Loop Corrections in the Effective Field Theory of Large-Scale Structure

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M.Sc. Theoretical and Mathematical Physics

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LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
Technische Universität München
MAX-PLANCK-INSTITUT FÜR PHYSIK
TMP Elite Master Course Theoretical and Mathematical Physics

What is the EFTofLSS?

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Large-scale structure

Late-time distribution of matter and galaxies

- Tracer of primordial fluctuations + information on other dynamical stages of cosmic history

- **3D surveys** $\longrightarrow$ ultra-precision cosmology

  [BOSS, Euclid, DESI, LSST, SphereX, …]

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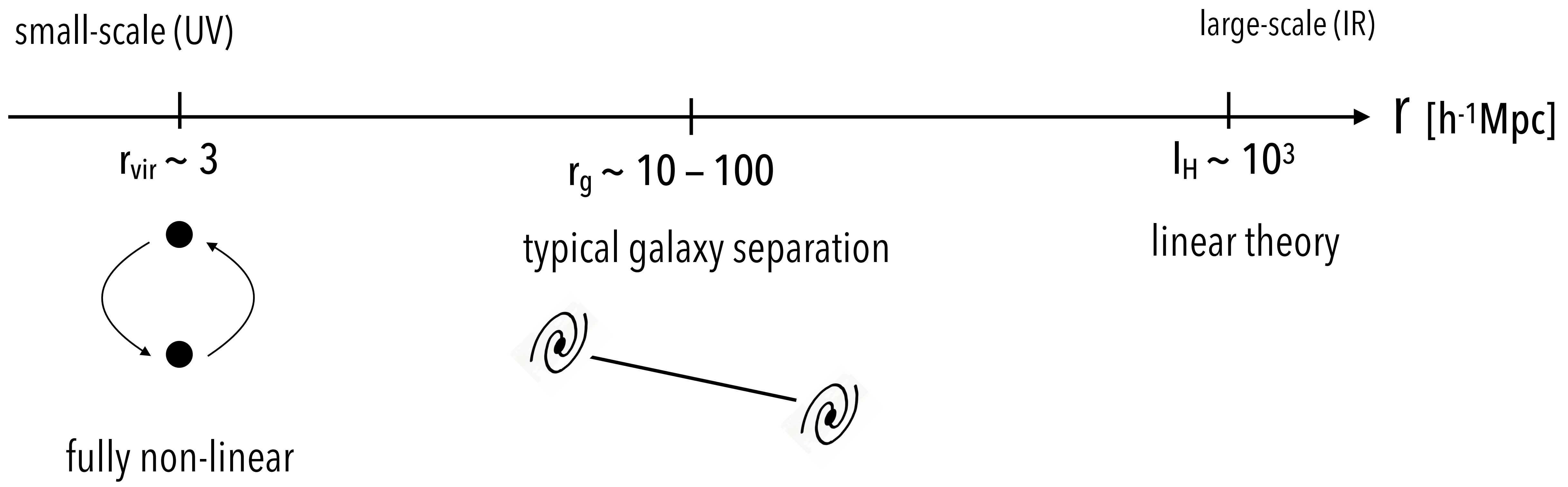
We need accurate theoretical model



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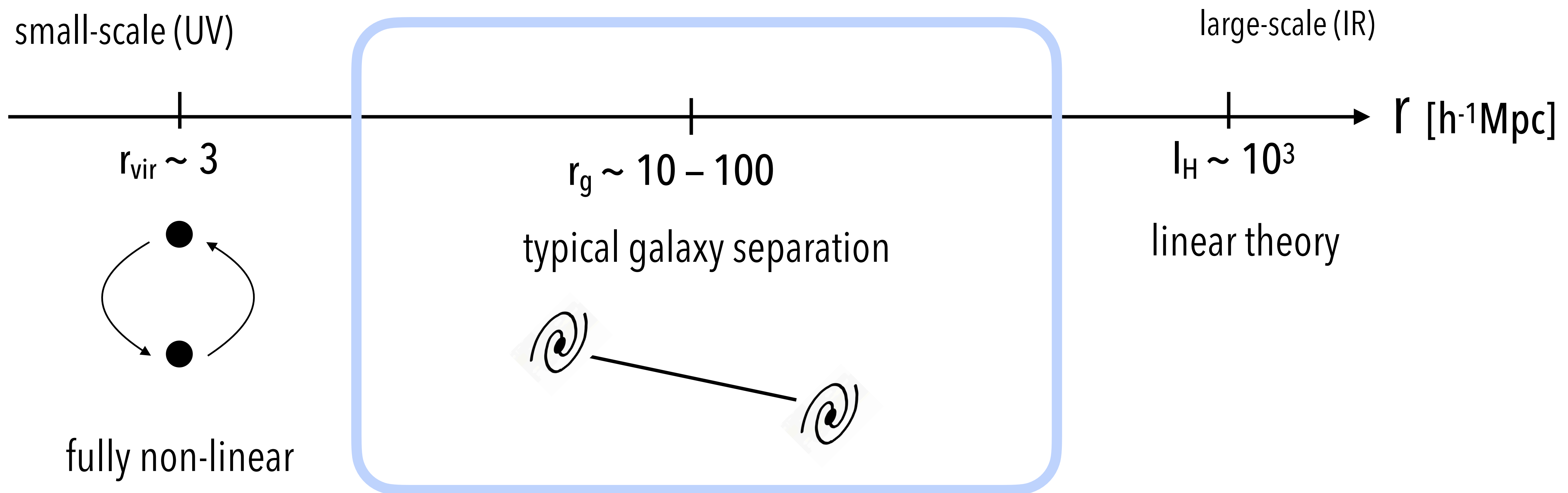

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separation of scales ⟶

EFTofLSS

[Baumann, Nicolis, Senatore, Zaldarriaga, '12]
[Carrasco, Hertzberg, Senatore , '12] + many later works

Galaxies / DM halos as perturbations on top of a homogeneous and isotropic background

Non-ideal effective fluid description

$$\delta_\Lambda(\tau, \mathbf{x}) \equiv \frac{\rho(\tau, \mathbf{x})}{\bar{\rho}(\tau)} - 1 \qquad \theta^i_\Lambda(\tau, \mathbf{x}) \qquad + \text{UV counterterms}$$

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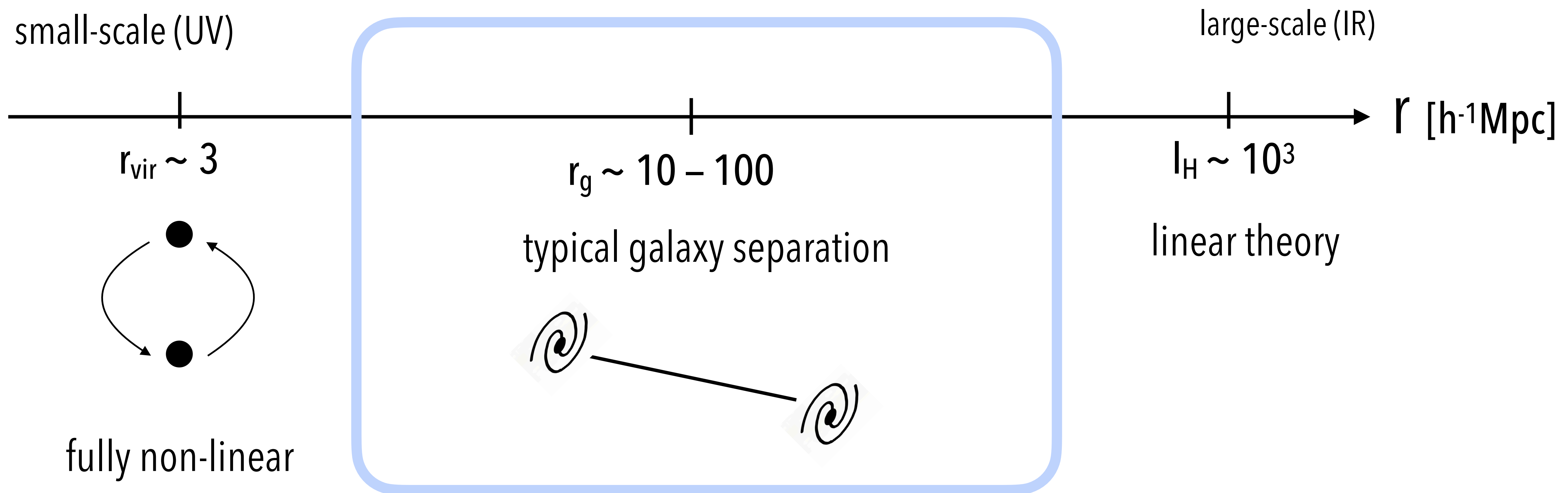

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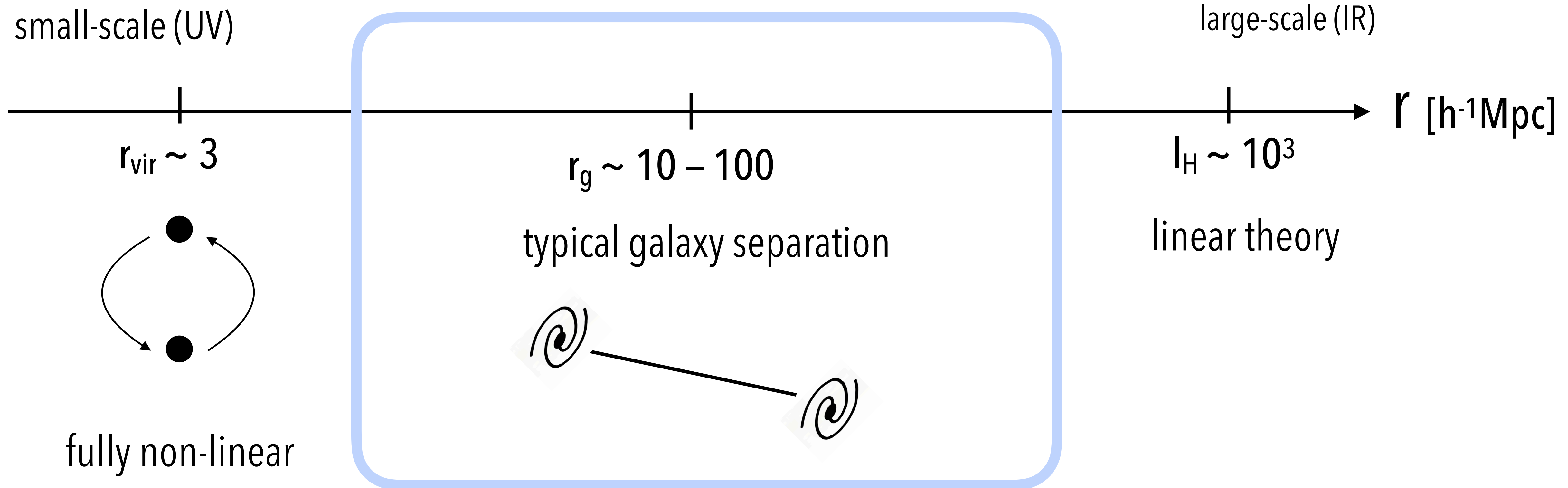
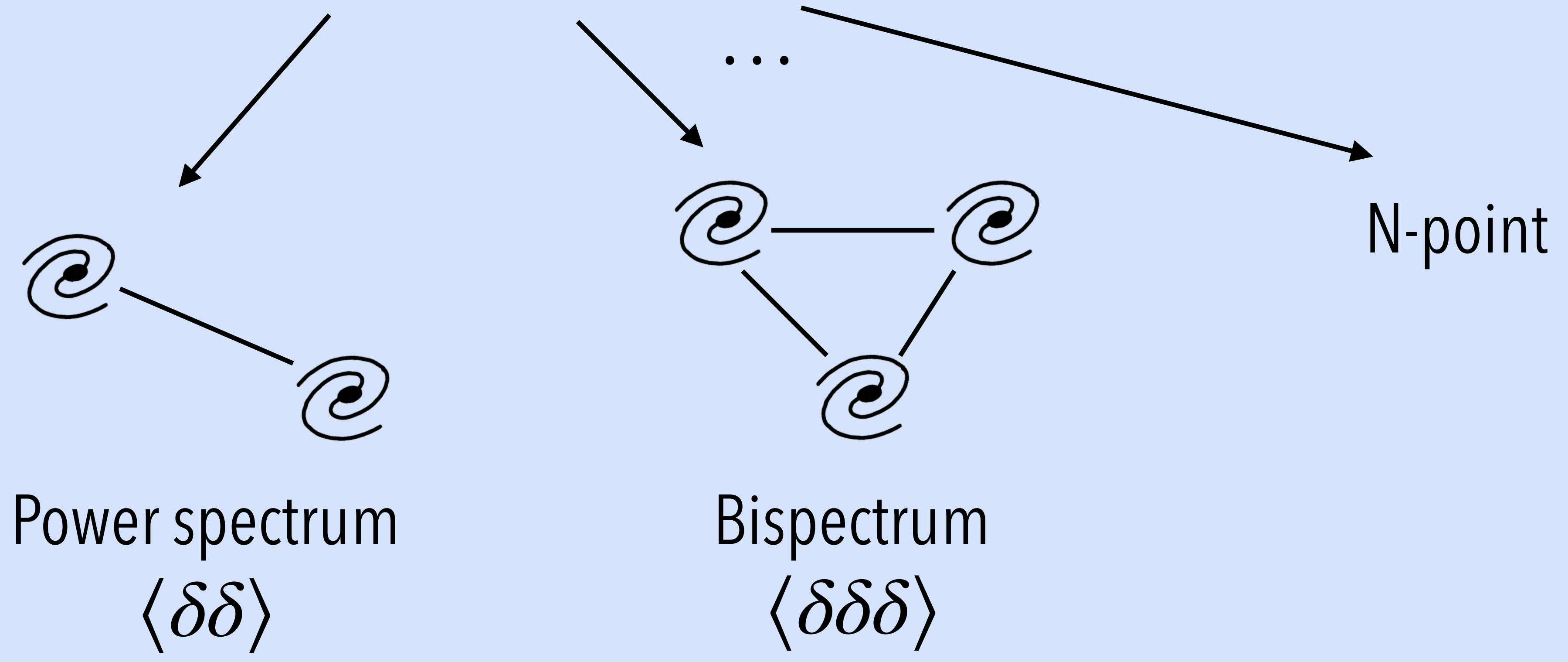


Correlation functions



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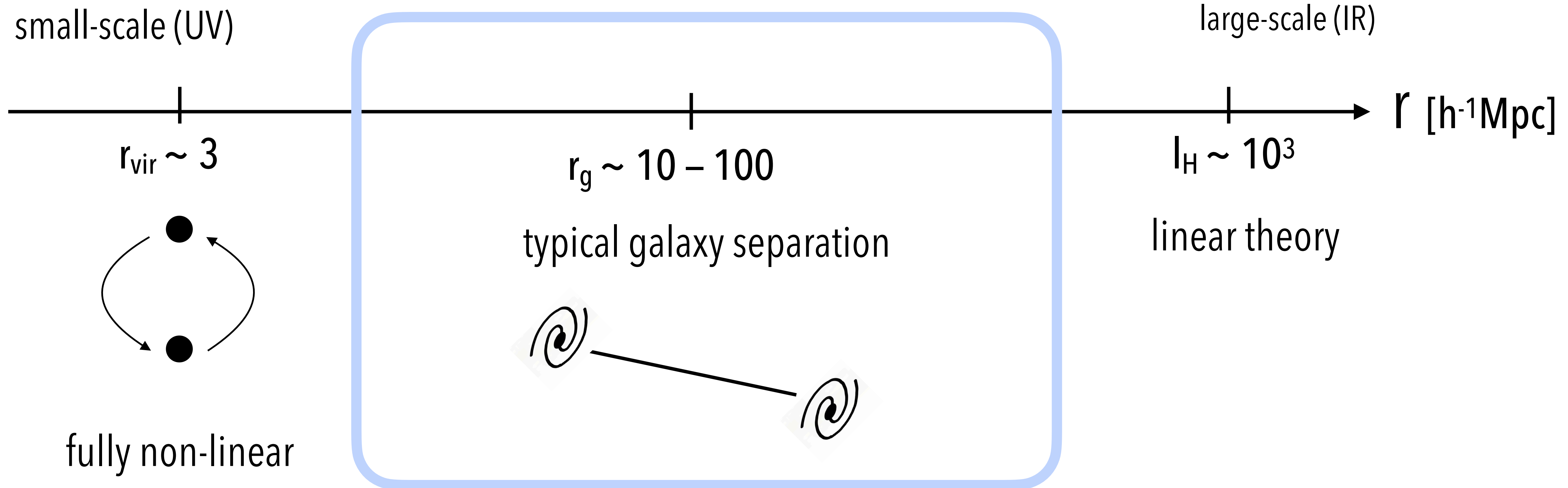

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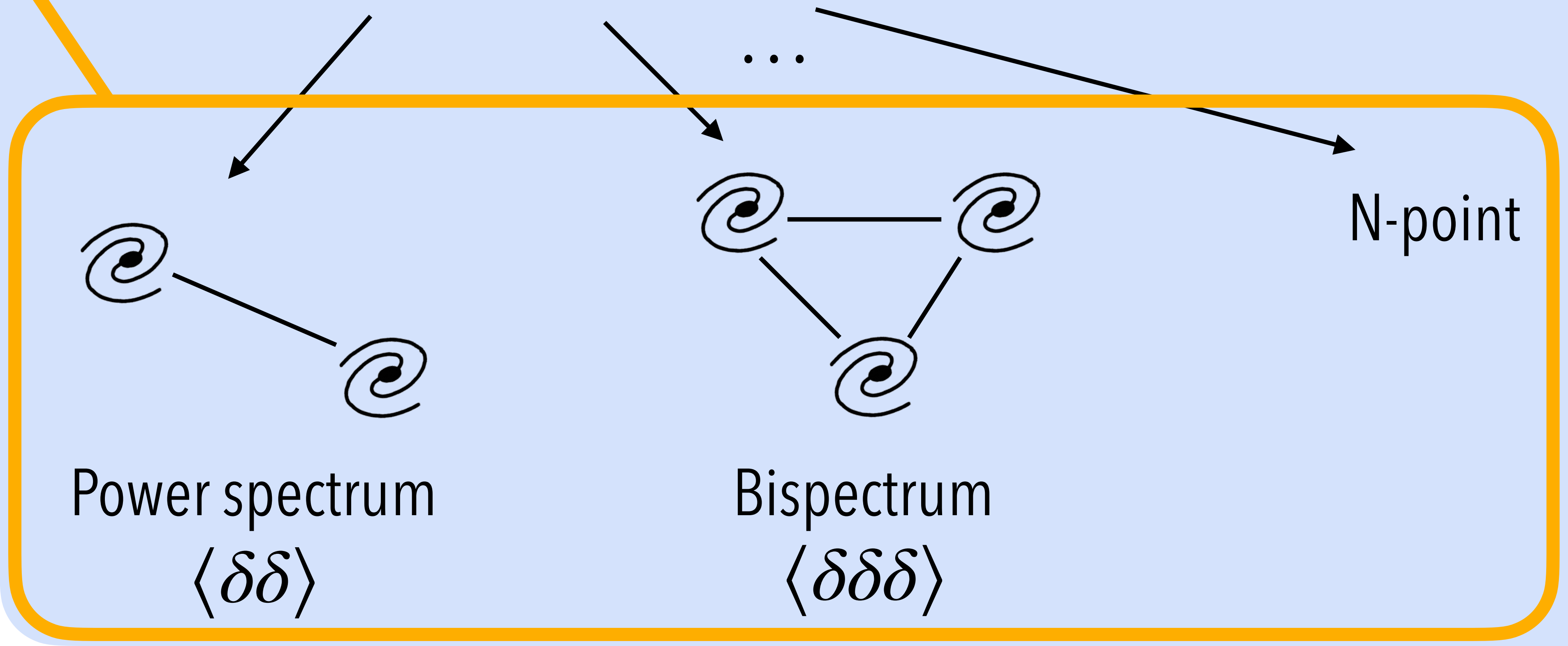

Can we use QFT methods to compute them?

Dark matter power spectrum in the EFTofLSS

[Scoccimarro, '96; Bernardeau, Colombi, Gatzañga, Scoccimarro, '02; Carrasco, Hertzberg, Senatore, '12; Pajer, Zaldarriaga, '13; Blas, Garny, Ivanov, Sibiryakov, '16; Philcox, '20; Ivanov, '22…]

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$$P(\tau, k) = D^2(\tau)\, P_{\rm lin}(k) \;+\; D^4(\tau)\, P_{1-\rm loop}(k) \;+\; D^6(\tau)\, P_{2-\rm loop}(k) \;+\; \ldots \;+\; \text{counterterms}$$

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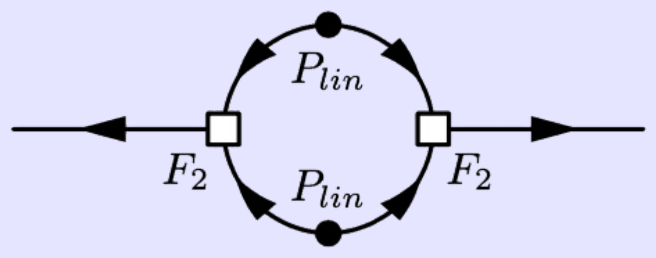

$$= \int \frac{d^3\mathbf{q}}{(2\pi)^3}\, F_2^2(\mathbf{q} - \mathbf{k}, \mathbf{q})\, P_{\rm lin}(q)\, P_{\rm lin}(|\,\mathbf{k} - \mathbf{q}\,|)$$

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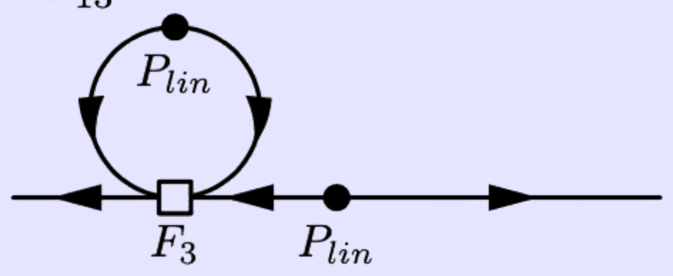

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[images from Simonović et al, 2017]

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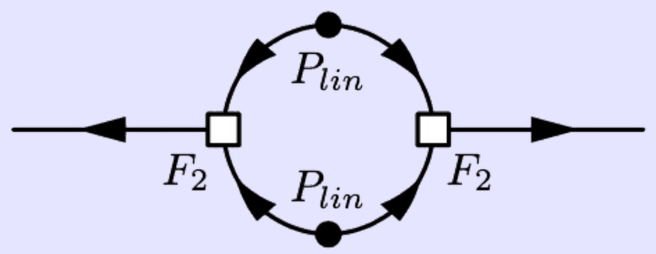

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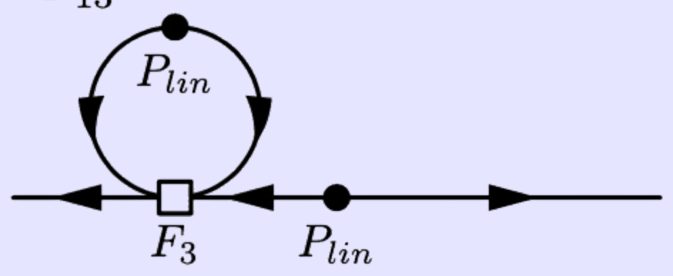

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rational functions!
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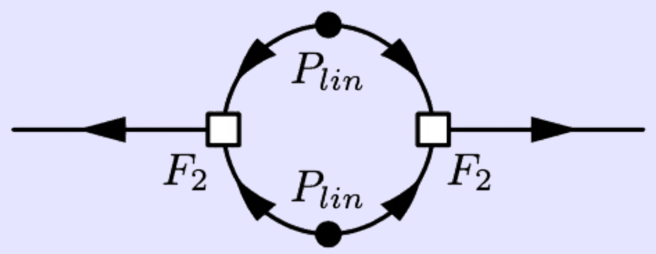

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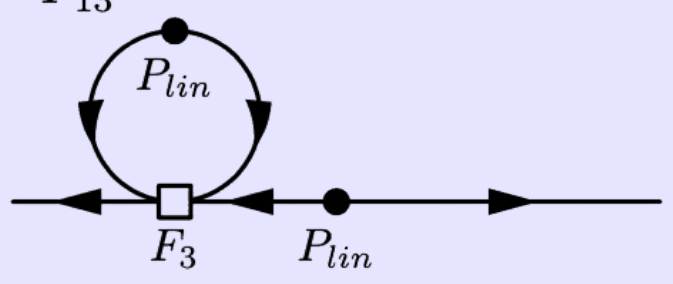

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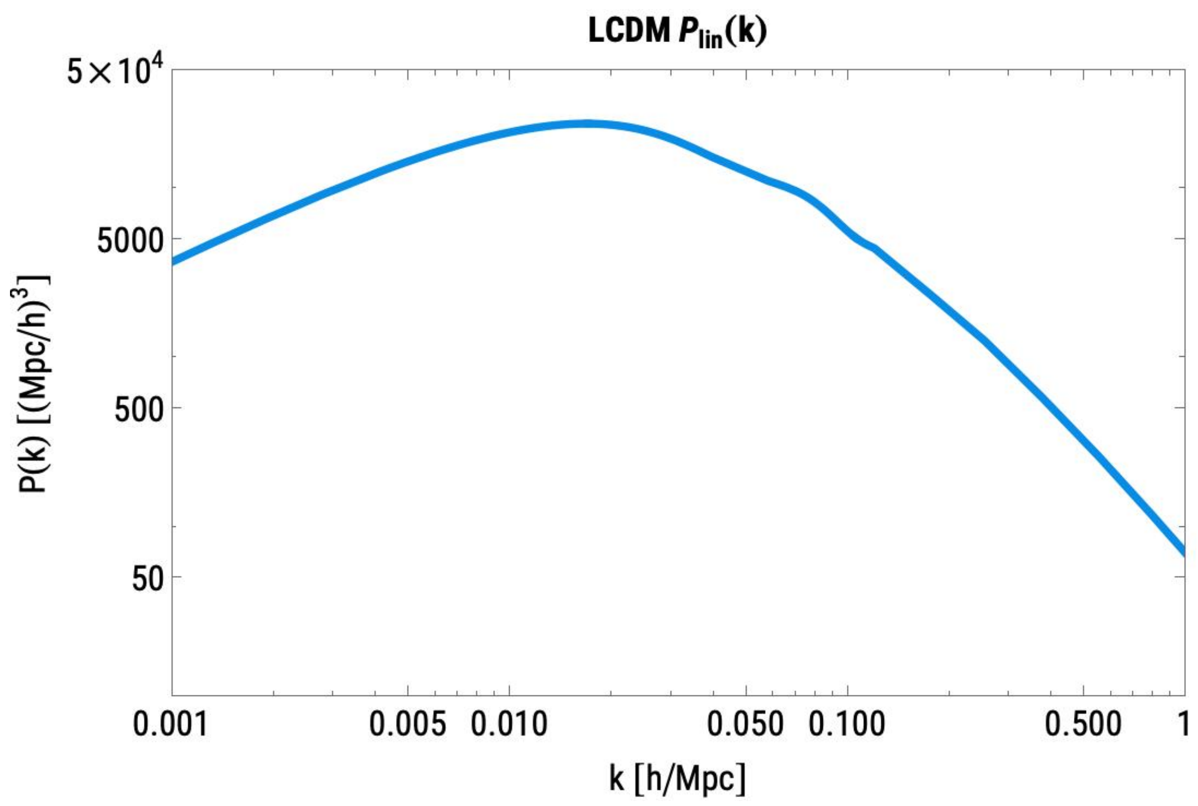

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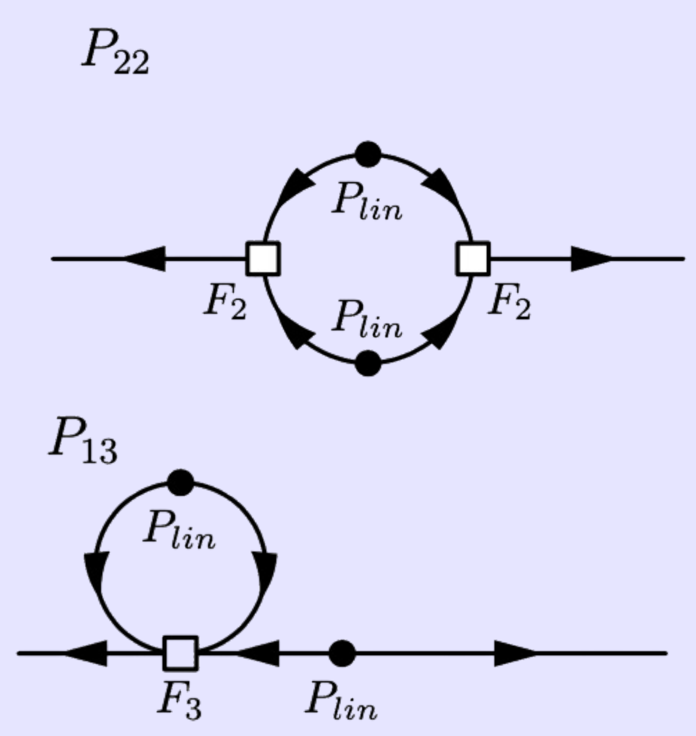

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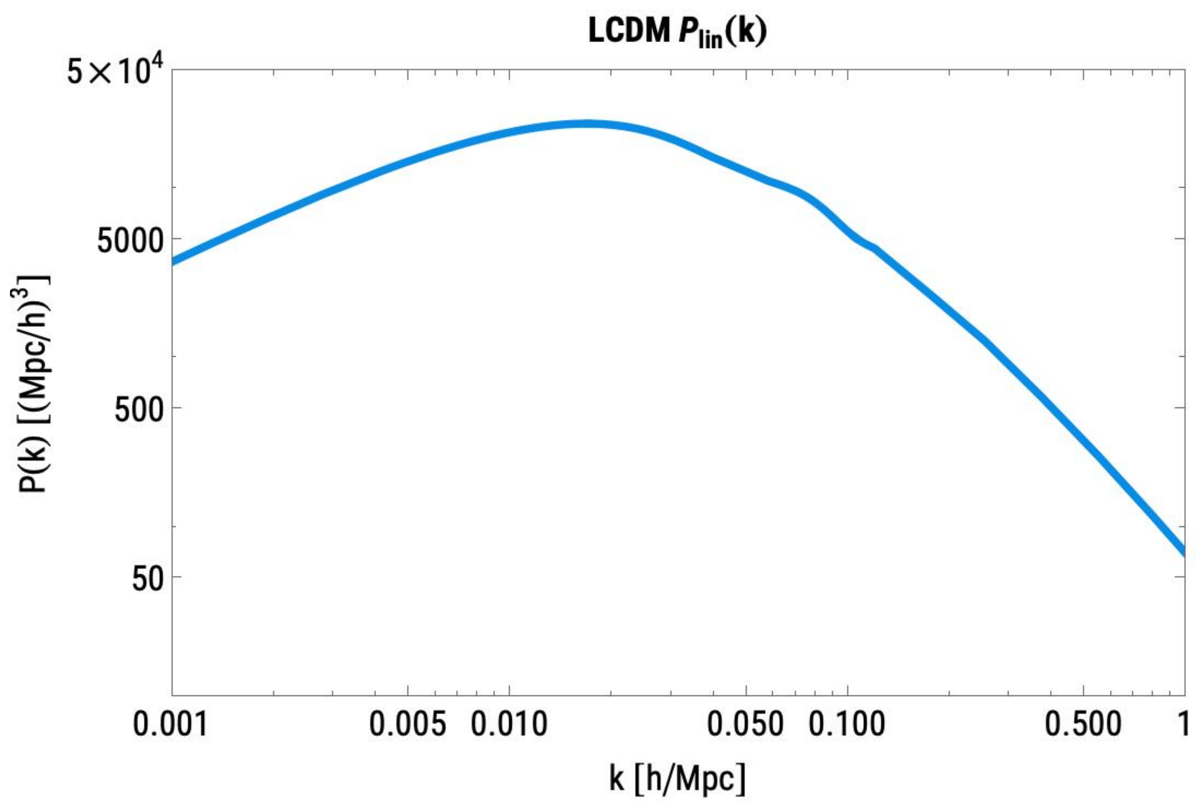

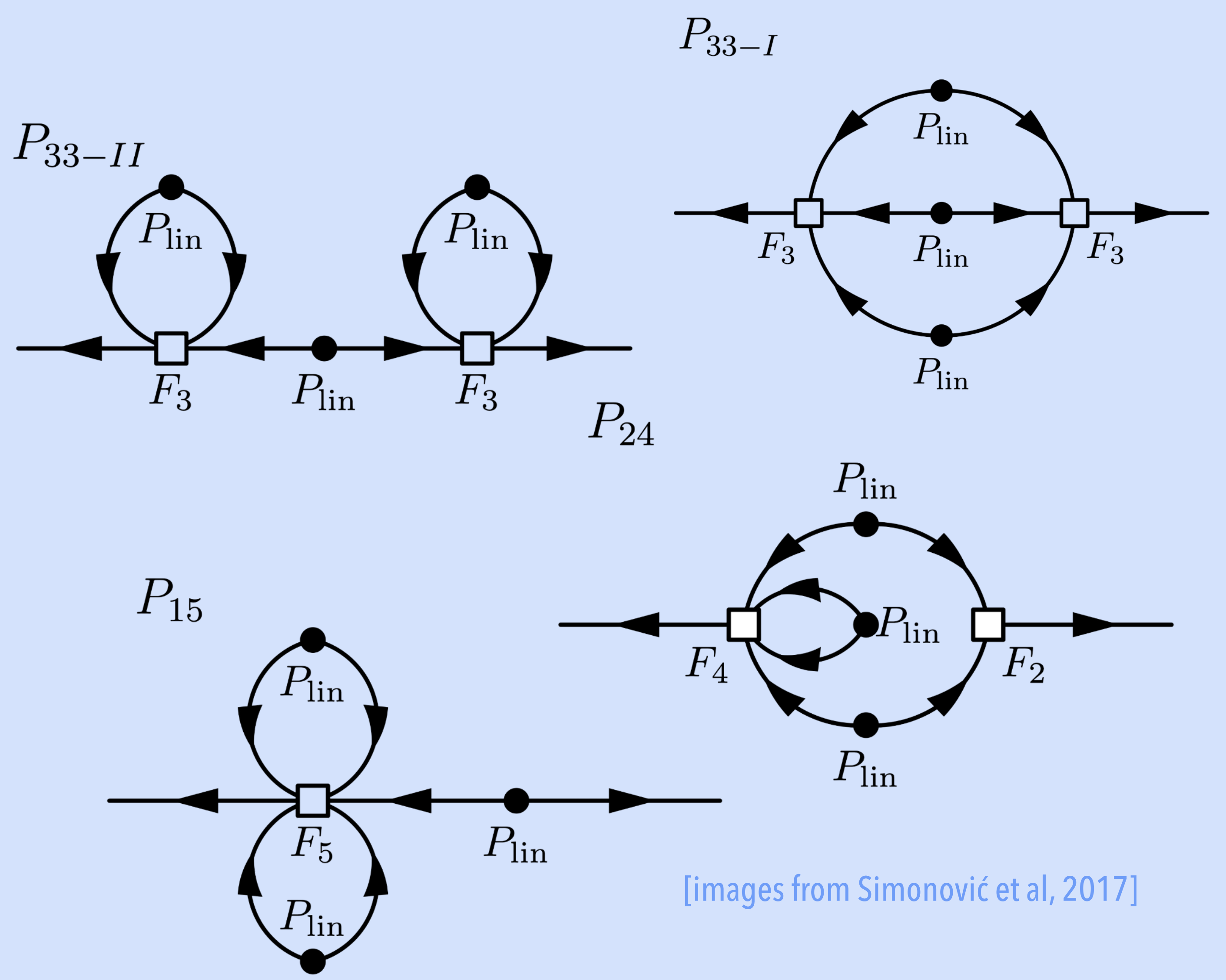

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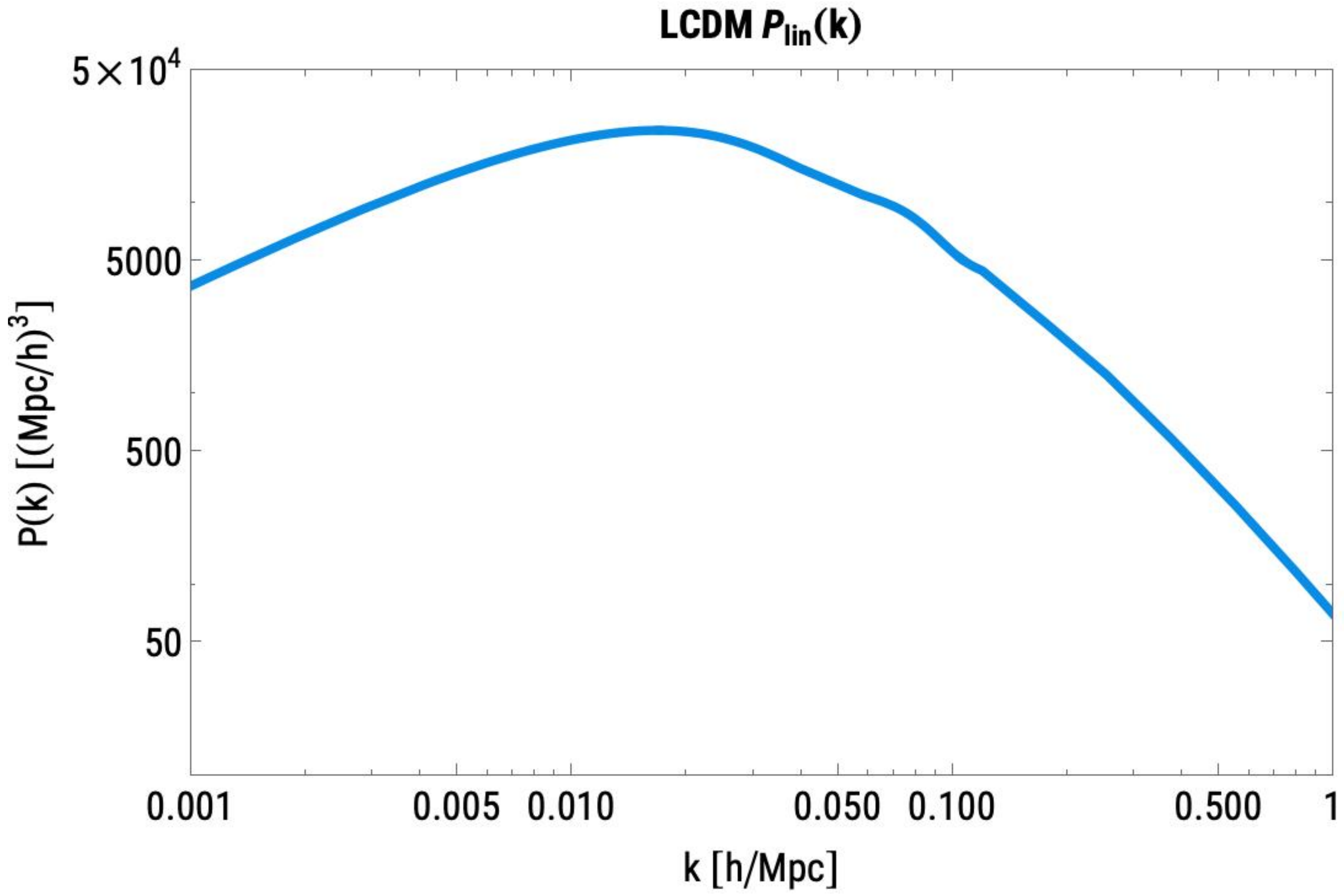
Efficient evaluation of loop corrections

$P_{lin}(k)$ has a lot of features ⟶ numerical integration is inefficient.

Solution:
approximate $P_{lin}(k)$ onto set of smoother functions and solve more, but easier loop integrals.



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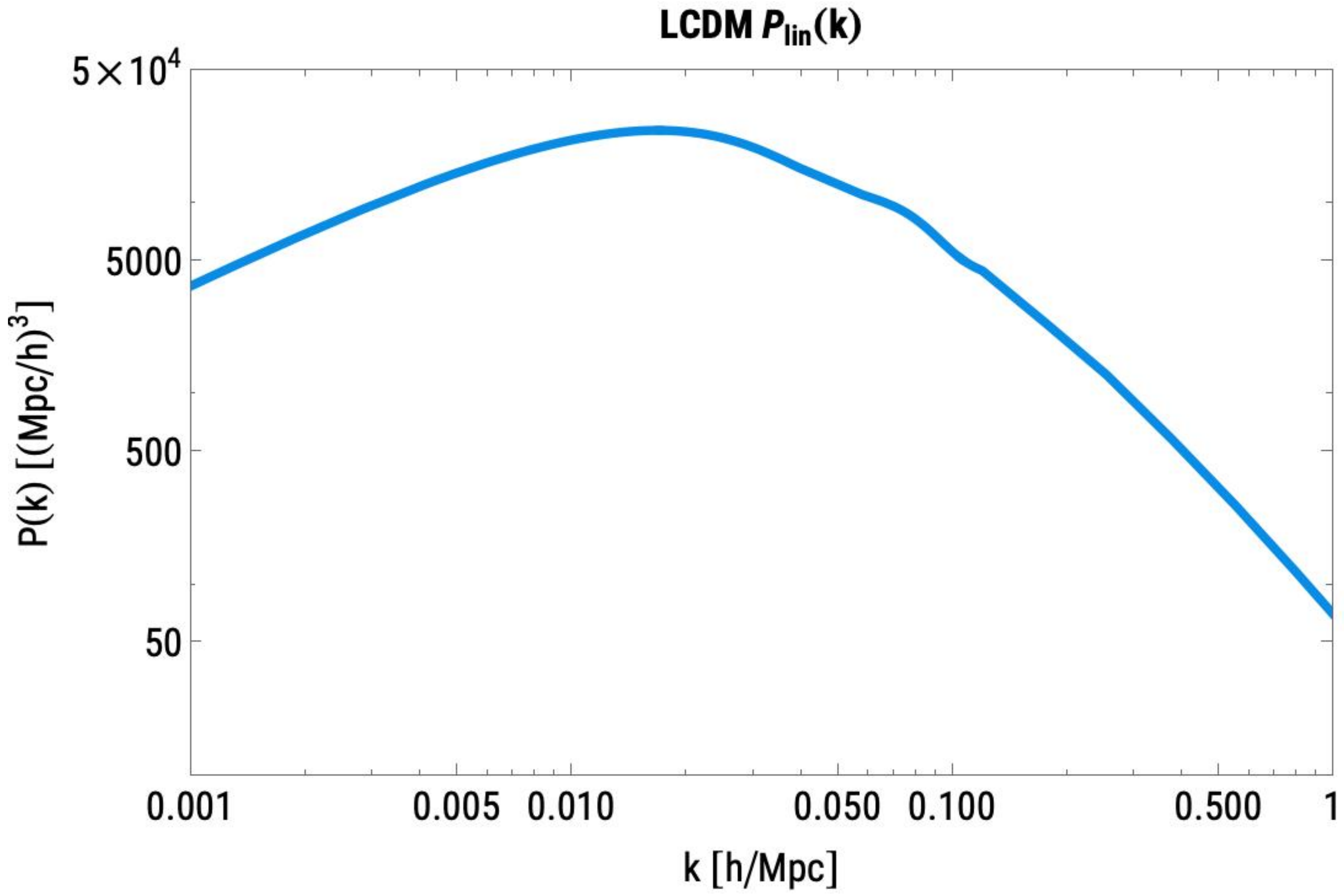

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There are various prescriptions

FFTLog

[Simonović et al, '17]

State of the art

Power spectrum 1-loop

Bispectrum 1-loop

(fully analytic)

Massive propagators

[Anastasiou et al, '23; '25]

State of the art

Power spectrum 1-loop (analytic)

Power spectrum 2-loop (numerical)

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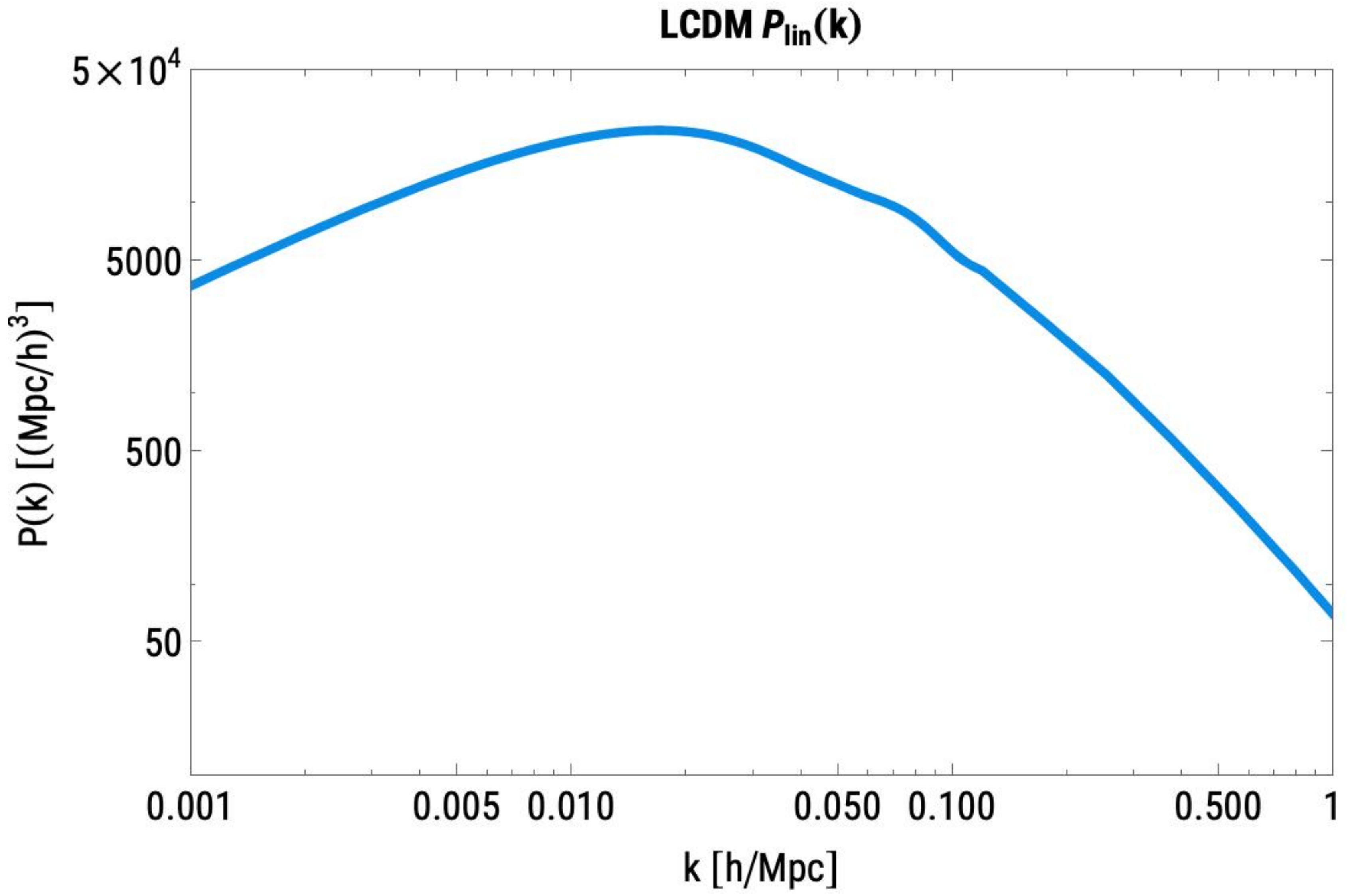

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let's compute power spectrum at 2-loops *analytically*

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The two-loop power spectrum using FFTLog

FFTLog prescription

[Simonović et al, '17]

Decompose $P_{lin}(k)$ onto power laws

$$P_{\rm lin}\left(k_n\right) = \sum_m c_m\, k_n^{\nu + i\eta_m}$$

Discrete Mellin Transform (FFTLog)

[Hamilton, '00]

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Cosmological loops decompose onto QFT-like integrals with analytic solution

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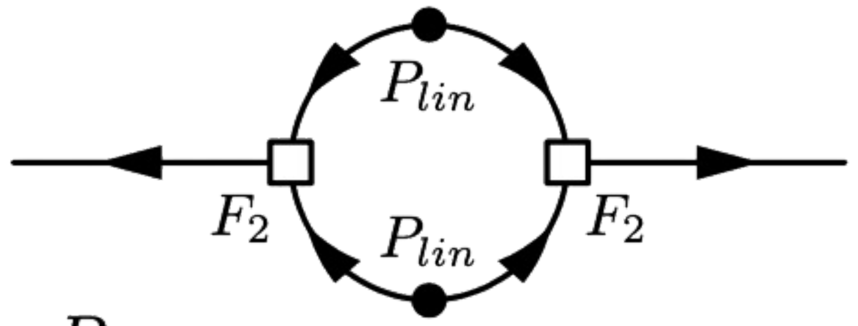
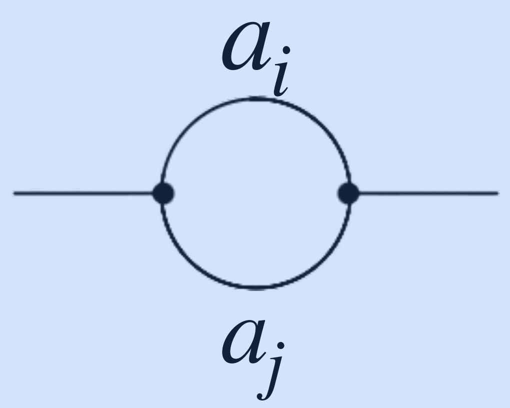


$$= \int_{\mathbf{q}} F_2^2(\mathbf{q}-\mathbf{k},\mathbf{q})\, P_{\rm lin}(q)\, P_{\rm lin}(|\mathbf{k}-\mathbf{q}|) \simeq \sum_{ij} A_{ij} \int_{\mathbf{q}} \frac{1}{q^{2a_i}\,|\mathbf{k}-\mathbf{q}|^{2a_j}}$$



a_i are complex!

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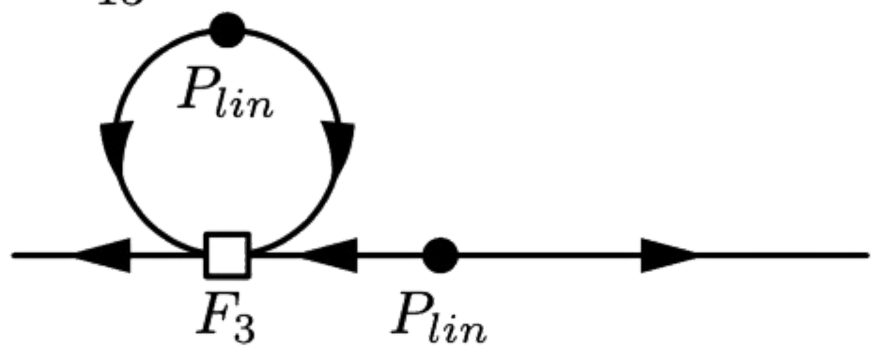
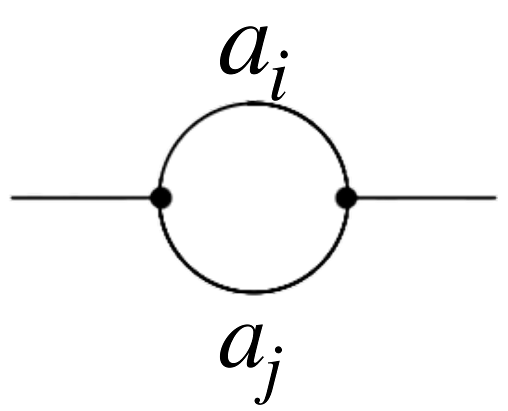
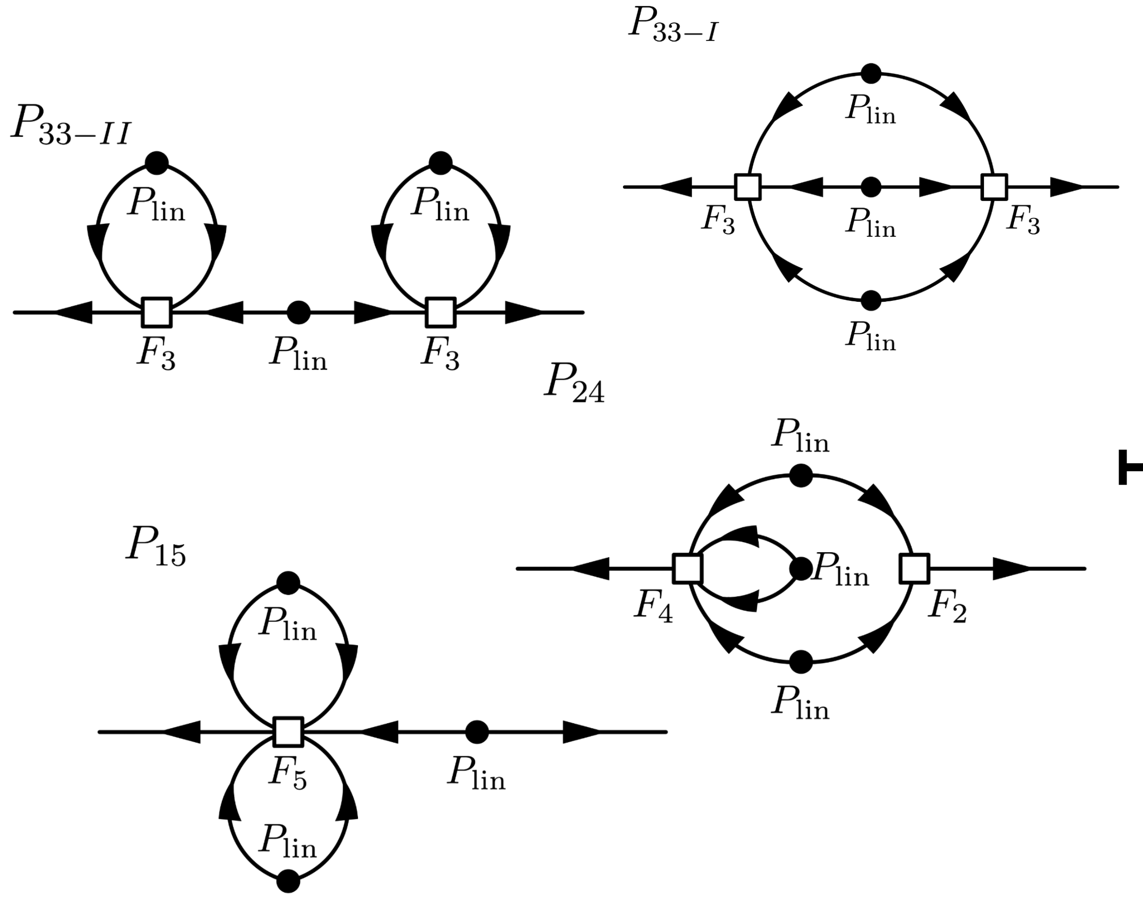


$$\simeq \sum_{ij} B_{ij}$$

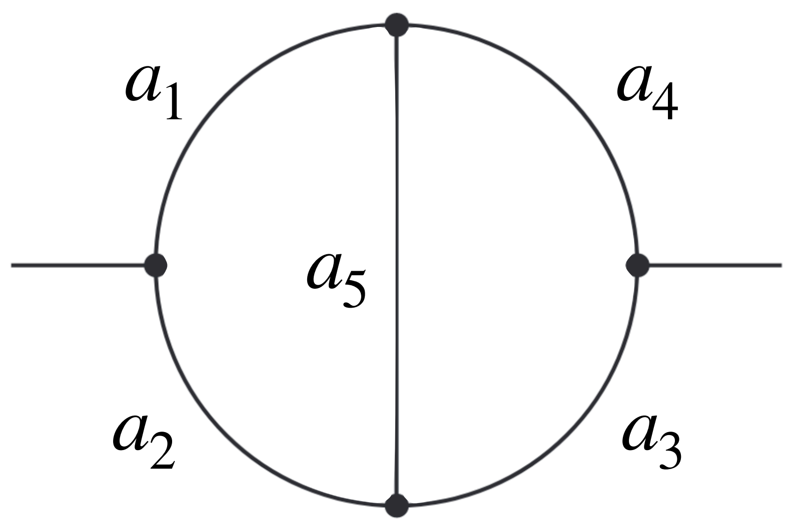


2-loop power spectrum: the massless kite integral



FFTLog $\longmapsto$

$$\sum_{a_1,\ldots,a_5} T_{a_1\ldots a_5}$$



$$\equiv I(a_1,\ldots,a_5,D=3)$$

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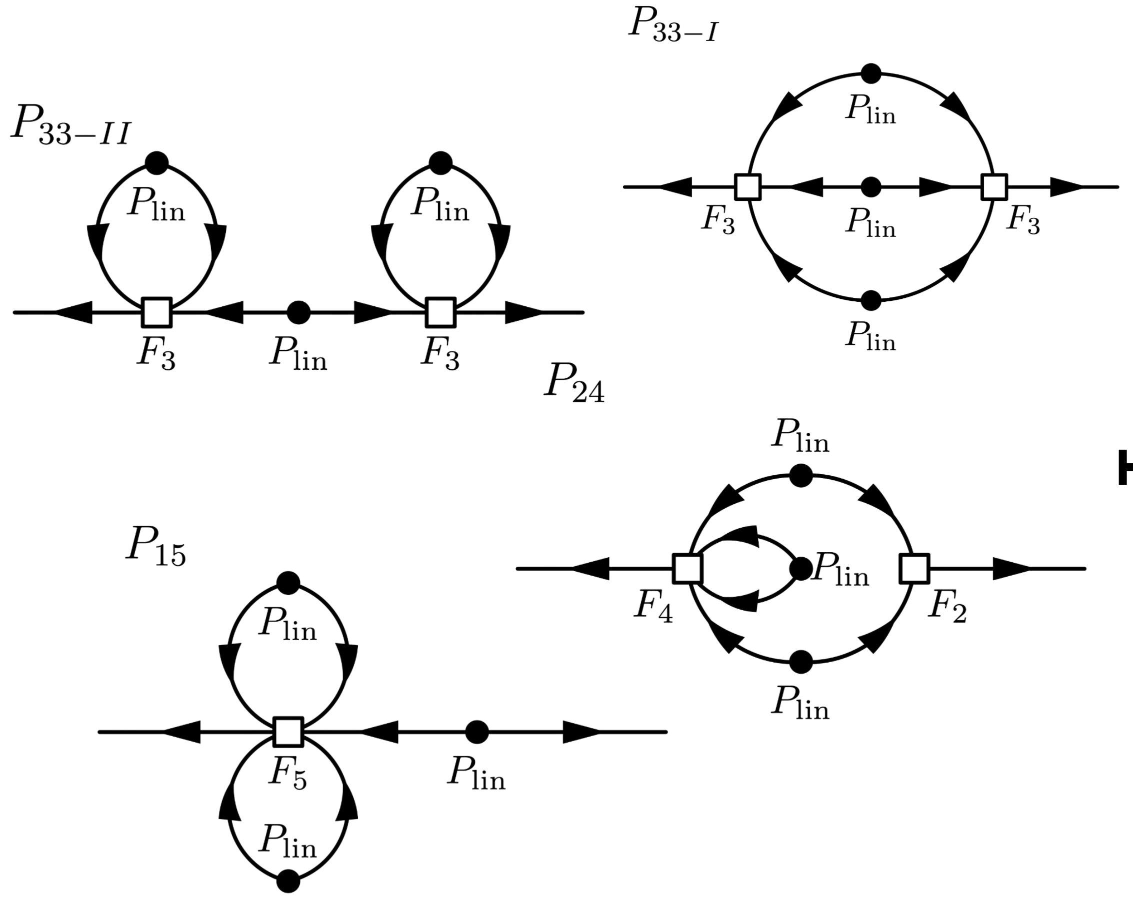

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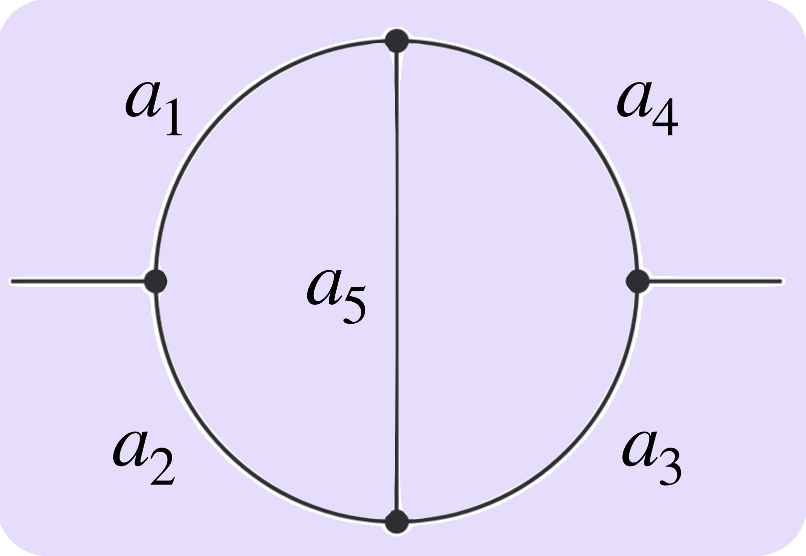

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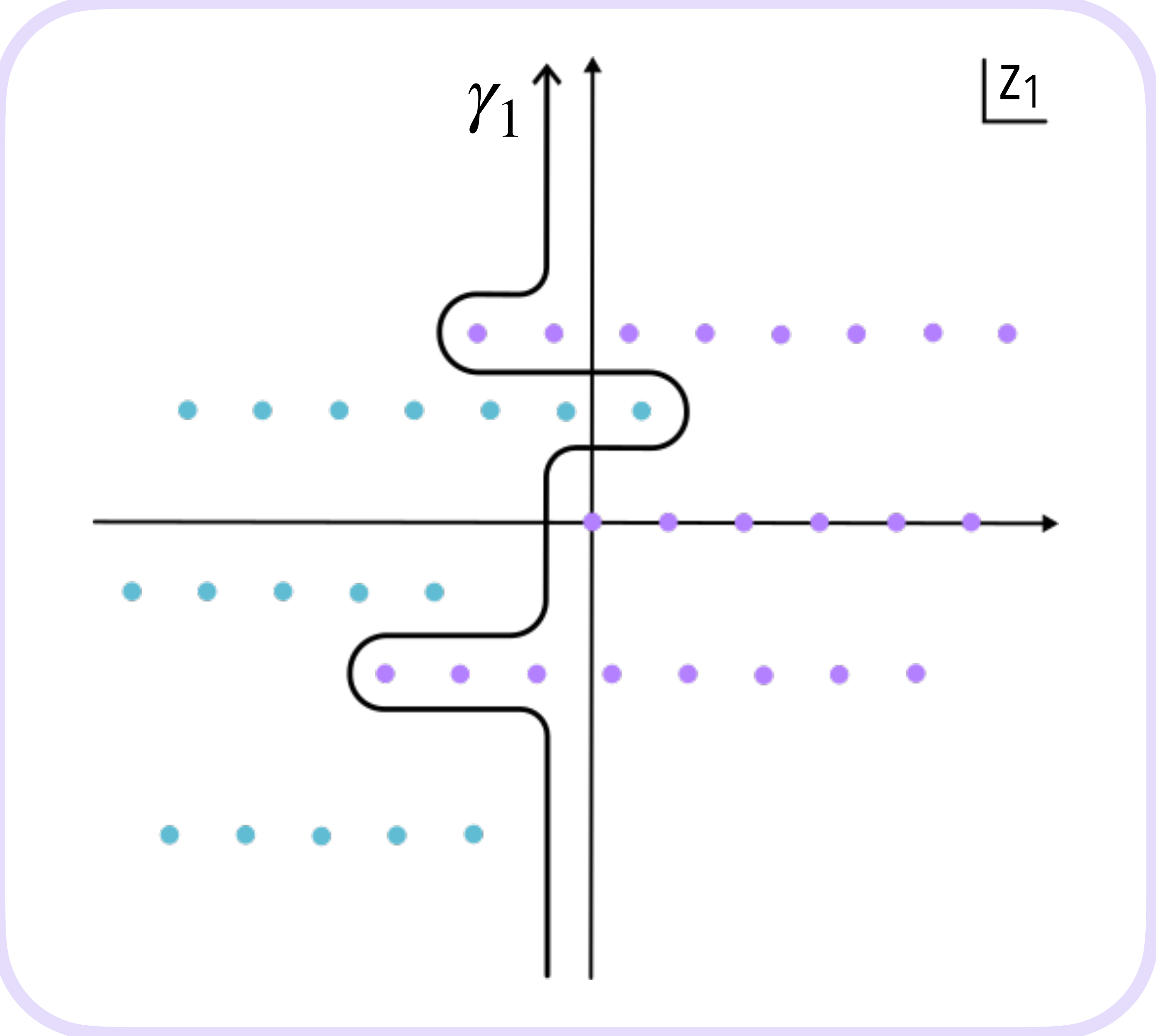
1) Evaluation of the Master Integral

2-fold Mellin-Barnes representation

$$I(a_i;D) = \int_{\gamma_1}\int_{\gamma_2} dz_1 dz_2 \frac{\prod \Gamma(A_j + B_j z_1 + C_j z_2)}{\prod \Gamma(E_j + F_j z_1 + G_j z_2)}$$



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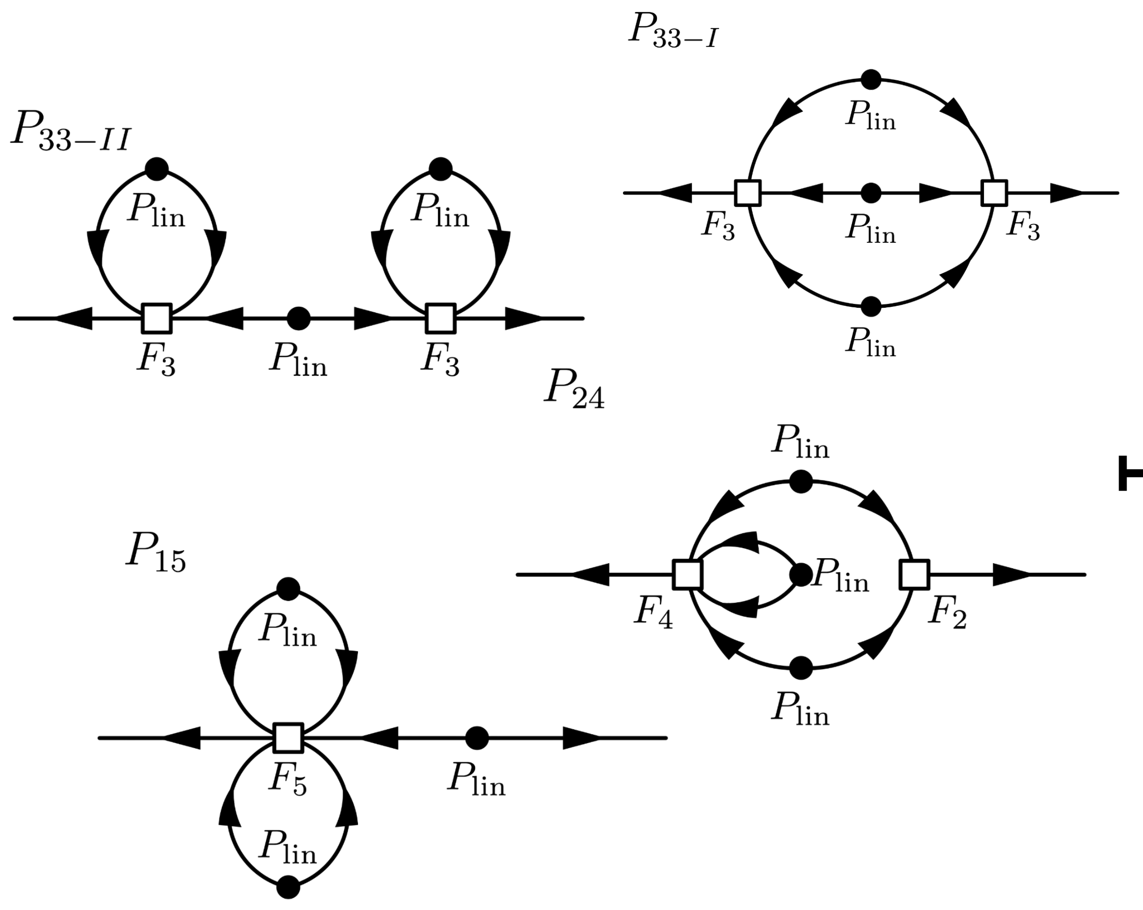

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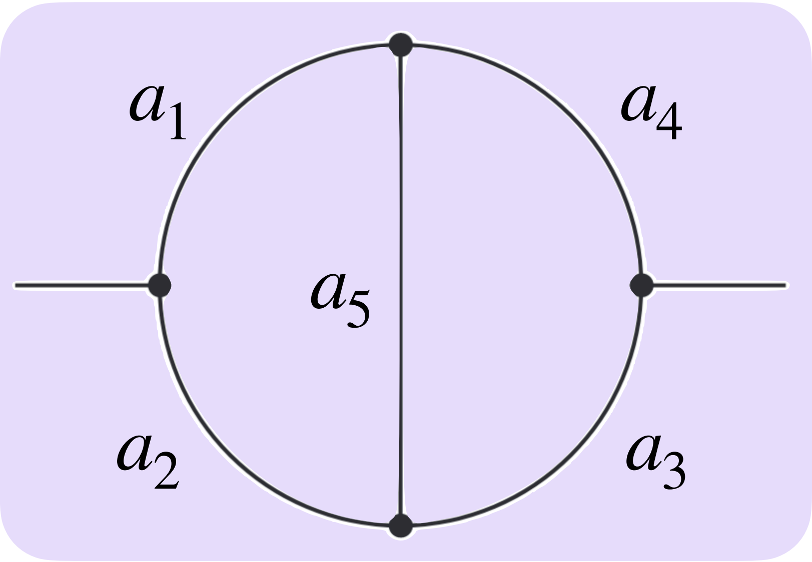

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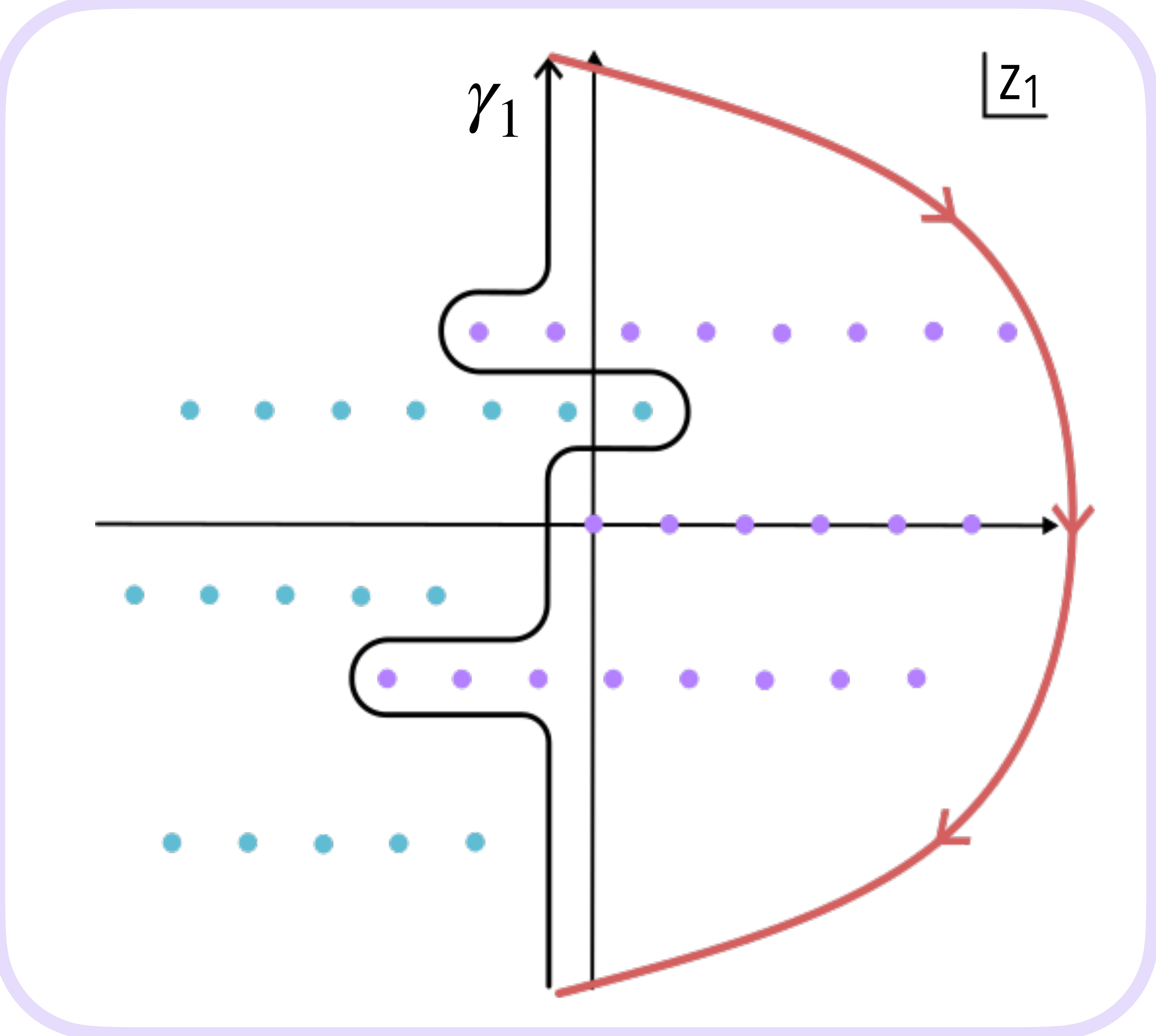
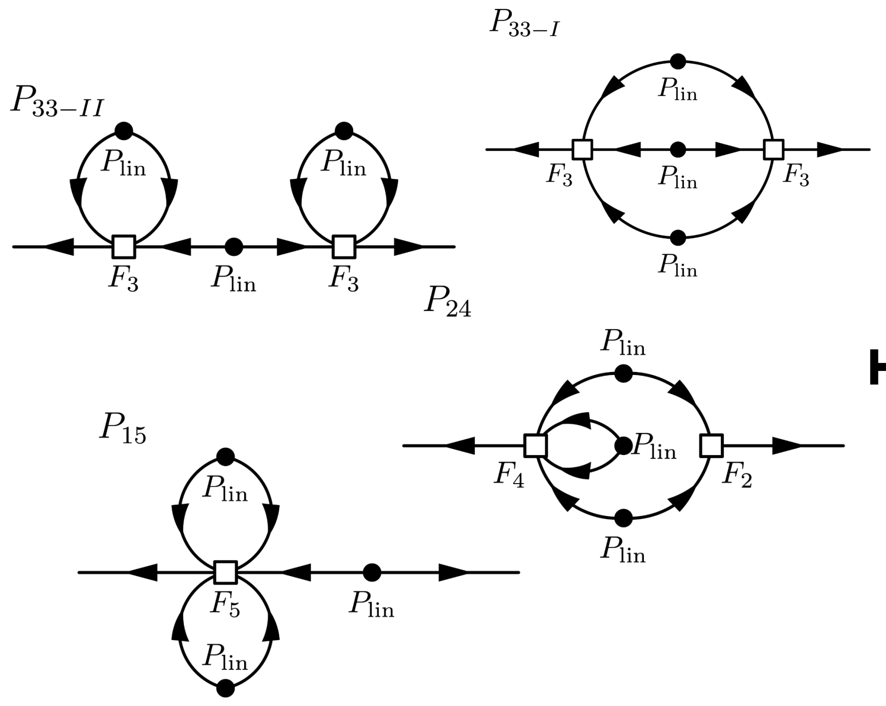
↓

Double sum representation

$$\sum_n^\infty \frac{\prod_i \Gamma(\dots)}{\prod_j \Gamma(\dots)}\, {}_4F_3(1)$$



2-loop power spectrum: towards the full result



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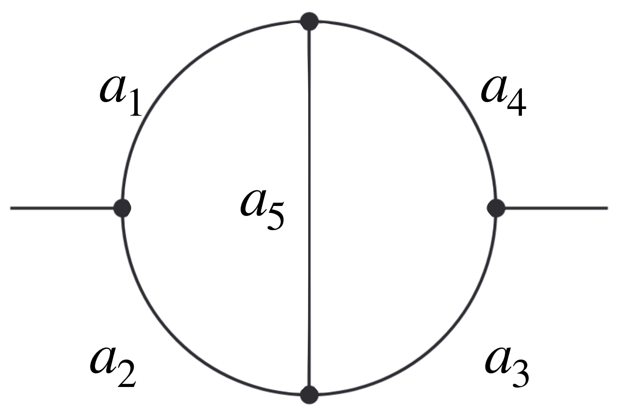
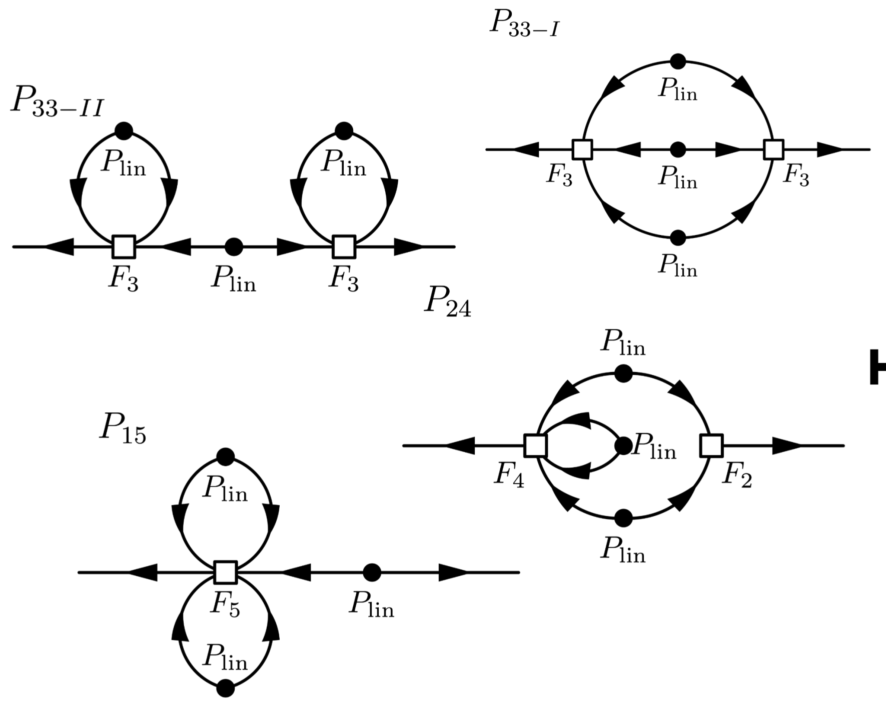


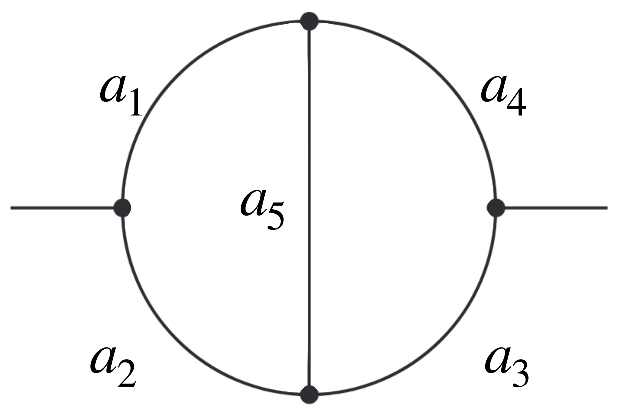
2) Full cosmological contributions

→ $\mathcal{O}(10^7)$ integral configurations per diagram

2-loop power spectrum: towards the full result



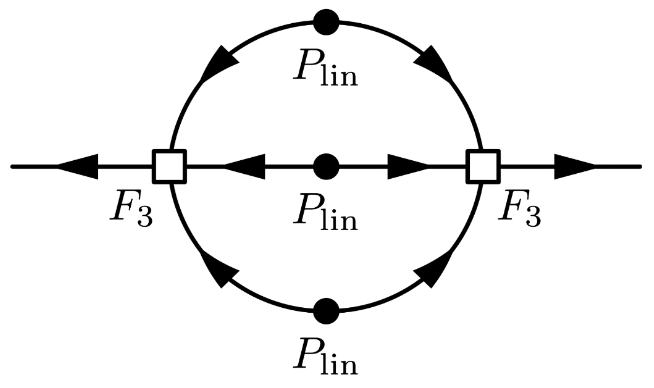
$$\longmapsto \sum_{a_1,\ldots,a_5} T_{a_1\ldots a_5}$$

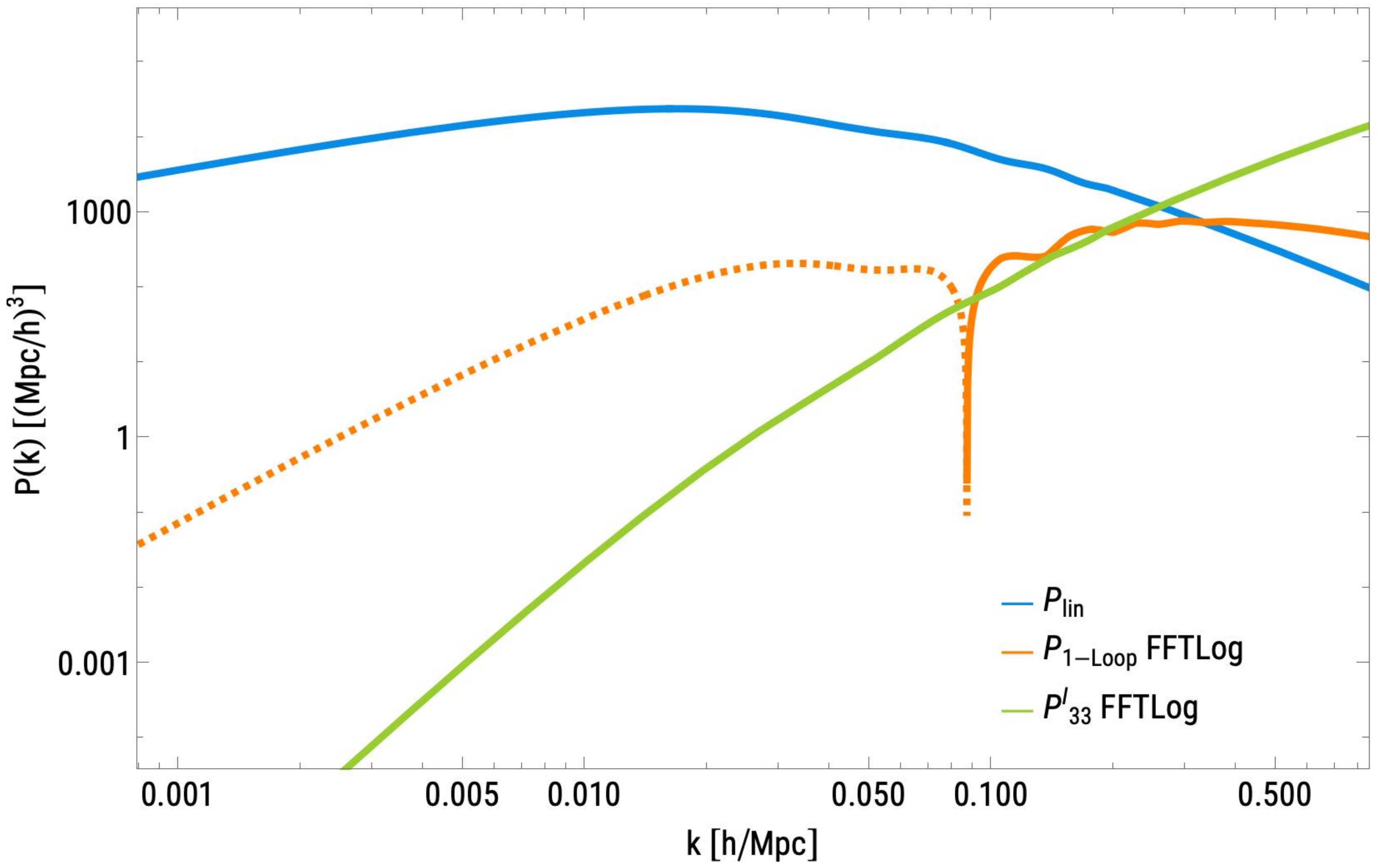


2) Full cosmological contributions

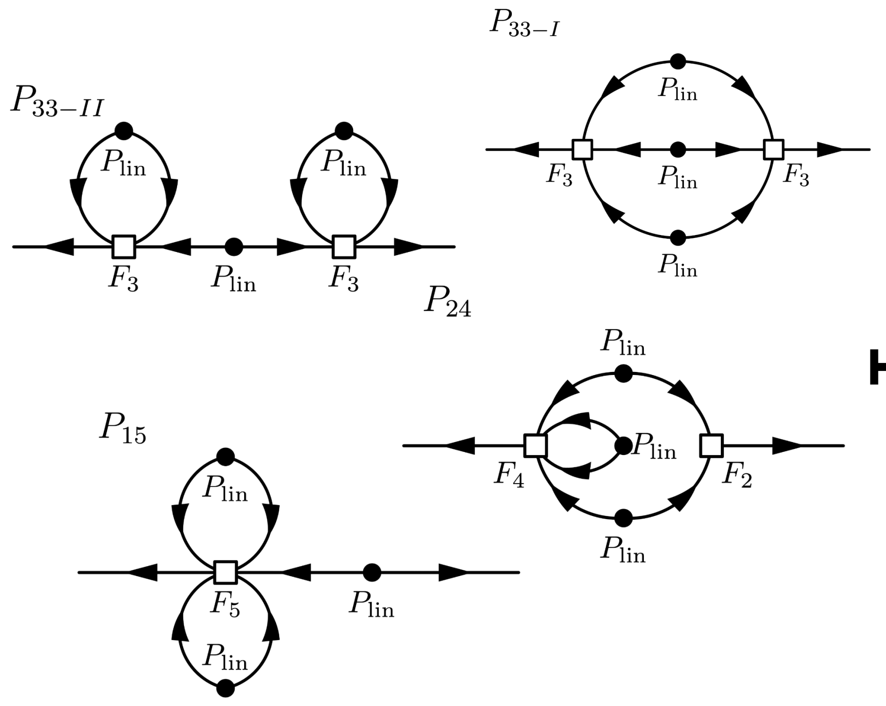
→ $\mathcal{O}(10^7)$ integral configurations per diagram

$\mathbf{P^{I}_{33}}$ contribution





2-loop power spectrum: towards the full result



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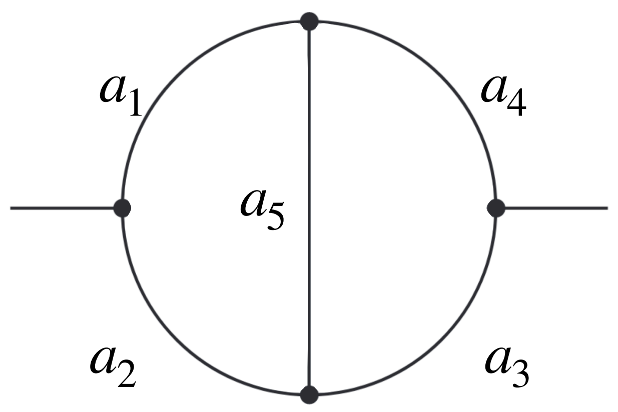

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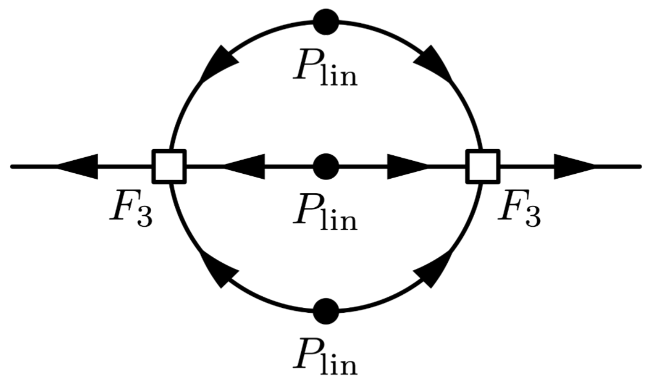
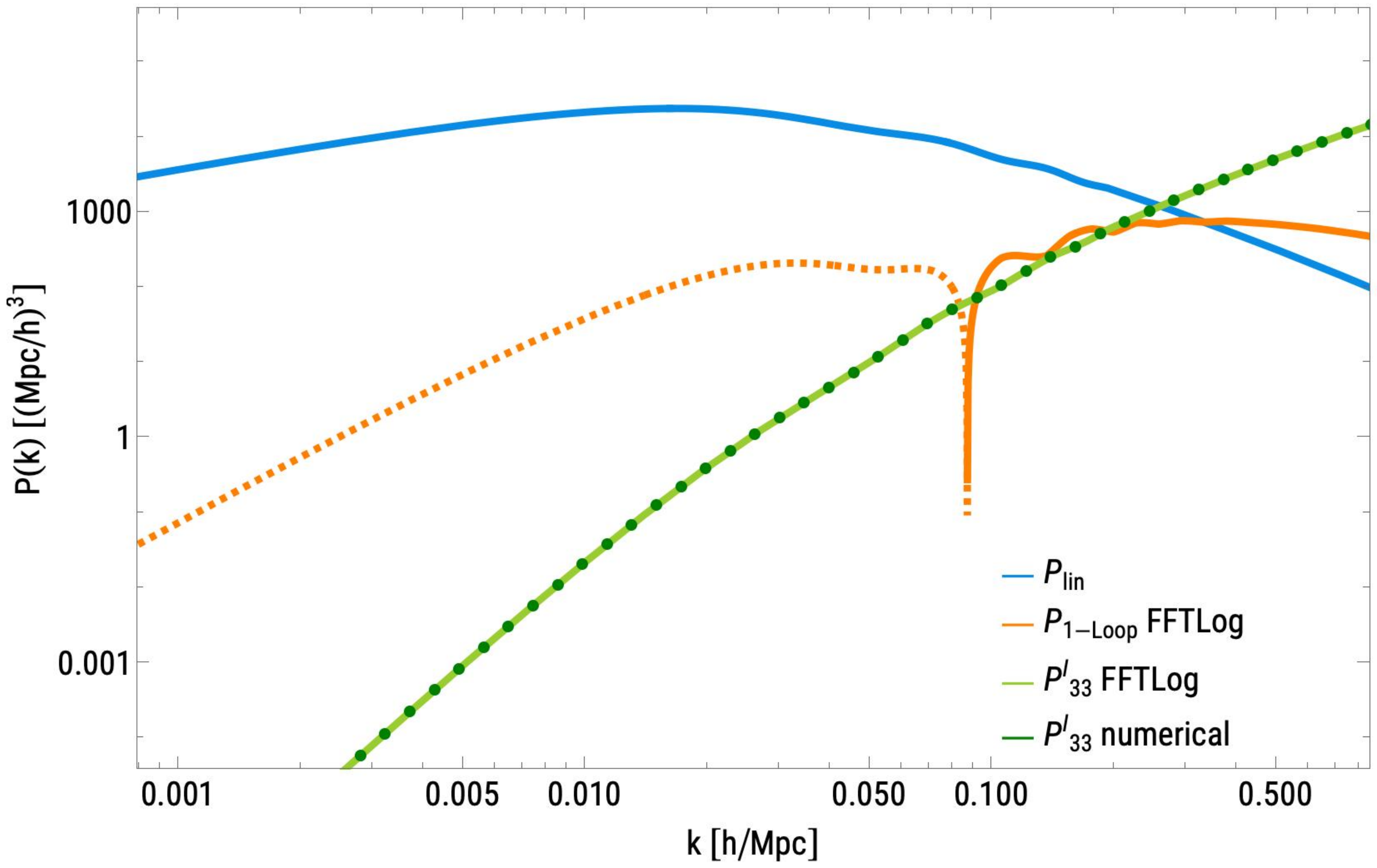
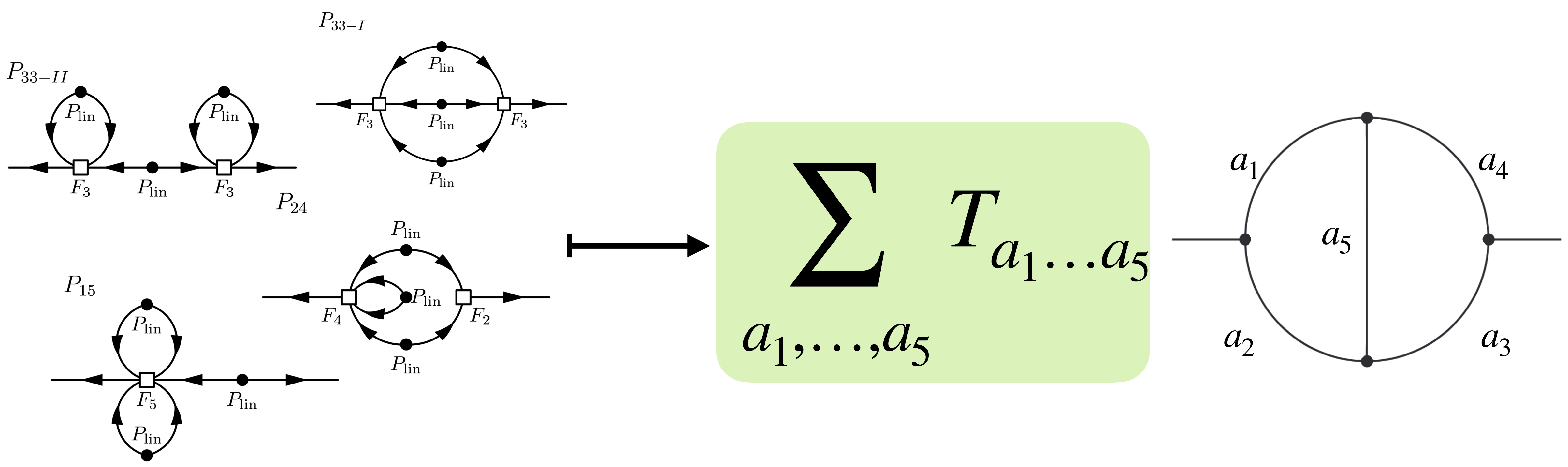




2-loop power spectrum: towards the full result



2) Full cosmological contributions

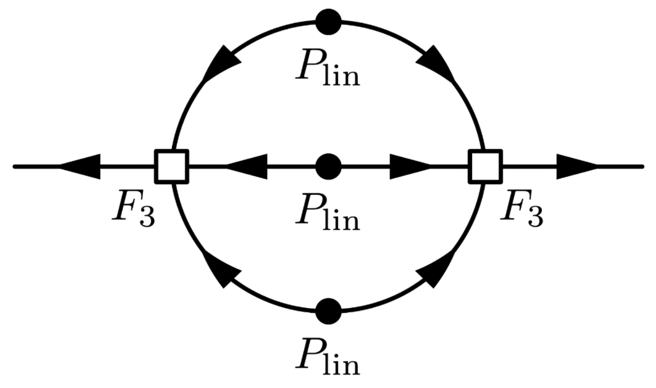
→ $\mathcal{O}(10^7)$ integral configurations per diagram

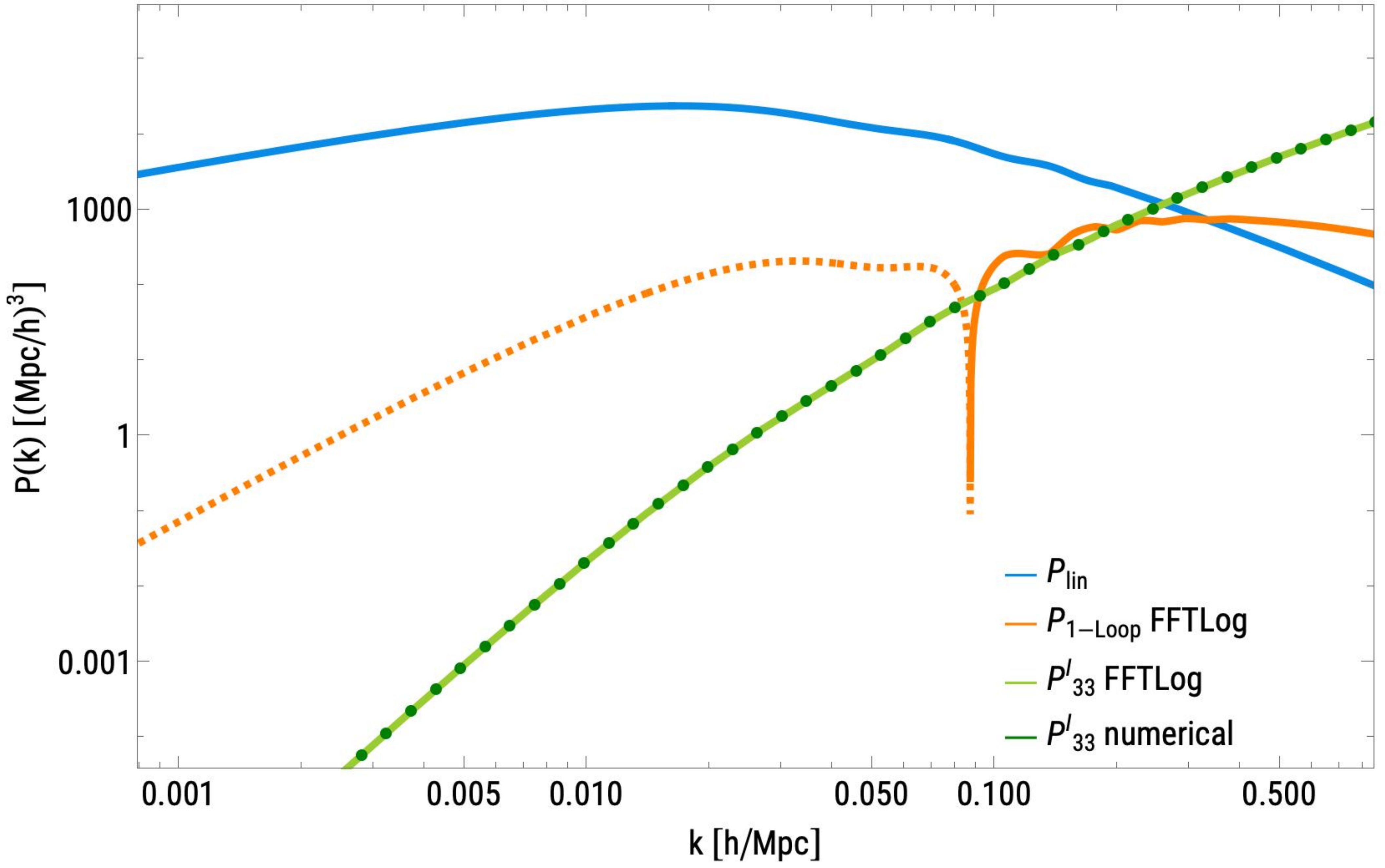
The next steps

→ analytic continuation to full parameter domain

→ systematic inclusion of **EFT counterterms**

P^{I}_{33} contribution





Outlook

1) Automation of full 2-loop power spectrum pipeline (improving convergence and handling of imaginary parts)

2) Extend to **higher loops** and **legs** [Steele, Baldauf, '21; Anastasiou, Favorito, Lewandowski, Senatore, Zheng, '25]

3) Higher loops for **galaxy correlation functions** [Desjacques, Jeong, Schmidt, '19; Bakx, Chisari, Vlah, '25]

4) FFTLog decomposition of **numerical bases** for $P_{lin}(k)$ [Bakx, Chisari, Vlah, '25]

Thank you!

Backup slides

Cold dark matter in the Newtonian limit

[Carrasco et al, '12]

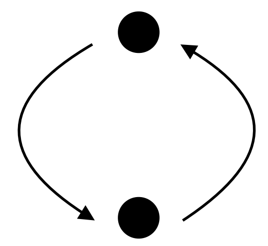
small-scale (UV)

Newtonian approximation

large-scale (IR)

r [h^{-1}Mpc]

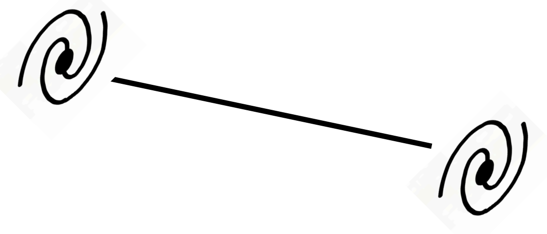
$r_{vir} \sim 3$



fully non-linear

$r_g \sim 10 - 100$

typical galaxy separation



$l_H \sim 10^3$

linear theory

Poisson $\quad \delta \sim \nabla^2 \Phi$

Boltzmann for collisionless massive particles $\quad \frac{d}{d\tau} f(\tau, \mathbf{x}, \mathbf{p})\,|_{\text{traj}} = 0$

Boltzmann hierarchy

$$\rho(\vec{x}, t) = \frac{m}{a^3} \int d^3p \; f(\vec{x}, \vec{p}) = \frac{m}{a^3} \sum_n \delta^{(3)}(\vec{x} - \vec{x}_n) \; ,$$

$$\pi^i(\vec{x}, t) = \frac{1}{a^4} \int d^3p \; p^i f(\vec{x}, \vec{p}) = \frac{m}{a^3} \sum_n v_n^i \delta^{(3)}(\vec{x} - \vec{x}_n) \; ,$$

$$\sigma^{ij}(\vec{x}, t) = \frac{1}{m a^5} \int d^3p \; p^i p^j f(\vec{x}, \vec{p}) = \sum_n \frac{m}{a^3} v_n^i v_n^j \delta^{(3)}(\vec{x} - \vec{x}_n) \; .$$

Integrate out UV physics

$$\mathcal{O}_l(\vec{x}, t) = [\mathcal{O}]_\Lambda (\vec{x}, t) = \int d^3x' \, W_\Lambda(\vec{x} - \vec{x}') \mathcal{O}(\vec{x}')$$

$$\int d^3p \; p^{i_1} \dots p^{i_n} \left[\frac{Df}{Dt} \right]_\Lambda (\vec{x}, \vec{p}) = 0$$

Equations of motion in real space, including first order counterterms

$$\nabla^2 \phi_l = \frac{3}{2} H_0^2 \Omega_m \frac{a_0^3}{a} \delta_l + \dots \; ,$$

$$\dot{\delta}_l = -\frac{1}{a} \partial_i \left((1 + \delta_l) v_l^i \right) \; ,$$

$$\dot{v}_l^i + H v_l^i + \frac{1}{a} v_l^j \partial_j v_l^i + \frac{1}{a} \partial^i \phi_l = -\frac{1}{a} c_s^2 \partial^i \delta_l + \frac{3}{4} \frac{c_{sv}^2}{H a^2} \partial^2 v_l^i + \frac{4 c_{bv}^2 + c_{sv}^2}{4 H a^2} \partial^i \partial_j v_l^j - \Delta J^i +$$

Standard Perturbation Theory (SPT)

[Bernardeau et al, '02]

Equations of motion in Fourier space, NO counterterms

$$\frac{\partial \tilde{\delta}(\mathbf{k},\tau)}{\partial \tau} + \tilde{\theta}(\mathbf{k},\tau) = -\int \mathrm{d}^3\mathbf{k}_1\, \mathrm{d}^3\mathbf{k}_2\, \delta_\mathrm{D}(\mathbf{k}-\mathbf{k}_{12})\alpha(\mathbf{k}_1,\mathbf{k}_2)\tilde{\theta}(\mathbf{k}_1,\tau)\tilde{\delta}(\mathbf{k}_2,\tau)\ ,$$

$$\frac{\partial \tilde{\theta}(\mathbf{k},\tau)}{\partial \tau} + \mathscr{H}(\tau)\tilde{\theta}(\mathbf{k},\tau) + \frac{3}{2}\,\Omega_m \mathscr{H}^2(\tau)\tilde{\delta}(\mathbf{k},\tau) = -\int \mathrm{d}^3\mathbf{k}_1\, \mathrm{d}^3\mathbf{k}_2\, \delta_\mathrm{D}(\mathbf{k}-\mathbf{k}_{12}) \times \beta(\mathbf{k}_1,\mathbf{k}_2)\tilde{\theta}(\mathbf{k}_1,\tau)\tilde{\theta}(\mathbf{k}_2,\tau)$$

$$\alpha(\mathbf{k}_1,\mathbf{k}_2) \equiv \frac{\mathbf{k}_{12}\cdot\mathbf{k}_1}{k_1^2}, \qquad \beta(\mathbf{k}_1,\mathbf{k}_2) \equiv \frac{k_{12}^2(\mathbf{k}_1\cdot\mathbf{k}_2)}{2k_1^2k_2^2}$$

Perturbative solution for the density field

$$\delta(\tau,\mathbf{k}) = \sum_{n=1}^{\infty} D^n(\tau)\,\delta^{(n)}(\mathbf{k})$$

$$D(a) = \frac{5}{2}\Omega_m^0 \mathscr{H}_0^2 \frac{\mathscr{H}}{a}\int_0^a \frac{da'}{\mathscr{H}^3}$$

$$\delta^{(n)}(\mathbf{k}) = \left[\prod_{i=1}^{n}\int d^3\mathbf{q}_i\right] \delta_D\left(\mathbf{k}-\mathbf{q}_{1\ldots n}\right) F_n\left(\mathbf{q}_1,\ldots,\mathbf{q}_n\right)\prod_{i=1}^{n}\delta^{(1)}\left(\mathbf{q}_i\right)$$

Non-linear growth induces mode coupling

$$F_n(\mathbf{q}_1,\ldots,\mathbf{q}_n) = \sum_{m=1}^{n-1}\frac{G_m(\mathbf{q}_1,\ldots,\mathbf{q}_m)}{(2n+3)(n-1)}[(2n+1)\alpha(\mathbf{k}_1,\mathbf{k}_2)F_{n-m}(\mathbf{q}_{m+1},\ldots,\mathbf{q}_n) + 2\beta(\mathbf{k}_1,\mathbf{k}_2)G_{n-m}(\mathbf{q}_{m+1},\ldots,\mathbf{q}_n)]\ ,$$

$$G_n(\mathbf{q}_1,\ldots,\mathbf{q}_n) = \sum_{m=1}^{n-1}\frac{G_m(\mathbf{q}_1,\ldots,\mathbf{q}_m)}{(2n+3)(n-1)}[3\alpha(\mathbf{k}_1,\mathbf{k}_2)F_{n-m}(\mathbf{q}_{m+1},\ldots,\mathbf{q}_n) + 2n\beta(\mathbf{k}_1,\mathbf{k}_2)G_{n-m}(\mathbf{q}_{m+1},\ldots,\mathbf{q}_n)]$$

(where $\mathbf{k}_1 \equiv \mathbf{q}_1+\cdots+\mathbf{q}_m$, $\mathbf{k}_2 \equiv \mathbf{q}_{m+1}+\cdots+\mathbf{q}_n$, $\mathbf{k} \equiv \mathbf{k}_1+\mathbf{k}_2$ and $F_1 = G_1 \equiv 1$).

gaussian!
can apply Wick's theorem

$$F_2(\mathbf{q}_1,\mathbf{q}_2) = \frac{5}{7} + \frac{1}{2}\frac{\mathbf{q}_1\cdot\mathbf{q}_2}{q_1q_2}\left(\frac{q_1}{q_2}+\frac{q_2}{q_1}\right) + \frac{2}{7}\frac{(\mathbf{q}_1\cdot\mathbf{q}_2)^2}{q_1^2q_2^2}$$

$$G_2(\mathbf{q}_1,\mathbf{q}_2) = \frac{3}{7} + \frac{1}{2}\frac{\mathbf{q}_1\cdot\mathbf{q}_2}{q_1q_2}\left(\frac{q_1}{q_2}+\frac{q_2}{q_1}\right) + \frac{4}{7}\frac{(\mathbf{q}_1\cdot\mathbf{q}_2)^2}{q_1^2q_2^2}$$

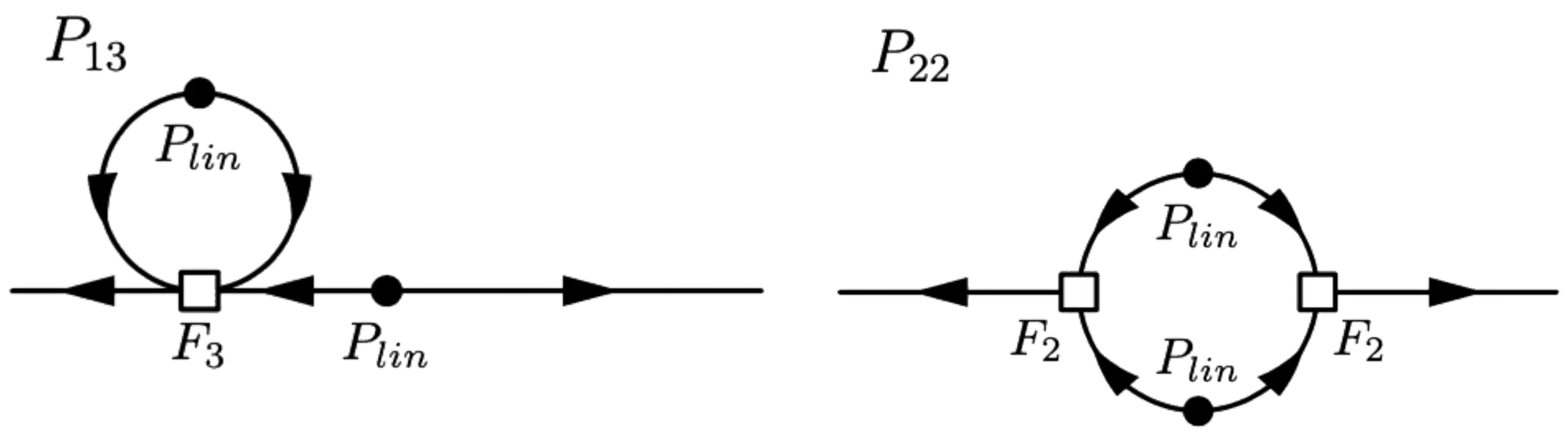
SPT loop contributions for the DM power spectrum

[Simonović et al, '17; Ivanov, '23]

Power spectrum at NLO

$$\langle \delta_{\mathbf{k}} \delta_{\mathbf{k}'} \rangle = \langle (\delta_{\mathbf{k}}^{(1)} + \delta_{\mathbf{k}}^{(2)} + \delta_{\mathbf{k}}^{(3)})(\delta_{\mathbf{k}'}^{(1)} + \delta_{\mathbf{k}'}^{(2)} + \delta_{\mathbf{k}'}^{(3)}) \rangle$$
$$= \langle \delta_{\mathbf{k}}^{(1)} \delta_{\mathbf{k}'}^{(1)} \rangle + \langle \delta_{\mathbf{k}}^{(2)} \delta_{\mathbf{k}'}^{(2)} \rangle + 2 \langle \delta_{\mathbf{k}}^{(3)} \delta_{\mathbf{k}'}^{(1)} \rangle$$
$$= (2\pi)^3 \delta_D^{(3)}(\mathbf{k} + \mathbf{k}')(P_{11} + P_{13} + P_{22}),$$

$$P_{22}(k) = \langle \delta_{\mathbf{k}}^{(2)} \delta_{\mathbf{k}'}^{(2)} \rangle' = 2 \int_{\mathbf{q}} [F_2(\mathbf{q} - \mathbf{k}, \mathbf{q})]^2 P_{11}(q) P_{11}(|\mathbf{k} - \mathbf{q}|),$$
$$P_{13}(k) = 2 \langle \delta_{\mathbf{k}}^{(1)} \delta_{\mathbf{k}'}^{(2)} \rangle' = 6 P_{11}(k) \int_{\mathbf{q}} F_3(\mathbf{k}, \mathbf{q} - \mathbf{q}) P_{11}(q).$$



Power spectrum at NNLO

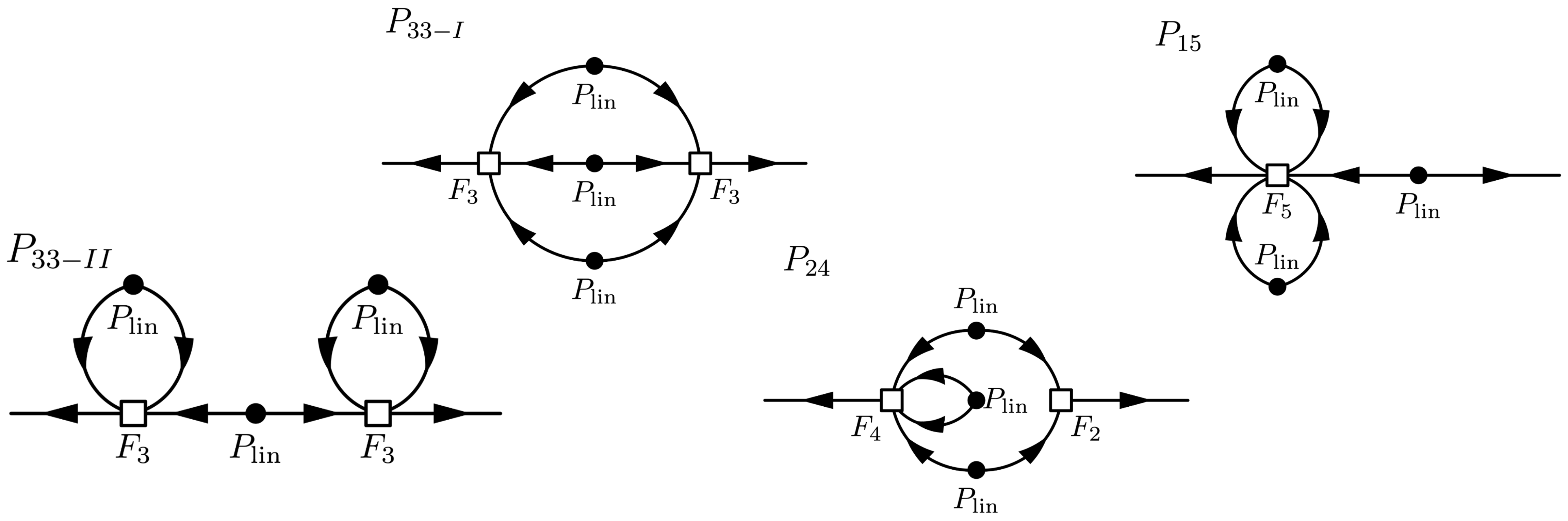
$$P_{2-\text{loop}}(k, \tau) = D^4(\tau)[P_{33}^{I}(k) + P_{33}^{II}(k) + P_{24}(k) + P_{15}(k)]$$

$$P_{33}^{I}(k) = 9 P_{\text{lin}}(k) \int_{\boldsymbol{q}} F_3(\boldsymbol{k}, \boldsymbol{q}, -\boldsymbol{q}) P_{\text{lin}}(q) \int_{\boldsymbol{p}} F_3(-\boldsymbol{k}, \boldsymbol{p}, -\boldsymbol{p}) P_{\text{lin}}(p),$$

$$P_{33}^{II}(k) = 6 \int_{\boldsymbol{q}} \int_{\boldsymbol{p}} F_3(\boldsymbol{q}, \boldsymbol{p}, \boldsymbol{k} - \boldsymbol{q} - \boldsymbol{p}) F_3(-\boldsymbol{q}, -\boldsymbol{p}, \boldsymbol{q} + \boldsymbol{p} - \boldsymbol{k}) P_{\text{lin}}(q) P_{\text{lin}}(p) P_{\text{lin}}(|\boldsymbol{k} - \boldsymbol{q} - \boldsymbol{p}|),$$

$$P_{24}(k) = 24 \int_{\boldsymbol{q}} \int_{\boldsymbol{p}} F_2(\boldsymbol{q}, \boldsymbol{k} - \boldsymbol{q}) F_4(\boldsymbol{p}, -\boldsymbol{p}, -\boldsymbol{q}, \boldsymbol{q} - \boldsymbol{k}) P_{\text{lin}}(q) P_{\text{lin}}(p) P_{\text{lin}}(|\boldsymbol{k} - \boldsymbol{q}|),$$

$$P_{15}(k) = 30 P_{\text{lin}}(k) \int_{\boldsymbol{q}} \int_{\boldsymbol{p}} F_5(\boldsymbol{k}, \boldsymbol{q}, -\boldsymbol{q}, \boldsymbol{p}, -\boldsymbol{p}) P_{\text{lin}}(q) P_{\text{lin}}(p).$$



Perturbative solutions including counterterms

[Baldauf et al, '15; Ivanov, '23]

Equations of motion in real space, a(t) time dependence

$$\mathscr{H}^2\left[-a^2\partial_a^2+\frac{3}{2}(\Omega_m-2)a\partial_a+\frac{3}{2}\Omega_m\right]\delta_{\mathbf{k}}=\mathscr{H}^2\mathscr{S}_\beta+\mathscr{H}\partial_a(a\mathscr{H}\mathscr{S}_\alpha)\,,$$

$$\mathscr{H}^2\left[a^2\partial_a^2+\left(4-\frac{3}{2}\Omega_m\right)a\partial_a+\left(2+\frac{\partial_a^2\mathscr{H}}{\mathscr{H}}+\left(4-\frac{3}{2}\Omega_m\right)\frac{\partial_a\mathscr{H}}{\mathscr{H}}-3\Omega_m\right)\right]\theta_{\mathbf{k}}$$

$$=-\partial_a(a\mathscr{H}^2\mathscr{S}_\beta)-\frac{3}{2}\Omega_m\mathscr{H}^2\mathscr{S}_\alpha\,,$$

$$\mathscr{S}_\alpha=-\int_{\mathbf{q}_1\mathbf{q}_2}\delta_D^{(3)}(\mathbf{k}-\mathbf{q}_{12})\alpha(\mathbf{q}_1,\mathbf{q}_2)\theta_{\mathbf{q}_1}\delta_{\mathbf{q}_2}$$

$$\mathscr{S}_\beta=-\int_{\mathbf{q}_1\mathbf{q}_2}\delta_D^{(3)}(\mathbf{k}-\mathbf{q}_{12})\beta(\mathbf{q}_1,\mathbf{q}_2)\theta_{\mathbf{q}_1}\theta_{\mathbf{q}_2}$$

Green's function

$$G_\delta(a,a')=H(a-a')\frac{2}{5}\frac{1}{\mathscr{H}_0^2\Omega_m^{(0)}}\frac{D_+(a')}{a'}\left(\frac{D_-(a)}{D_-(a')}-\frac{D_+(a)}{D_+(a')}\right)$$

Including counterterms

$$\mathscr{H}^2\mathscr{S}_\beta\to\mathscr{H}^2\mathscr{S}_\beta+\tau_\theta$$

$$\tau_\theta\equiv\partial^i\left[\frac{1}{\rho_\ell}\partial^j\sigma_{ij}\right]=(c_s^2\partial^2\delta+\partial^iJ_i)$$

$$\delta_{\mathbf{k}}^{(\sigma)}=\int da'G_\delta(a,a')\tau_\theta(a')$$

NLO solution including counterterms

$$\delta_{\mathbf{k}}^{\rm NL}=\delta_{\mathbf{k}}^{(1)}+\delta_{\mathbf{k}}^{(2)}+\delta_{\mathbf{k}}^{(3)}+\delta_{\mathbf{k}}^{\rm stress.}+\delta_{\mathbf{k}}^{\rm stoch.}$$

SPT solutions

effective pressure

$$\delta_{\mathbf{k}}^{\rm stress.}=-\gamma_\Lambda k^2\delta_{\mathbf{k}}^{(1)}\equiv-\int da'G_\delta(a,a')k^2c_s^2(a')\delta_{\mathbf{k}}^{(1)}(a')$$

only autocorrelates

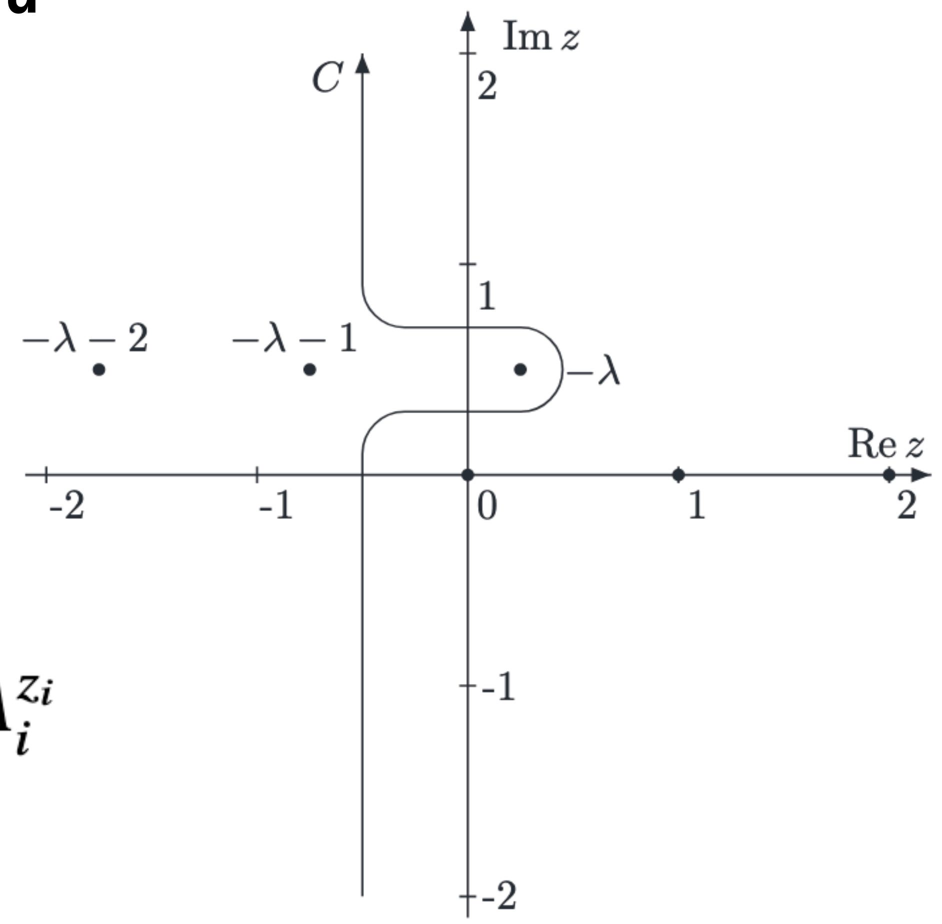
$$\langle\delta^{\rm stoch.}(\mathbf{k}')\delta^{\rm stoch.}(\mathbf{k})\rangle'=P_J(k)=R^7_{\rm stoch.}k^4+\ldots$$

Mellin-Barnes integrals and special functions

[Smirnov, '06; Dubovyk et al, '22]

Master Mellin-Barnes formula

$$\frac{1}{(A+B)^{\lambda}} = \frac{1}{\Gamma(\lambda)}\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dz\Gamma(\lambda+z)\Gamma(-z)A^{z}B^{-\lambda-z}$$



$$\frac{1}{(A_1+\ldots+A_n)^{\lambda}} = \frac{1}{\Gamma(\lambda)}\frac{1}{(2\pi i)^{n-1}}\int_{c-i\infty}^{c+i\infty} dz_2\cdots\int_{c-i\infty}^{c+i\infty} dz_n \prod_{i=2}^{n} A_i^{z_i}$$
$$\times A_1^{-\lambda-z_2-\ldots-z_n}\Gamma(\lambda+z_2+\ldots+z_n)\prod_{i=2}^{n}\Gamma(-z_i).$$

Generalized hypergeometric functions

$${}_pF_q(a_1,\ldots,a_p;b_1,\ldots,b_q;z) = \sum_{n=0}^{\infty}\frac{(a_1)_n\cdots(a_p)_n}{(b_1)_n\cdots(b_q)_n}\frac{z^n}{n!} =$$
$$= \frac{\Gamma(b_1)\cdots\Gamma(b_q)}{\Gamma(a_1)\cdots\Gamma(a_p)}\sum_{n=0}^{\infty}\frac{\Gamma(n+a_1)\cdots\Gamma(n+a_p)}{\Gamma(n+b_1)\cdots\Gamma(n+b_q)}\frac{z^n}{n!}$$

MB representation for the massless 2-loop kite integral

$$I(a_i;D) = \frac{1}{\Gamma(a_1)\Gamma(a_2)\Gamma(a_5)\Gamma(D-a_{125})}\int_{\gamma_1}\int_{\gamma_2} dz_1 dz_2 \frac{\Gamma(-z_1)\Gamma(-z_2)\Gamma(D/2-a_3+z_1)\Gamma(D/2-a_4+z_2)\Gamma(a_5+z_1+z_2)\Gamma(-a_{15}+D/2-z_1)\Gamma(-a_{25}+D/2-z_2)\Gamma(a_{34}-D/2-z_1-z_2)\Gamma(a_{125}-D/2+z_1+z_2)}{\Gamma(a_3-z_1)\Gamma(a_4-z_2)\Gamma(-a_{34}+D+z_1+z_2)}$$