Anisotropic Hydrodynamics from Gauge/Gravity Duality

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IMPRS Workshop – 02/05/2011

based on arXiv:1011.5912
in collaboration with J. Erdmenger and P. Kerner

Motivation

Gauge/Gravity Duality:

- novel method to study strongly coupled theories
- simple calculations to determine the dynamics of these systems
- prediction $\frac{\eta}{s} = \frac{1}{4\pi}$,
  good agreement with measurements of Quark-Gluon-Plasma (RHIC)

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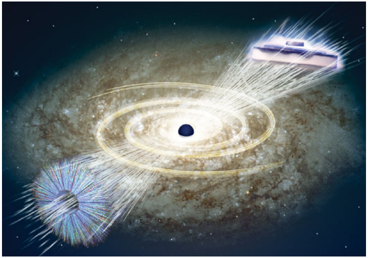
Anisotropic Systems:

- more than one distinct shear mode
- deviation of $\frac{\eta}{s}$ from $\frac{1}{4\pi}$ in holographic systems
  $\Rightarrow$ step towards applying gauge/gravity duality to real-world systems

Outline

Gauge/Gravity Duality

- classical gravity in asymptotically AdS space
- dual field theory on the conformally flat boundary of the AdS space



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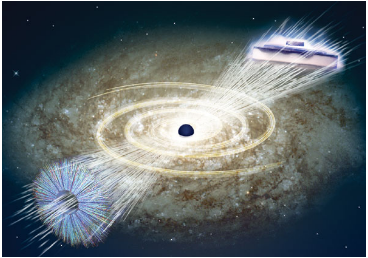

field theory	$\Leftrightarrow$	gravity theory
strongly coupled	$\Leftrightarrow$	weakly coupled
finite temperature	$\Leftrightarrow$	black hole
finite chemical potential	$\Leftrightarrow$	$\langle A_t \rangle \neq 0$
spontaneous breaking of global symmetries	$\Leftrightarrow$	hairy black holes

Hydrodynamics – Overview

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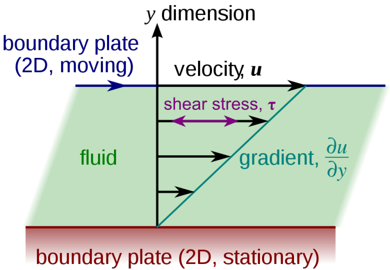
- constitutive equations

$$T^{\mu\nu} = T^{\mu\nu}_{\text{eq.}} + \Pi^{\mu\nu} \text{ and } J^{\mu} = J^{\mu}_{\text{eq.}} + \Upsilon^{\mu}$$

$$\text{with } \Pi_{ij} \sim \eta\left(\partial_i u_j + \partial_j u_i\right)$$

What is the Shear Viscosity?

- measure of the momentum diffusion transverse to the momentum



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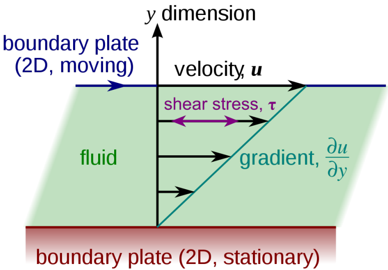

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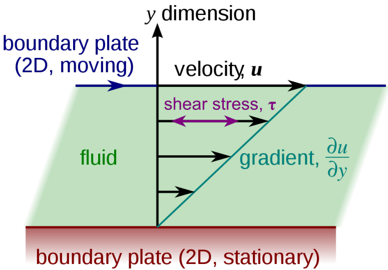

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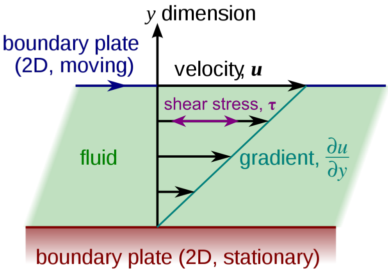

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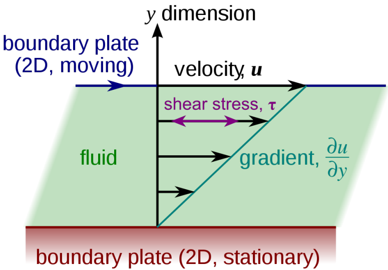
- transversely isotropic systems:
  2 independent shear viscosities



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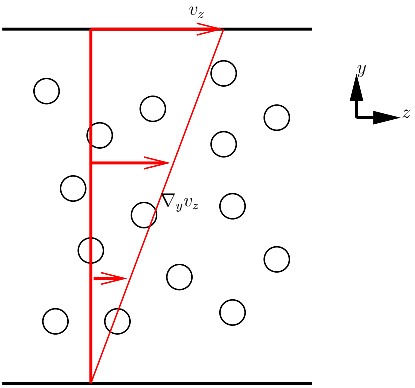

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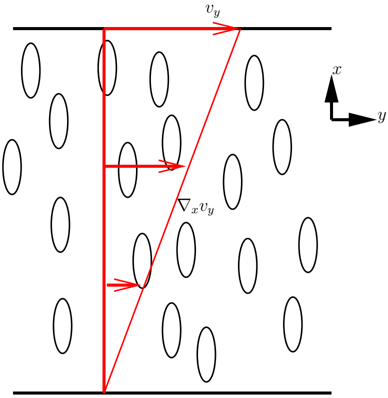

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- using $s \sim T^3$

$$\Rightarrow \frac{\eta}{s} \sim \frac{1}{\lambda^2}$$

Shear Viscosity from Gauge/Gravity Duality

- universal property of strongly coupled isotropic field theories with classical, two-derivative gravity dual

$$\frac{\eta}{s} = \frac{1}{4\pi} \simeq 0.079$$

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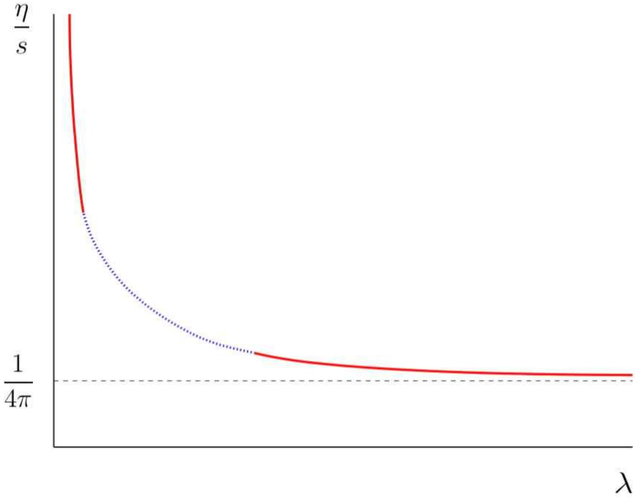

Mateos

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- conjecture by Kovtun, Son and Starinets:

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

for all substances found in Nature

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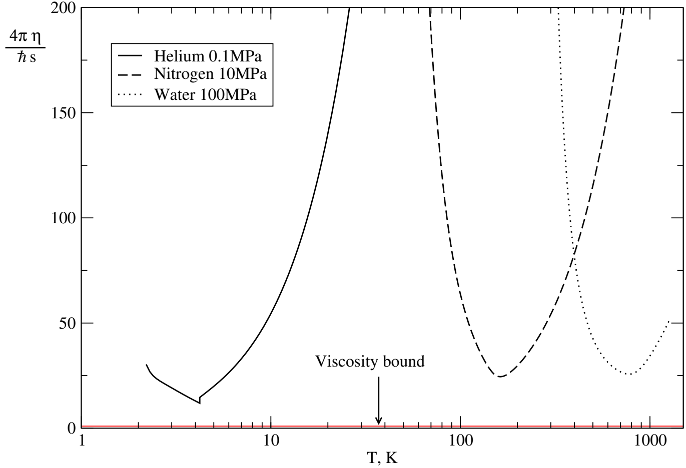

Son *et al.*

Holographic System with Spontaneous Broken Rotational Symmetry

- setup: $SU(2)$ Einstein-Yang-Mills theory in $(4+1)$-dimensional asymptotically AdS space
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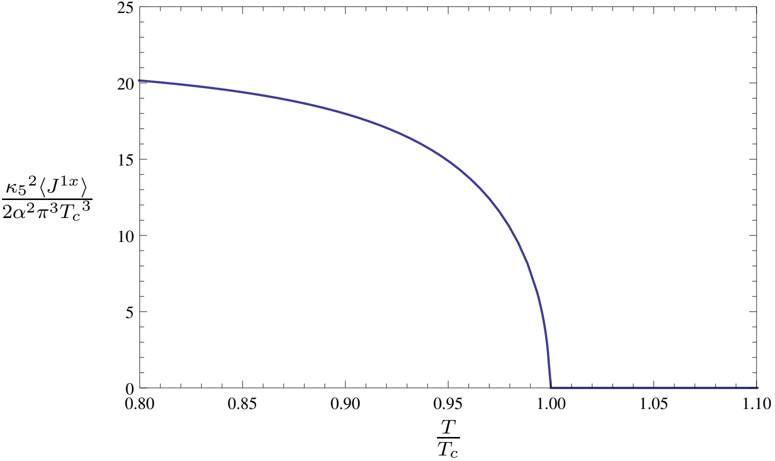

Erdmenger *et al.*

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$\Rightarrow$ rotational symmetry spontaneously broken

- considering fluctuations with small frequencies

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$\Rightarrow$ 2 distinct shear viscosities: η_{yz} and η_{xy}

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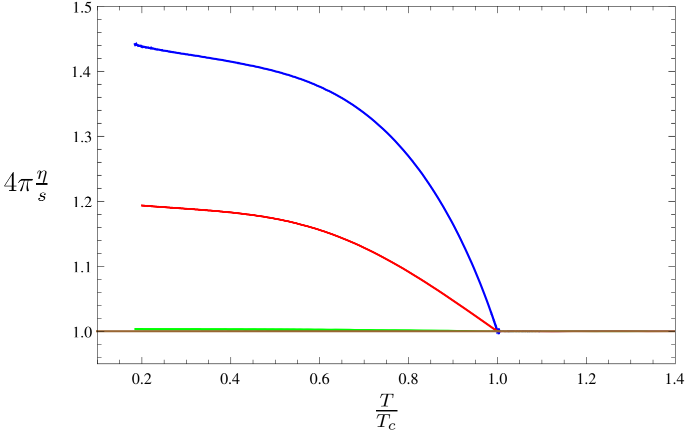
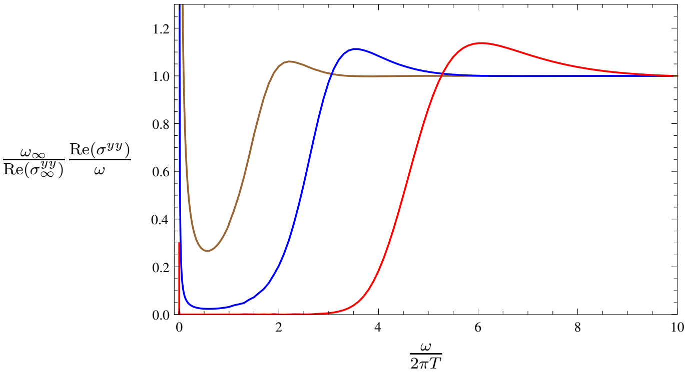


colour coding: $\frac{\eta_{yz}}{s} = \frac{1}{4\pi}$ and $\frac{\eta_{xy}}{s}$ depending on the amount of symmetry breaking (blue > red > green)

Conductivity

$$J^y = \sigma^{yy} E_y$$



colour coding: $T = 1.6\, T_c$, $T = 0.9\, T_c$, $T = 0.5\, T_c$

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- zero temperature behaviour of transport coefficients

Thank you!

Equations

- $SU(2)$ Einstein-Yang-Mills theory:

$$S = \frac{1}{2{\kappa_5}^2} \int \mathrm{d}^5x \left[R - \Lambda - \frac{\alpha^2}{2} F^a_{MN} F^{a\,MN} \right]$$

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$$\mathrm{d}s^2 = -N(r)\sigma(r)^2 \mathrm{d}t^2 + \frac{r^2}{f(r)^4}\mathrm{d}x^2 + r^2 f(r)^2 \left(\mathrm{d}y^2 + \mathrm{d}z^2\right) + \frac{1}{N(r)}\mathrm{d}r^2$$

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- equations of motion:

$$R_{MN} - 4g_{MN} = \alpha^2 \left(T_{MN} - \frac{1}{3} {T_P}^P g_{MN} \right) ,$$

$$\text{with:} \quad T_{MN} = F^a_{PM} F^{a\,P}{}_N - \frac{1}{4} F^a_{PQ} F^{a\,PQ} g_{MN}$$

$$\nabla_M F^{a\,MN} = -\varepsilon^{abc} A^b_M F^{c\,MN}$$

Fluctuations

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- real-time retarded Green's functions

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- transport coefficients using Kubo formulas:

$$\eta_{xy} = -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\,(G^{xy,xy}(\omega,0))$$

$$\sigma_{yy} = -i\frac{G^{3y,3y}}{\omega}$$