Precision studies of radiative effects

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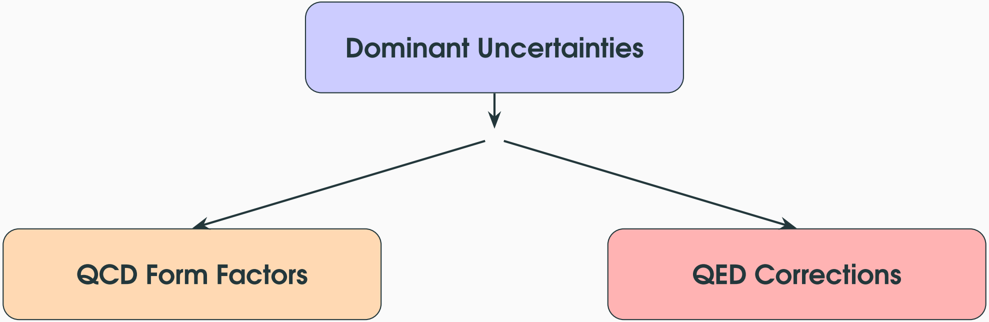
Motivation

- Heavy flavour decays (e.g. B, D mesons) provide a powerful laboratory to test the Standard Model



- The main focus of this work is to improve upon the Form Factor modelling in Sherpa and implement full NLO QED corrections

Form factors

Form factor

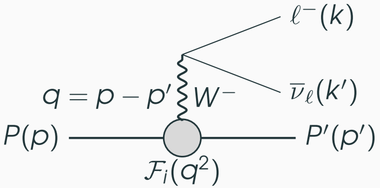


Figure 1: $P \to P' \, \ell \, \nu$ semileptonic decay described by form factors

1. What's a form factor?
   - QCD object which cannot be described perturbatively

Form factor

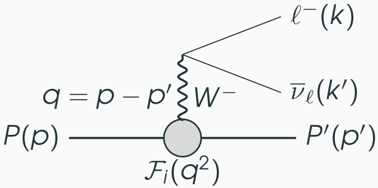


Figure 1: $P \to P' \, \ell \, \nu$ semileptonic decay described by form factors

1. What's a form factor?
   - QCD object which cannot be described perturbatively
2. How can it be calculated?
   - Particular models, LQCD or QCD sum rules

Form factor

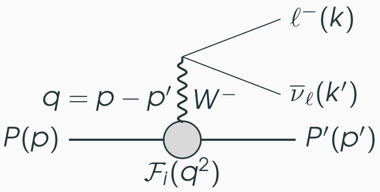


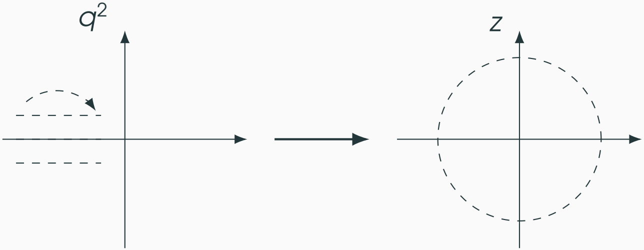
Figure 1: $P \to P' \, \ell \, \nu$ semileptonic decay described by form factors

1. What's a form factor?
   - QCD object which cannot be described perturbatively
2. How can it be calculated?
   - Particular models, LQCD or QCD sum rules
3. What would be the best approach?
   - Several pheno studies I did, suggests a model-independent approach like z-expansion

Model Independent Parameterisation

z-expansion idea

- Conformally map the domain where $\mathcal{F}_i(q^2)$ is analytic in complex q^2 to $|z| < 1$, then use a Taylor series:



$$z(q^2) = \frac{\sqrt{t_{\mathrm{cut}} - q^2} - \sqrt{t_{\mathrm{cut}}}}{\sqrt{t_{\mathrm{cut}} - q^2} + \sqrt{t_{\mathrm{cut}}}}$$

$$\mathcal{F}_i(q^2) = \sum_k a_k \, z(q^2)^k$$

z-expansion for pseudo-scalar to pseudo-scalar

$$\langle P_2(p')|\overline{q}_2\gamma^\mu q_1|P_1(p)\rangle = f_+(q^2)\left[(p+p')^\mu - \frac{m_1^2-m_2^2}{q^2}q^\mu\right] + f_0(q^2)\frac{m_1^2-m_2^2}{q^2}q^\mu$$

- BCL (Bourrely-Caprini-Lellouch) parametrisation explains data most accurately

$$f_+(q^2) = \frac{1}{1-q^2/m_{\text{pole}}^2}\sum_{n=0}^{N-1} a_n\left[z^n - \frac{n}{N}(-1)^{n-N}z^N\right]$$

$$f_0(q^2) = \sum_{n=0}^{N-1} b_n z^n$$

z-expansion for pseudo-scalar to vector

$$\langle V(p',\epsilon^*)|\mathcal{J}^\mu|P(p)\rangle = \frac{2V(q^2)}{m_P+m_V}\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu p_\rho p'_\sigma - i(m_P+m_V)A_1(q^2)\epsilon^{*\mu}$$
$$+ i\frac{A_2(q^2)}{m_P+m_V}(\epsilon^*\cdot q)(p+p')^\mu$$
$$+ i\frac{2m_V}{q^2}A_0(q^2)(\epsilon^*\cdot q)q^\mu$$

- Different z-expansions are used to fit the data (e.g. BGL for $B \to D^*$ and BSZ for $B \to \rho$)

Fit to data and QED effects

Results for $B \to D$

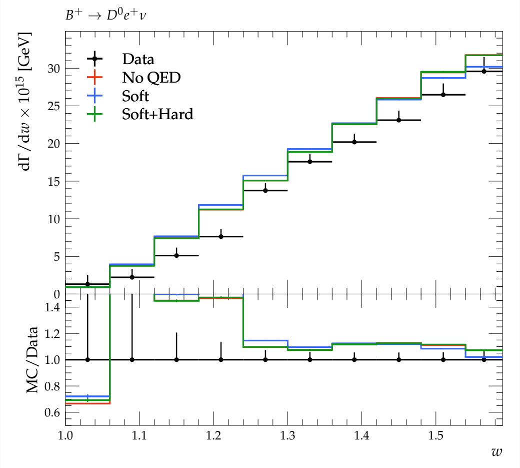


Figure 2: Electron spectrum

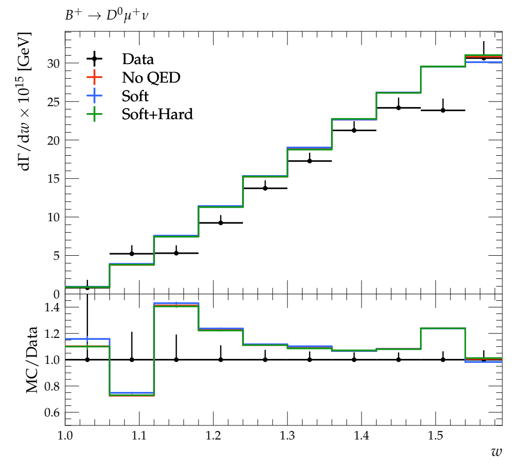


Figure 3: Muon spectrum

Results for $B \to D^*$ lepton spectrum

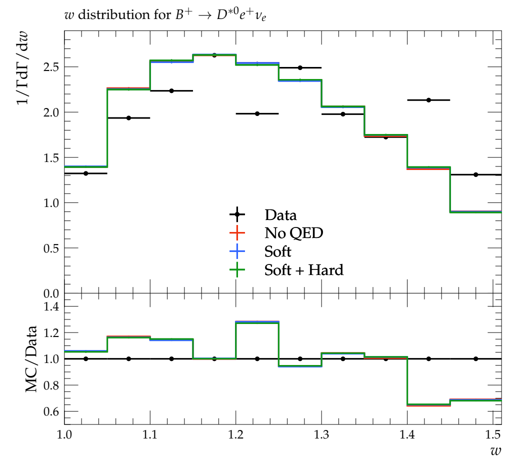


Figure 4: Electron spectrum

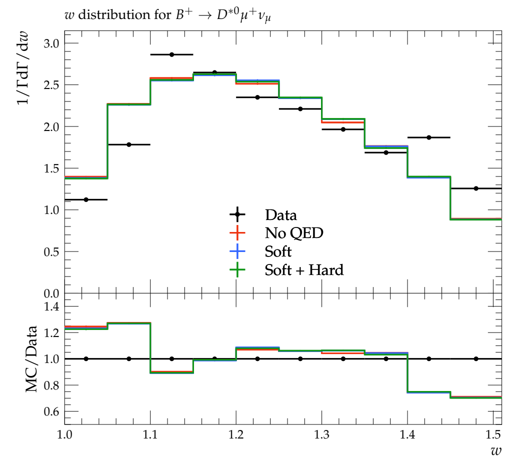


Figure 5: Muon spectrum

Results for $B \to D^*$ angular distributions

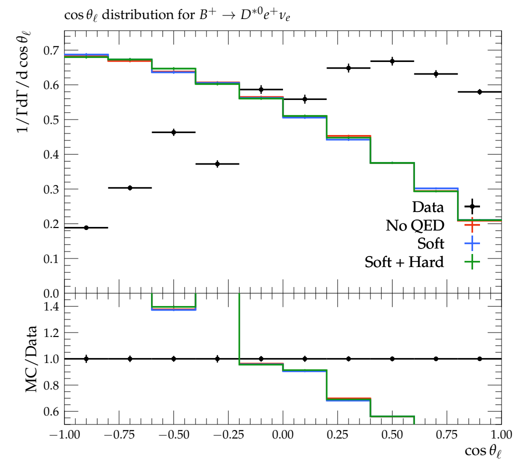


Figure 6: Electron helicity angle distribution

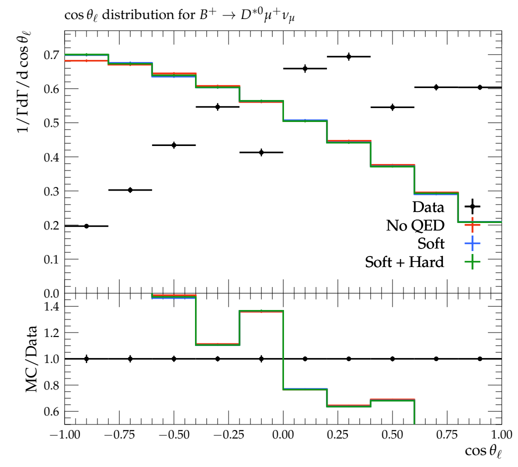


Figure 7: Muon helicity angle distribution

Next-to-leading order QED

Possible real emission diagrams

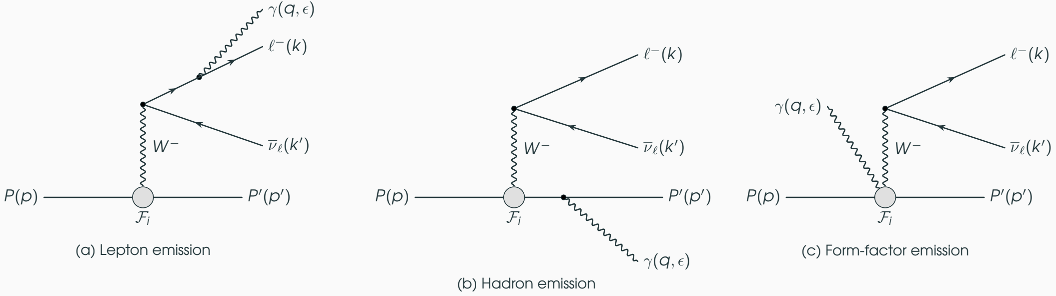
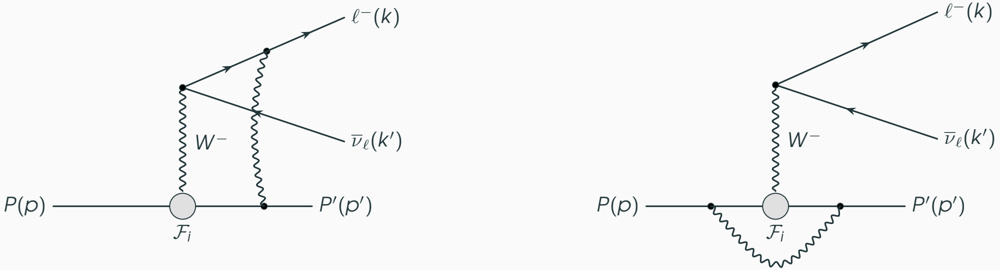


Figure 8: Real emission diagrams

- The vertex emission is a bit funny!
- Either we use OpenLoops or we just impose Ward Identity to figure out this piece



(a) Virtual lepton–hadron

(b) Virtual hadron–hadron

Figure 9: Virtual emission diagrams

- Unlikely to get a photon of high enough virtuality to resolve meson structure
- Scalar QED works best!

Implementation in MC code

Yennie-Frautschi-Suura (YFS) formalism

- YFS-Formalism allows to correct any leading order process to any order within the scope of QED-effects

$$
\begin{aligned}
\Gamma =& \frac{1}{2M}\sum_{n_R}\frac{1}{n_R!}\int d\Phi_{p_f} d\Phi_k (2\pi)^4\delta^4\left(\sum p_i - \sum p_f - \sum k\right) e^{2\alpha(B+\tilde{B}(\Omega))} \\
&\times \prod_{i=1}^{n_R}\tilde{S}(k_i)\Theta(k,\Omega)\left[\tilde{\beta}_0^0 + \tilde{\beta}_0^1 + \sum_{i=1}^{n_R}\frac{\tilde{\beta}_1^1(k_i)}{\tilde{S}(k_i)}\right] + \mathcal{O}(\alpha^2),
\end{aligned}
$$

- All IR poles are absorbed into the YFS Form Factor
- The $\beta's$ are IR subtracted hard ME's

Case study: $B_s^0 \to \mu^+\mu^-$

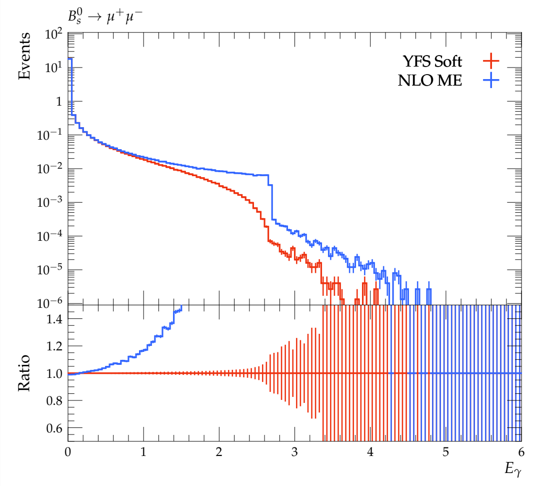


Figure 10: Total radiative energy loss

Virtual emission

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i C_i(\mu)\mathcal{O}_i(\mu)$$

$$\mathcal{M}_{vir} = \mathcal{M}_{born}(1 + \delta_{vertex}^{bare} + \delta C_{10} + \delta f)$$

- The IR poles are handled by the YFS form factor
- Need a renormalisation scheme; we use $\overline{\text{MS}}$ scheme
- Remember to use theoretical inputs ONLY!

Two potential questions worth pursuing

Form factor shift

- In real emission for semi-leptonic decay, $\mathcal{F}_i(q^2)$ object is dependent on the momentum transferred
- Why it matters? Possible migration between different q^2 bins, so a distortion in shape
- A naive taylor expansion gives,

$$\mathcal{F}_i(q_\gamma^2) \approx \mathcal{F}_i(q^2) + (q_\gamma^2 - q^2)\frac{d\mathcal{F}_i(q^2)}{dq^2}$$

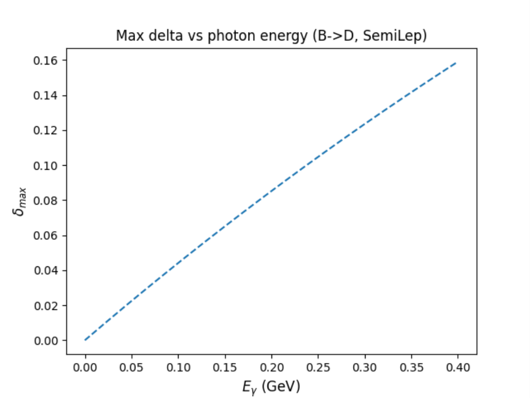
- In soft limit, constant form factor is a very good approximation

Do the terms beyond leading order matter?

Closer look into kinematics

Let's look at the following variable for a single event;

$$\delta_{soft} = \mathcal{F}_i(q_\gamma^2) - \mathcal{F}_i(q^2) = (q_\gamma^2 - q^2)\frac{d\mathcal{F}_i(q^2)}{dq^2}$$



Possible solution

$$\mathcal{F}_i(q_\gamma^2) = \mathcal{F}_i(q^2 + \Delta)$$

$$\implies \mathcal{F}_i(q_\gamma^2) = \sum_{n=0}^{n=\infty} \frac{\Delta^n}{n!} \partial_{q^2}^n \mathcal{F}_i(q^2)$$

$$\implies \mathcal{F}_i(q_\gamma^2) = e^{\Delta \partial_{q^2}} \mathcal{F}_i(q^2)$$

- The operator formalism presents a good mathematical structure to the YFS
- But we have to figure out the MC algorithm to implement it
- Why this idea is so interesting? To my knowledge no MC generator has ever done this, so something to look forward to!

Problem with experimental parameters

- If we were to perform our calculations using experimental inputs, we have to keep in mind that they already include QED effects
- So we need to define some kind of subtraction method; so that we can perform QED dressing without any overlaps
- For $\pi \to \tau\nu$, I have come up with the following,

$$\Delta\Gamma_{NLO} = \int d\Phi_2 |\mathcal{M}_{vir+ct}|^2 + \int d\Phi_3 |\mathcal{M}_{real}|^2 = 0$$

Current and future work

In progress

- Exact ME calculations
- Understanding the YFS code in detail
- Creating a separate Form factor module in Sherpa

Future work

- Implement all calculations in the YFS code
- Understand the problem behind the angular distribution within Sherpa and fix it
- If time permits, try to study non-leptonic decays
- Last but not least, complete writing my thesis!

List of References

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[4] C Glenn Boyd, Benjamin Grinstein, and Richard F Lebed. **"Precision corrections to dispersive bounds on form factors".** In: *Physical Review D* 56.11 (1997), p. 6895.

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[7] KS Kuzmin, NM Levashko, and MI Krivoruchenko. **“Electromagnetic nucleon form factors in the extended vector meson dominance model”.** In: *Physical Review D* 111.1 (2025), p. 013004.

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Thank you for your attention!

Questions?

Backup Slides

Light Cone Sum Rules (LCSR)

- To put it simpply, LCSR aims to express Form factors in terms of light cone structure of hadrons
- Let's consider $B \to Dl\overline{\nu_l}$,

$$\Pi^\mu(p,q) = i\int d^4x\, e^{iq\cdot x}\langle D(p')|T\{\overline{c}(x)\gamma^\mu b(x), j_B^\dagger(0)\}|0\rangle, \qquad p' = p - q$$

$$\Pi^\mu(p,q) = \frac{\langle D(p')|\overline{c}\gamma^\mu b|B(p_B)\rangle\langle B(p_B)|j_B^\dagger|0\rangle}{m_B^2 - p_B^2} + \cdots, \qquad p_B = p + q$$

$$\langle B(p_B)|j_B^\dagger|0\rangle = f_B m_B^2$$

$$\langle D(p')|\overline{c}\gamma^\mu b|B(p_B)\rangle = F_+(q^2)(p_B + p')^\mu + F_-(q^2)(p_B - p')^\mu$$

Light Cone Sum Rules (LCSR)

$$\Pi^\mu(p,q) = \int_0^1 du\, T^\mu(u,q^2)\,\phi_D(u)$$

$$\frac{1}{m_B^2 - p_B^2} \longrightarrow e^{-m_B^2/M^2}$$

$$F(q^2) \sim \frac{1}{f_B}\int_0^{u_0} du\, T(u,q^2)\,\phi_D(u)\, e^{-s(u)/M^2}$$

z-expansion for form factors

- Instead of using momentum tranferred q^2, it uses a variable $z(q^2)$ to improve series convergence

$$z(q^2, t_{cut}, t_0) = \frac{\sqrt{t_{cut} - q^2} - \sqrt{t_{cut} - t_0}}{\sqrt{t_{cut} - q^2} + \sqrt{t_{cut} - t_0}}$$

$$t_+ = (m_H + m_L)^2$$

$$t_- = (m_H - m_L)^2$$

$$t_0 = t_+ \left(1 - \sqrt{1 - \frac{t_-}{t_+}}\right) \text{ or } \; t_-$$

BGL for $B \to D$

$$f_i(z) = \frac{1}{P_i(z)\,\phi_i(z)} \sum_{j=0}^{\infty} a_{i,j}\, z^j$$

$$P_i(z) = \prod_p \frac{z - z_p}{1 - z z_p}$$

$$z_p(M_p, N) = \frac{\sqrt{(1+r)^2 - \frac{M_p^2}{M_B^2}} - \sqrt{4Nr}}{\sqrt{(1+r)^2 - \frac{M_p^2}{M_B^2}} + \sqrt{4Nr}}$$

BGL for $B \to D$

$$\phi_g = 16r^2\sqrt{\frac{n_I}{3\pi\chi_1^-(0)}}\frac{(1+z)^2(1-z)^{-1/2}}{[(1+r)(1-z)+2\sqrt{r}(1+z)]^4}$$

$$\phi_f = \frac{4r}{M_B^2}\sqrt{\frac{n_I}{3\pi\chi_1^+(0)}}\frac{(1+z)(1-z)^{3/2}}{[(1+r)(1-z)+2\sqrt{r}(1+z)]^4}$$

$$\phi_{F_1} = \frac{4r}{M_B^3}\sqrt{\frac{n_I}{6\pi\chi_1^+(0)}}\frac{(1+z)(1-z)^{5/2}}{[(1+r)(1-z)+2\sqrt{r}(1+z)]^5}$$

$$\phi_{F_2} = 8\sqrt{2}\,r^2\sqrt{\frac{n_I}{\pi\chi_1^L(0)}}\frac{(1+z)^2(1-z)^{-1/2}}{[(1+r)(1-z)+2\sqrt{r}(1+z)]^4}$$

BSZ for $B \to \rho$

$$F_i(q^2) = \frac{1}{1 - q^2/m_{R,i}^2} \sum_{k=0}^{2} \alpha_k^i \left[z(q^2) - z(0)\right]^k$$

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$