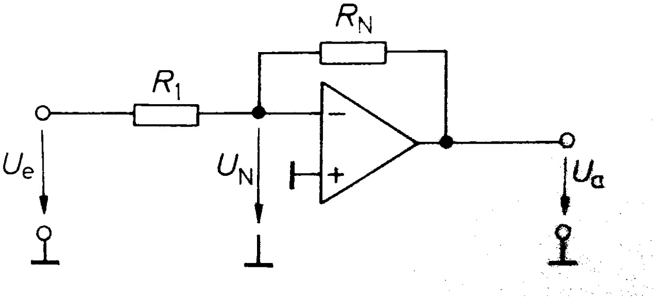
- Amplification is positive.
- Value of the amplification is fully determined by the choice of R_N and R_1.

Voltage follower



- $U_a = U_e$.
- Small output impedance, i.e., behaves like a voltage source.
- Use of this circuit as an impedance converter.

Inverting amplifier



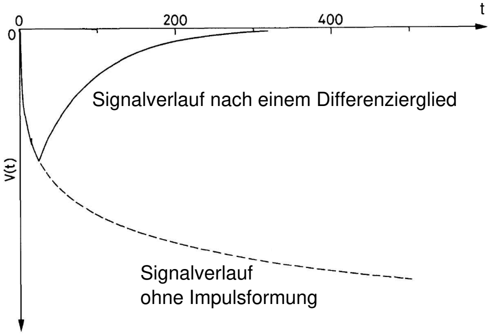
$$U_P = U_N = 0.$$

$$\Rightarrow \quad \underline{U_a} = R_N \cdot I_N = R_N(-I_1) = -R_N \frac{U_e}{R_1} = \underline{-\frac{R_N}{R_1} U_e}.$$

- Amplification is negative.
- Value of the amplification is fully determined by the choice of R_N and R_1.

Pulse shaping

Introductory Example: Signal Pulse of a Cylindrical Drift Tube

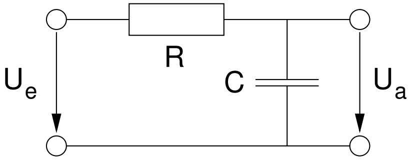


Pulse shaping with a differentiator

- Retains the information of the signal start time.
- Significantly reduces the dead time of the tube compared to the case without pulse shaping.

Low-pass and high-pass filters

Low-Pass

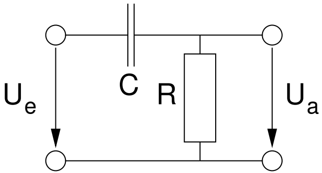


$$U_a = \frac{\frac{1}{i\omega C}}{R + \frac{1}{i\omega C}} U_e = \frac{1}{1 + i\omega RC} U_e.$$

$\omega \to 0$: $U_a \to U_e$.

$\omega \to \infty$: $U_a \to 0$.

High-Pass



$$U_a = \frac{R}{R + \frac{1}{i\omega C}} U_e = \frac{1}{1 + \frac{1}{i\omega RC}} U_e.$$

$\omega \to 0$: $U_a \to 0$.

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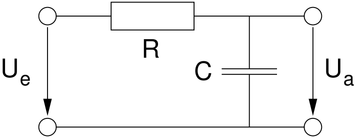
Recapitulation of the previous lecture

Low-pass and high-pass filters

Low-Pass



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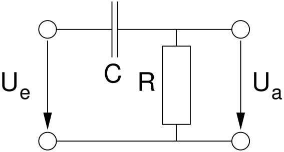
High-Pass



$$U_a = \frac{1}{1 + \frac{1}{i\omega RC}} U_e.$$

3dB Cutoff Frequency

$$\frac{1}{|1 + i\omega RC|^2} = \frac{1}{2} \Leftrightarrow \omega = \frac{1}{RC}.$$

$\omega \gg \frac{1}{RC}$: $U_a \approx \frac{1}{i\omega RC} U_e = \frac{\hat{U}_e(\omega)}{i\omega RC} e^{i\omega t}$, so
$U_a \approx \frac{1}{RC} \int U_e dt.$
Integrating above the cutoff frequency.

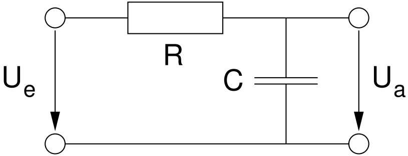
3dB Cutoff Frequency

$$\frac{1}{\left|1 + \frac{1}{i\omega RC}\right|^2} = \frac{1}{2} \Leftrightarrow \omega = \frac{1}{RC}$$

$\omega \ll \frac{1}{RC}$:
$U_a \approx i\omega RC U_e = i\omega RC \hat{U}_e e^{i\omega t}$, so
$U_a \approx RC \frac{dU_e}{dt}$.
Differentiating above the cutoff

Recapitulation of the previous lecture

Behaviour of a low pass filter



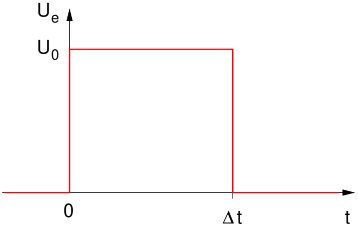
1st possibility: Use of complex impedances and a Fourier transformation from the frequency to the time domain.

2nd possibility: Solving the following differential equation.

$$U_a = \frac{Q}{C} \Rightarrow \frac{dU_a}{dt} = \frac{1}{C} I.$$

$$U_e = U_R + U_a = R \cdot I + U_a = RC\frac{dU_a}{dt} + U_a.$$

Low pass: behaviour with a rectangular pulse



$$U_e(t) = \begin{cases} U_0 & (t \in [0, \Delta t]), \\ 0 & \text{otherwise.} \end{cases}$$

$t \leq 0$: $0 = RC\frac{dU_a}{dt} + U_a$, hence $U_a = 0$.

$t \in (0, \Delta t)$: $$U_0 = RC\frac{dU_a}{dt} + U_a$$

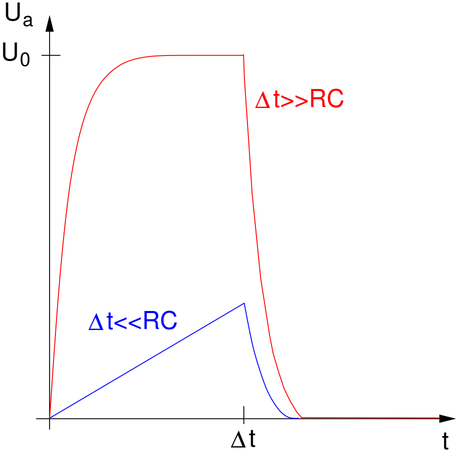
$$\Leftrightarrow \quad U_0 - U_a = RC\frac{dU_a}{dt} \quad \Leftrightarrow \quad \int_0^t \frac{1}{RC}dt' = \int_0^{U(t)} \frac{dU_a}{U_0 - U_a}$$

$$\Leftrightarrow \quad -\frac{t}{RC} = \ln\frac{U_0 - U_a(t)}{U_0} \quad \Leftrightarrow \quad e^{-\frac{1}{RC}t} = \frac{U_0 - U_a(t)}{U_0}$$

$$\Leftrightarrow \quad U_a(t) = U_0(1 - e^{-\frac{1}{RC}t}).$$

$t \geq \Delta t$: $\frac{dU_a}{dt} = -\frac{1}{RC}U_a$, hence $U_a(t) = U_a(\Delta t)e^{-\frac{1}{RC}(t-\Delta t)}$.

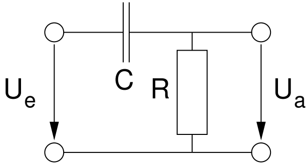
Low pass: behaviour with a rectangular pulse



$\Delta t \gg RC$: $U_a(t \to \Delta t - 0) \approx U_0$.

$\Delta t \ll RC$: $U_a(t) \approx U_0 \frac{t}{RC}$ for $t \in (0, \Delta t)$.

Behaviour of a high pass filter



$$U_a = R \cdot I = RC\frac{d(U_e - U_a)}{dt} = RC\frac{dU_e}{dt} - RC\frac{dU_a}{dt}.$$

Choose U_e as before, as a rectangular pulse.

$t \leq 0$: $U_a(t) = 0$.

$t \in (0, \Delta t)$: $U_a(t) = -RC\frac{dU_a}{dt}$, hence $U_a(t) = U_a(0)e^{-\frac{t}{RC}} = U_0 e^{-\frac{t}{RC}}$.

$\epsilon \to 0 + 0$: $t \in [\Delta t, \Delta t + \epsilon)$: $U_e(t) = U_0\left(1 - \frac{t-\Delta t}{\epsilon}\right)$, hence $\frac{dU_e}{dt} = -\frac{U_0}{\epsilon}$.

$$U_a + \frac{RC}{\epsilon}U_0 = -RC\frac{dU_a}{dt}$$

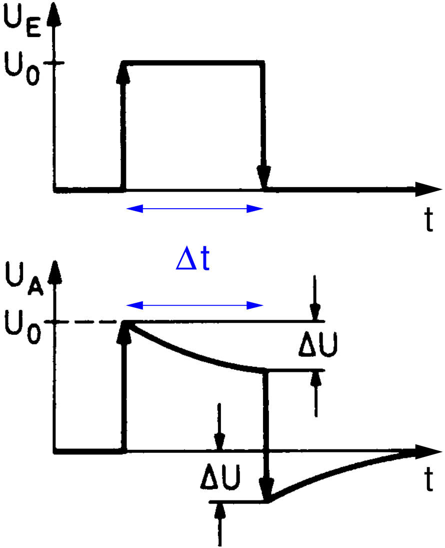
$$\Leftrightarrow \quad \epsilon U_a + RCU_0 = -\epsilon RC\frac{dU_a}{dt}$$

$$\underset{\epsilon\to 0}{\Leftrightarrow} \quad U_0 = -\epsilon\frac{dU_a}{dt}, \; U_0\epsilon = -\epsilon\left[U_a(\Delta t + \epsilon) - U_a(\Delta t)\right]$$

$$\Leftrightarrow \quad U_a(\Delta t + \epsilon) = U_a(\Delta t) - U_0 = U_0\left(e^{-\frac{\Delta t}{RC}} - 1\right)$$

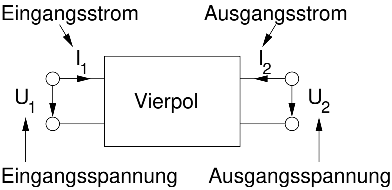
$t > \Delta t$: $U_a(t) = U_0\left(e^{-\frac{\Delta t}{RC}} - 1\right)e^{-\frac{t-\Delta t}{RC}}$.

Low pass: behaviour with a rectangular pulse



Bipolar pulse shaping possible with a high pass.

Four-pole equations



Low-pass, high-pass, and similar circuits with a total of four connections are called four-poles. Using so-called four-pole equations, one can easily calculate the behavior of circuits composed of many four-poles.

Two of the four quantities are freely selectable, the other two depend on these. For example, $U_1 = U_1(I_1, I_2)$, $U_2 = U_2(I_1, I_2)$.

$$
\begin{aligned}
dU_1 &= \left.\frac{\partial U_1}{\partial I_1}\right|_{I_2} dI_1 + \left.\frac{\partial U_1}{\partial I_2}\right|_{I_1} dI_2, \\
dU_2 &= \left.\frac{\partial U_2}{\partial I_1}\right|_{I_2} dI_1 + \left.\frac{\partial U_2}{\partial I_2}\right|_{I_1} dI_2.
\end{aligned}
$$

If the four-pole consists only of linear, passive components, then even $\frac{\partial U_k}{\partial I_\ell} = \frac{U_k}{I_\ell}$ holds.

Chains of four-poles

For the calculation of the behavior of a chain of four-poles, the chain form is useful, where the input or output variables are expressed as functions of the output or input variables:

$$
\begin{aligned}
dU_1 &= \left.\frac{\partial U_1}{\partial U_2}\right|_{I_2} dU_2 + \left.\frac{\partial U_1}{\partial I_2}\right|_{U_2} dI_2, \\
dI_1 &= \left.\frac{\partial I_1}{\partial U_2}\right|_{I_2} dU_2 + \left.\frac{\partial I_1}{\partial I_2}\right|_{U_2} dI_2.
\end{aligned}
$$

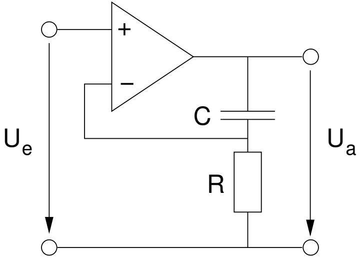
$$
d\begin{pmatrix} U_1 \\ I_1 \end{pmatrix} = A \cdot d\begin{pmatrix} U_2 \\ I_2 \end{pmatrix}.
$$

To obtain the behavior of a four-pole consisting of a chain of four-poles, one only needs to multiply the production of the matrices A_k of the individual four-poles with each other.

Pulse shaping with low and high pass filters

For pulse shaping of detector signals, one connects low and high pass filters of different time constants (RC) in series. To separate the passes, an operational amplifier with capacitive coupling of the signals can be used.

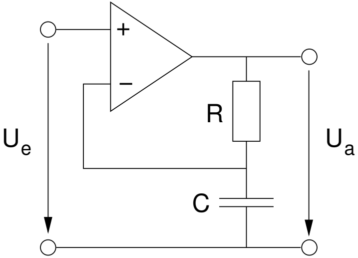
Low Pass



$$U_a = \left(1 + \frac{1}{i\omega RC}\right) U_e.$$

Amplification of low frequencies.

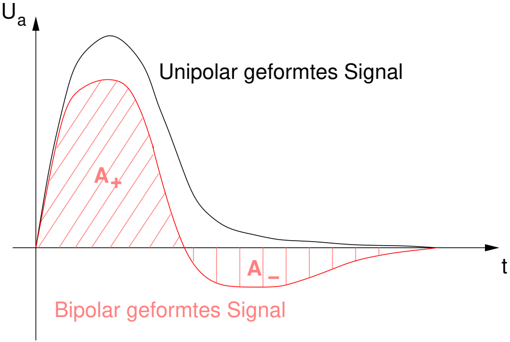
High Pass



$$U_a = (1 + i\omega RC)\, U_e.$$

Amplification of higher frequencies.

Unipolar and bipolar pulse shaping



$$|A_+| = |A_-|$$

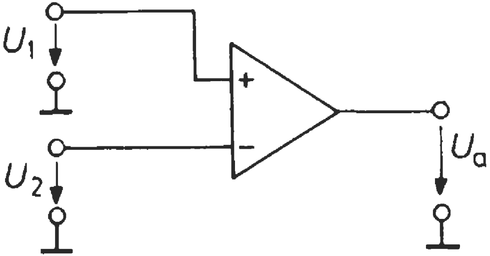
Disadvantage of unipolar signal shapes: Drift of the pulse baseline due to the superposition of successive pulses at high signal rates.

Remedy for this problem: Use of bipolar pulse shaping, which on average does not shift the pulse baseline.

From analog to digital signals

Operational amplifiers as comparators

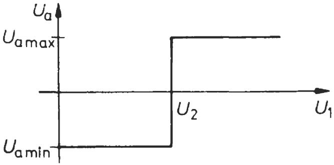
- An operational amplifier saturates when $|U_P - U_N|$ exceeds a small range of values.
- Comparators are operational amplifiers where this range has been chosen very small.



In the ideal case:

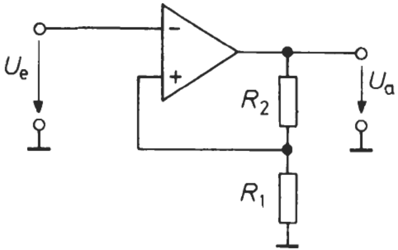
$$U_a = \begin{cases} U_{a,\text{max}} & \text{for } U_1 > U_2, \\ U_{a,\text{min}} & \text{for } U_1 < U_2. \end{cases}$$

Characteristic curve:



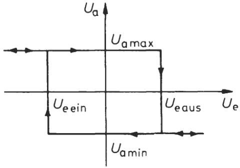
- A Schmitt trigger is a comparator where the turn-on and turn-off levels do not coincide.
- A comparator saturates when $U_P \neq U_N$.

Inverting Schmitt Trigger



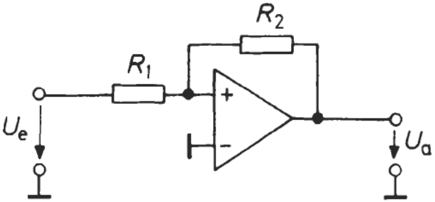
- Turn-on level: $U_{e,\text{on}} = \frac{R_1}{R_1+R_2} U_{a,\text{min}}$.
- Turn-off level: $U_{e,\text{off}} = \frac{R_1}{R_1+R_2} U_{a,\text{max}}$.
- The difference between turn-on and turn-off levels is called the hysteresis.

Transfer characteristic:

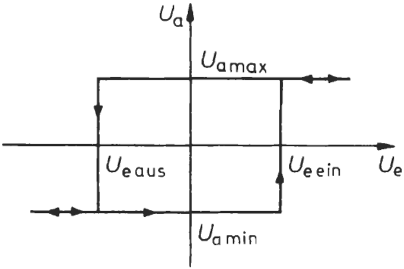


- Turn-on level: $U_{e,\text{on}} = -\frac{R_1}{R_2} U_{a,\text{min}}$.
- Turn-off level: $U_{e,\text{off}} = -\frac{R_1}{R_2} U_{a,\text{max}}$.

Circuit



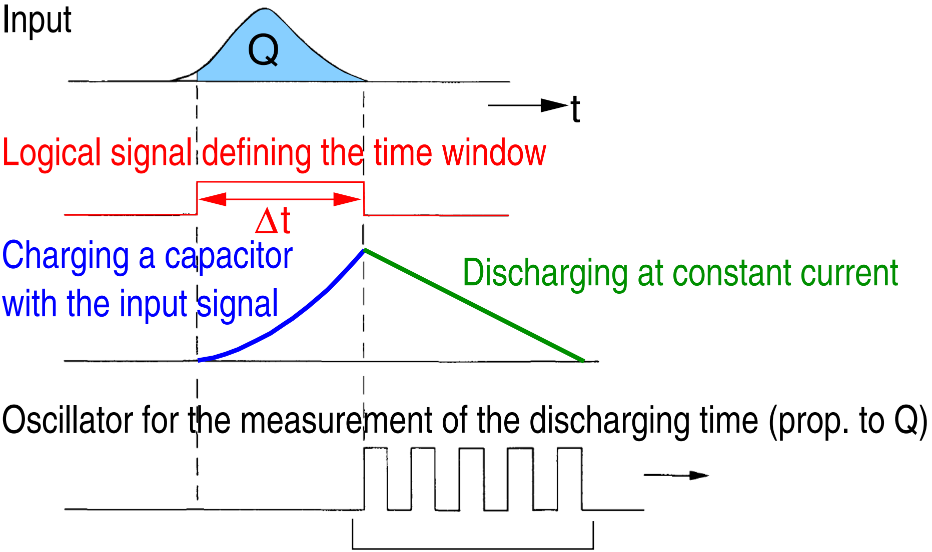
Transfer characteristic:



Two basic types of analog-to-digital converters are distinguished.

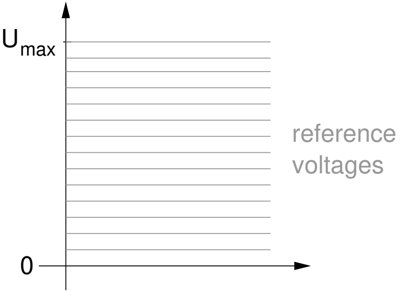
- Charge-sensing analog-to-digital converter
  Measurement of
  $$Q := \int_{t_0}^{t_0+\Delta t} I(t)dt$$
  and conversion of the measured value into an integer.
- Amplitude sensing analog-to-digital converter
  Measurement of the peak value of a signal $U(t)$ in the interval $[t_0, t_0 + \Delta t]$ and conversion of the measured value into an integer.

Wilkinson's method for charge measurement



Number of oscillations=direct measure of the discharging time

Weighing method for signal amplitude measurement



Division of the dynamic range of the analog-to-digital converter into a series of comparison voltages.
Conversion of the results of the voltage comparisons into a bit pattern.

Time-to-Digital Converter (TDC)

Analog signal $\rightarrow$ Comparator $\rightarrow$ Logic signal $\rightarrow$ Time measurement

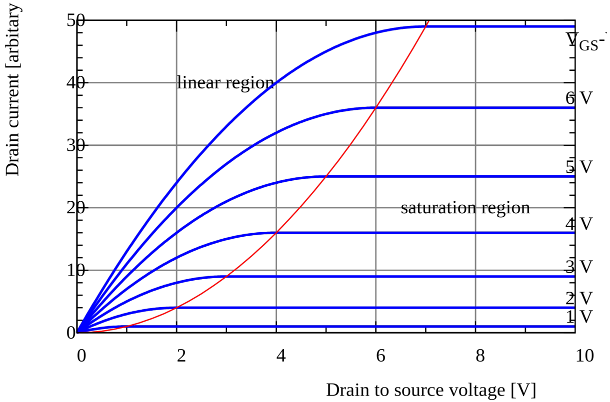
Simplest approach to time measurement

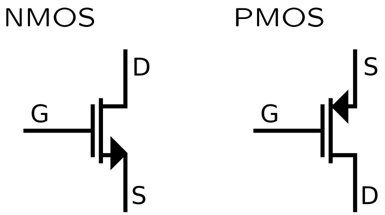
- Clock generator with a period T smaller than the desired time measurement accuracy.
- Continuous counting of clock cycles. Use a counter with n bits such that $2^n \cdot T >$(time intervals to be measured).
- Record at which clock cycles n_{Start} and n_{Stop} the start and stop signals have arrived.
  $t_{Start} - t_{Stop}$ is then measured as $n_{Start} - n_{Stop}$.
  If the counter overflows, one must use $n_{Start} - n_{Stop} + 1$.

Components for processing digital/logical signals

Logic families

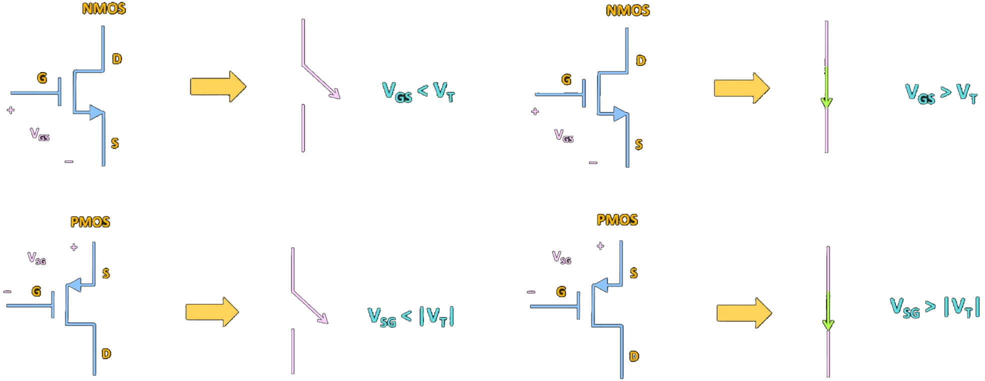
- As mentioned earlier there are different definitions of logic signal levels related to different so-called “logic families”.
- Still in use today (or “popular”):
  - Transistor-transistor logic (TTL) using bipolar transistors.
  - Emitter coupled logic (ECL) using bipolar transistors.
  - Complementary metal oxide semicondutor logic (CMOS) using MOSFETs.



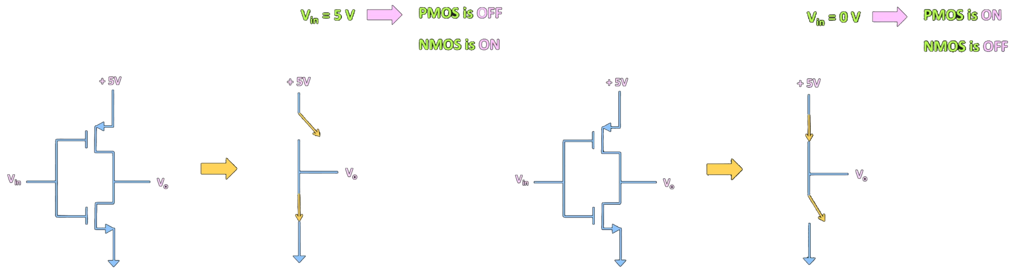


- MOSFETs are operated in saturation mode for logic gates.

MOSFETs as switches

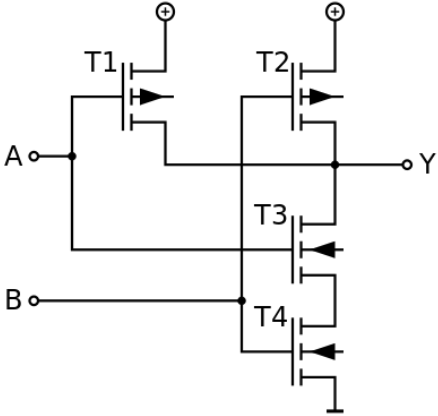


CMOS inverter

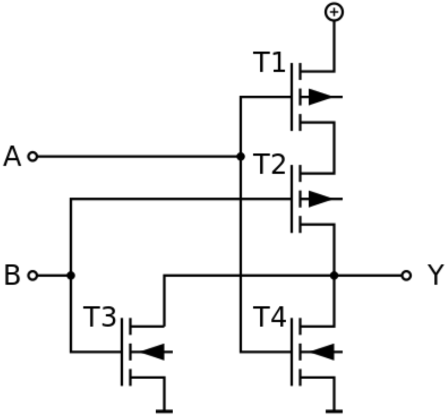


CMOS NAND and NOR

$Y = \overline{A\ AND\ B}$



$Y = \overline{A\ OR\ B}$



Two States: logical 0 and logical 1.

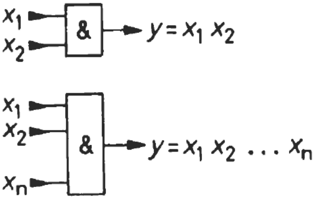
Logical Basic Functions

- Conjunction: $y = x_1 \wedge x_2 = x_1 \cdot x_2 = x_1 x_2$.
- Disjunction: $y = x_1 \vee x_2 = x_1 + x_2$.
- Negation: $y = \bar{x}$.

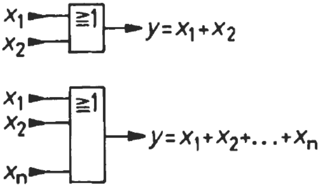
Rules of Calculation

Kommutatives Gesetz: $x_1x_2 = x_2x_1$	$x_1 + x_2 = x_2 + x_1$
Assoziatives Gesetz: $x_1(x_2x_3) = (x_1x_2)x_3$	$x_1 + (x_2 + x_3) = (x_1 + x_2) + x_3$
Distributives Gesetz: $x_1(x_2 + x_3) = x_1x_2 + x_1x_3$	$x_1 + x_2x_3 = (x_1+x_2)(x_1+x_3)$
Absorptionsgesetz: $x_1(x_1 + x_2) = x_1$	$x_1 + x_1x_2 = x_1$
Tautologie: $xx = x$	$x + x = x$
Gesetz für die Negation $x\overline{x} = 0$	$x + \overline{x} = 1$
Doppelte Negation: $\overline{(\overline{x})} = x$	
De Morgans Gesetz: $\overline{x_1x_2} = \overline{x}_1 + \overline{x}_2$	$\overline{x_1 + x_2} = \overline{x}_1\overline{x}_2$
Operationen mit 0 und 1: $x \cdot 1 = x$, $x \cdot 0 = 0$, $\overline{0} = 1$	$x + 0 = x$, $x + 1 = 1$, $\overline{1} = 0$

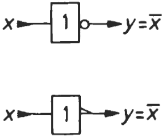
Conjunction
AND Gate



Disjunction
OR Gate



Negation
NOT Gate



Method of disjunctive normal form

To establish more complex logical functions, one can use the so-called disjunctive normal form.

n input variables $x_1, \ldots, x_n$. 1 output variable y.

1. Set up a table listing all possible input values along with the desired output value. This table is also called a truth table.
2. Identify all rows in the truth table where $y = 1$.
3. For each of these rows, form the conjunction of all input variables; for $x_k = 1$, substitute x_k, otherwise $\bar{x}_k$.
4. The sought function is obtained by forming the disjunction of all found product terms.

Example of exclusive OR

Truth Table

Row	x_1	x_2	y	
1	1	1	0	
2	1	0	1	$\rightarrow x_1 \cdot \bar{x}_2 =: K_2$
3	0	1	1	$\rightarrow \bar{x}_1 \cdot x_2 =: K_3$
4	0	0	0	

Result

$$y = K_2 + K_3 = (x_1 \cdot \bar{x}_2) + (\bar{x}_1 \cdot x_2).$$