Study of Quantum Entanglement in the $\Upsilon(4S) \rightarrow B^0\bar{B}^0$ system at Belle II

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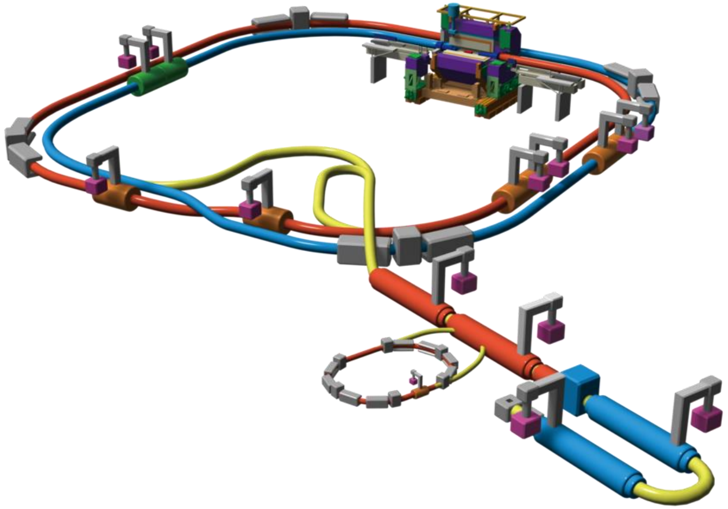
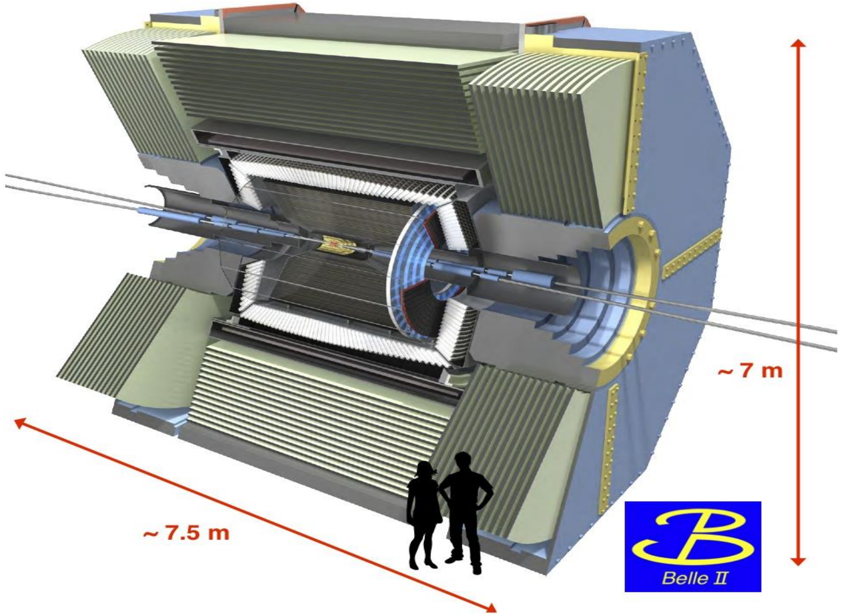
Belle II Experiment

SuperKEKB Accelerator & Belle II Detector

- High energy experiment located in Tsukuba, Japan
- Detector *Belle II* designed for precision measurements at high luminosities
- Energy asymmetric e^+e^-- collider *SuperKEKB*
- Collision at resonance energy for $\Upsilon(4S) = 10.58\ \mathrm{GeV}$
- $\Upsilon(4S)$ decays almost exclusively in B^+B^- & $\boldsymbol{B^0\bar{B}^0}$ $(b\bar{q})$





What is Quantum Entanglement?

- Two particles' states are completely dependent on each other
- Both particles are described in one inseparable wave function
- Measuring one particle's state instantaneously determines the state of the other
- In case of $\Upsilon(4S) \rightarrow B_1^0 \bar{B}_2^0$ after pair production:

$$\rightarrow |\,\psi\,\rangle = \frac{1}{\sqrt{2}} \left(|\, B_1^0 \bar{B}_2^0 \,\rangle - \left|\, \bar{B}_1^0 B_2^0 \right\rangle \right)$$

- Until this wave function collapses, they are described as orthogonal states

Motivation for QM Tests at *Belle II*

Key Questions:

- Can we measure Quantum Mechanical properties, e.g. Entanglement at *Belle II*?
- Are hadrons at high energies entangled like photons & atoms?

Consequences:

- Deviations could hint to rare Standard Model process or New Physics
- Deviations would introduce bias in Belle II measurements

Neutral B Oscillation

- Flavor eigenstates $B^0/\bar{B}^0 \neq$ Mass eigenstates
- B^0 can transition (oscillate) to $\bar{B}^0$ over time
- Oscillation probability is given by

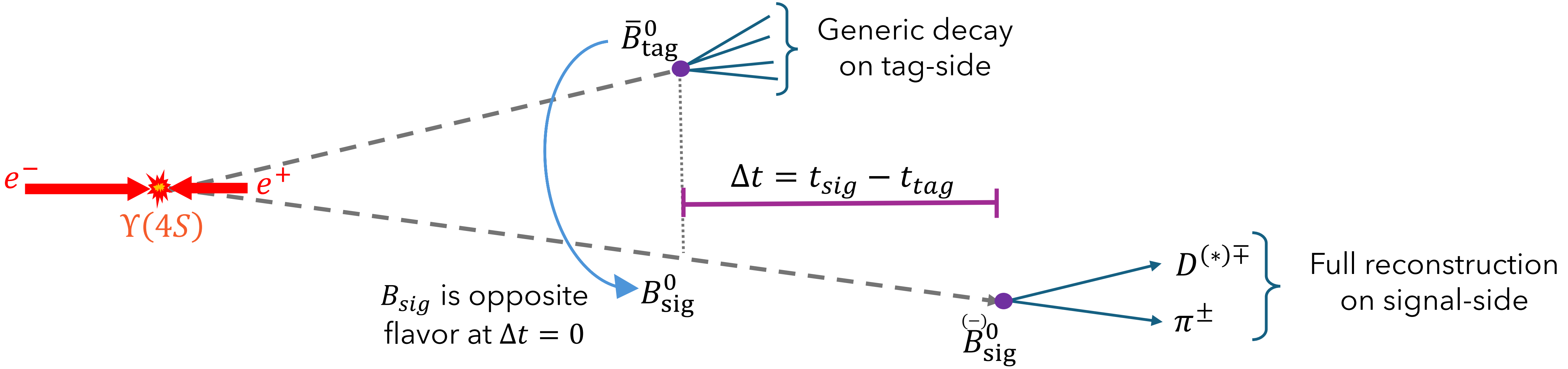
$$P(B^0 \to \bar{B}^0;\ t) \propto\ e^{-t/\tau}(1 - \cos\Delta m t)$$

- Osc. Freq. Δm is the mass difference betw. the two mass eigenstates

Time evolution of $B^0\bar{B}^0$ pairs

- Entangled state $|\,\psi\,\rangle = \frac{1}{\sqrt{2}}\left(\,|\,B^0_{\rm tag}\bar{B}^0_{\rm sig}\,\rangle - |\,\bar{B}^0_{\rm tag}B^0_{\rm sig}\,\rangle\,\right)$
- Wave function collapses at $\Delta t = 0$ with decay of $B_{\rm tag}$ (here: $B_{\rm tag}(\Delta t = 0) = \overline{B^0}$)
- B_{sig} starts oscillation at $\Delta t = 0$:

$$P(B^0 \to \bar{B}^0; \Delta t) \propto\ e^{-\frac{\Delta t}{\tau}}(1 - \cos\Delta m\Delta t)$$



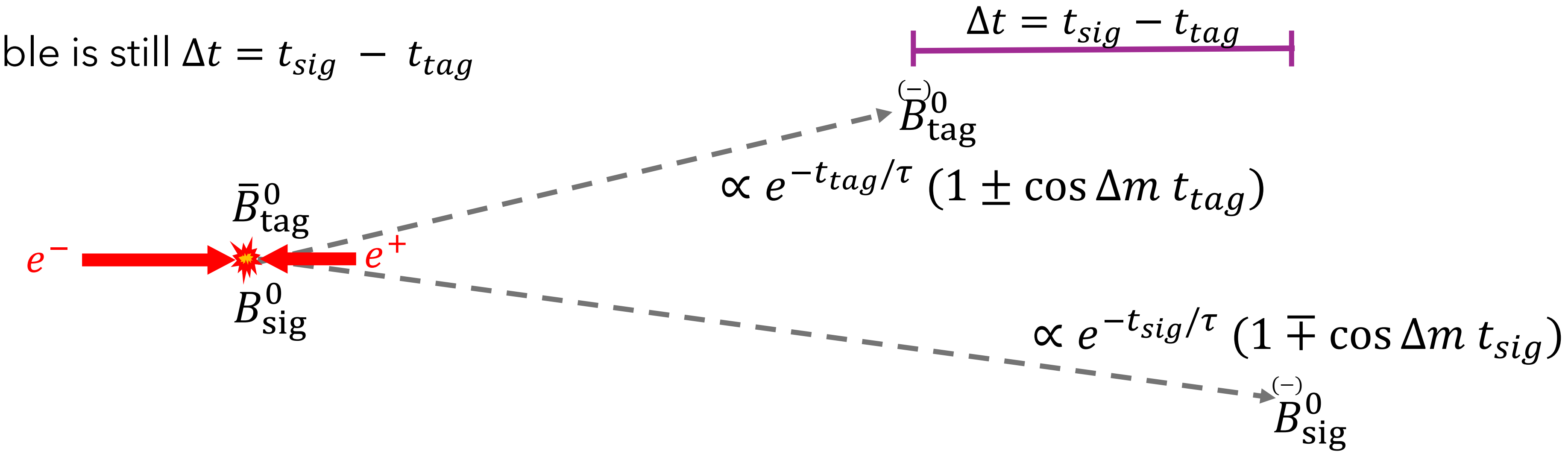
Generic decay on tag-side

B_{sig} is opposite flavor at $\Delta t = 0$

Full reconstruction on signal-side

How do spontaneously disentangled $B^0\bar{B}^0$ evolve?

- After pair production in $\Upsilon(4S) \to B^0\bar{B}^0$ in orthogonal states, wave function collapses immediately
- Independent time evolution starts from opposite states
- B_{tag} decays at t_{tag}, B_{sig} decays at t_{sig}
- Observable is still $\Delta t = t_{sig} - t_{tag}$



Resulting Δt – distribution: $P_{SD}(\Delta t) = \frac{1}{8} e^{-\Delta t/\tau}\,(2 \mp f_{cos}\cdot\cos\Delta m\Delta t \pm f_{sin}\cdot\sin\Delta m|\Delta t|\,)$

Partial Spontaneous Disentanglement

Standard Case:

$$\boxed{P_{QM}(\Delta t) \propto e^{-\Delta t/\tau}\,(1 \mp \cos \Delta m \Delta t)}$$

Spontaneous Disentanglement (SD):

Independent oscillation from $t = 0$ of both Bs starting from orthogonal states

$$\boxed{P_{SD}(\Delta t) \propto e^{-\Delta t/\tau}\,(2 \mp f_{cos} \cdot \cos \Delta m \Delta t \pm f_{sin} \cdot \sin \Delta \mathrm{m} |\Delta t|)}$$

Partial Spontaneous Disentanglement (PSD):

- Same as SD model with a fraction $\boldsymbol{(\zeta)}$ of disentangled pairs

$$P_{PSD}(\zeta;\,\Delta t) = (1-\zeta)\cdot \boxed{P_{QM}\,(\Delta t)} + \zeta \cdot \boxed{P_{SD}(\Delta t)} \qquad \zeta \in [0, 1]$$

- Last measured value: $\boldsymbol{\zeta = 0.029 \pm 0.059}$ – compatible with full entanglement, but cannot exclude $\zeta \sim \mathcal{O}(10\%)$ at 90% CL (*Belle*, 2007)

Disentanglement fraction ζ is our key physics parameter!

Analysis Strategy

- Monte Carlo Data (Signal & Generic)
- Reconstruction
- Model Detector Response & Background
- Δt - fit with physics parameter ζ
- Study of systematic uncertainties

Goals:

i. Implement a model sensitive to disentanglement fraction ζ

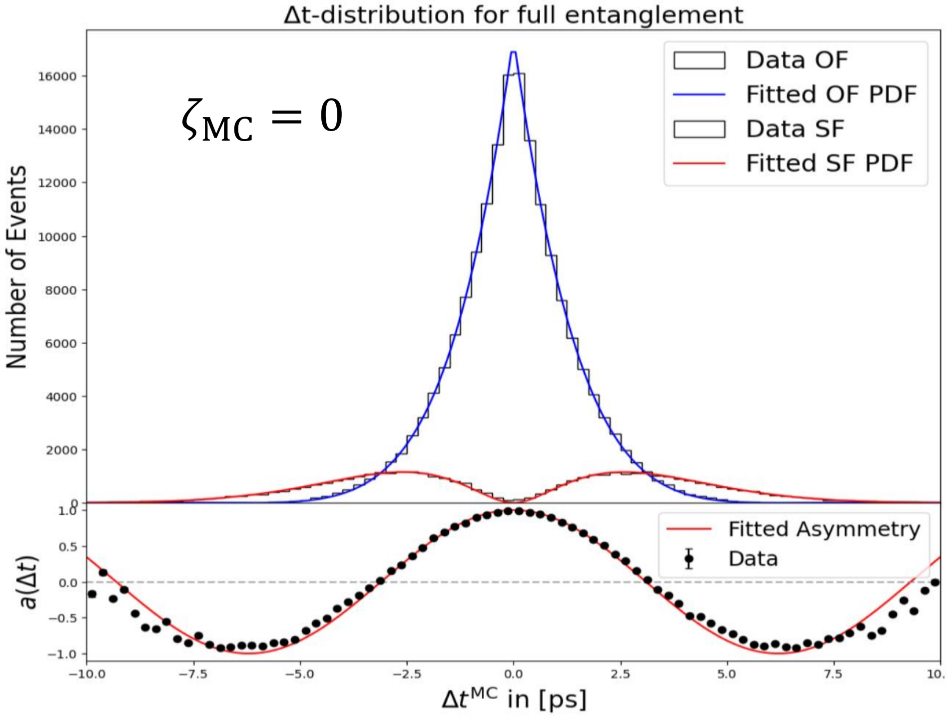
ii. Measure disentanglement fraction ζ

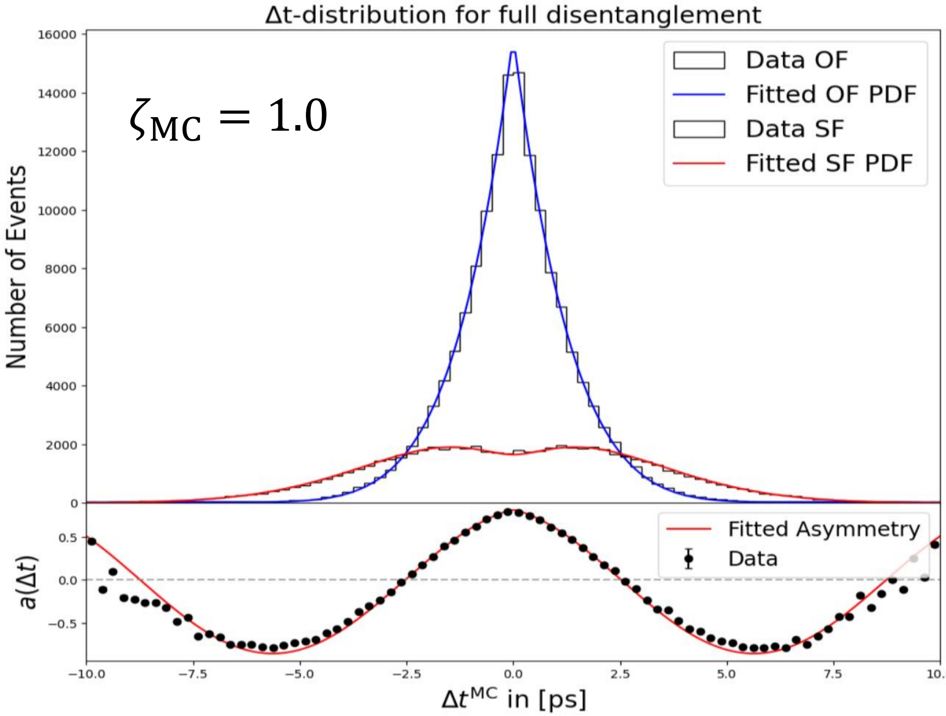
Monte Carlo simulation

- Development of new EvtGen (event generator) model for simulating Partial Disentanglement (now availbable in official Belle II analysis software)
- Signal Channel: *Flavor specific* $B^0 \rightarrow D^{(*)-}\pi^+$ modes
- Signal MC production with detector sim. (Geant4) & Final State Radiation
- Samples generated with different $\zeta_{\mathrm{MC}} = 0, \ldots, 1.0$

Opposite Flavor (OF)
$B^0\bar{B}^0$ & $\bar{B}^0 B^0$

Same Flavor (SF)
$B^0 B^0$ / $\bar{B}^0\bar{B}^0$

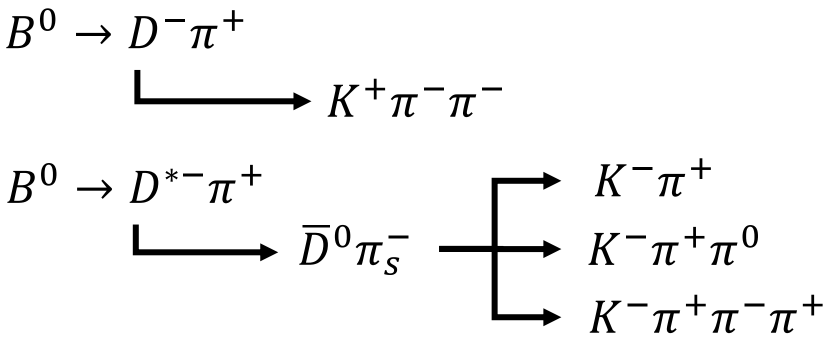




Reconstruction & Event Selection

Signal-side reconstruction

- Fully reconstructed in 4 channels
- Kinematic conditions (M_{bc} & ΔE) on B candidate
- Decay chain fit to get signal-side vertex → input to Δt



$B^0 \to D^-\pi^+$

$\quad \rightarrow K^+\pi^-\pi^-$

$B^0 \to D^{*-}\pi^+$

$\quad \rightarrow \bar{D}^0\pi_s^- \rightarrow K^-\pi^+ \,/\, K^-\pi^+\pi^0 \,/\, K^-\pi^+\pi^-\pi^+$

Tag-side reconstruction

- Inclusive reconstruction for Flavor tagging & vertexing
- Tag-side vertex from primary vertices
- Combined with signal-side vertex to compute Δt
- Rest of Event tracks are used for flavor tagging
- Flavor Tagger assigns *mistag* probability (w) to every event
- Mistag dilutes Δt – distribution

  → similar looking effect as disentanglement!

Detector Effects

- Modeled by *<u>event-by-event Resolution Function</u>* $R(\delta\Delta t; \sigma_{\Delta t})$
- $R(\delta\Delta t; \sigma_{\Delta t})$ depends on *residuals* $\delta\Delta t = \Delta t - \Delta t^{\mathrm{MC}}$
- Resolution function consists of multiple gaussian and exponential tails
- Implemented analytically by convolution with Physics PDF

  → $R(\delta\Delta t; \sigma_{\Delta t}) \otimes P_{PSD}(\Delta t^{\mathrm{MC}})$
- Advantage:
  - Numerical stability
  - Full control & understanding of resolution function

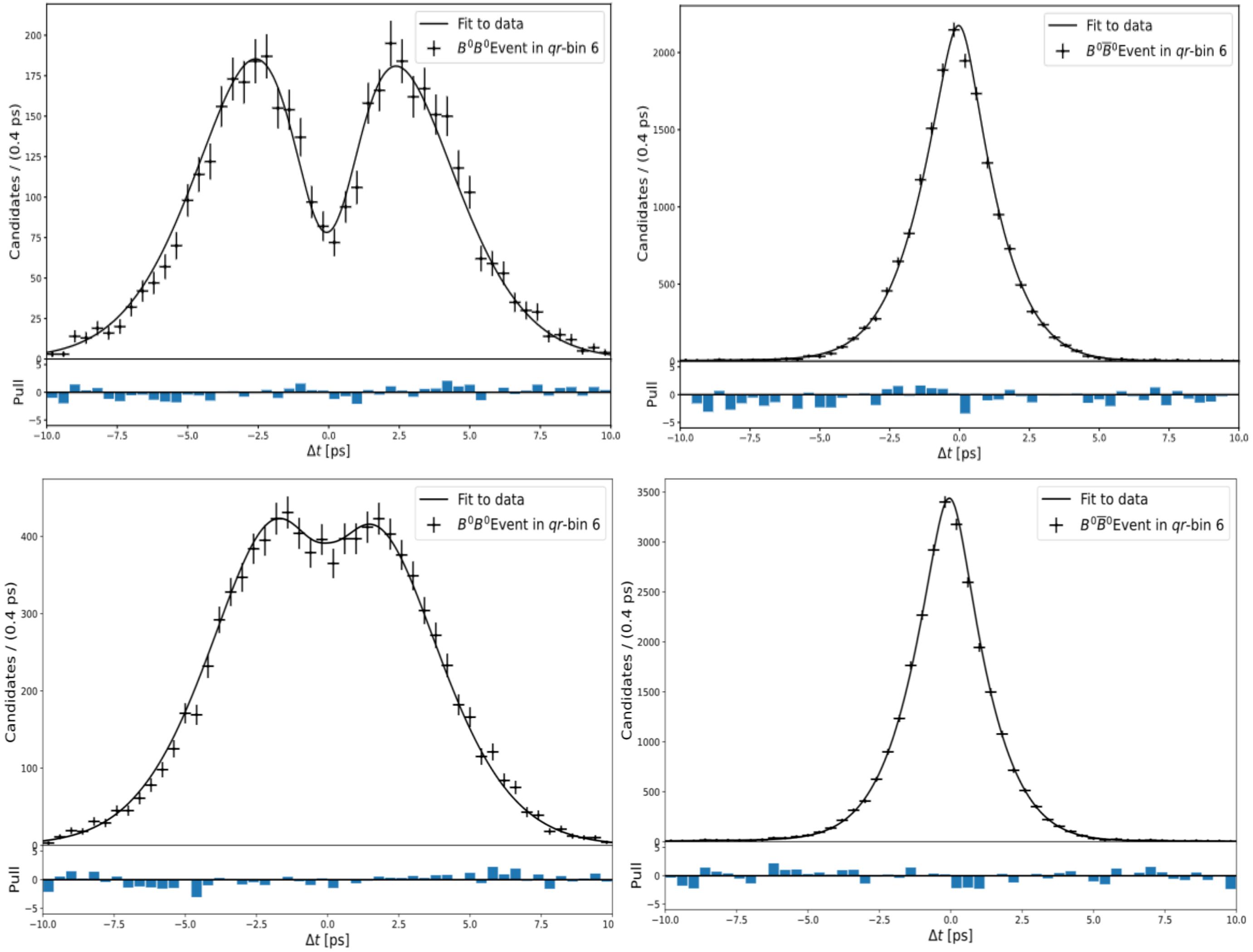
Single Δt fits on Signal MC ($\sim 1\,\mathrm{ab}^{-1}$)

Fit on complete observed Δt – distribution

- Unbinned maximum likelihood fit
- ζ, mistag w and main parameters from resolution function free
- Fits on samples with different ζ_{MC}
- Fit values consistent with input!

$$\zeta_{MC} = 0\,\% \rightarrow \zeta_{fit} = (1.5 \pm 3.2)\%$$

$$\zeta_{MC} = 60\,\% \rightarrow \zeta_{fit} = (58.1 \pm 2.2)\%$$

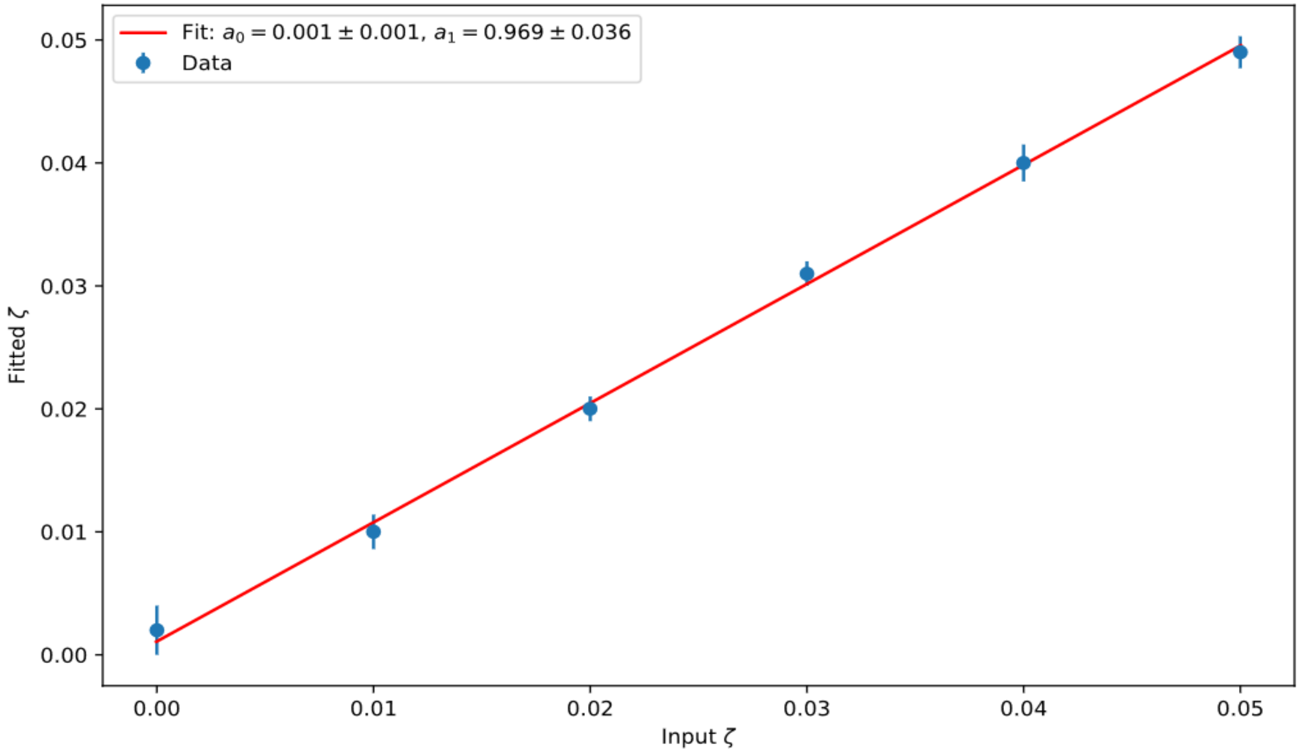


Robustness of the Fitter

Pure Toy MC Study

- Toy Samples drawn directly from PDF for different values of ζ_{MC} (Input)
- For each sample a fit is performed and we average the fitted ζ_{fit} values
- Plot shows linearity of averaged fitter yields
- Fitter confirmed to be robust under stat. fluctuactions

Linearity of fitter with pure Toy MC



Challenge: Correlation between ζ & mistag w

- Signature for disentanglement has a Cross-Talk with mistag w

  → Mistag dilutes time-dependent distribution by factor $|1-2w|$
- Mistag w is highly correlated with disentanglement fraction ζ
- Mistag is a free parameter in the fit
- Fixing mistag w to calibrated value increases sensitivity on ζ significantly ($\sim 5\ \% \rightarrow 1\ \%$)
- BUT mistag is calibrated using the same $B^0 \rightarrow D^{(*)-}\pi^+$ mode & entanglement assumption
- Suggested ways to reduce the $\zeta - w$ cross talk
  a) Flavor tag using Lepton- & fast hadron tag condition
  b) Improve tag-vertex resolution

}

Trade-off with
losing statistics

Summary

- First test of Quantum Entanglement at *Belle II*

  → Successfull proof of concept
- Developed and tested fit framework for determination of ζ
- Validated fitter with toy studies
- Evaluated systematic uncertainties: $\sigma_{syst} = 3.0\ \%$
- Fit on generic MC sample ($\sim 500\ \mathrm{fb}^{-1}$):

  → statistical Unc.: $\sigma_{stat} = 4.5\ \%$
- Overall uncertainties expected to be smaller than previous measurement ($\Delta\zeta \approx 6\ \%$, *Belle*)

Outlook

- Finished analysis in internal Belle II review
- Future project: Further bring down statistical uncertainties by including $B^0 \to D^{(*)-} l^+ \nu$ modes

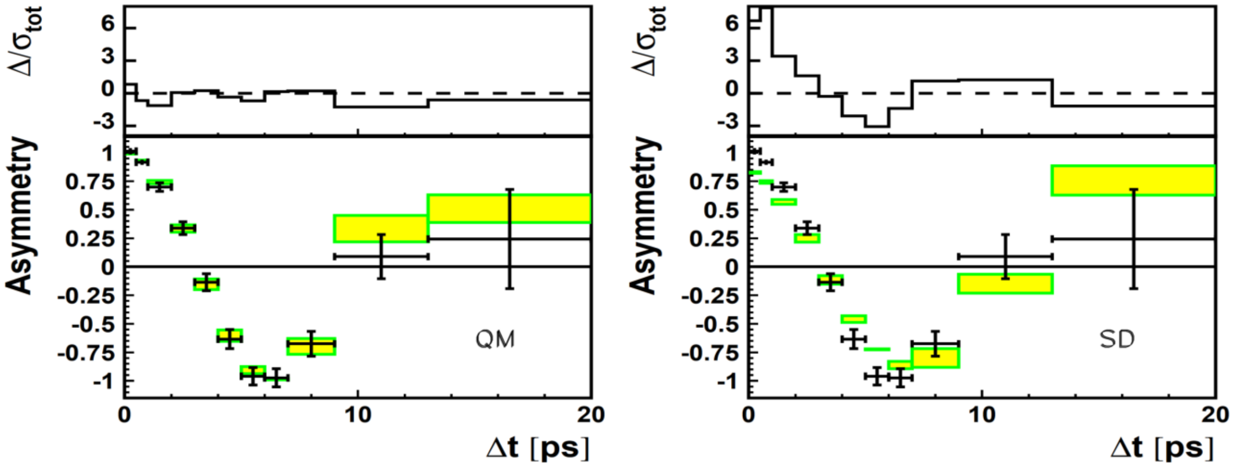
  → Higher Branching fraction than hadronic mode

  → Also flavor-specific mode

Backup Slides

Previous Study at *Belle* (2007)

- Use semi-leptonic modes $B^0 \rightarrow D^- l^+ \nu_l$, ($\sim 180\ \mathrm{fb}^{-1}$ *Belle* data)
- Only use events with leptonic tags & highest purity of tagging efficiency (~8k signal events)
- Deconvolution with Single Value Decomposition (SVD) to get rid of detector effects → Pure physics
- Disentanglement models compared in time-dep. Asymmetry $a(\Delta t)$
- Reported value of $\boldsymbol{\zeta = 0.029 \pm 0.059}$ – compatible with full entanglement, but cannot exclude $\zeta \sim \mathcal{O}(10\%)$ at 90% CL
- Caveat: Mistag - ζ correlation unaccounted (w fixed to MC), i.e. phase shift by mistag no considered



A .Go et al. (arXiv:quant-ph/0702267)

Comparing different cases

Standard Case (QM):

- $\boxed{P_{QM}(\Delta t) \propto \; e^{-\Delta t/\tau} \, (1 \mp \cos \Delta m \Delta t)}$

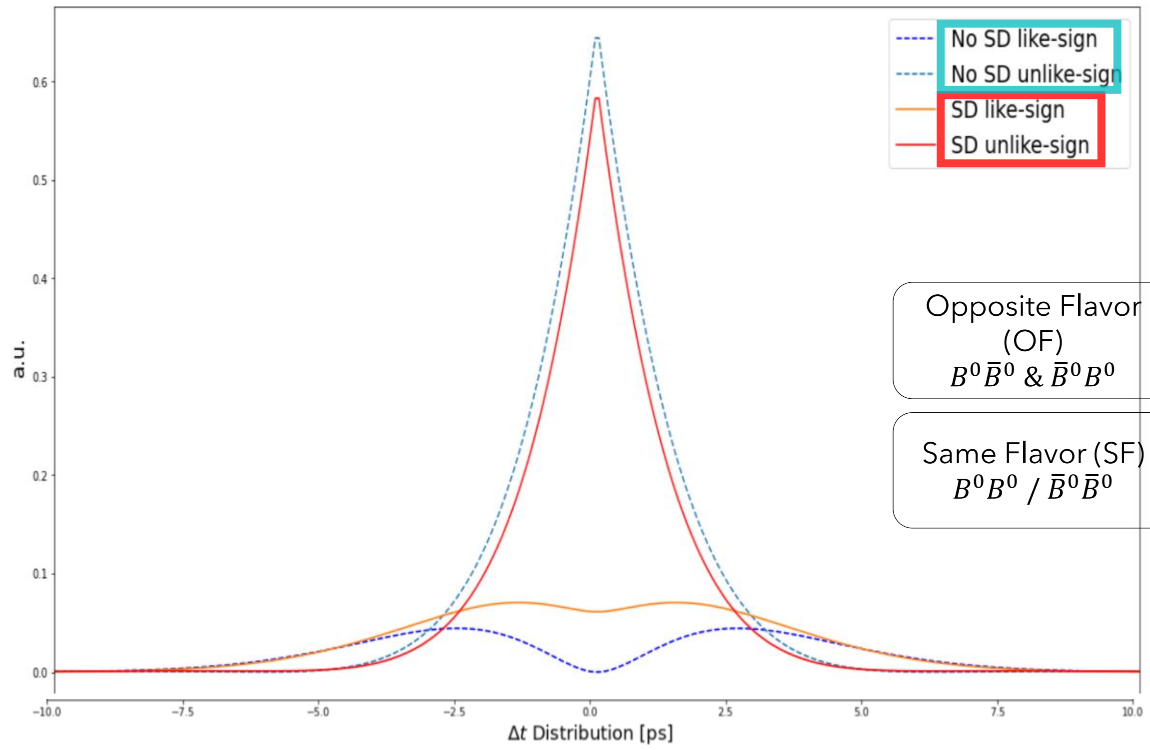
Spontaneous Disentanglement (SD):

- Independent oscillation from $t = 0$ of both Bs starting from orthogonal states
- $\boxed{P_{SD}(\Delta t) \propto e^{-\Delta t/\tau} \, (2 \mp f_{cos} \cdot \cos \Delta m \Delta t \pm f_{sin} \cdot \sin \Delta \mathrm{m} |\Delta t|)}$

Partial Spontaneous Disentanglement (PSD):

- Same as SD model with a fraction **($\boldsymbol{\zeta}$)** of disentangled pairs
- $P_{PSD}(\zeta;\, \Delta t) = (1 - \zeta) \cdot \boxed{P_{QM}\,(\Delta t)} + \zeta \cdot \boxed{P_{SD}(\Delta t)} \;\; \zeta \in [0, 1]$
- Last measured value: $\boldsymbol{\zeta = 0.029 \pm 0.059}$ – compatible with full entanglement, but cannot exclude $\zeta \sim \mathcal{O}(10\%)$ at 90% CL (Belle 2007)

Decay time difference Δt - distribution



Opposite Flavor (OF)
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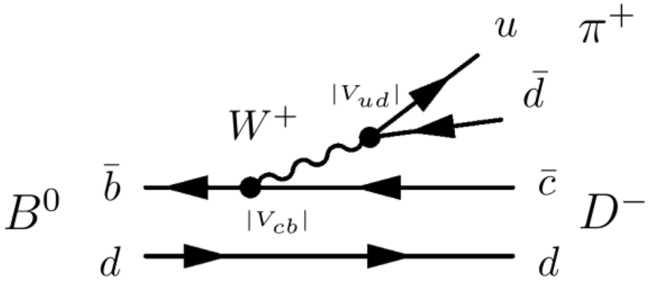
Same Flavor (SF)
$B^0 B^0$ / $\bar{B}^0 \bar{B}^0$

Disentanglement fraction ζ is our key physics parameter!

Choice of $B^0 \to D^{(*)-}\pi^+$ Channel

1. Self-tagging decay – final state particles ‘tag’ flavor of decaying B
    - $B^0 \to D^{(*)-}\pi^+$
    - $\bar{B}^0 \to D^{(*)+}\pi^-$
2. No CP Interference
    - Dominated by single tree amplitude (see Feynman diagram)
    - No mixing-induced CP violating terms
3. Relatively large BF & no neutral final state particles
    - Good signal yield & well understood systematics
    - Good vertexing → precise time resolution



Selection Criteria

Signal-side reconstruction

- $\varDelta E \in [-0.10, 0.25]$ GeV for $\Delta E = E_B^* - E_{beam}^*$, with $E_{beam}^{*2} = \sqrt{s}/2$
- $M_{bc} \in [5.2,\ 5.3]$ GeV for $M_{bc} = \sqrt{E_{beam}^{*2} - p_B^2}$
- $M_{D^\pm} \in [1.810, 1.895]$ GeV for $D^\pm$ candidates
- $Q \in [4.6, 7.0]$ MeV for $Q = M(D^{*-}) - M(D^0) - M(\pi^-)$ for $D^{*\pm}$ candidates
- $D^{(*)\pm}$ are combined with primary $\pi^\mp$ to form the B candidate
- Polar angle θ within CDC acceptance range
- $dz < 3.0$ cm and $dr < 0.5$ cm

Tag-side reconstruction

- $dz < 2.0$ cm and $dr < 0.5$ cm
- Tracks must have hits in PXD, SVD and CDC
- Track momentum over 50 MeV/c for beam background suppression

Resolution Function

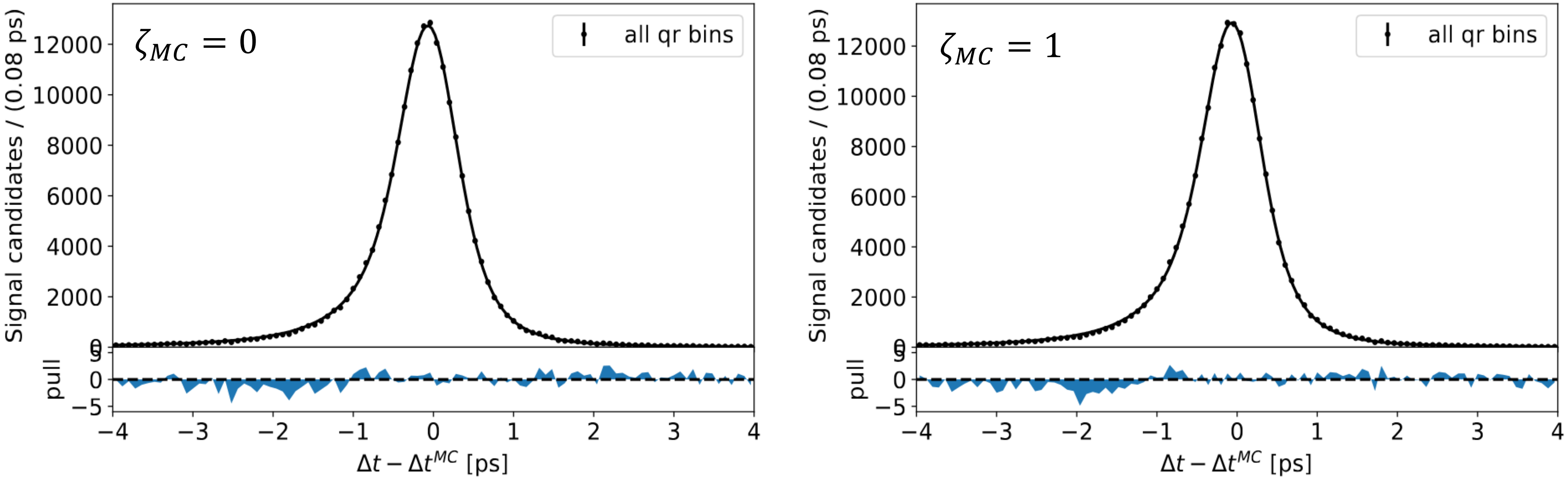
$$
\begin{aligned}
R_{\text{core}}(\delta\Delta t;\sigma_{\Delta t}) =& (1-f_{\text{tail}})\cdot G(\delta\Delta t;\mu_{\text{main}}\cdot\sigma_{\Delta t}, s_{\text{main}}\cdot\sigma_{\Delta t}) \\
& + f_{\text{tail}}(1-f_{\text{exp}})\cdot G(\delta\Delta t;\mu_{\text{tail}}\cdot\sigma_{\Delta t}, s_{\text{tail}}\cdot\sigma_{\Delta t}) \\
& + f_{\text{tail}}\cdot f_{\text{exp}}\cdot G(\delta\Delta t;\mu_{\text{tail}}\cdot\sigma_{\Delta t}, s_{\text{tail}}\cdot\sigma_{\Delta t}) \\
& \otimes ((1-f_{\text{R}})\text{exp}_{\text{L}}(\delta\Delta t/c\cdot\sigma_{\Delta t}) + f_{\text{R}}\text{exp}_{\text{R}}(-\delta\Delta t/c\cdot\sigma_{\Delta t}))
\end{aligned}
$$

$$R_{\text{OL}}(\delta\Delta t;\sigma) = G(\delta\Delta t, 0, \sigma_{\text{OL}})$$

1. Double Gaussian for detector resolution – dep. on $\sigma_{\Delta t}$
2. Exponential tails for detached D vertices – dep. on $\sigma_{\Delta t}$
3. Broad Gaussian for outlier effects independentent of $\sigma_{\Delta t}$

Resolution function parameters fixed through Residual fits

- Residual fit on $\delta\Delta t = \Delta t - \Delta t^{\mathrm{MC}}$
- independent of flavor & disentanglement effects
- Nuisance parameters calibrated through this fit



Toy MC Studies

Main Goal: Asses bias in modelling and the fitting procedure

with figure of merit: $\mathrm{Pull} = \frac{x_i - \mu_i}{\sigma_i}$

Pure Toy MC Method

- Goal: Test for PDF's behavior under stat. fluctuations
- Generate samples (replicas) directly from PDF
- Fit same PDF to replica samples (N ~1000)
- Check for consistency using pulls

Bootstrap Method

- Goal: Direct test on fitting process
- Adv: less exposure to systematic uncertainties stemming from the PDF
- Bootstrap samples from Monte Carlo
- Repeat fitting process on bootstrap samples (N ~1000)

Background Modeling - ΔE fits

Signal ΔE

- $B^0 \to D^{(*)-}\pi^+$
- Modelled by Crystal Ball with tails to the left and right

$B^0 \to D^- K^+$ Background

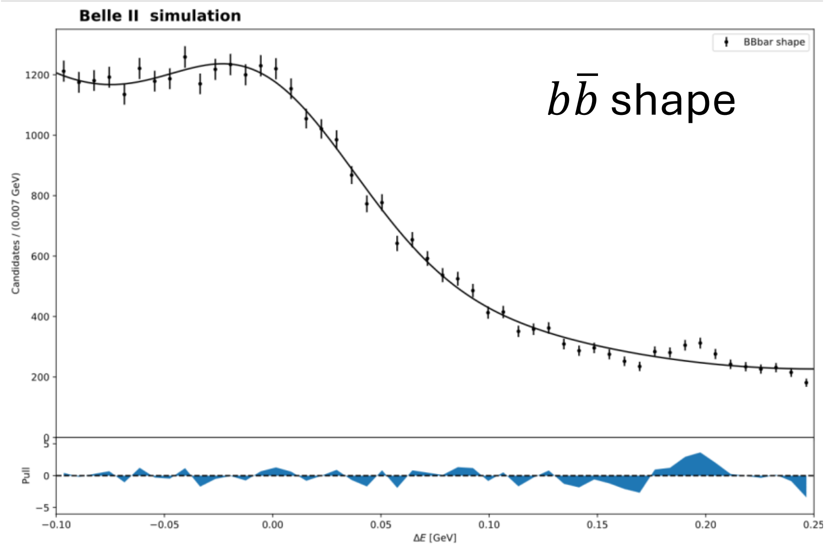
- From $K \leftrightarrow \pi$ misidentification
- Peaking background (shifted)
- Kinematics are very similar to signal
- Modelled by Crystal Ball with tails to the left and right

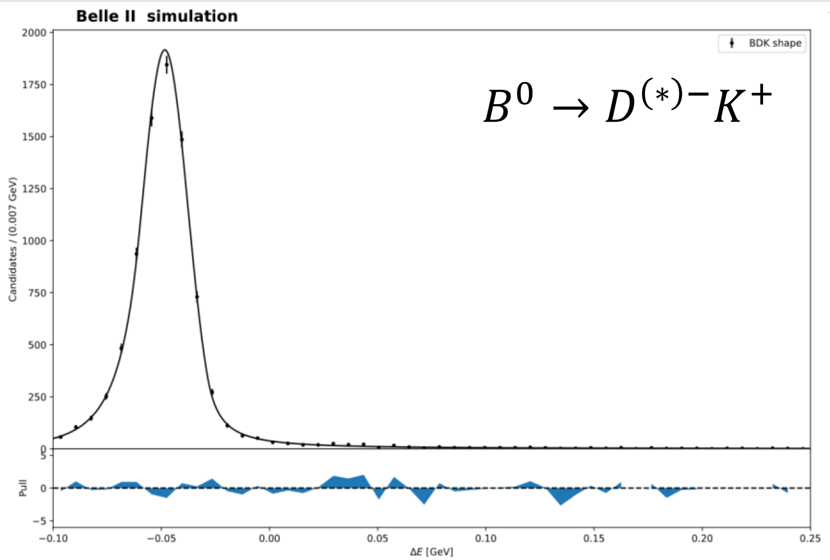
$b\bar{b}$ - Background

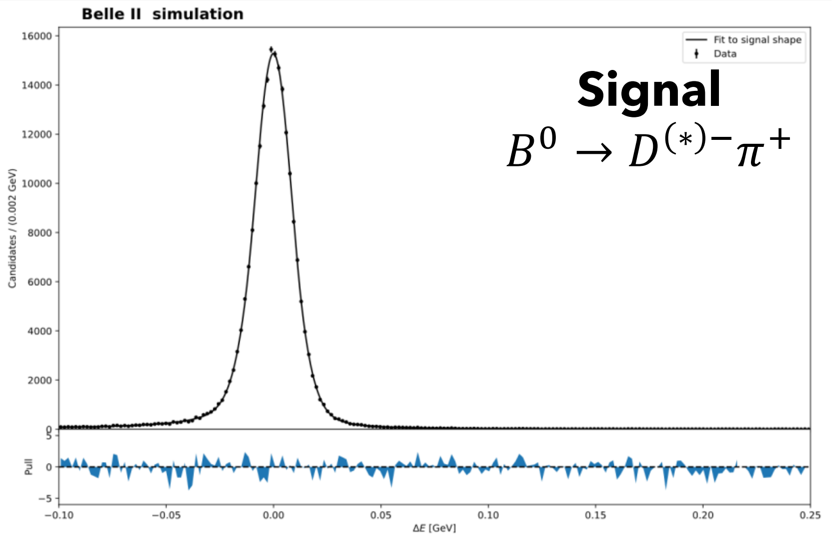
- Wrong reconstrution
- Modeled by 2nd order polynomial & Gaussian

Continuum background ($e^+e^- \to q\bar{q}$)

- jet-like topology (Back to back)
- Broad Smooth ΔE modeled by exponential







Systematic Uncertainties

Analysis Bias:

- Imperfections & correlations between variabls not accounted in fit model
- Bootstrap MC – Linearity Fit

τ & Δm_d input bias

- Parameters fixed to PDG values in fit (same as ones used for MC production)
- Input parameter limits in fit: $\mu_i \pm 1\sigma_\mathrm{i}$ and average extracted ζ value

Background modeling

- Test how model depency of ΔE fits bias ζ
- ΔE shapes for different backgrounds ($q\bar{q}, b\bar{b}$, BDK) modeled by different functions
- Resulting sWeights are used as inputs for Δt fits → ζ fit results are averaged

Tag-Side Interference

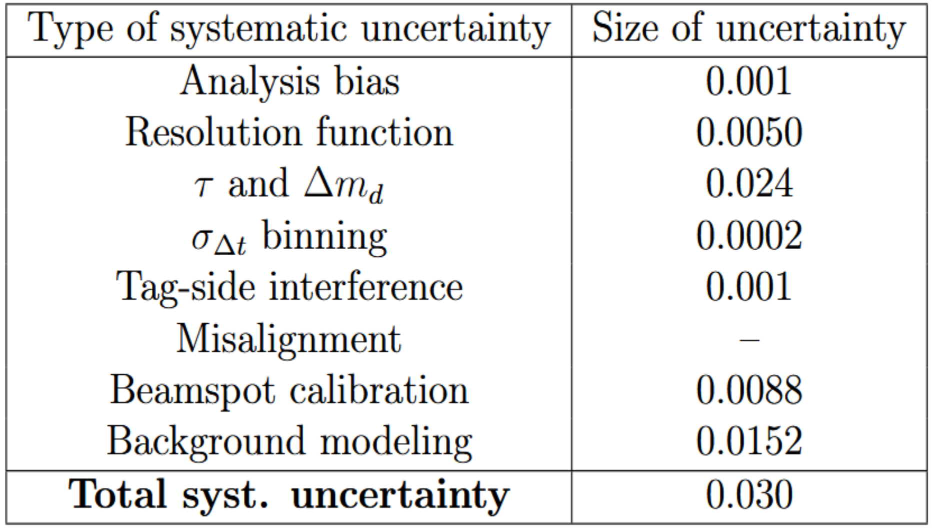
- Different amplitudes for TSI are included in fit model
- Test how this additional $f_{TSI} \cdot \sin \Delta m \Delta t$ term affects ζ

Beam Spot

- Beam spot calibration has limited precision (time variation,statistics, intrinsic systematic uncertainties)

Resolution Function

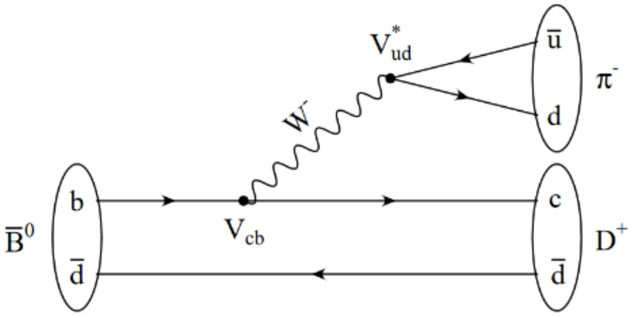
- Test how fixing nuisance parameters to MC via residual fit bias ζ
- Floating one (or combination of) nuisance params in fit → See how ζ behaves



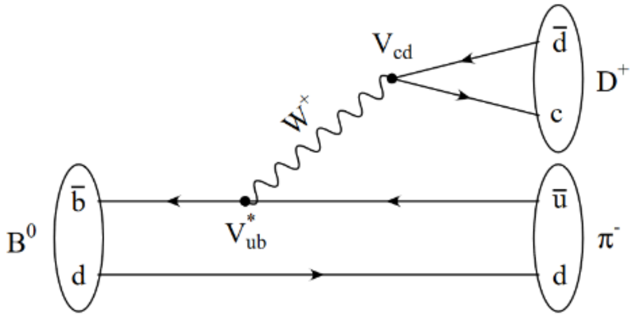
Type of systematic uncertainty	Size of uncertainty
Analysis bias	0.001
Resolution function	0.0050
τ and Δm_d	0.024
$\sigma_{\Delta t}$ binning	0.0002
Tag-side interference	0.001
Misalignment	–
Beamspot calibration	0.0088
Background modeling	0.0152
Total syst. uncertainty	0.030

Tag-Side Interference

- $\bar{B}^0 \to D^+\pi^-$ suffers from Tag-Side Interference
- Same final state can be reached through $B^0 \to D^+\pi^-$ can be reached through Doubly CKM Suppressed tree topology
- Relative branching ratio $|V_{ud}^{*}V_{cb}/V_{ub}^{*}V_{cd}| \approx 2\ \%$
- Introduces another $f_{TSI} \cdot \sin \Delta m \Delta t$ term with f_{TSI}, the amplitude → Asymmetric phase shift in time-dep. asymmetry



CKM favored $\overline{B^0} \to D^+\pi^-$ decay



Doubly CKM suppr. $B^0 \to D^+\pi^-$ decay

Ways disentanglement can be/seem broken

Soft photon emission

- A soft photon ($\sim 40\ \mathrm{MeV}$) is emitted in $\Upsilon(4S) \rightarrow B^0\bar{B}^0\gamma$, taking Spin-1 from $\Upsilon(4S)$ away
- Although wave function of $B^0\bar{B}^0$ is in $C = -1$ and in due to angular momentum conservation in orthogonal states, mesons are not entangled
- No interference with $\Upsilon(4S) \rightarrow B^0\bar{B}^0$, since not same final state

Superposition of $C = +1$

- Highly suppressed (theoretical)
- Interference since same final state
- If $C = +1$ and $C = -1$ would be generated 50/50, same signature as full disentanglement