Towards heavy-quark pair production at NNLO+PS accuracy at lepton colliders

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Introduction and motivation

We consider $e^+e^- \to Q\bar{Q}$ and $e^+e^- \to Q\bar{Q}\,j$ @ NLO

- Add processes into `POWHEG-BOX-V2`
- **Target**: use `MiNLO` for heavy quarks
- what has been done: $H \to b\bar{b}$ massless (NNLO+PS) [1]
- Interesting by itself
- Applications for future colliders $(e^+e^- \to t\,\bar{t}) \sim$ FCC-ee like
- `POWHEG` applied to $e^+e^- \to Q\bar{Q}$ $\oplus$ our results on $Q\bar{Q}\,j$ @NLO
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Infrared divergencies of QCD

- QCD has IR divergences
- IR div. cancel through KLN theorem $\rightarrow$ well defined theory

- massless QCD $\rightarrow$ soft + collinear
- massive QCD $\rightarrow$ only soft
- can distinguish "g" and "Q" massive ($g \parallel Q$ not degenerate)

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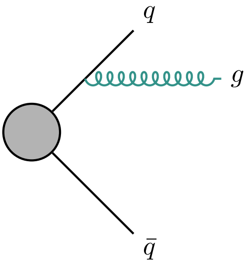
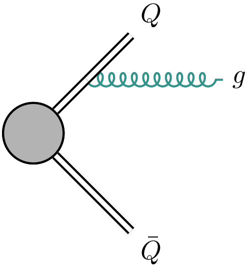
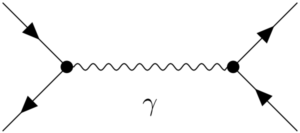




Consider $e^+e^- \xrightarrow{\gamma} Q\bar{Q}$



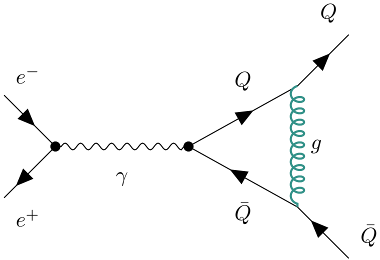
Useful quantities

$$\mu_Q \equiv \frac{m_Q}{\sqrt{s}} \qquad v \equiv \sqrt{1 - 4\mu_Q^2}$$

Leading order:

$$\sigma_B = \frac{3 \cdot 2^{2\epsilon-3} e^4 \pi^{\epsilon-1}\, Q_q^2\, s^{-\epsilon-1} v^{1-2\epsilon} \left(v^2 + 2\epsilon - 3\right) \Gamma(2-\epsilon)}{(2\epsilon-3)\Gamma(2-2\epsilon)}$$

Virtual contributions [2] [3]

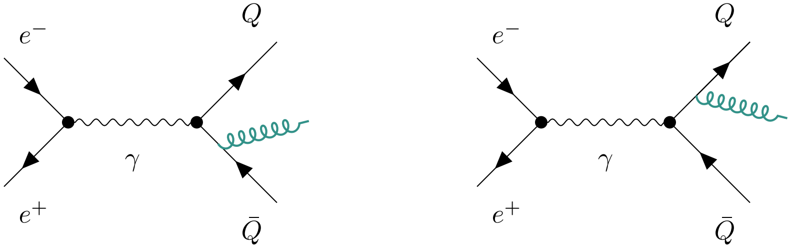


- renormalize external legs → $Z_Q \sim 1 + \alpha_s/\epsilon \qquad (Z_q = 0)$
- Functions $A_0,\, B_0,\, C_0$ → ex. $A_0(m_Q^2),\, B_0(m_Q^2, 0, m_Q^2)$
- For finite mass, only one (soft) pole

$$\sigma_V \propto -\frac{1}{\epsilon}\left[2 + \frac{1+v^2}{v}\log\left(\frac{1-v}{1+v}\right)\right] + \text{finite}$$

- Massless limit $v \to 1$, log is divergent → big logs

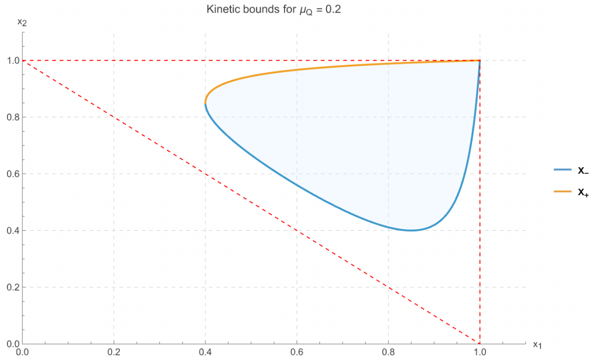
Real contributions



- Three body phase-space $\rightarrow x_i \equiv 2\,E_i/\sqrt{s}$
- Masses reduce kinematical region $\quad x_1 \in [2\mu_Q,\, 1] \quad x_2 \in [x_-,\, x_+]$

$$x_+ = \frac{(2 - x_1)\left(2\mu_Q^2 - x_1 + 1\right) + \sqrt{x_1^2 - 4\mu_Q^2}(1 - x_1)}{2\left(\mu_Q^2 - x_1 + 1\right)}$$

Real contributions



- Masses cure collinear divergences
- Still need to subtract soft ones

POWHEG

[4]

- Use `POWHEG` $\rightarrow$ general and semi-automated framework
- Implement code for $e^+e^- \rightarrow Q\bar{Q}$ and $Q\bar{Q}\,j$
- `POWHEG` inputs: Born, Real and Virtual M.E. squared + Phase-space
- FKS subtraction is performed fully automatically

- `POWHEG` master eq.

$$d\sigma = d\Phi_n\,\bar{B}(\Phi_n) \otimes \text{PS}_{\texttt{POWHEG}}$$

Our concern *so far*[1]:

$$\bar{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \sum_\alpha \left[\int d\Phi_{\text{rad}}\, R(\Phi_{n+1}) \right]_\alpha$$

[1] Not looked at Parton Shower, future work.

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FKS subtraction method (massless)

- Distributions in final state variables $\to$ natural NLO formalism
- Emitted parton k

$$\xi \equiv 2\,k^0/\sqrt{s} \qquad y \equiv \cos\theta_{qg} \qquad\qquad (\oplus\,\phi)$$

- Soft div. $\xi \to 0$, collinear div. $y \to 1$

$$[dk] = \frac{s^{1-\epsilon}}{(4\pi)^{d-1}}\,\xi^{1-2\epsilon}\,(1-y^2)^{-\epsilon}\,d\xi\,dy\,d\Omega_{d-2}$$

- Mappings $x_i = x_i(\xi, y)$
- $\bar{\mathcal{R}}$ finite in IR limits

$$\bar{\mathcal{R}} \equiv \xi^2(1-y)\mathcal{R}$$

FKS subtraction method (massless)

- Distributions highlight IR limits

$$\xi^{-1-2\epsilon} = -\frac{1}{2\epsilon}\delta(\xi) + \left(\frac{1}{\xi}\right)_+ - 2\epsilon\left(\frac{\log\xi}{\xi}\right)_+ + \mathcal{O}(\epsilon^2) \qquad (\oplus\, y)$$

- Counter-terms

$$\hat{\mathcal{R}} \equiv \frac{1}{\xi}\left(\frac{1}{\xi}\right)_+ \left(\frac{1}{1-y}\right)_+ \bar{\mathcal{R}}$$

- Integration of $\hat{\mathcal{R}}$ gives finite contributions

FKS in the massive case [6]

- Extend FKS mappings to the massive case
- Different limits of ξ FKS variable $\xi \in [0,\, \xi^{\text{max}}]$

$$\xi^{\text{max}}(y \,|\, \mu_Q) \equiv \frac{1 - 4\mu_Q^2}{1 + \mu_Q \sqrt{\left(4\mu_Q^2 - 1\right) y^2 + 1} - 2\mu_Q^2}$$

- $\mathcal{R}$ singular for soft limit $\xi \to 0$
- $\mathcal{R}$ **not anymore** singular for $y \to 1$

$$\int [dk]\, \mathcal{R} \equiv \mathcal{R}^s + \mathcal{R}^{\text{count}}$$

- Core of the subtraction

$$\mathcal{V} + \int [dk]\, \mathcal{R} = \underbrace{\mathcal{V} + \mathcal{R}^s}_{\text{analytic pole cancellation}} + \mathcal{R}^{\text{count}}$$

NLO cross section for $e^+e^- \xrightarrow{\gamma} Q\bar{Q}$ [2]

- Attach phase-space $d\Phi$
- NLO total cross section for $e^+e^- \xrightarrow{\gamma} Q\bar{Q}$

$$\sigma_{\mathsf{NLO}} = \sigma_{\mathsf{LO}}\, C_{\mathsf{F}}\, \frac{\alpha_s}{2\pi} \left[\frac{3}{2} + 18\mu_Q^2 + \mathcal{O}(\mu_Q^4 \log \mu_Q^2)\right]$$

- Checked `POWHEG` with analytical cross section

- Differential distributions $\rightarrow$ `POWHEG` $\oplus$ `OpenLoops` [7]
- "Checked" `OpenLoops` results with analytical M.E. [8]

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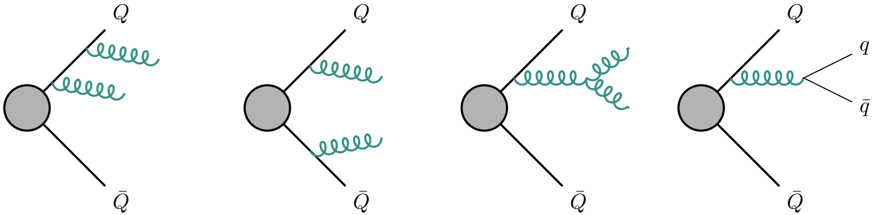
Starting point: $Q\bar{Q}\,j$ @NLO



- $e^+e^- \to Q\bar{Q}\,j$ @ NLO is the **baseline** for `MiNLO`
- Build exclusive jets with Durham e^+e^- generalized k_T alg. (`FastJet` [9])
- Distance for jet algorithm

$$d_{ij} = 2\,\min(E_i^2, E_j^2)(1-\cos\theta_{ij}) \qquad\qquad y_{ij} = d_{ij}/s$$

- "Relevant useful" obs to study regions where large logs develop: $\{y_{12},\, y_{23}\}$[2]

[2] Resolution scale from $n+1 \to n$ jets.

Setup for $e^+e^- \to Q\bar{Q}\,j$ @NLO

Consider three scenarios

$$
\begin{aligned}
\text{FCC}: \{m_Q = 172.5\,,\ \sqrt{s} = 500\}\ \text{GeV} \quad &\to \quad \mu_Q = 0.34\\
\text{LEP}: \{m_Q = 10.0\,,\ \sqrt{s} = 200\}\ \text{GeV} \quad &\to \quad \mu_Q = 0.05\\
\text{TH}: \{m_Q = 10.0\,,\ \sqrt{s} = 1000\}\ \text{GeV} \quad &\to \quad \mu_Q = 0.01
\end{aligned}
$$

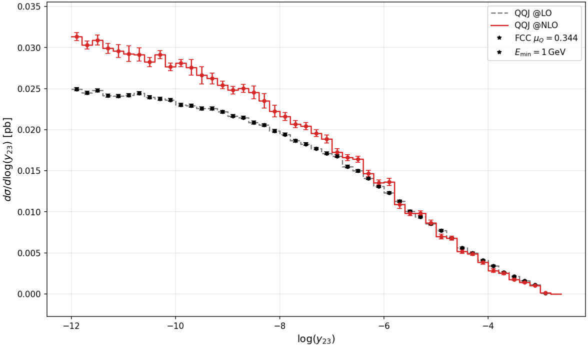
- Two types of logs

$$L_Q \equiv \log\left(\mu_Q^2\right) \qquad\qquad L_y = \log(1/y_{23})$$

- FCC: L_Q not big (same scale)   TH: big L_Q logs (massless)
- Will need to resum L_y in the low energy region $y_{23} \to 0$

Results for $e^+e^- \to Q\bar{Q}\, j$ @NLO

PRELIMINARY



Hard region should be well described by the Fixed Order computation:

$$\frac{1}{\sigma_{\mathsf{LO}}}\frac{d\sigma^{\mathsf{LO}}}{d\log(y_{23})} \sim \alpha_s\, C_{\mathsf{F}} \left[2(L_Q+2)\, L_y + L_Q(L_Q+1) \right]$$

Results for $e^+e^- \to Q\bar{Q}\,j$ @NLO

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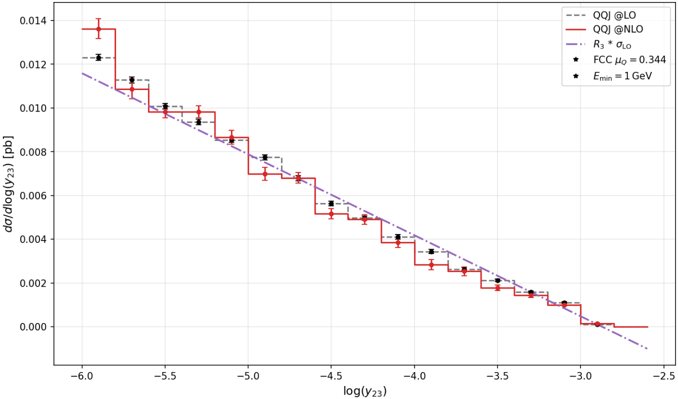

- Hard region "well described" (preliminary) by Fixed Order
- One of the few cases where resummation with masses is known [10]

MiNLO (Multi-scale improved NLO) [12] [13]

- Consider NLO prediction for final state F $(F = Q\bar{Q}$ for us)
- Add 1 jet $\rightarrow F + j$ has IR div. $\rightarrow$ energy cut E_{min} on jet

- Multi-scale: $\{\sqrt{s},\, y_{23}\}$ $\oplus$ m_Q
- So far, MiNLO only with massless emitters[3]

- MiNLO combines $F \oplus F + j$ @NLO in a smooth way
- Jet can now go to IR limits
- NLO accuracy for both $Q\bar{Q}$ and $Q\bar{Q}\,j$
- Obtain[4] $\sigma_{\text{NLO}}(Q\bar{Q})$ running $Q\bar{Q}\,j$ code

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MiNLO

- Modify POWHEG $\bar{B}$ with Sudakov $\oplus$ appropriate scales for couplings

$$y_{23} = k_t^2/s \qquad \alpha_s = \alpha_s(k_t^2) \qquad \Delta_g = \Delta_g(y_{23})$$

- Sudakov factors $\to$ cure IR div. in regions where third jet gets unresolved
- Master MiNLO formula

$$\bar{B}_{\texttt{MiNLO}}(\Phi_{Q\bar{Q}g}) = \alpha_s\, \Delta_g^2 \left[B \cdot \left(1 - 2\Delta_g^{(1)} \right) + V + \int d\Phi_{\text{rad}}\, R \right]$$

- Extend MiNLO to massive case
- Masses add a lot of features to NLO comput. $\to$ IR singularities less bad
- Masses change log structure of the theory $\to$ resummation less obvious !

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Conclusions and future work

- Prepared a solid setup in POWHEG for $e^+e^- \to Q\bar{Q}$ and $Q\bar{Q}\,j$
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Next steps and goals to reach by the end of the Master's Thesis:

- Set up code for MiNLO $\to$ resummed plots for y_{23}
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- need Δ_g factors, with mass terms
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Thank you for your attention

MiNLO original massless formulation

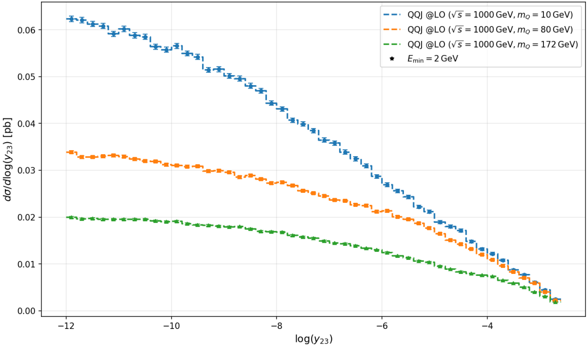
Some details about MiNLO (massless).

- A certain logarithmic accuracy is required (at least underlying logs)
- MiNLO cures divergences without spoiling FO NLO accuracy
- All LL, all NLL
- A piece of NNLL $\rightarrow B_2$ (ex. $H \rightarrow b\bar{b}$)

For the massive case:

- Need to understand log accuracy needed
- At LL, there is one log less

Mass effects on y_{23}



- Look at the effects of mass logs on y_{23}

$$L_Q = \log\left(\mu_Q^2\right)$$

- Massless case has big L_Q → dominant in y_{23} low energy region

Heavy flavours in jets

- We build jets from massive quarks
- Recent literature for flavoured jet-algorithms $\rightarrow$ tag flavour (b-jets)
- Issue: if massless computation $\rightarrow$ jet alg. IR-UNsafe[5]
- There are ways to fix it (WIP from the community)

- We are aware of this result
- We use a **massive** computation $\rightarrow$ no issue

[5] After a certain order.

POWHEG details

POWHEG master eq.

$$d\sigma = d\Phi_n\, \bar{B}(\Phi_n) \otimes \mathsf{PS}_{\texttt{POWHEG}}$$

Fixed order (NLO):

$$\bar{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \sum_{\alpha} \left[\int d\Phi_{\mathsf{rad}}\, R(\Phi_{n+1}) \right]_{\alpha}$$

For the Parton Shower (NLO+PS):

$$\Delta(\Phi_n\,,\, p_T) \equiv \exp\left\{ -\int \frac{d\Phi_{\mathsf{rad}}\, R(\Phi_{n+1})\, \theta(k_T(\Phi_{n+1}) - p_T)}{B(\Phi_n)} \right\}$$

$$\mathsf{PS}_{\texttt{POWHEG}} = \Delta(\Phi_n\,,\, p_T^{\mathsf{min}}) + \Delta(\Phi_n\,,\, k_T(\Phi_{n+1}))\, \frac{R(\Phi_{n+1})}{B(\Phi_n)}\, d\Phi_{\mathsf{rad}}$$

Details about massive FKS

Consider $\bar{Q} \to \bar{Q} + g$, FKS massive param. is given by

$$x_{\bar{Q}} = -\frac{2\left(-\xi\,|y|\sqrt{(\xi-1)^2+\mu_Q^2\left(4\xi+\xi^2\left(y^2-1\right)-4\right)}+\xi^2-3\xi+2\right)}{4\xi+\xi^2\left(y^2-1\right)-4}$$

Build `POWHEG` Jacobian $\mathcal{P}(\xi,\, y\,|\,\mu_Q)$ and define

$$g^{(d)}(\xi,\, y\,|\,\mu_Q) \equiv \mathcal{R}^{(d)}(\xi,\, y)\cdot\mathcal{P}(\xi,\, y)$$

Now separe contributions

$$\mathcal{R}^s \sim \int_{-1}^{1} dy\,(1+y)^{-\epsilon}\,(1-y)^{-1-\epsilon}\,\int_0^{\xi^{\mathsf{max}}(y)} d\xi\left[-\frac{\xi_{\mathsf{m}}^{-2\epsilon}}{2\epsilon}\,\delta(\xi)\right] g^{(d)}(\xi,\, y)$$

$$\mathcal{R}^{\mathsf{count}} \sim \int_{-1}^{1} dy\,\frac{1}{1-y}\,\int_0^{\xi^{\mathsf{max}}(y)} d\xi\left(\frac{1}{\xi}\right)_+ g^{(4)}(\xi,\, y)$$

Full glory $\sigma_{\mathsf{NLO}}(e^+e^- \xrightarrow{\gamma} Q\bar{Q})$ [8]

Total cross section with full mass effects

$$\sigma_{\mathsf{NLO}} = \sigma_{\mathsf{LO}}\, C_{\mathsf{F}}\, \frac{\alpha_s}{2\pi}\, f(\mu_Q)$$

$$\begin{aligned} f(\mu_Q) = \frac{3}{2}(6\mu_Q^2+1)\sqrt{1-4\mu_Q^2} - 4\sqrt{1-4\mu_Q^2}(2\mu_Q^2+1)\left[\log\left(\frac{1}{\mu_Q^2}-4\right) - \log(\mu_Q)\right] \\ - 4(7\mu_Q^4 + 2\mu_Q^2 - 3)\log\left(\frac{\sqrt{1-4\mu_Q^2}+1}{2\mu_Q}\right) \\ + 4(1-4\mu_Q^4)\left\{2\log\left(\frac{\sqrt{1-4\mu_Q^2}+1}{2\mu_Q}\right)\left[-\log\left(\frac{1}{\mu_Q^2}-4\right) + 3\log\left(\frac{\sqrt{1-4\mu_Q^2}+1}{2\mu_Q}\right)\right.\right. \\ \left.\left. + \log(\mu_Q)\right] + 2\mathsf{Li}_2\left(\frac{1}{4}\left(\sqrt{\frac{1}{\mu_Q^2}-4} - \frac{1}{\mu_Q}\right)^2\right) + \mathsf{Li}_2\left(\frac{1}{16}\left(\sqrt{\frac{1}{\mu_Q^2}-4} - \frac{1}{\mu_Q}\right)^4\right)\right\} \end{aligned}$$

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