From **Perturbative Unitarity** to **CP Violation:**

Exploring **Extended Scalar Sectors**

Carolina T. Lopes

IMPRS Recruiting Workshop

Outline of the talk

Motivation

Perturbative Unitarity for Models with Singlet and Doublet Scalars

Probing New Physics via CP Violation in hVV' Couplings

Conclusions

Motivation

- The Standard Model (SM) is highly successful but does not address certain phenomena
- Many Beyond-Standard-Model (BSM) theories introduce **extra scalar fields** to address these limitations

Motivation

Theoretical Consistency

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Unitarity → **Probability Conservation**

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SM	**2HDM**	**SM + S**
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Unitarity → **Probability Conservation**

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SM | **2HDM** | **SM + S** | **...**

But there was still no framework that worked for **any number of singlets and doublets...**

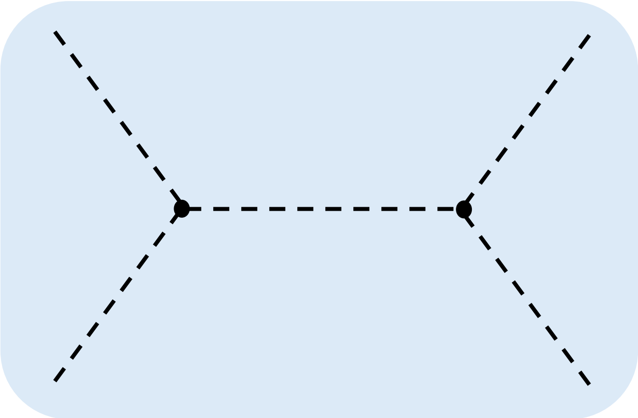
→ **done** by **Carolina T. Lopes, André Milagre and João P. Silva,** ***arXiv:2510.02434v2 [hep-ph]***

Perturbative Unitarity

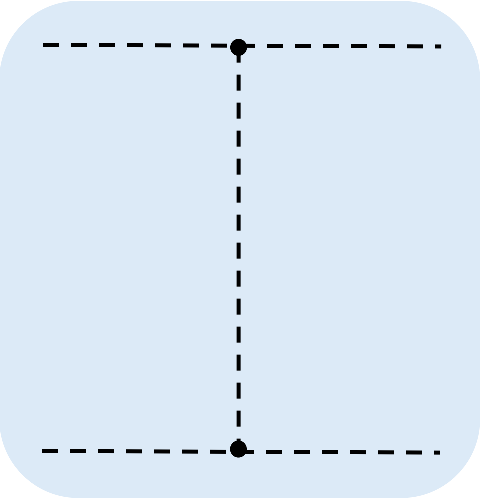
$2 \to 2$ *scattering*

Let's consider a $2 \to 2$ scattering process between complex scalar fields, with flavour indices a, b, c, d,

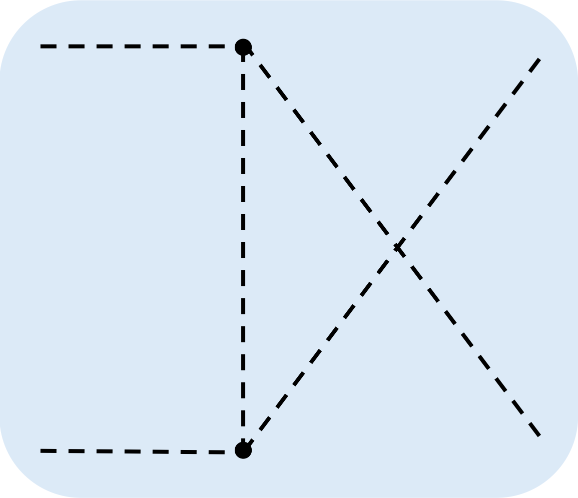
$$A_a B_b \to C_c D_d$$



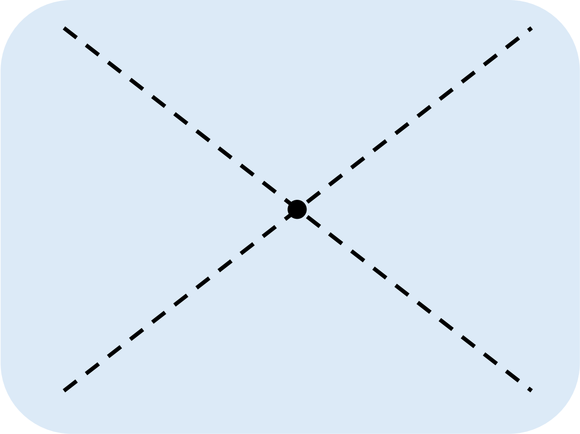
s - channel



t - channel



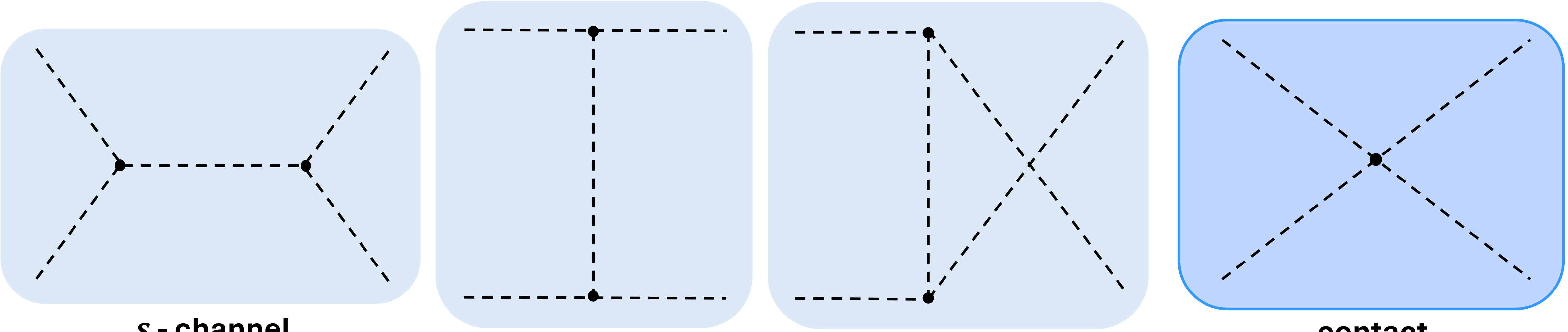
u - channel



contact

Perturbative Unitarity

Amplitude of the 2 $\rightarrow$ 2 scattering processes



s - channel

t - channel

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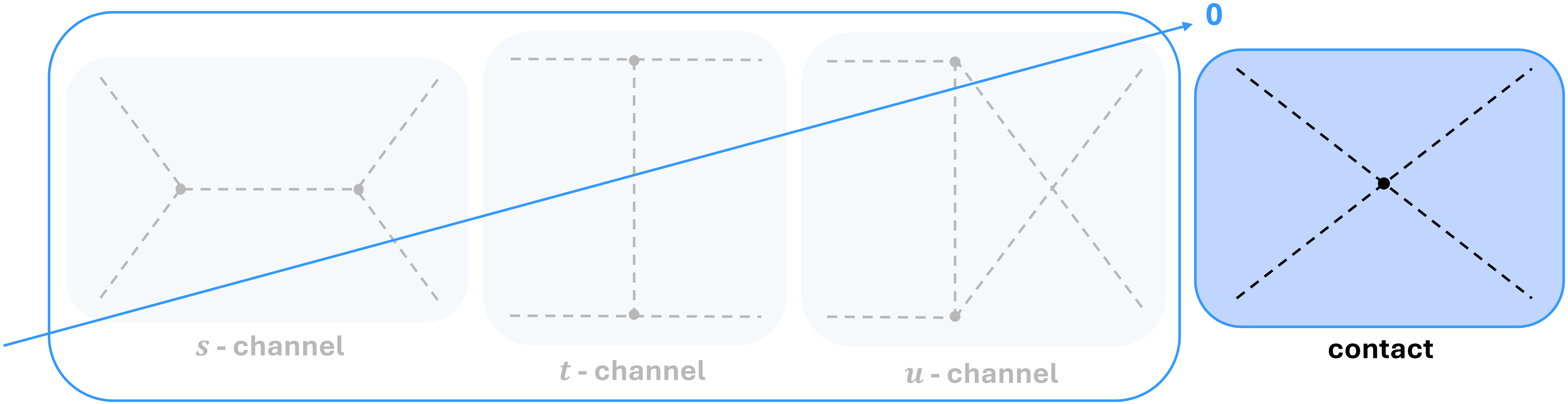
contact

High-energy limit

$\mathrm{s}, \mathrm{t}, \mathrm{u} \rightarrow \infty$

Perturbative Unitarity

Amplitude of the $2 \rightarrow 2$ *scattering processes*



High-energy limit

$s, t, u \rightarrow \infty$

Amplitude – tree-level & high-energy limit

Consequently, in the **high-energy limit**, only the **quartic interactions** involving the external scalars contribute to the tree-level amplitude

$$\mathcal{M}_{A_a B_b \to C_c D_d} = -\frac{\partial^4 V_4}{\partial A_a \partial B_b \partial C_c^* \partial D_d^*}$$

Perturbative Unitarity

Partial wave decomposition

The amplitude can be written as a sum over angular momentum components:

$$\mathcal{M}(\cos\theta) = 16\pi \sum_{J=0}^{\infty} a_J (2J + 1) P_J(\cos\theta)$$

Partial Wave Expansion

J - Total orbital angular momentum of the final state;
θ - Scattering angle;

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Partial Waves

Perturbative Unitarity

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Perturbative Unitarity

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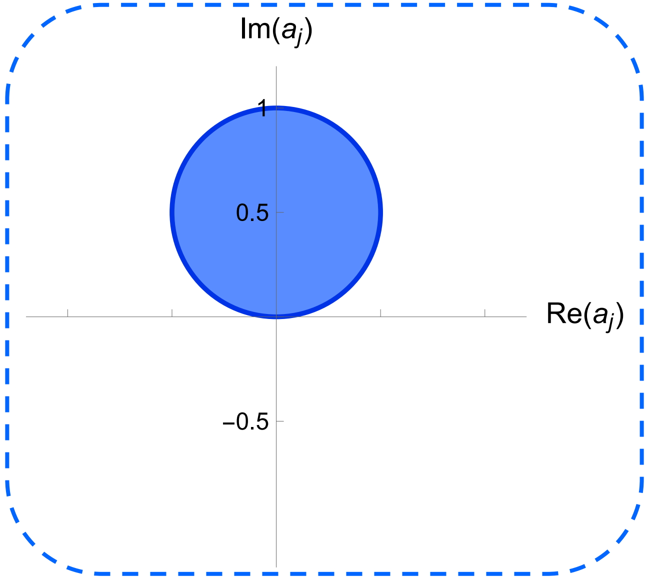

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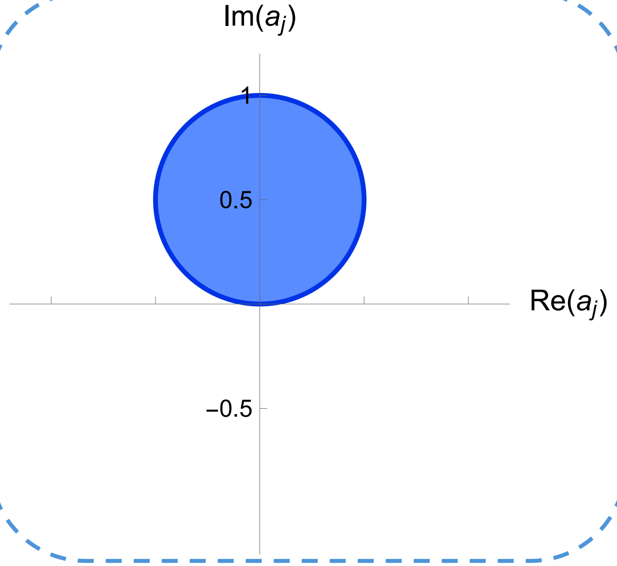

$$|a_J| \leq 1$$

$$|\mathrm{Re}\{a_J\}| \leq \frac{1}{2}$$

$$0 \leq \mathrm{Im}\{a_J\} \leq 1$$

Perturbative Unitarity

Partial wave decomposition

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Tree-level $\Longrightarrow$ $a_J \in \mathbb{R}$

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Perturbative Unitarity

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Perturbative Unitarity

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Tree-level unitarity $\Longrightarrow$ $|a_J| \leq \frac{1}{2}$

- In the **high-energy** limit, $\mathcal{M}(\cos\theta)$ is independent of $\theta \Longrightarrow$ $a_J = \frac{2J+1}{32\pi} \mathcal{M} \int_{-1}^{1} P_J(\cos\theta)\, d\cos\theta$
- Given that $P_0(\cos\theta) = 1$ and $\int_{-1}^{1} P_m(\cos\theta) P_n(\cos\theta)\, d\cos\theta = \frac{2}{2n+1}\delta_{mn} \Longrightarrow$ $\boldsymbol{a_0}$ gives the strongest bound

Perturbative Unitarity

Unitarity Bounds

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$$16\pi|a_0| = \left| N_{ab} N_{cd} \frac{\partial^4 V_4}{\partial A_a \partial B_b \partial C_c^* \partial D_d^*} \right| \leq 8\pi$$

$$N_{ij} \equiv \frac{1}{\sqrt{2^{\delta_{ij}}}}$$

Symmetry factor for identical particles

Perturbative Unitarity

The Model

$SU(2)_L \times U(1)_Y$

$\Phi_i = \left(\phi_i^+, \phi_i^0\right)^T, \qquad i = 1, \ldots, \boldsymbol{n_D}$

$SU(2)$ doublets with $Y = \frac{1}{2}$

$\varphi_i^+, \qquad i = 1, \ldots, \boldsymbol{n_c}$

$SU(2)$ singlets with $Y = 1$

$\chi_i\,, \qquad i = 1, \ldots, \boldsymbol{n_n}$

$SU(2)$ singlets with $Y = 0$

For $2 \to 2$ scattering, in the **high-energy limit**, the most general renormalizable **quartic part** of the scalar potential:

$$
\begin{aligned}
V \supset V_4 = {} & \lambda_{ab,cd}\left(\Phi_a^\dagger \Phi_b\right)\left(\Phi_c^\dagger \Phi_d\right) + \alpha_{ab,cd}(\varphi_a^- \varphi_b^+)(\varphi_c^- \varphi_d^+) + \beta_{ab,cd}(\chi_a \chi_b)(\chi_c \chi_d) \\
& + \delta_{ab,cd}(\Phi_a^\dagger \Phi_b)(\varphi_c^- \varphi_d^+) + \gamma_{ab,cd}(\Phi_a^\dagger \Phi_b)(\chi_c \chi_d) + \zeta_{ab,cd}\,(\varphi_a^- \varphi_b^+)(\chi_c \chi_d) \\
& + \kappa_{ab,cd}\,(\Phi_a^T \sigma_2 \Phi_b)(\varphi_c^- \chi_d) + \kappa_{ab,cd}^*\,\left(\Phi_b^\dagger \sigma_2 \Phi_a^*\right)(\varphi_c^+ \chi_d)
\end{aligned}
$$

Conserved Quantum Numbers

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$$

- Quantum numbers corresponding to the symmetries of the underlying theory remain **conserved**
- Therefore, we can constrain the possible initial and final states
  - **Coupled-channel analysis ⇒ coupled-channel matrix**
  - **Unitarity bound:** largest eigenvalue $\leq 8\pi$

Our approach aims to take advantage of all three conserved quantities: $\boldsymbol{Q, Y, T}$

Perturbative Unitarity

Basis of States, using $\boldsymbol{Q}, \boldsymbol{Y}$ and $\boldsymbol{T}$

It is sufficient to apply partial-wave unitarity bounds to the scattering matrices constructed from the states listed in this table:

$\lvert \boldsymbol{Q}, \boldsymbol{Y}, \boldsymbol{T} \rangle$	**State**	**Conditions**	**Dimensionality**
$\lvert 2, 2, 0 \rangle$	$\varphi_i^+ \varphi_j^+$	$i \leq j$	$n_c(n_c + 1)/2$
$\left\lvert 2, \frac{3}{2}, \frac{1}{2} \right\rangle$	$\phi_i^+ \varphi_j^+$	-	$n_D n_c$
$\lvert 2, 1, 1 \rangle$	$\phi_i^+ \phi_j^+$	$i \leq j$	$n_D(n_D + 1)/2$
$\lvert 1, 1, 0 \rangle$	$\{\phi_{[i}^+ \phi_{j]}^0,\ \varphi_i^+ \chi_j\}$	$\{i < j, -\}$	$n_D(n_D - 1)/2 + n_n n_c$
$\left\lvert 1, \frac{1}{2}, \frac{1}{2} \right\rangle$	$\{\phi_i^+ \chi_j,\ \phi_i^{0*} \varphi_j^+\}$	-	$n_D(n_n + n_c)$
$\lvert 1, 0, 1 \rangle$	$\phi_i^+ \phi_j^{0*}$	-	n_D^2
$\lvert 0, 0, 0 \rangle$	$\{\Phi_i \Phi_j^*,\ \varphi_i^+ \varphi_j^-,\ \chi_i \chi_j\}$	$\{-, -, i \leq j\}$	$n_D^2 + n_c^2 + n_n(n_n + 1)/2$

Perturbative Unitarity

Mathematica Notebook: ***BounDS***

To automate the generation of scattering matrices, compute their eigenvalues, and ensure these values remain below 8π, we have developed a `Mathematica` notebook, **BounDS**

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The **user** simply inputs:

Step 1 Specify the values n_D, n_c, and n_n

e.g.

```
nD = 2; nc = 0; nn = 0;
```

Step 2 Additional flavor and/or generalized CP symmetries (abelian and non-abelian)

e.g.

```
nSym = 1; Sym[1] = {Φ[1] → Φ[1], Φ[2] → Exp[Ia] Φ[2]};
```

Perturbative Unitarity

Mathematica Notebook: ***BounDS***

To automate the generation of scattering matrices, compute their eigenvalues, and ensure these values remain below 8π, we have developed a `Mathematica` notebook, **BounDS**

BounDS outputs:

The **quartic part** of the scalar potential and the **scattering matrices**

e.g.

$$(\boldsymbol{\Phi}_1{}^{\dagger}\cdot\boldsymbol{\Phi}_1)\,\lambda_{11,11} + (\boldsymbol{\Phi}_2{}^{\dagger}\cdot\boldsymbol{\Phi}_2)\,\lambda_{22,22} + 2\,(\boldsymbol{\Phi}_1{}^{\dagger}\cdot\boldsymbol{\Phi}_1)\,(\boldsymbol{\Phi}_2{}^{\dagger}\cdot\boldsymbol{\Phi}_2)\,\lambda_{11,22} + 2\,(\boldsymbol{\Phi}_1{}^{\dagger}\cdot\boldsymbol{\Phi}_2)\,(\boldsymbol{\Phi}_2{}^{\dagger}\cdot\boldsymbol{\Phi}_1)\,\lambda_{12,21}$$

e.g.

$$(\phi_1{}^{+}\cdot\phi_1{}^{+} \quad \phi_1{}^{+}\cdot\phi_2{}^{+} \quad \phi_2{}^{+}\cdot\phi_2{}^{+})$$

$$\begin{pmatrix} 2\lambda_{11,11} & 0 & 0 \\ 0 & 2\,(\lambda_{11,22} + \lambda_{12,21}) & 0 \\ 0 & 0 & 2\lambda_{22,22} \end{pmatrix}$$

Closed-form expressions for the eigenvalues of the scattering matrices whenever possible

e.g.

$2\lambda_{11,11}$

$2\,(\lambda_{11,22} + \lambda_{12,21})$

$2\lambda_{22,22}$

Perturbative Unitarity

Unitarity Bounds in the **Standard Model**

Recall:

$$16\pi|a_0| = \left| N_{ab} N_{cd} \frac{\partial^4 V_4}{\partial A_a \partial B_b \partial C_c^* \partial D_d^*} \right| \leq 8\pi$$

The **largest eigenvalue** will give the strongest bound

$$|6\lambda_{11,11}| \leq 8\pi \rightarrow |\lambda_{11,11}| \leq \frac{4\pi}{3}$$

In the SM:

$$\lambda_{11,11} = \frac{M_H^2}{2v^2}$$

Perturbative Unitarity

*Unitarity Bounds in the **Standard Model***

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$$\lambda_{11,11} = \frac{M_H^2}{2v^2} \rightarrow M_H \leq \sqrt{\frac{8\pi}{3}}v \sim \mathbf{712\ GeV}$$

The well-known bound on the Higgs mass!

CP Violation

Probing New Physics via CP Violation in hVV' Couplings

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  - ***Direct probes*** of the CP structure of the coupling in htt production at ATLAS and CMS

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  - Sensitivity is driven primarily by ***CP-odd observables*** involving final-state polarizations

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  - Sensitivity is driven primarily by ***CP-odd observables*** involving final-state polarizations

Our interest lies in what current and proposed ***indirect searches*** can reveal about a ***CP-odd htt coupling***

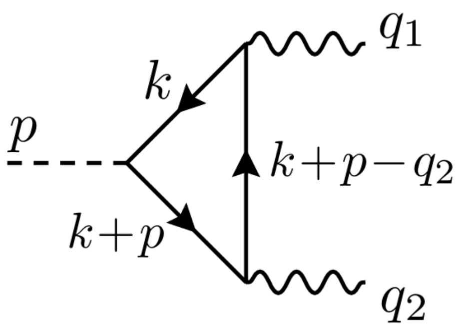
CP Violation

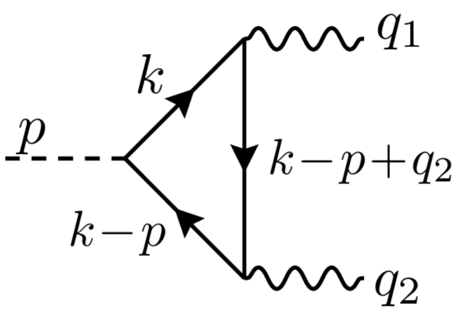
CP Violation in $h \to ZZ, h \to Z\gamma$ and $h \to \gamma\gamma$

- Assuming the CP Violating contribution arises from a ***complex top Yukawa coupling:***

$$\mathcal{L}_{Yuk} \supset \frac{m_t}{v} \overline{\psi_t} \left(\kappa_t + i\gamma_5 \tilde{\kappa}_t\right) \psi_t$$

- We can extract the CP- odd part of the amplitude corresponding to the following processes:





For all three decay channels, assuming a top in the loop

CP Violation

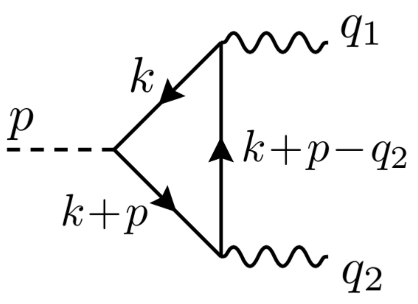
CP Violation in $h \to ZZ, h \to Z\gamma$ and $h \to \gamma\gamma$

The amplitude of each diagram can be written as

$$\mathcal{M}_i^{\mu\nu} = \int \frac{d^4k}{(2\pi)^4} \frac{N_i^{\mu\nu}}{D_0 D_1 D_2}$$

where $i = 1, 2$ labels the two diagrams and

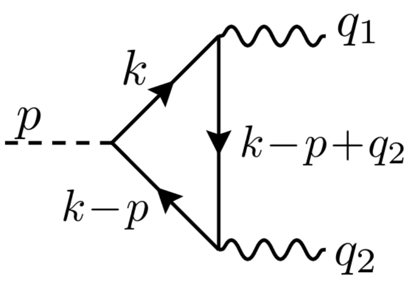
$$D_0 = k^2 - m^2$$
$$D_1 = (k+p)^2 - m^2$$
$$D_2 = (k+p-q_2)^2 - m^2$$



1st Diagram

$$D_0 = k^2 - m^2$$
$$D_1 = (k+p)^2 - m^2$$
$$D_2 = (k+p-q_2)^2 - m^2$$

if $k \to -k$ in $N_2^{\mu\nu}$



2nd Diagram

CP Violation

CP Violation in $h \to ZZ, h \to Z\gamma$ and $h \to \gamma\gamma$

We will express our results in terms of the ***scalar Passarino-Veltman functions***

$$B_0(r_{10}^2, m_0^2, m_1^2) = (2\pi\mu)^\epsilon \frac{i}{\pi^2} \int d^d k \, \frac{1}{(k^2 - m_0^2)\left((k+r_1)^2 - m_1^2\right)}$$

$$C_0(r_{10}^2, r_{12}^2, r_{20}^2, m_0^2, m_1^2, m_2^2) = (2\pi\mu)^\epsilon \frac{i}{\pi^2} \int d^d k \, \frac{1}{(k^2 - m_0^2)\left((k+r_1)^2 - m_1^2\right)\left((k+r_2)^2 - m_2^2\right)}$$

$$r_{ij} = \left(r_i - r_j\right)^2 \quad \forall i,j = 0, \dots, n-1$$

$$r_1 = p = q_1 + q_2, \; r_2 = p - q_2 = q_1$$

CP Violation

Amplitude in $h \rightarrow ZZ$

$$\mathcal{M}^{\mu\nu}(odd) = \frac{2i m_t^2 M_Z^2 \tilde{\kappa}_t}{\pi^2 v^3 (M_H^2 - 4M_Z^2)} \epsilon^{\mu\nu\alpha\beta} q_{1\alpha} q_{2\beta} \times$$
$$\times \left[4 g_A^2 \left[B_0(M_H^2, m_t^2, m_t^2) - B_0(M_Z^2, m_t^2, m_t^2)\right] + \left(g_A^2 M_H^2 - g_V^2 (M_H^2 - 4M_Z^2)\right) C_0(M_Z^2, M_Z^2, M_H^2, m_t^2, m_t^2, m_t^2)\right]$$

Amplitude in $h \rightarrow Z\gamma$

$$\mathcal{M}^{\mu\nu}(odd) = -\frac{ie\, g_V\, m_t^2\, M_Z\, Q_t\, \tilde{\kappa}_t}{\pi^2 v^2} \epsilon^{\mu\nu\alpha\beta} q_{1\alpha} q_{2\beta}\, C_0(0, M_Z^2, M_H^2, m_t^2, m_t^2, m_t^2)$$

Amplitude in $h \rightarrow \gamma\gamma$

$$\mathcal{M}^{\mu\nu}(odd) = -\frac{ie^2\, m_t^2\, Q_t^2\, \tilde{\kappa}_t}{2\pi^2\, v} \epsilon^{\mu\nu\alpha\beta} q_{1\alpha} q_{2\beta}\, C_0(0,0, M_H^2, m_t^2, m_t^2, m_t^2)$$

CP Violation

Heavy-top limit

$$x_t = 4m_t^2/M_H^2$$
$$y_t = 4m_t^2/M_Z^2$$

CP Violation

Heavy-top limit

$h \to ZZ$

$$\mathcal{M}^{\mu\nu}\,(\mathrm{odd}) \simeq -\frac{i\,M_Z^2\,\tilde{\kappa}_t}{\pi^2 v^3 (M_H^2 - 4M_Z^2)}\,\epsilon^{\mu\nu\alpha\beta}\,q_{1\alpha}q_{2\beta}$$
$$\times \left[4g_A^2\left[\frac{M_H^2 - M_Z^2}{3} + \frac{M_H^4 - M_Z^4}{30\,m_t^2}\right] - \left(g_A^2 M_H^2 - g_V^2(M_H^2 - 4M_Z^2)\right)\left(1 + \frac{M_H^2}{12m_t^2} + \frac{M_Z^2}{6m_t^2}\right)\right]$$

$h \to Z\gamma$

$$\mathcal{M}^{\mu\nu}_{(\mathrm{odd})} \simeq \frac{ie\,g_V\,M_Z\,Q_t\,\tilde{\kappa}_t}{2\pi^2 v^2}\,\epsilon^{\mu\nu\alpha\beta}\,q_{1\alpha}q_{2\beta}\left(1 + \frac{M_H^2}{12m_t^2} + \frac{M_Z^2}{12m_t^2}\right)$$

$h \to \gamma\gamma$

$$\mathcal{M}^{\mu\nu}_{(\mathrm{odd})} \simeq \frac{ie^2 Q_t^2\,\tilde{\kappa}_t}{2\pi^2 v}\,\epsilon^{\mu\nu\alpha\beta}\,q_{1\alpha}q_{2\beta}\left(1 + \frac{M_H^2}{12m_t^2}\right)$$

Conclusions

- Scalar extensions are theoretically well-motivated and widely studied
- Consistency checks, like ***perturbative unitarity,*** are crucial

→ **First Project:** **Carolina T. Lopes, André Milagre and João P. Silva,** *arXiv:2510.02434v2 [hep-ph]*

- Extended scalar sectors naturally allow ***CP violation***
- Complex couplings, such as a top Yukawa, generate loop-level CP-odd effects

→ **Second Project:** with **Yosef Nir, João P. Silva**

- Higgs decays provide a clean window to test these new physics signatures

THANK YOU!

EXTRA SLIDES

Motivation

Requirements

For any number of SU(2) singlet and doublet scalars:

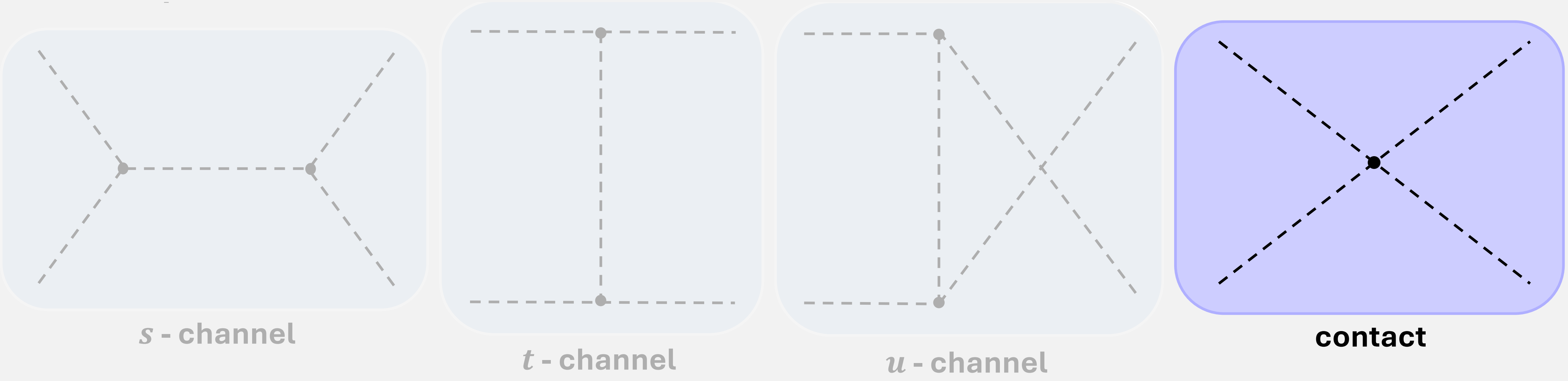
Vacuum stability → **case-by-case**

Boundedness from below → **case-by-case**

S, T, U oblique parameters → **done** by **W. Grimus, L. Lavoura, O.M. Ogreid and P. Osland**, *arXiv:0802.4353v1* ***[hep-ph]***

Partial-Wave Unitarity Bounds

Amplitude



$$i\mathcal{M} = i\mathcal{M}_s + i\mathcal{M}_t + i\mathcal{M}_u + i\mathcal{M}_c$$

SM and @ tree-level

High-energy limit ($s \gg M_h^2$) → 0

$$\mathcal{M}_s \propto \frac{M_h^2}{M_Z^2}\frac{1}{s - M_h^2} \quad \mathcal{M}_t \propto \frac{M_h^2}{M_Z^2}\frac{1}{t - M_h^2} \quad \mathcal{M}_u \propto \frac{M_h^2}{M_Z^2}\frac{1}{u - M_h^2} \quad \mathcal{M}_c \propto \frac{M_h^2}{M_Z^2}$$

The Model

Potential

$SU(2)_L \times U(1)_Y$

For 2 → 2 scattering, in the high-energy limit, we want to build the most general renormalizable quartic part of the scalar potential :

$SU(2)_L$ invariant | $U(1)_Y$ invariant

$\Phi_i \sim \left(2, \frac{1}{2}\right)$ | $\Phi_i^\dagger \sim \left(\bar{2}, -\frac{1}{2}\right)$ | $\varphi_i^+ \sim (1,1)$ | $\varphi_i^- \sim (1,-1)$ | $\chi_i \sim (1,0)$

$2 \otimes 2 = 3 \oplus 1$ → $\Phi_i^T \sigma_2 \Phi_j + h.c.$ | $Y = \pm 1$ | $SU(2)_L$ invariant but **not** $U(1)_Y$ invariant

$\bar{2} \otimes 2 = 3 \oplus 1$ → $\Phi_i^\dagger \Phi_j$ | $Y = 0$ | $SU(2)_L \times U(1)_Y$ invariant

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$\varphi_i^+ \varphi_j^-$ | $Y = 0$ | $SU(2)_L \times U(1)_Y$ invariant

$\varphi_i^+ \chi_j + h.c.$ | $Y = \pm 1$ | $SU(2)_L$ invariant but **not** $U(1)_Y$ invariant

$\chi_i \chi_j$ | $Y = 0$ | $SU(2)_L \times U(1)_Y$ invariant

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$\Phi_i^T \sigma_2 \Phi_j + h.c.$	$Y = \pm 1$	$SU(2)_L$ invariant but **not** $U(1)_Y$ invariant
$\varphi_i^+ \chi_j + h.c.$	$Y = \pm 1$	$SU(2)_L$ invariant but **not** $U(1)_Y$ invariant

The Model

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$\Phi_i^T \sigma_2 \Phi_j + h.c.$	$Y = \pm 1$	$SU(2)_L$ invariant but **not** $U(1)_Y$ invariant
$\varphi_i^- \chi_j + h.c.$	$Y = \mp 1$	$SU(2)_L$ invariant but **not** $U(1)_Y$ invariant

↓

$(\Phi_i^T \sigma_2 \Phi_j)(\varphi_i^- \chi_j) + h.c.$	$Y = 0$	$SU(2)_L \times U(1)_Y$ invariant

The Model

Potential

$SU(2)_L \times U(1)_Y$

For 2 → 2 scattering, in the high-energy limit, we want to build the most general renormalizable quartic part of the scalar potential :

$\Phi_i \sim \left(2, \frac{1}{2}\right)$ $\Phi_i^\dagger \sim \left(\bar{2}, -\frac{1}{2}\right)$ $\varphi_i^+ \sim (1,1)$ $\varphi_i^- \sim (1,-1)$ $\chi_i \sim (1,0)$

$\Phi_i^\dagger \Phi_j \Phi_k^\dagger \Phi_l$ $\Phi_i^\dagger \Phi_j \, \varphi_k^+ \varphi_l^-$ $\varphi_i^+ \varphi_j^- \varphi_k^+ \varphi_l^-$ $\Phi_i^\dagger \Phi_j \chi_k \chi_l$ $\varphi_i^+ \varphi_j^- \chi_k \chi_l$ $\chi_i \chi_j \chi_k \chi_l$

$(\Phi_i^T \sigma_2 \Phi_j)(\varphi_i^- \chi_j) + h.c.$

$SU(2)_L \times U(1)_Y$ invariant

The Model

Hermiticity

$$V \supset V_4 = \lambda_{ab,cd}(\Phi_a^\dagger \Phi_b)(\Phi_c^\dagger \Phi_d) + \alpha_{ab,cd}(\varphi_a^- \varphi_b^+)(\varphi_c^- \varphi_d^+) + \beta_{ab,cd}(\chi_a \chi_b)(\chi_c \chi_d)$$
$$+ \delta_{ab,cd}(\Phi_a^\dagger \Phi_b)(\varphi_c^- \varphi_d^+) + \gamma_{ab,cd}(\Phi_a^\dagger \Phi_b)(\chi_c \chi_d) + \zeta_{ab,cd}(\varphi_a^- \varphi_b^+)(\chi_c \chi_d)$$
$$+ \kappa_{ab,cd}(\Phi_a^T \sigma_2 \Phi_b)(\varphi_c^- \chi_d) + \kappa^*_{ab,cd}(\Phi_b^\dagger \sigma_2 \Phi_a^*)(\varphi_c^+ \chi_d)$$

- Not all couplings are independent,

Hermiticity + Field Swap $\Longrightarrow$

$$\lambda_{ab,cd} = \lambda^*_{ba,dc} = \lambda_{cd,ab}$$
$$\alpha_{ab,cd} = \alpha^*_{ba,dc} = \alpha_{cd,ab} = \alpha_{ad,cb}$$
$$\beta_{ab,cd} = \beta^*_{(ab,cd)} = \beta_{(ab,cd)}$$
$$\delta_{ab,cd} = \delta^*_{ba,dc}$$
$$\gamma_{ab,cd} = \gamma^*_{ba,cd} = \gamma_{ab,dc}$$
$$\zeta_{ab,cd} = \zeta^*_{ba,cd} = \zeta_{ab,dc}$$
$$\kappa_{ab,cd} = -\kappa_{ba,cd}$$

Any permutation

The Model

Hermiticity

$$
\begin{aligned}
V \supset V_4 = {} & \lambda_{ab,cd}\left(\Phi_a^\dagger \Phi_b\right)\left(\Phi_c^\dagger \Phi_d\right) + \alpha_{ab,cd}(\varphi_a^- \varphi_b^+)(\varphi_c^- \varphi_d^+) + \beta_{ab,cd}(\chi_a \chi_b)(\chi_c \chi_d) \\
& + \delta_{ab,cd}(\Phi_a^\dagger \Phi_b)(\varphi_c^- \varphi_d^+) + \gamma_{ab,cd}(\Phi_a^\dagger \Phi_b)(\chi_c \chi_d) + \zeta_{ab,cd}(\varphi_a^- \varphi_b^+)(\chi_c \chi_d) \\
& + \kappa_{ab,cd}(\Phi_a^T \sigma_2 \Phi_b)(\varphi_c^- \chi_d) + \kappa_{ab,cd}^* \left(\Phi_b^\dagger \sigma_2 \Phi_a^*\right)(\varphi_c^+ \chi_d)
\end{aligned}
$$

- Not all couplings are independent,

Hermiticity + Field Swap $\Longrightarrow$

$$
\begin{aligned}
\lambda_{ab,cd} &= \lambda_{ba,dc}^* = \lambda_{cd,ab} \\
\alpha_{ab,cd} &= \alpha_{ba,dc}^* = \alpha_{cd,ab} = \alpha_{ad,cb} \\
\beta_{ab,cd} &= \beta_{(ab,cd)}^* = \beta_{(ab,cd)} \\
\delta_{ab,cd} &= \delta_{ba,dc}^* \\
\gamma_{ab,cd} &= \gamma_{ba,cd}^* = \gamma_{ab,dc} \\
\zeta_{ab,cd} &= \zeta_{ba,cd}^* = \zeta_{ab,dc} \\
\kappa_{ab,cd} &= -\kappa_{ba,cd}
\end{aligned}
$$

Any permutation

$\Longrightarrow$

$$
\begin{gathered}
\lambda_{aa,bb},\ \lambda_{ab,ba} \in \mathbb{R} \\
\alpha_{aa,bb} = \alpha_{ab,ba} \in \mathbb{R} \\
\beta_{ab,cd} \in \mathbb{R} \\
\delta_{aa,bb} \in \mathbb{R} \\
\gamma_{aa,cd} \in \mathbb{R} \\
\zeta_{aa,cd} \in \mathbb{R} \\
\kappa_{aa,cd} = 0
\end{gathered}
$$

The Model

Basis of States, first using only $\boldsymbol{Q}$ and $\boldsymbol{Y}$

$\vert\boldsymbol{Q},\boldsymbol{Y}\rangle$	State	Conditions	Dimensionality
$\vert\mathbf{2},\mathbf{2}\rangle$	$\varphi_i^+\,\varphi_j^+$	$i \leq j$	$n_c(n_c+1)/2$
$\left\vert\mathbf{2},\frac{\mathbf{3}}{\mathbf{2}}\right\rangle$	$\phi_i^+\varphi_j^+$	-	$n_D n_c$
$\vert\mathbf{2},\mathbf{1}\rangle$	$\phi_i^+\phi_j^+$	$i \leq j$	$n_D(n_D+1)/2$
$\left\vert\mathbf{1},\frac{\mathbf{3}}{\mathbf{2}}\right\rangle$	$\phi_i^0\varphi_j^+$	-	$n_D n_c$
$\vert\mathbf{1},\mathbf{1}\rangle$	$\{\phi_i^+\phi_j^0,\ \varphi_i^+\chi_j\}$	-	$n_D^2+n_n n_c$
$\left\vert\mathbf{1},\frac{\mathbf{1}}{\mathbf{2}}\right\rangle$	$\{\phi_i^+\chi_j,\ \phi_i^{0*}\varphi_j^+\}$	-	$n_D(n_n+n_c)$
$\vert\mathbf{1},\mathbf{0}\rangle$	$\phi_i^+\phi_j^{0*}$	-	n_D^2
$\vert\mathbf{0},\mathbf{1}\rangle$	$\phi_i^0\phi_j^0$	$i \leq j$	$n_D(n_D+1)/2$
$\left\vert\mathbf{0},\frac{\mathbf{1}}{\mathbf{2}}\right\rangle$	$\{\phi_i^0\chi_j,\ \phi_i^-\varphi_j^+\}$	-	$n_D(n_n+n_c)$
$\vert\mathbf{0},\mathbf{0}\rangle$	$\{\phi_i^+\phi_j^-,\ \phi_i^0\phi_j^{0*},\ \varphi_i^+\varphi_j^-,\ \chi_i\chi_j\}$	$\{-,-,-,i \leq j\}$	$2n_D^2+n_c^2+n_n(n_n+1)/2$

The Model

Basis of States, first using only $\boldsymbol{Q}$ and $\boldsymbol{Y}$

$$\mathcal{M}_{A_a B_b \to C_c D_d} = -\frac{\partial^4 V_4}{\partial A_a \partial B_b \partial C_c^* \partial D_d^*} \rightarrow$$

$$\mathcal{M}[\phi_a^+ \varphi_b^+ \to \phi_c^+ \varphi_d^+] = \mathcal{M}[\phi_a^0 \varphi_b^+ \to \phi_c^0 \varphi_d^+] = \delta_{ca,db}$$
$$\mathcal{M}[\phi_a^+ \phi_b^+ \to \phi_c^+ \phi_d^+] = \mathcal{M}\left[\phi_a^0 \phi_b^0 \to \phi_c^0 \phi_d^0\right] = 2\lambda_{ca,db} + 2\lambda_{da,cb}$$
$$\mathcal{M}[\phi_a^+ \chi_b \to \phi_c^+ \chi_d] = \mathcal{M}[\phi_a^0 \chi_b \to \phi_c^0 \chi_d] = 2\gamma_{ca,bd}$$
$$\mathcal{M}[\phi_a^+ \chi_b \to \phi_c^{0*} \varphi_d^+] = \mathcal{M}[\phi_a^0 \chi_b \to \phi_c^- \varphi_d^+] = 2i\kappa_{ca,db}$$
$$\mathcal{M}[\phi_a^{0*} \varphi_b^+ \to \phi_c^{0*} \varphi_d^+] = \mathcal{M}[\phi_a^- \varphi_b^+ \to \phi_c^- \varphi_d^+] = \delta_{ac,bd}$$

- Not all scattering amplitudes are independent ⇒ perturbative unitarity bounds from scattering involving $\left|\mathbf{1}, \frac{3}{2}\right\rangle$, $|\mathbf{0}, \mathbf{1}\rangle$ and $\left|\mathbf{0}, \frac{1}{2}\right\rangle$ are redundant, since they are identical to the ones derived for $\left|\mathbf{2}, \frac{3}{2}\right\rangle$, $|\mathbf{2}, \mathbf{1}\rangle$ and $\left|\mathbf{1}, \frac{1}{2}\right\rangle$

The Model

Basis of States, using $\boldsymbol{Q}, \boldsymbol{Y}$ and $\boldsymbol{T}$

It is sufficient to apply partial-wave unitarity bounds to the scattering matrices constructed from the states listed in this table:

$\lvert \boldsymbol{Q}, \boldsymbol{Y}, \boldsymbol{T} \rangle$	**State**	**Conditions**	**Dimensionality**
$\lvert \mathbf{2}, \mathbf{2}, \mathbf{0} \rangle$	$\varphi_i^+ \varphi_j^+$	$i \leq j$	$n_c(n_c+1)/2$
$\left\lvert \mathbf{2}, \frac{\mathbf{3}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}} \right\rangle$	$\phi_i^+ \varphi_j^+$	-	$n_D n_c$
$\lvert \mathbf{2}, \mathbf{1}, \mathbf{1} \rangle$	$\phi_i^+ \phi_j^+$	$i \leq j$	$n_D(n_D+1)/2$
$\lvert \mathbf{1}, \mathbf{1}, \mathbf{0} \rangle$	$\{\phi_{[i}^+ \phi_{j]}^0,\ \varphi_i^+ \chi_j\}$	$\{i < j, -\}$	$n_D(n_D-1)/2 + n_n n_c$
$\left\lvert \mathbf{1}, \frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}} \right\rangle$	$\{\phi_i^+ \chi_j,\ \phi_i^{0*} \varphi_j^+\}$	-	$n_D(n_n + n_c)$
$\lvert \mathbf{1}, \mathbf{0}, \mathbf{1} \rangle$	$\phi_i^+ \phi_j^{0*}$	-	n_D^2
$\lvert \mathbf{0}, \mathbf{0}, \mathbf{0} \rangle$	$\{\Phi_i \Phi_j^*,\ \varphi_i^+ \varphi_j^-,\ \chi_i \chi_j\}$	$\{-, -, i \leq j\}$	$n_D^2 + n_c^2 + n_n(n_n+1)/2$

Scattering Matrices and Eigenvalues

Recall:

$$16\pi|a_0| = \left|N_{ab}N_{cd}\frac{\partial^4 V_4}{\partial A_a \partial B_b \partial C_c^* \partial D_d^*}\right| \leq 8\pi$$

Matrix Element

$\left|2, \frac{3}{2}, \frac{1}{2}\right\rangle$

$$16\pi\, a_0[\phi_a^+\varphi_b^+ \to \phi_c^+\varphi_d^+] = \delta_{ca,db}$$

$|1, 0, 1\rangle$

$$16\pi\, a_0[\phi_a^+\phi_b^{0*} \to \phi_c^+\phi_d^{0*}] = 2\lambda_{ca,bd}$$

$|2, 2, 0\rangle$

$$16\pi\, a_0[\varphi_a^+\varphi_b^+ \to \varphi_c^+\varphi_d^+] = 4\, N_{ab}N_{cd}\,\alpha_{ca,db}$$

$|2, 1, 1\rangle$

$$16\pi\, a_0[\phi_a^+\phi_b^+ \to \phi_c^+\phi_d^+] = 2N_{ab}N_{cd}\left(\lambda_{ca,db} + \lambda_{da,cb}\right)$$

$\left|1, \frac{1}{2}, \frac{1}{2}\right\rangle$

$$16\pi\, a_0[\phi_a^{0*}\varphi_b^+ \to \phi_c^{0*}\varphi_d^+] = \delta_{ac,db}$$
$$16\pi\, a_0[\phi_a^+\chi_b \to \phi_c^+\chi_d] = 2\,\gamma_{ca,bd}$$
$$16\pi\, a_0[\phi_a^+\chi_b \to \phi_c^{0*}\varphi_d^+] = 2i\,\kappa_{ca,db}$$

$|0, 0, 0\rangle$

$$16\pi\, a_0[\varphi_a^+\varphi_b^- \to \varphi_c^+\varphi_d^-] = 4\,\alpha_{ba,cd}$$
$$16\pi\, a_0[\chi_a\chi_b \to \chi_c\chi_d] = 24\, N_{ab}N_{cd}\,\beta_{ab,cd}$$
$$16\pi\, a_0[\varphi_a^+\varphi_b^- \to \chi_c\chi_d] = 2N_{cd}\,\zeta_{ba,cd}$$
$$16\pi\, a_0[\Phi_a\Phi_b^* \to \varphi_c^+\varphi_d^-] = \sqrt{2}\,\delta_{ba,cd}$$
$$16\pi\, a_0[\Phi_a\Phi_b^* \to \chi_c\chi_d] = 2\sqrt{2}\,N_{cd}\,\gamma_{ba,cd}$$
$$16\pi\, a_0[\Phi_a\Phi_b^* \to \Phi_c\Phi_d^*] = 4\lambda_{ba,cd} + 2\lambda_{ca,bd}$$

$|1, 1, 0\rangle$

$$16\pi\, a_0[\phi_{[a}^+\phi_{b]}^0 \to \phi_{[c}^+\phi_{d]}^0] = 2\lambda_{ca,db} - 2\lambda_{dacb}$$
$$16\pi\, a_0[\varphi_a^+\chi_b \to \varphi_c^+\chi_d] = 2\,\zeta_{ca,bd}$$
$$16\pi\, a_0[\phi_{[a}^+\phi_{b]}^0 \to \varphi_c^+\chi_d] = 2\sqrt{2}\,i\,\kappa_{ba,cd}$$

Unitarity Bounds in the Standard Model

Recall:

$$16\pi|a_0| = \left|N_{ab}N_{cd}\frac{\partial^4 V_4}{\partial A_a \partial B_b \partial C_c^* \partial D_d^*}\right| \leq 8\pi$$

Matrix Element

Scattering Matrices and Eigenvalues:

$M_{\vert 2,1,1\rangle}$	$2\lambda_{11,11}$
$M_{\vert 1,0,1\rangle}$	$2\lambda_{11,11}$
$M_{\vert 0,0,0\rangle}$	$6\lambda_{11,11}$

→ The **largest eigenvalue** will give the strongest bound

$$|6\lambda_{11,11}| \leq 8\pi \rightarrow |\lambda_{11,11}| \leq \frac{4\pi}{3}$$

Unitarity Bounds in the Standard Model

Recall:

$$16\pi|a_0| = \left|N_{ab}N_{cd}\frac{\partial^4 V_4}{\partial A_a \partial B_b \partial C_c^* \partial D_d^*}\right| \leq 8\pi$$

Matrix Element

The **largest eigenvalue** will give the strongest bound

$$|6\lambda_{11,11}| \leq 8\pi \rightarrow |\lambda_{11,11}| \leq \frac{4\pi}{3}$$

In the SM:

$$\lambda_{11,11} = \frac{M_H^2}{2v^2} \rightarrow M_H \leq \sqrt{\frac{8\pi}{3}}v \sim \mathbf{712\ GeV}$$

The well-known bound on the Higgs mass!

Unitarity Bounds in the Standard Model

Recall:

$$16\pi|a_0| = \left|N_{ab}N_{cd}\frac{\partial^4 V_4}{\partial A_a \partial B_b \partial C_c^* \partial D_d^*}\right| \leq 8\pi$$

Matrix Element

In the 2HDM

$\|Q,Y,T\rangle$	State	Dimensionality
$\|2,1,1\rangle$	$\{\phi_1^+\phi_1^+, \phi_1^+\phi_2^+, \phi_2^+\phi_2^+\}$	3
$\|1,0,1\rangle$	$\{\phi_1^+\phi_1^{0*}, \phi_1^+\phi_2^{0*}, \phi_2^+\phi_1^{0*}, \phi_2^+\phi_2^{0*}\}$	4
$\|1,1,0\rangle$	$\phi_{[1}^+\phi_{2]}^0$	1
$\|0,0,0\rangle$	$\{\Phi_1\Phi_1^*, \Phi_1\Phi_2^*, \Phi_2\Phi_1^*, \Phi_2\Phi_2^*\}$	4

Just **one additional doublet** has made the matrices much **larger!**