Concepts of Experiments at Future Colliders II

PD Dr. Oliver Kortner

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Recapitulation of the previous lecture

Shockley-Ramo theorem for a cylindrical drift tube

Theorem

The induced current I by a given electrode due to the movement of a charge q equals

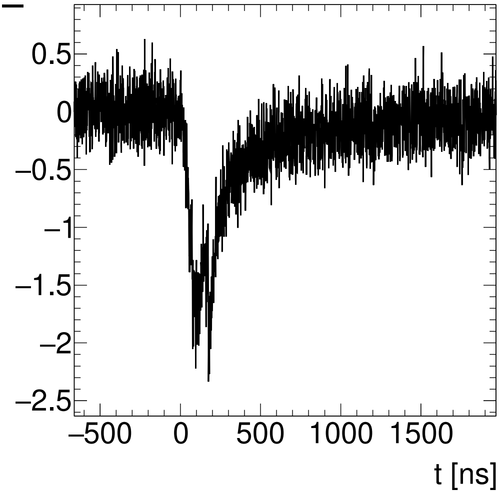
$$I = E_v q v$$

where v is the instantaneous velocity of the charge and E_v is the component in the direction v of that electric field which would exist at the charge's position under the following circumstances: charge removed, given electrode raised to unite potential, all other electrodes grounded.

Consequences

- Avalanche electrons give a large, but very short current because of their small drift distance to the anode wire.
- Ions give currents over a longer time interval. As they are created close to the anode wire, I is initally large and becomes smaller with the drift towards the tube wall.

Introductory example: cylindrical drift tube



- Particle detectors provide current or voltage pulses, which contain information about particle passage or deposited energy.
- To obtain this information, they must be processed electronically.

Analog and digital signals

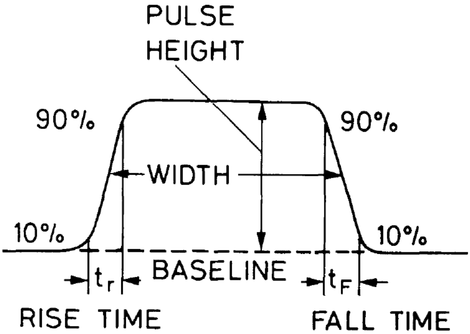
Analog signal: Information contained in the continuous variation of electrical signal properties, e.g., pulse height, pulse duration, or pulse shape.

Digital signal: Information stored in discrete form.

Example. TTL (Transistor-Transistor Logic):
Logical 0: Signal between 0 and 0.8 V.
Logical 1: Signal between 2 V and 5 V.

Advantage of a digital signal: No information loss with small signal disturbances.

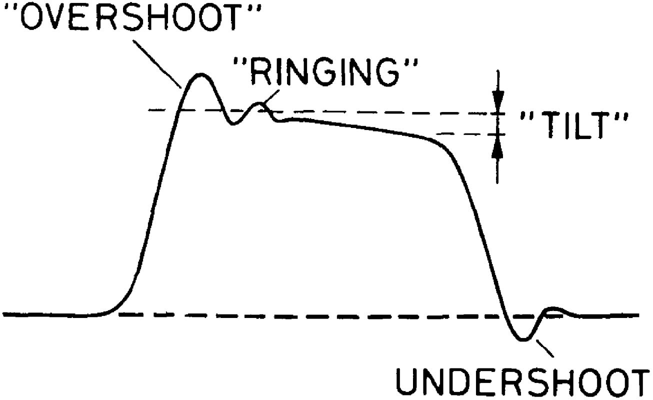
Characteristics of a signal pulse



Slow Signal: $t_A \gtrsim$100 ns.

Fast Signal: $t_A \lesssim$1 ns.

Deformed rectangular pulse



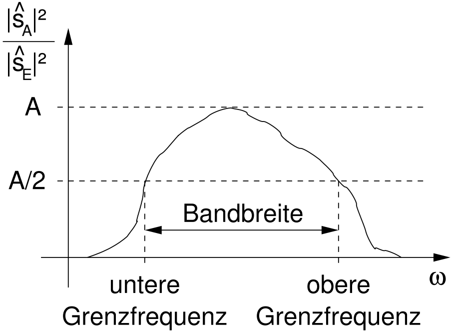
Attenuation and bandwidth

Attenuation

$$\text{Attenuation [dB]} := 10 \cdot \log_{10}\left(\frac{|\hat{s_A}|^2}{|\hat{s_E}|^2}\right).$$

$$-3 \text{ dB} = 10 \cdot \log_{10}\left(\frac{|\hat{s_A}|^2}{|\hat{s_E}|^2}\right) \Leftrightarrow \frac{|\hat{s_A}|^2}{|\hat{s_E}|^2} = 10^{-\frac{3}{10}} = \frac{1}{2}.$$

Bandwidth



Passive electronic components – Ohmic resistance

Drude's model of electrical conduction in metals

Metals are electrical conductors. In an ideal conductor, the conduction electrons experience no resistance. In a real conductor, they collide with the atomic nuclei.

Assumptions

- Neglect of interaction between the conduction electrons.
- Free electron motion between collisions with atomic nuclei.
  - Non-accelerated motion in between collisions.
- Elastic collisions between conduction electrons and atomic nuclei. The conduction electrons are not heated by the collisions.

Recapitulation of the previous lecture

Electron movement in the Drude model

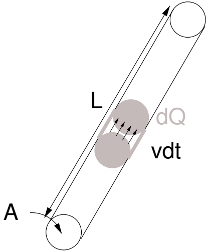
Equation of motion of a conduction electron:

$$m_e \cdot \frac{d\vec{v}}{dt} = -e\vec{E}.$$

τ: Average time between two collisions off atoms.

$$< \vec{v} >= -\frac{e}{m_e}\vec{E} \cdot \tau + \underbrace{< \vec{v}_0 >}_{=0\ (in\ \ therm.\ \ equ.)} = -\frac{e}{m_e}\tau \cdot \vec{E}.$$

n: Conduction electron density.
L: Length of the real conductor.
A: Cross section of the real conductor.



$$dQ = -n \cdot e|\vec{v}| \cdot dt \cdot A \ \Leftrightarrow\ I = \frac{dQ}{dt} = -nev \cdot A = \frac{ne^2\tau}{m_e} \cdot A \cdot E.$$

Hence

$$\vec{j} = \frac{ne^2\tau}{m_e} \cdot \vec{E} =: \sigma \cdot \vec{E}.$$

σ: electric conductivity.

Recapitulation of the previous lecture

Ohm's law

Voltage between the ends of the conductor:

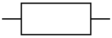
$$U = L \cdot \underbrace{E}_{=\frac{I}{\sigma \cdot A}} = \frac{L}{\sigma \cdot A} \cdot I =: R \cdot I \text{ (Ohm's Law).}$$

Ohmic resistance

$$R = \frac{L}{\sigma \cdot A} =: \rho \cdot \frac{L}{A}.$$

ρ: specific resistance (unit: Ωcm).

Schematic symbols for an ohmic resistance:



(DIN)

(USA)

Passive electronic components – capacitance

$$C = \frac{Q}{U} \;\Rightarrow\; \text{No current flow at DC voltage.}$$

Current flow at AC voltage:

$$\frac{dU}{dt} = \frac{\frac{dQ}{dt}}{C} = \frac{I}{C}.$$

Transition to frequency representation:

$$U(t) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{\infty} \hat{U}(\omega) e^{i\omega t} d\omega, \;\; I(t) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{\infty} \hat{I}(\omega) e^{i\omega t} d\omega.$$

$$\frac{dU}{dt} = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{\infty} i\omega \hat{U}(\omega) e^{i\omega t} d\omega = \frac{I(t)}{C} = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{\infty} \frac{1}{C} \hat{I}(\omega) e^{i\omega t} d\omega,$$

leading to $i\omega \hat{U}(\omega) = \frac{1}{C}\hat{I}(\omega)$, thus $\boxed{\hat{U}(\omega) = \frac{1}{i\omega C}\hat{I}(\omega)}$.

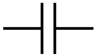
Recapitulation of the previous lecture

Capacitance – impedance and schematic symbol

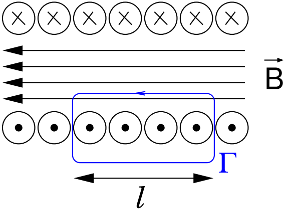
$$\boxed{\hat{U}(\omega) = \frac{1}{i\omega C}\hat{I}(\omega).}$$

Impedance: $Z_C = \frac{1}{i\omega C}$.

Schematic symbol:



Reminder: Field inside an ideal coil



$\frac{dN}{dl}$: Number of turns per unit length.
Ampére's law:

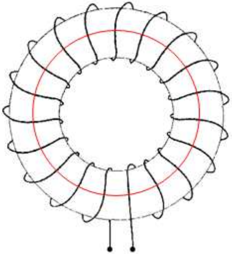
$$\oint_\Gamma \vec{B} \cdot d\vec{s} = l \cdot B = \mu_0 \cdot I \cdot \frac{dN}{dl} \cdot l.$$

$$B = \mu_0 \frac{dN}{dl} \cdot I =: \frac{1}{A} L \cdot I.$$

A: Cross-sectional area of the coil.
L: Inductance.

Recapitulation of the previous lecture

Ideal toroidal coil



- B exists only inside the coil.
- If the coil is made of an ideal conductor, $\vec{E}$ inside the conductor is 0. Otherwise, an infinitely large current would flow through the conductor.

$\Rightarrow$ $U_{ab} = 0$.

- With alternating current, because $\frac{dI}{dt} \neq 0$, $\frac{\partial B}{\partial t} \neq 0$, resulting in a non-zero electromotive force.

$$\text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t}.$$

$$U_{ab} = \oint \vec{E} \cdot d\vec{s} = \int_A \text{curl } \vec{E} d\vec{A} = -\int_A \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A} = -\frac{\partial}{\partial t} B \cdot A = -\frac{\partial}{\partial t} \frac{1}{A} LIA = -L\frac{dI}{dt}.$$

In the frequency domain, we have $\boxed{\hat{U}(\omega) = -i\omega L \hat{I}(\omega)}$.

Recapitulation of the previous lecture

Inductance – impedance and circuit symbol

$$\boxed{\hat{U}(\omega) = -i\omega L \hat{I}(\omega).}$$

Impedance: $Z_L = -i\omega L$.

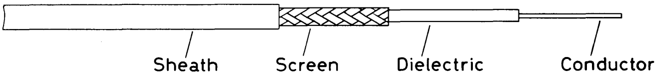
Circuit Symbol:  (DIN)

 (USA)

Remark. In the frequency domain, the behavior of a circuit containing the mentioned passive elements can be calculated in a similar manner to a circuit containing ohmic resistances, by using impedances.

Signal transmission

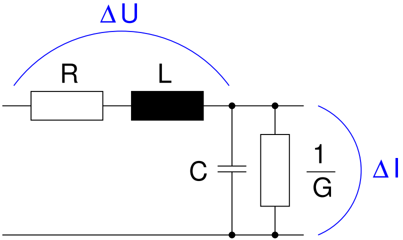
Explanatory example: signal transmission via a coaxial cable



Due to their shielding, coaxial cables do not emit electromagnetic waves. However, they can intercept electromagnetic interference from the surroundings through their shielding.

Signal propagation in a coaxial cable

Equivalent circuit diagram for a Δz length segment of a coaxial cable



R, L, C, $\frac{1}{G}$ represent resistance, inductance, capacitance, and conductance per uni t length, respectively.

In an ideal cable, R and G are both equal to 0.

Derivation of the general wave equation for a coaxial cable

$$\Delta U = -(R \cdot \Delta z) \cdot I - (L \cdot \Delta z) \cdot \frac{\partial I}{\partial t}.$$

$$\Delta I = -\left(\frac{1}{G} \cdot \Delta z\right) \cdot U - (C \cdot \Delta z) \cdot \frac{\partial U}{\partial t}.$$

Dividing by Δz and taking the limit as $\Delta z \to 0$ yields

$$\frac{\partial U}{\partial z} = -R \cdot I - L \cdot \frac{\partial I}{\partial t},$$

$$\frac{\partial I}{\partial z} = -\frac{1}{G} \cdot U - C \cdot \frac{\partial U}{\partial t}.$$

Recapitulation of the previous lecture

Wave equation for a coaxial cable

$$\frac{\partial U}{\partial z} = -R \cdot I - L \cdot \frac{\partial I}{\partial t}, \quad \left| \frac{\partial}{\partial z} \cdot \right.$$
$$\frac{\partial I}{\partial z} = -\frac{1}{G} \cdot U - C \cdot \frac{\partial U}{\partial t}. \quad \left| \frac{\partial}{\partial t} \cdot \right.$$

$$\frac{\partial^2 U}{\partial z^2} = -R \cdot \frac{\partial I}{\partial z} - L \frac{\partial^2}{\partial z \partial t} I,$$
$$\frac{\partial^2}{\partial z \partial t} I = -\frac{1}{G} \cdot \frac{\partial U}{\partial t} - C \cdot \frac{\partial^2 U}{\partial t^2}.$$

$$\boxed{\frac{\partial^2 U}{\partial z^2} = LC \frac{\partial^2 U}{\partial t^2} + (LG + RC) \frac{\partial U}{\partial t} + RGU.}$$

Ideal cable: R=0, G=0. $\boxed{\frac{\partial^2 U}{\partial z^2} = LC \frac{\partial^2 U}{\partial t^2}}$

(Wave equation with $v = \frac{1}{\sqrt{LC}}$).

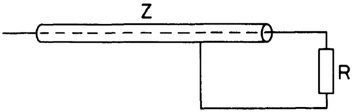
Properties of a coaxial cable

- In a real cable, G is very close to 0.
- In a real cable, $R \neq 0$ leads to dispersion. In practice, the cables used are usually so short that dispersion can be neglected, so $R = 0$ can be assumed.
- $L = \frac{\mu}{2\pi} \ln \frac{b}{a}$ [H/m], $C = \frac{2\pi\epsilon}{\ln \frac{b}{a}}$ [F/m].

  $$\Rightarrow v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\epsilon}}.$$

  Thus, the choice of dielectric determines v.
- Characteristic impedance: $Z := \frac{dU}{dI} = \sqrt{\frac{L}{C}}$.
  The characteristic impedance depends on the geometry of the cable, i.e., its inner and outer diameter as well as the dielectric used.

Reflections at the ends of the cables



$$U(t,x) = f(x - vt) + g(x + vt),$$

representing an incoming + reflected wave.

Input signal: U_E, I_E. $Z = \frac{U_E}{I_E}$.
Reflected signal: U_R, I_R, $Z = \frac{U_R}{I_R}$.
Voltage drop across the resistor R: $U_E + U_R$.
Current through R: $I_E - I_R$.

$$\Rightarrow R = \frac{U_E + U_R}{I_E - I_R} = \frac{U_E\left(1 + \frac{U_R}{U_E}\right)}{I_E\left(1 - \frac{I_R}{I_E}\right)} = Z\frac{1+\rho}{1-\rho}$$

with the reflection coefficient $\rho := \frac{U_R}{U_E} = \frac{I_R}{I_E}$. It holds $\rho = \frac{R-Z}{R+Z}$.

- Open cable: $R = \infty$. $\rho = 1$. Complete reflection at the cable end.
- Short-circuited cable: $R = 0$. $\rho = -1$. Reflection with opposite amplitude.
- Terminated cable: $R = Z$. $\rho = 0$. No reflection.

Processing of analog detector signals

- The analog signals from particle detectors are usually very small.

  Example: MDT drift tube filled with Ar/CO_2 (93:7) at 3 bar.

  $\frac{dE}{dx} = 7.5$ keV/cm$\hat{\approx} 7.5/0.03 = 250$ Electron ion pair/cm.

  At a gas gain of 20,000 this corresponds a total charge of only ~ 1 pC.

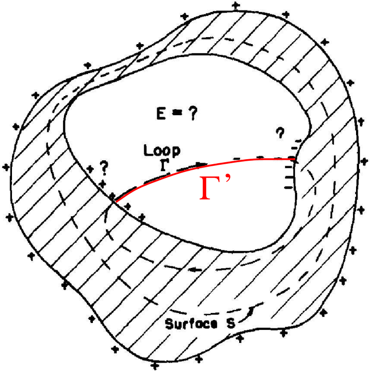
$\Rightarrow$ Protection of small signals by a Faraday cage.

$\Rightarrow$ Amplification of signals.

$\Rightarrow$ Transmission of unamplified signal over as short as possible distances.

A Faraday cage in electrostatics

- No electric field inside a conductors, otherwise there would be a current.
- The electric field in a region perfectly enclosed by a conducting cavity equals 0.
  Proof by contradiction.



(Fig.5-12 from Feynman lectures Vol 2)

If E were non-zero inside the cavity, there would be a path Γ' for which $\int_{\Gamma'} \vec{E} \cdot d\vec{s} \neq 0$.

Since $\vec{E} = 0$ inside the conductor, then $\oint_{\Gamma} \vec{E} \cdot d\vec{s} = \int_{\Gamma'} \vec{E} \cdot d\vec{s} \neq 0$, which contradicts rot $\vec{E} = 0$.

1. Equation of motion underlying the Drude model

$$m_e \frac{d\vec{v}}{dt} = -\frac{m_e}{\tau}\vec{v} - e\vec{E}.$$

Considering $\vec{E}(t,\vec{x}) = \vec{E}(\omega,\vec{x})e^{-i\omega t}$, then $\vec{v}(t,\vec{x}) = \vec{v}(\vec{x})e^{-i\omega t}$, and we obtain

$$\vec{v}(\vec{x}) = \frac{-e\tau}{m_e}\frac{1}{1-i\omega\tau}\vec{E}(\omega,\vec{x}),$$

leading to

$$\vec{j} = -ne\vec{v} = \frac{e^2\tau}{m_e}\frac{1}{1-i\omega\tau}\vec{E} =: \underbrace{\frac{\sigma_0}{1-i\omega\tau}}_{=:\sigma(\omega)}\vec{E}$$

2. Maxwell's equations for electromagnetic fields in conductors

$$\text{div}\,\vec{E} = 0. \quad \text{div}\,\vec{B} = 0. \quad \text{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}. \quad \text{rot}\,\vec{B} = \frac{1}{c^2 \epsilon_0}\vec{j} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}.$$

$$\text{rot}(\text{rot}\,\vec{E}) = \text{grad}(\underbrace{\text{div}\,\vec{E}}_{=0}) - \Delta \vec{E} = \text{rot}\left(-\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}\text{rot}\,\vec{B}.$$

Now, utilizing $\vec{j} = \sigma(\omega)\vec{E}$ for $\vec{E}(t,\vec{x}) = \vec{E}(\omega,\vec{k})e^{-i(\omega t - \vec{k}\cdot\vec{x})}$, we obtain

$$|\vec{k}|^2 = \frac{\omega^2}{c^2}\left[1 + i\frac{\sigma(\omega)}{\epsilon_0 \omega}\right].$$

$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau} \underset{\omega\tau \gg 1}{\longrightarrow} \frac{i\sigma_0}{\omega\tau}$, thus

$$|\vec{k}|^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\sigma_0}{\epsilon_0 \omega^2 \tau}\right) = \frac{\omega^2}{c^2}\left(1 - \frac{ne^2}{\epsilon_0 \omega^2}\right),$$

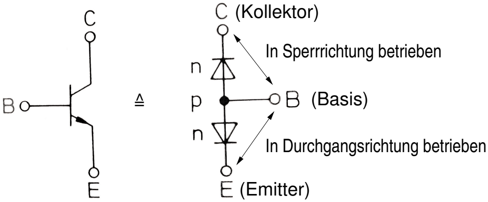
which is negative for $\omega < \frac{ne^2}{\epsilon_0}$. Then, $|\vec{k}|$ is imaginary and the electric field exponentially decreases with increasing penetration into the conductor.

Functioning of a Faraday cage in alternating fields

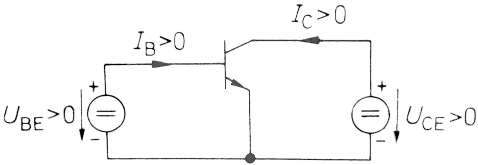
Conclusions

- Even alternating fields can be shielded by a Faraday cage if their frequency does not become too high.
- For example, choosing aluminium or brass as sufficiently thick material for the Faraday cage, one can shield fields up to the gigahertz range.

A bipolar transistor is an npn or pnp junction with 3 terminals.



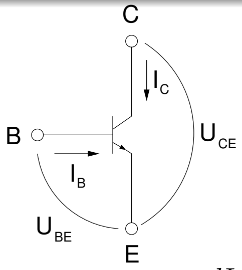
Polarity of an npn transistor



Increasing U_{BE} reduces the voltage between the base and collector, causing diode BC to conduct more and thus allowing more current to flow from the emitter than has flowed into the base.

Concept of small-signal amplification

- A bipolar transistor is a current amplifier with the current amplification $B = \frac{I_C}{I_B}$.
- The value of B depends on the values of the applied voltages.
- In practice, one is interested in the amplification of small signals. To achieve this, these small signals are superimposed on a DC voltage that sets the operating point of the transistor.
- Since B fluctuates from one transistor to another, the amplification is determined by the circuitry of the transistor, as explained in the following examples.



Goal: Amplification of small, time-varying signals.

$$dI_B = \left.\frac{\partial I_B}{\partial U_{BE}}\right|_{U_{CE}} \cdot dU_{BE} + \left.\frac{\partial I_B}{\partial U_{CE}}\right|_{U_{BE}} \cdot dU_{CE},$$

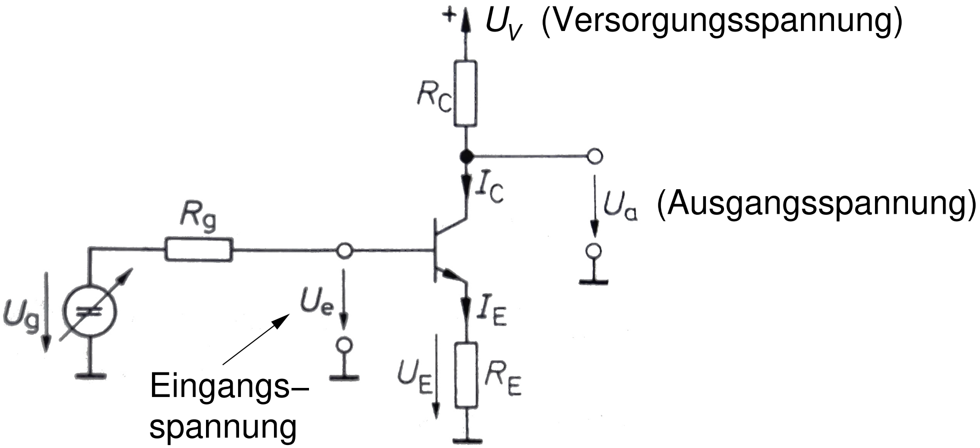
$$dI_C = \left.\frac{\partial I_C}{\partial U_{BE}}\right|_{U_{CE}} \cdot dU_{BE} + \left.\frac{\partial I_C}{\partial U_{CE}}\right|_{U_{BE}} \cdot dU_{CE}.$$

- $\frac{1}{r_{BE}} := \frac{\partial I_B}{\partial U_{BE}}|_{U_{CE}}$ is small. $\frac{\partial I_B}{\partial U_{CE}}|_{U_{BE}} \approx 0.$
- Slope $S := \frac{\partial I_C}{\partial U_{BE}}|_{U_{CE}}$ is large. $\frac{1}{r_{CE}} := \frac{\partial I_C}{\partial U_{CE}}|_{U_{BE}}$ is small.

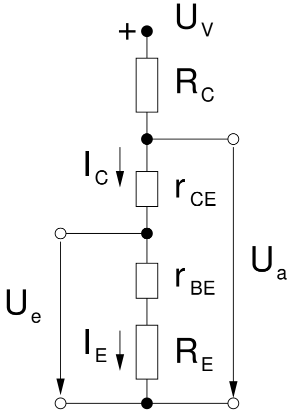
$$\Rightarrow dI_B = \frac{1}{r_{BE}} \cdot dU_{BE},$$

$$dI_C = S \cdot dU_{BE} + \frac{1}{r_{CE}} \cdot dU_{CE}.$$

1st Example: Emitter circuit with current feedback



Equivalent circuit for calculating the small-signal amplification $A := \frac{dU_a}{dU_e}$

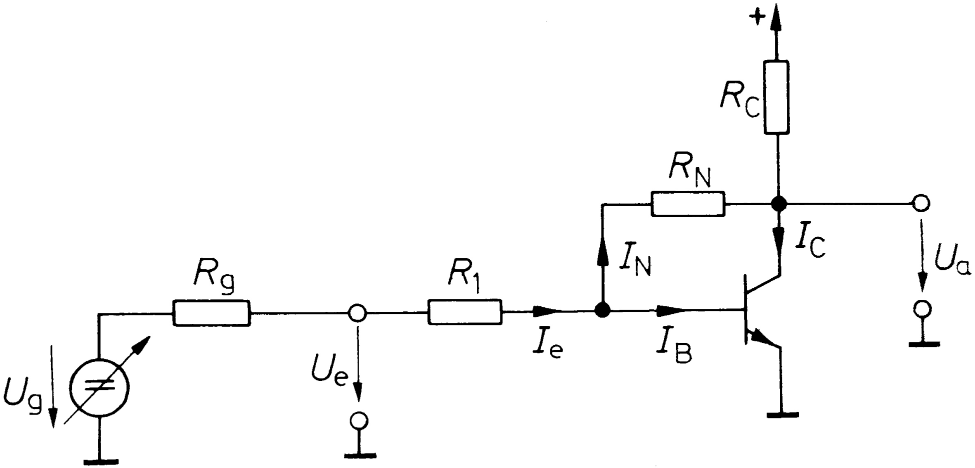


$$dI_E = \frac{dU_e}{r_{BE} + R_E} \underset{r_{BE} \ll R_E}{\approx} \frac{dU_e}{R_E}.$$

$$dI_C = \frac{d(U_V - U_a)}{R_C} \underset{dU_V = 0}{=} -\frac{dU_a}{R_C}.$$

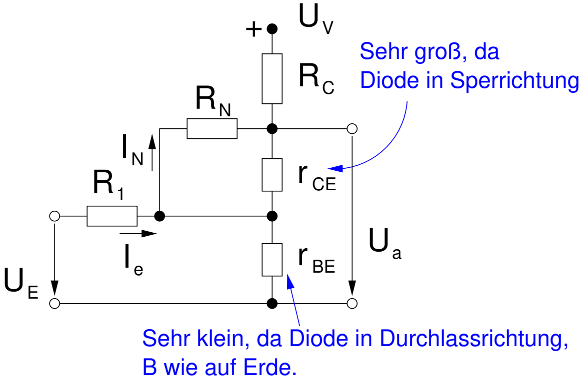
$$dI_E = dI_C \Rightarrow A = \frac{dU_a}{dU_e} = -\frac{R_C}{R_E}.$$

The circuit is inverting with a small-signal amplification that depends only on the configuration of the transistor, namely R_C and R_E.


+
R_C
R_N
I_C
I_N
U_a
R_g
R_1
I_e
I_B
U_e
U_g

Calculation of small-signal amplification

Equivalent circuit for calculating the small-signal amplification $A := \frac{dU_a}{dU_e}$



$$dU_e = R_1\,dI_e,\;\; dU_a = R_N\,dI_N = -R_N\,dI_E.$$

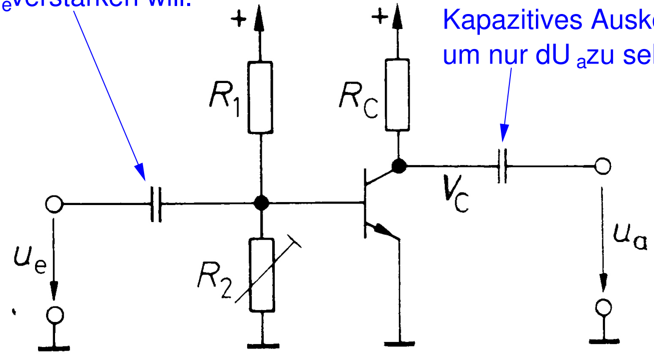
$$\Rightarrow A = \frac{dU_a}{dU_e} = \frac{-R_N}{R_1}.$$

The circuit is inverting with a small-signal amplification that depends only on the configuration of the transistor, namely R_N and R_1.

Operating point adjustment

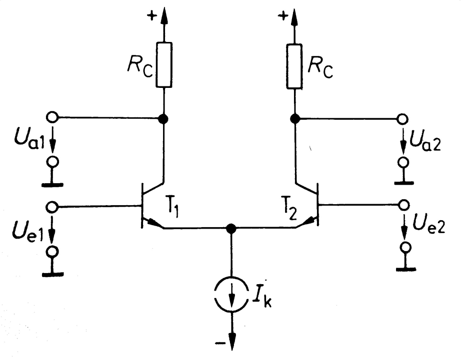
Kapazitives Einkoppeln des Signals, um den Arbeitspunkt nicht zu verschieben. Möglich, da man nur dU_e verstärken will.

Kapazitives Auskoppeln, um nur dU_a zu sehen.



Spannungsteiler zur Fest–legung des Arbeitspunktes des Transistors

Operation of a differential amplifier



- Constant current source at the emitter. $\Rightarrow dI_k = 0$.
- Internal resistance of the constant current source: r_k.

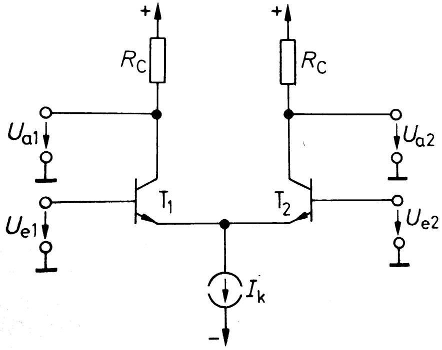
- $I_k = I_{C1} + I_{C2} \Rightarrow dI_{C1} = -dI_{C2}$.
- So $dU_{a1} = -dU_{a2}$.
- Also
  $dU_{e1} = dU_{BE1} = -dU_{BE2} = -dU_{e2}$.
- $U_D := U_{e1} - U_{e2}$.
  $dU_{e1} = d(U_{e1} - U_{e2} + U_{e2})$
  $= dU_D + dU_{e2} = dU_D - dU_{e1}$,
  thus $dU_D = \frac{1}{2} dU_{e1}$.

$\Rightarrow$ Differential amplification $A_D = \frac{dU_{a1}}{dU_D}$

$A_D = \frac{dU_{a1}}{2dU_{BE1}} = -\frac{1}{2} S(R_C || r_{CE})$.

Since S is large, A_D is also large.

Operation of a differential amplifier



- Constant current source at the emitter. $\Rightarrow dI_k = 0$.
- Internal resistance of the constant current source: r_k.

- $I_k = I_{C1} + I_{C2} \Rightarrow dI_{C1} = -dI_{C2}$.
- So $dU_{a1} = -dU_{a2}$.
- Also $dU_{e1} = dU_{BE1} = -dU_{BE2} = -dU_{e2}$.
- $U_D := U_{e1} - U_{e2}$.
  $dU_{e1} = d(U_{e1} - U_{e2} + U_{e2})$
  $= dU_D + dU_{e2} = dU_D - dU_{e1}$,
  thus $dU_D = \frac{1}{2} dU_{e1}$.

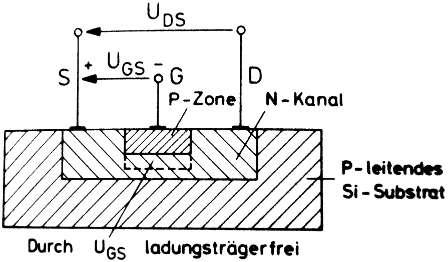
$\Rightarrow$ Differential amplification $A_D = \frac{dU_{a1}}{dU_D}$

$A_D = \frac{dU_{a1}}{2dU_{BE1}} = -\frac{1}{2} S(R_C || r_{CE})$.

Since S is large, A_D is also large.

Besides the differential amplification, there is also a much smaller common-mode amplification $A_{\mathrm{CM}} := \frac{dU_{a1}}{d(U_{e1}+U_{e2})/2} = -\frac{1}{2}\frac{R_C}{r_k}$, which immediately follows from the formula for the amplification of the emitter circuit with current feedback.

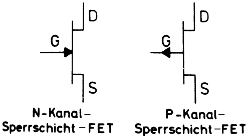
Construction of an n-channel junction field-effect transistor



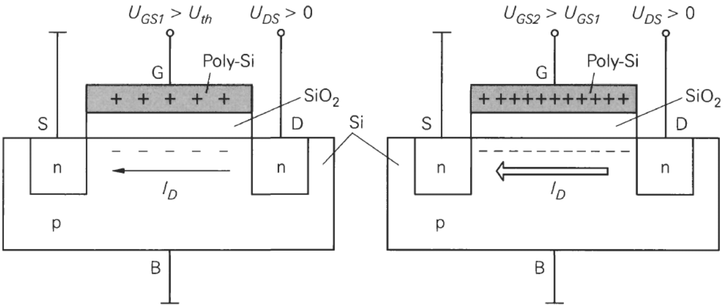
S: Source.

G: Gate.

D: Drain.



- Control of the size of the charge carrier-free zone via the value of the voltage U_{GS}.
- Thickness of the charge carrier-free zone determines the resistance between drain and source.
- Advantage of field-effect transistors over bipolar transistors: Lower power consumption, as the control is done via the applied electric field and not via a current.



- Structure forms a capacitor from gate terminal, dielectric, and bulk terminal.
- Application of positive voltage between gate and bulk charges the capacitor.
- Electric field causes migration of minority carriers (electrons in p-silicon) to the junction and recombination with majority carriers (defect electrons in p-silicon), known as “depletion“.
- Space charge region forms at the junction with negative space charge.
- At threshold voltage U_{th}, displacement of majority carriers becomes significant, limiting recombination.
- Accumulation of minority carriers results in near-inversion of p-doped substrate close to the oxide, known as strong inversion“
- Increased gate voltage induces band bending of conduction and valence bands at the junction in band model.
- Fermi level shifts closer to the conduction band than the valence band, inverting the semiconductor material.
- Formed thin n-type conducting channel connects source and drain n-regions, allowing charge carriers to flow (almost) unimpeded from source to drain.