Status of NNLO Calculations for DIS and Higgs Production

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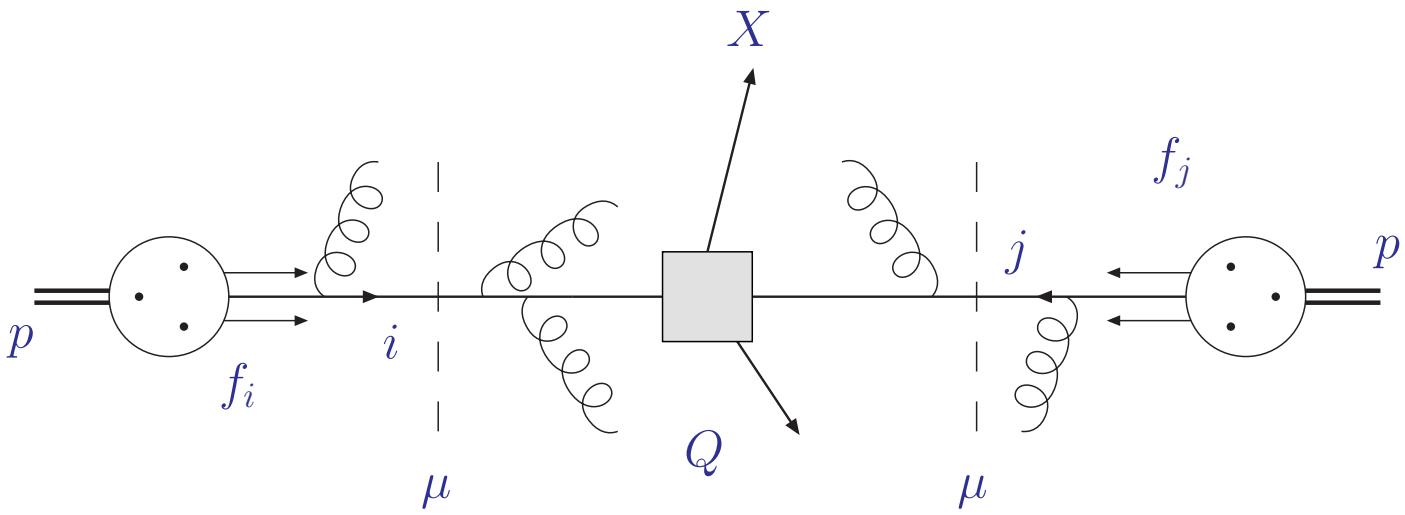
– Ringberg Workshop *New Trends in HERA Physics 2011*, Sep 27, 2011, Ringberg –

Past and Present

- HERA: deep structure of proton at highest Q^2 and smallest x
- LHC: Higgs boson search at highest energies $\sqrt{S} = 7$TeV

- Quantum Chromodynamics (QCD) ubiquitous at proton colliders
  - reliable understanding essential for precision and discovery physics

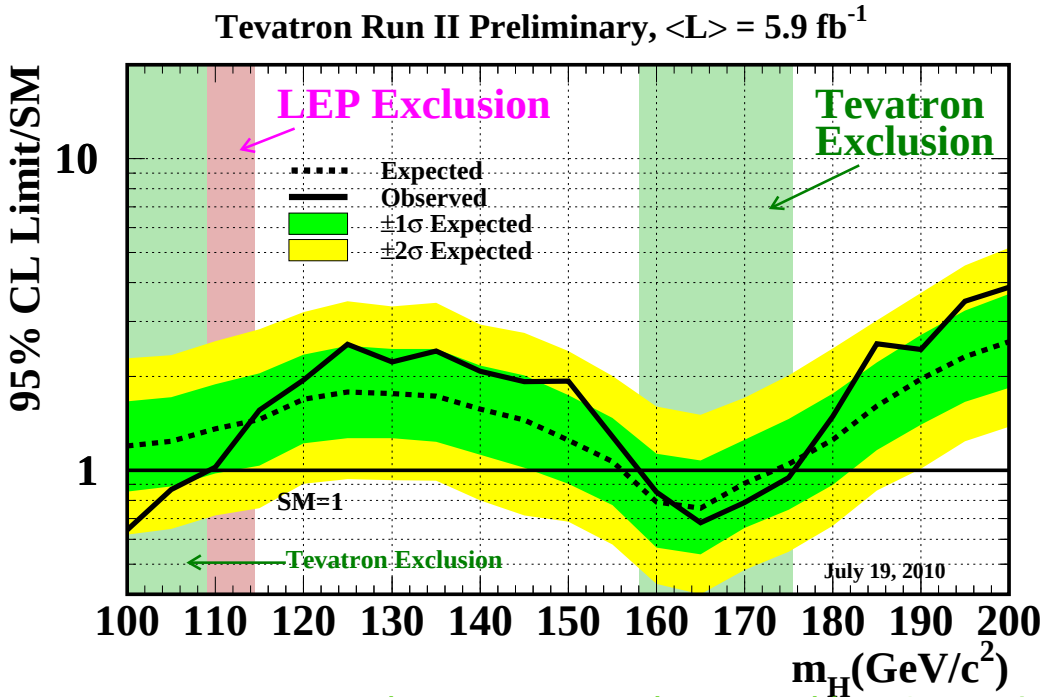
QCD factorization

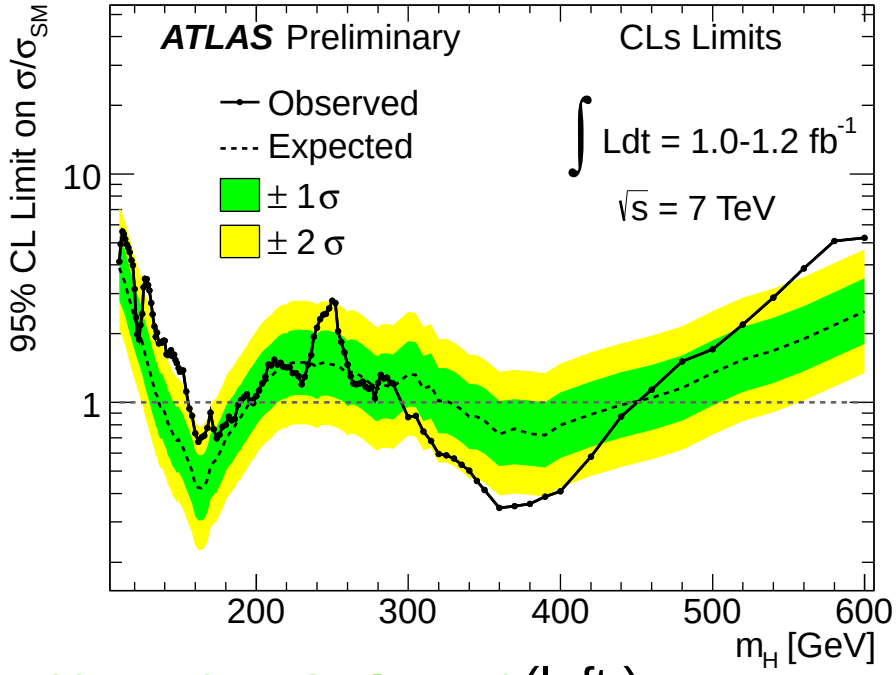


$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - known to NLO, NNLO, ... ($\mathcal{O}(\text{few}\%)$ theory uncertainty)
- Non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, ...

Why precision ?





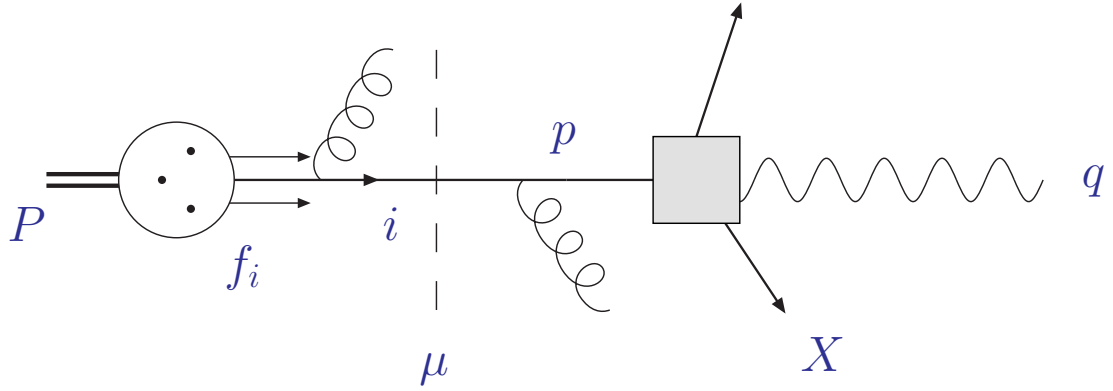
Tevatron New Phenomena & Higgs Working Group `http://tevnphwg.fnal.gov/` (left)

ATLAS coll. `ATLAS-CONF-2011-112` (right)

- Higgs sensitivity driven by large higher order corrections
- Measurement of Higgs properties (mass, couplings, ...) requires:
  - precise estimate of the cross-section
  - LO → NLO: reliable information on cross section
  - NLO → NNLO: improvement of theoretical uncertainities

What role for DIS ?

- Deep-inelastic scattering
  - constituent partons from proton interact at short distance
  - photon momentum $Q^2 = -q^2$, Bjorken's $x = Q^2/(2p \cdot q)$



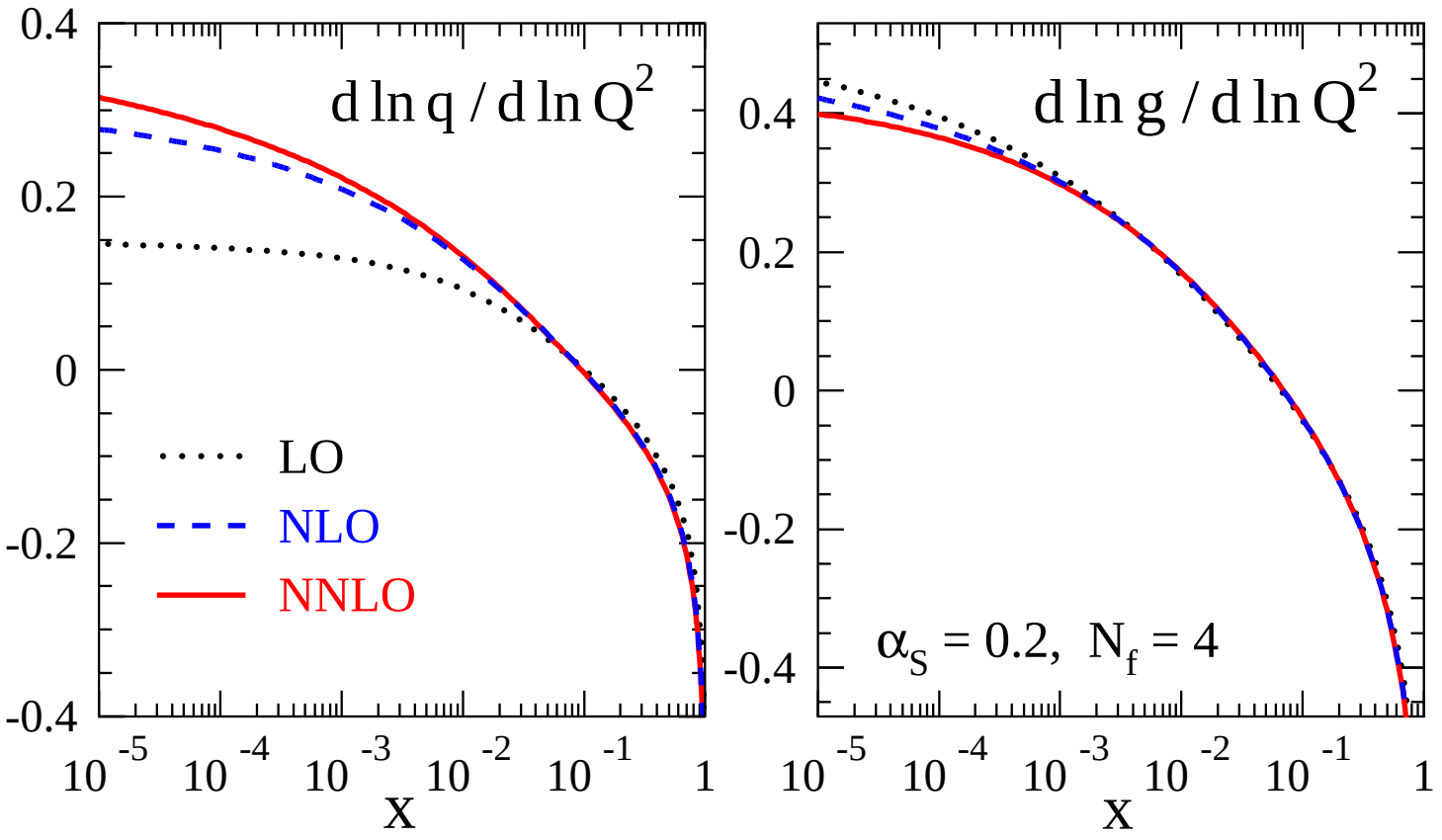
$$\sigma_{\gamma p \to X} = \sum_i f_i(\mu^2) \otimes \hat{\sigma}_{\gamma i \to X} \left(\alpha_s(\mu^2), Q^2, \mu^2, m_q^2 \right)$$

- Evolution equations for parton distributions f_i

$$\frac{d}{d \ln \mu^2} f_i(x, \mu^2) = \sum_k \left[P_{ik}(\alpha_s(\mu^2)) \otimes f_k(\mu^2) \right](x)$$

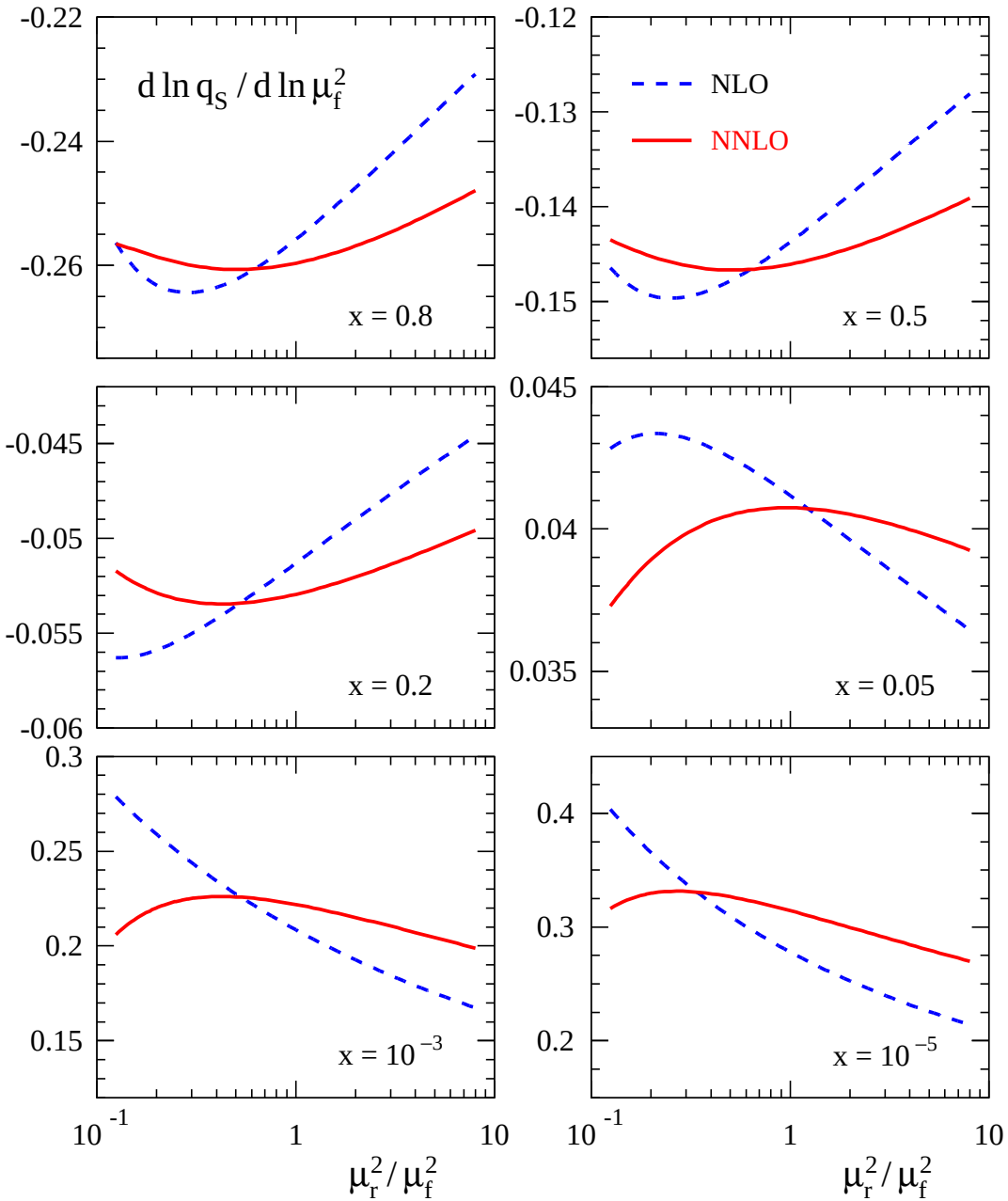
Perturbative stability of evolution

- Scale derivatives of quark and gluon distributions at $Q^2 \approx 30$ GeV^2

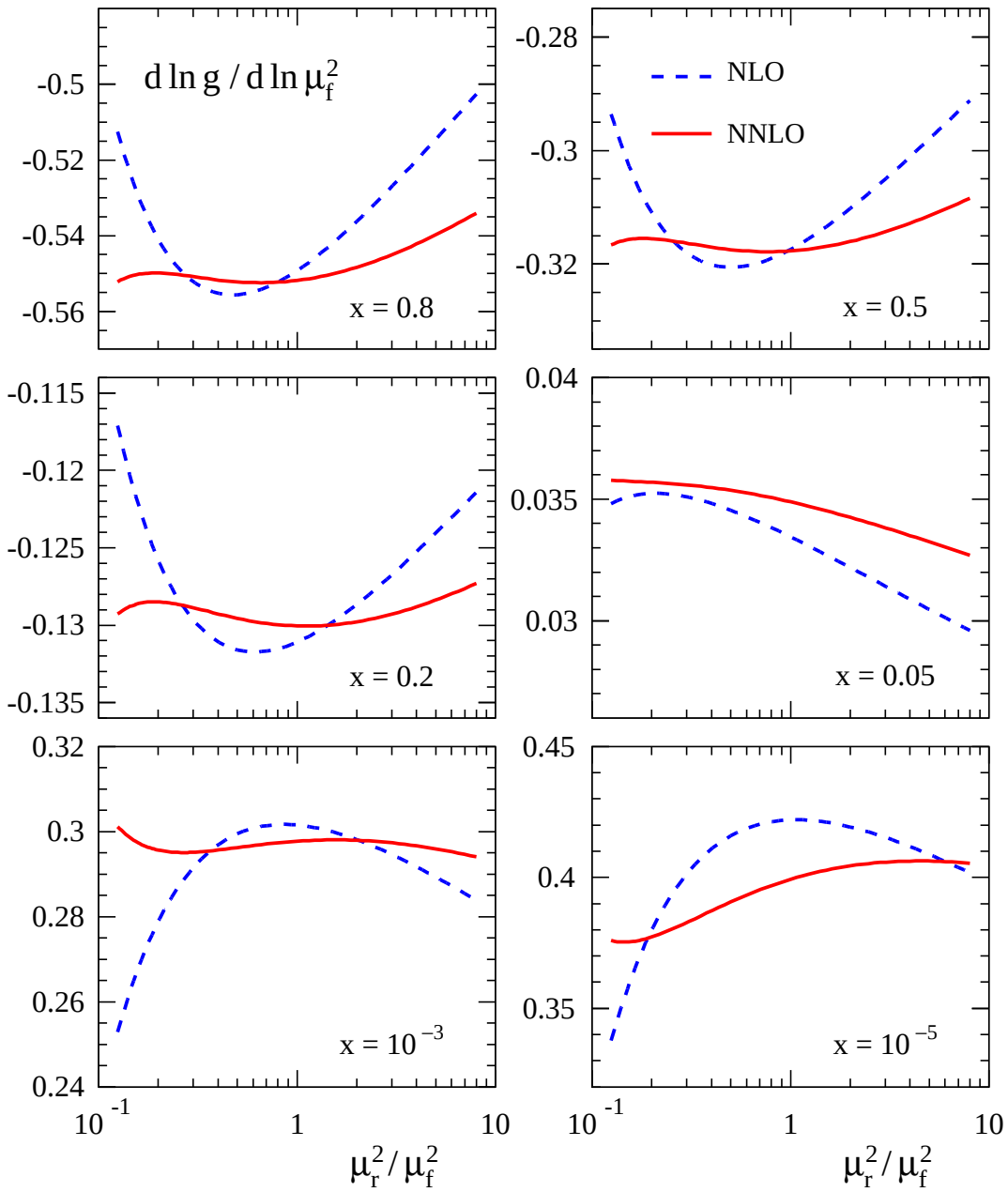


- Expansion very stable except for very small momenta $x \lesssim 10^{-4}$

  S.M. Vermaseren, Vogt '04



- Singlet-quark distribution:
  $\dot{q}_S \equiv d \ln q_S / d \ln \mu_f^2$
  renormalization scale μ_r dependence at NLO and NNLO



- Gluon distribution:
  $\dot{g} \equiv d\ln q_s / d\ln \mu_f^2$
  renormalization scale μ_r dependence at NLO and NNLO

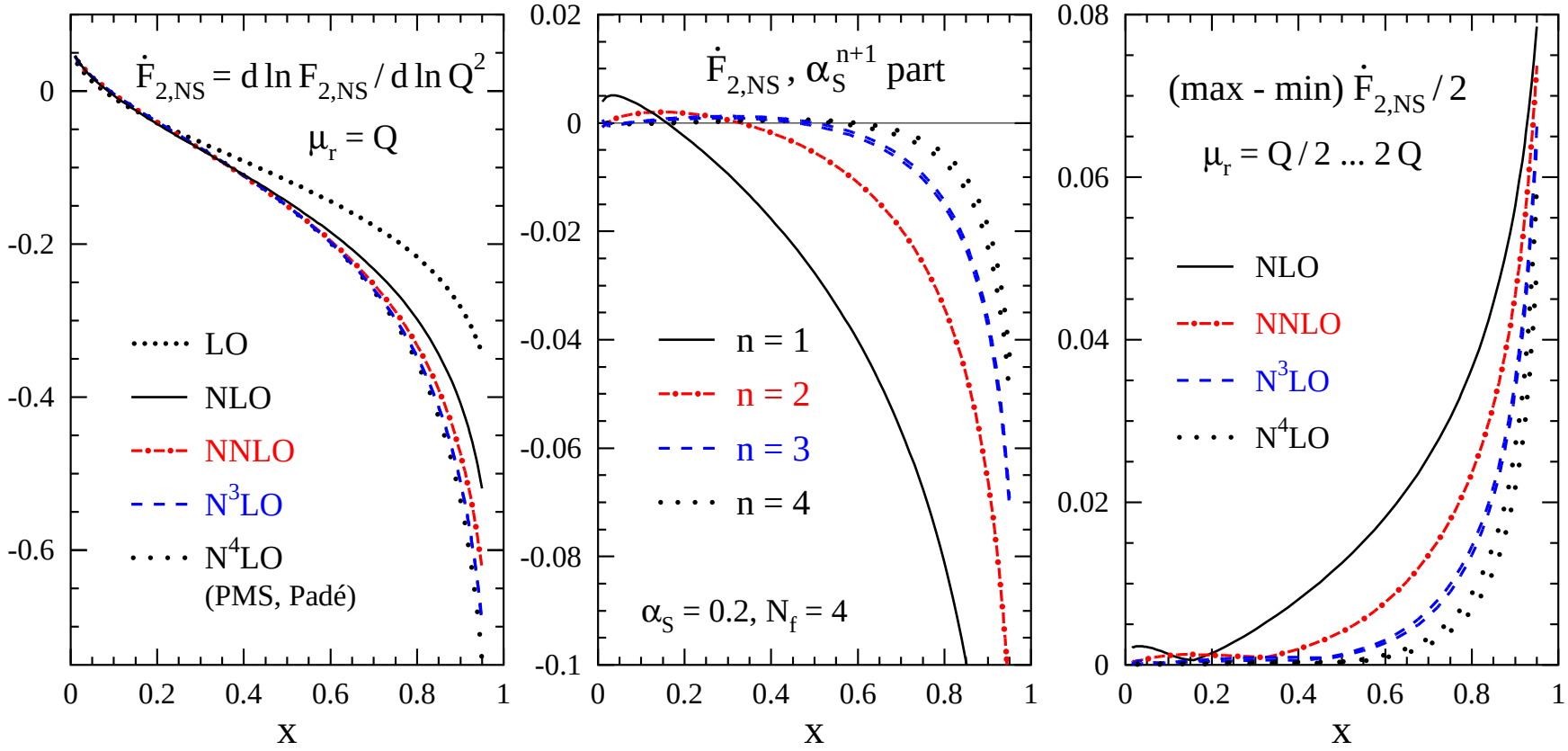
Status Quo for DIS

Light quarks

- Neglect "light quark" masses $m_u, m_d \ll \Lambda_{QCD}$ and $m_s < \Lambda_{QCD}$ in hard scattering process
  - scale-dependent u, d, s, g PDFs from mass singularities
- Structure functions at three loops (NNLO and N^{3}LO)
  - γ-exchange structure functions F_2, F_L S.M., Vermaseren, Vogt '05
  - charged-current structure function F_3 S.M., Vermaseren, Vogt '08
- Deep-inelastic jet production
  - $ep \to 1$ jet X inclusive production known to NLO [many people]

Structure function F_2 (non-singlet)

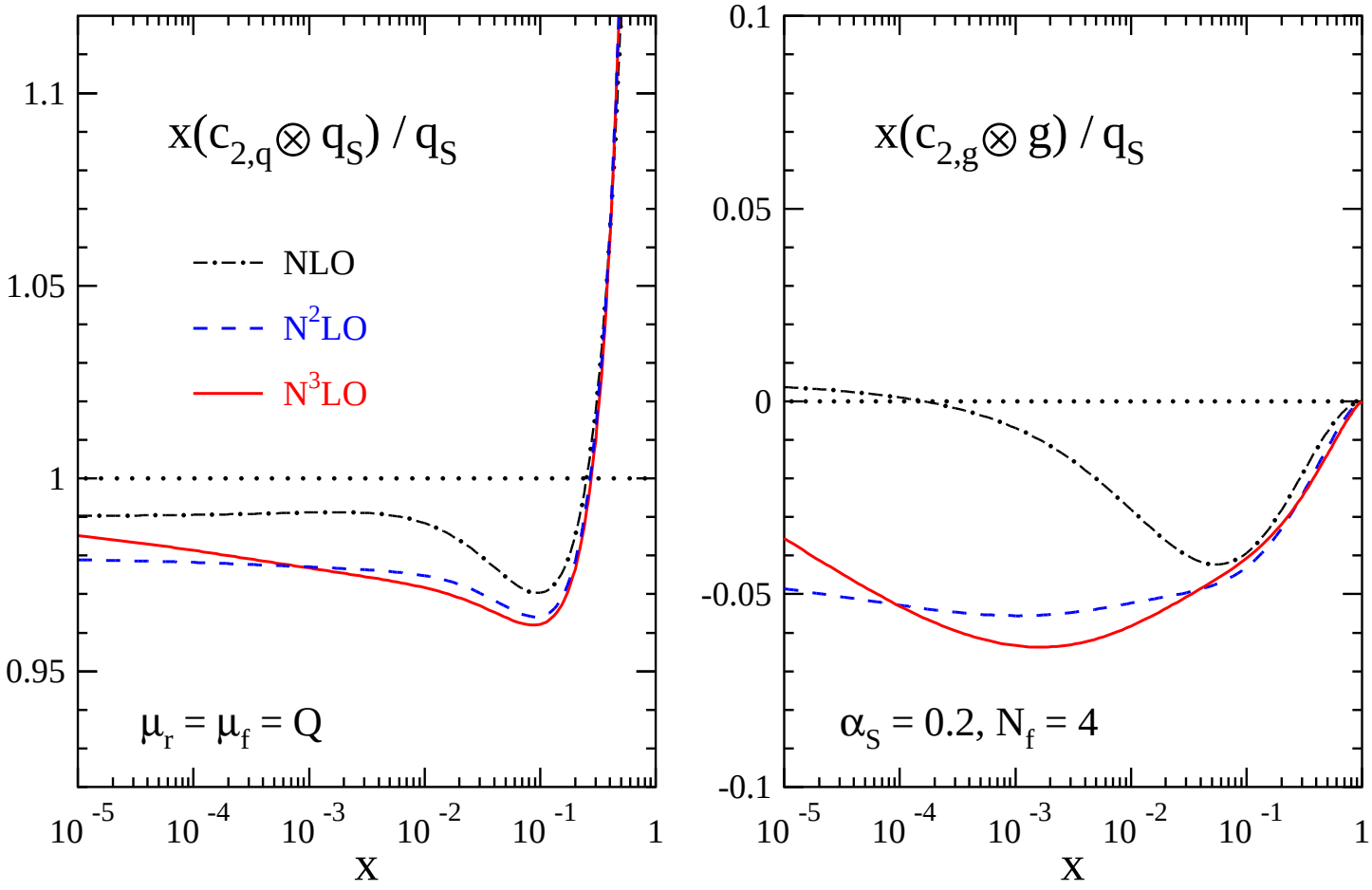
- Large-x convergence of perturbative series
  - approx. N^{3}LO structure functions



- Potential for 'gold-plated' determination of α_s
  - theory uncertainty $\Delta_{\rm pert.}\alpha_s < 1\%$

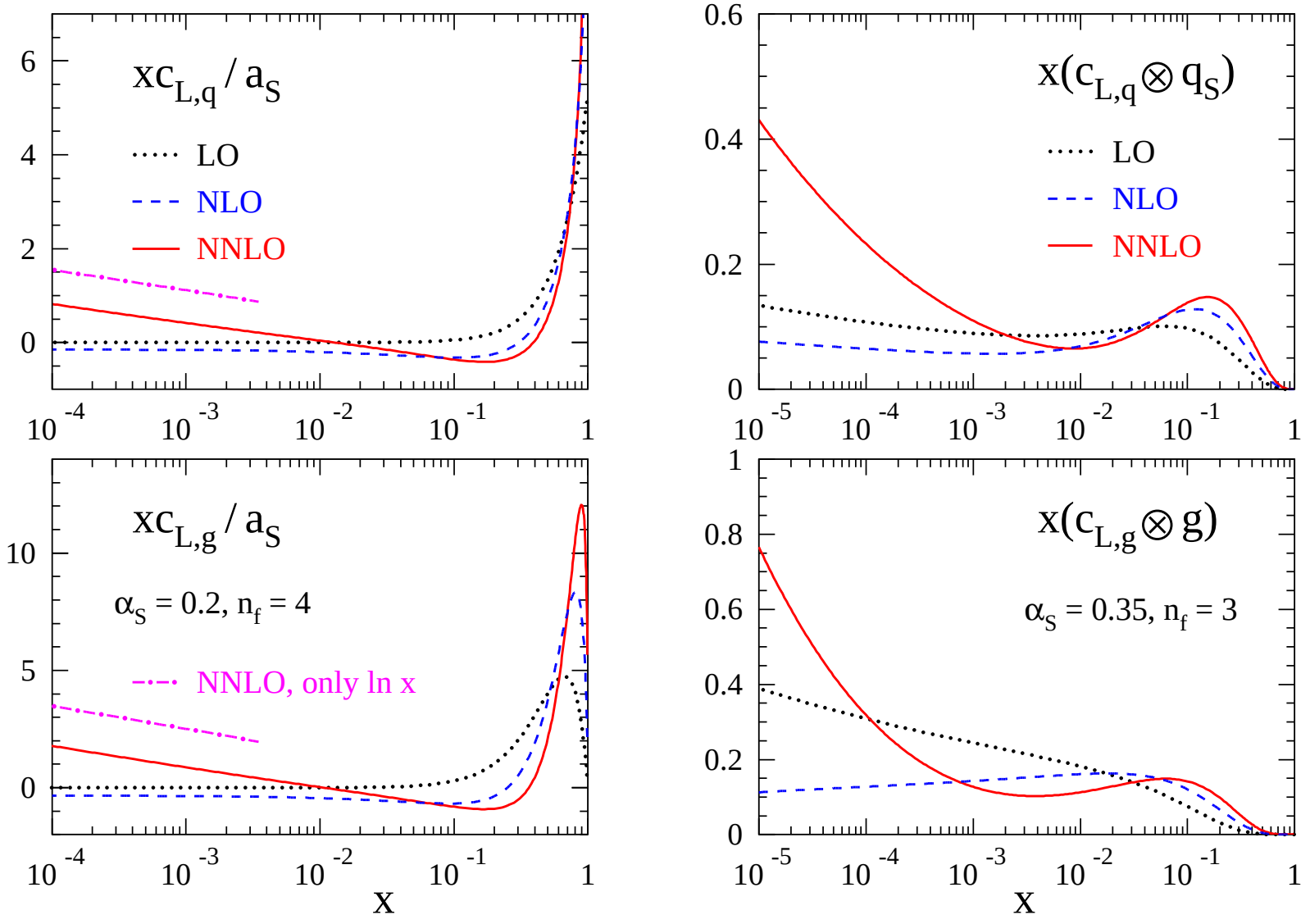
- At very large x: soft-gluon resummation S.M., Vermaseren, Vogt '05

Structure function F_2 (singlet)



- Perturbative expansion to N^{3}LO of the quark and gluon contribution
- Perturbative stability of F_2

Longitudinal structure function F_L



S.M., Vogt, Vermaseren hep-ph/0411112

Status Quo for DIS

Heavy quarks

- No mass singularities for $m_c, m_b, m_t \gg \Lambda_{QCD}$, no (evolving) PDFs
  - c and b PDFs for $Q \ggg m_c, m_b$ generated perturbatively
  - matching of two distinct theories
    $\longrightarrow n_f$ light flavors + heavy quark of mass m at low scales
    $\longrightarrow n_f + 1$ light flavors at high scales

Heavy-quark production in DIS

NLO QCD corrections

- NLO for charm structure function
  - neutral current Laenen, Riemersma, Smith, van Neerven '93
  - charged current Gottschalk '81; Glück, Kretzer, Reya '96

Threshold improvements beyond NLO

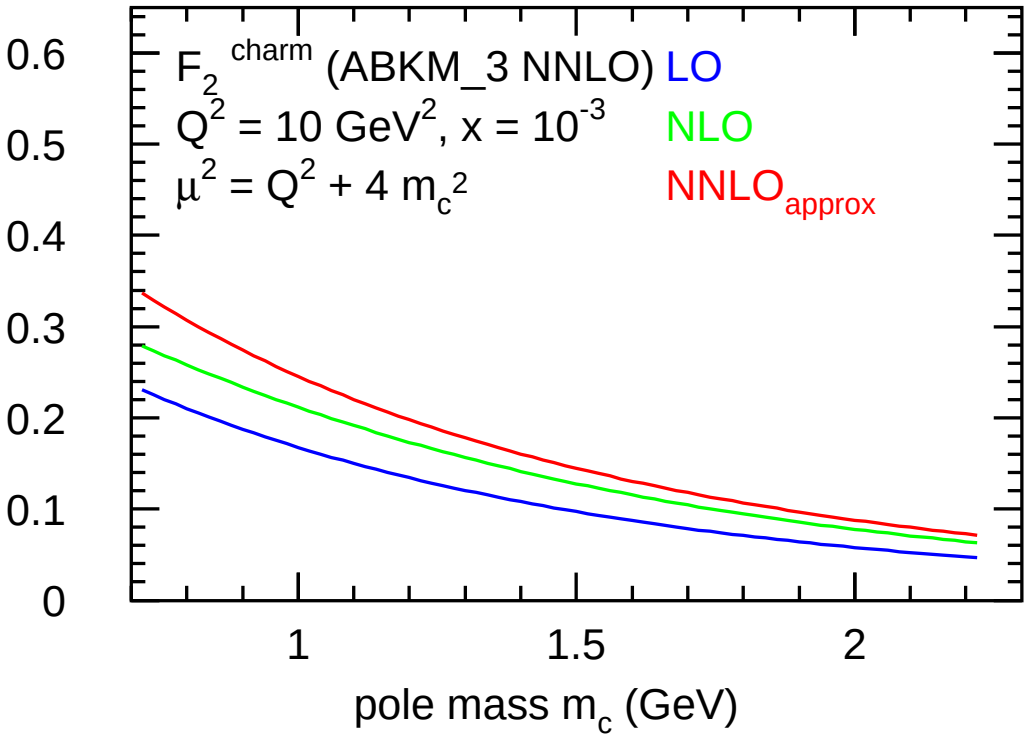
- Parton cross section close to threshold $s \simeq 4m^2$
  - Sudakov logarithms $\ln(\beta)$ with velocity of heavy quark $\beta = \sqrt{1 - 4m^2/s}$ at n^{th}-order: $\alpha_s^n \ln^{2n}(\beta) \longleftrightarrow \alpha_s^n \ln^{2n}(N)$
- Resummation in Mellin space (renormalization group equation) predicts fixed orders in perturbation theory
  - approximate expressions to NNLO
    Laenen, S.M. '98; Alekhin, S.M. '08; Lo Presti, Kawamura, S.M., Vogt '10

Asymptotics beyond NLO

- NNLO corrections at large $Q^2 \gg m^2$ Bierenbaum, Blümlein, Klein '09

Running quark masses in DIS

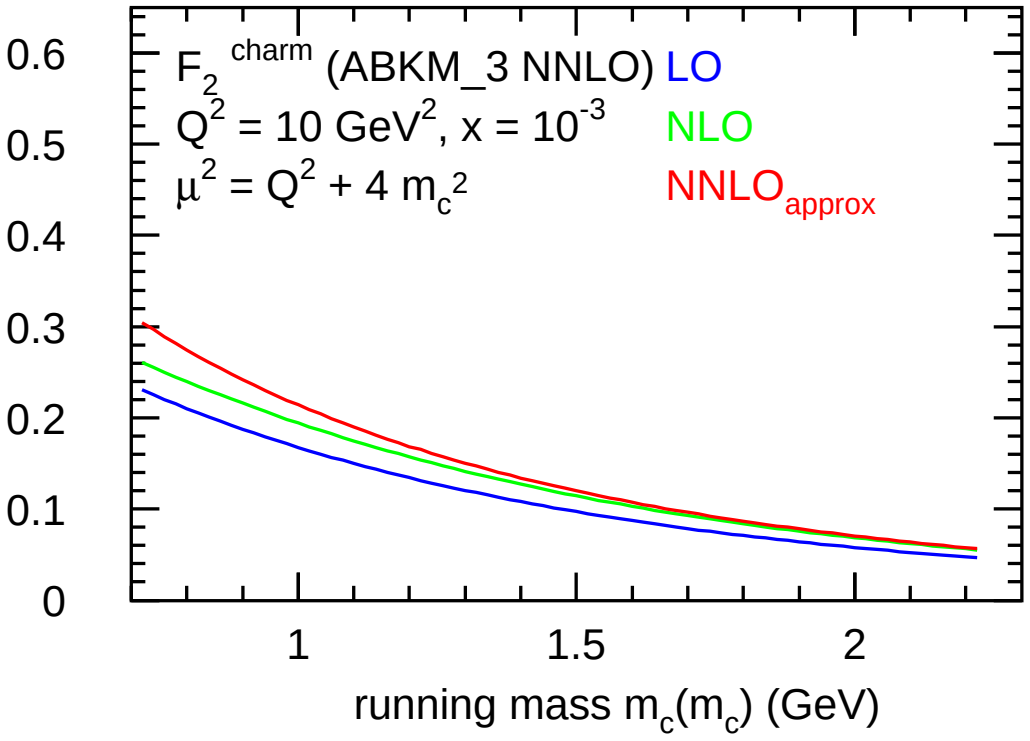
- Charm structure function



- Running quark masses in DIS
  - improved convergence
  - reduced scale dependence
- Comparison with pole mass scheme

Running quark masses in DIS

- Charm structure function



- Running mass
- Direct determination of $m_c(m_c)$
  Alekhin, S.M. '10

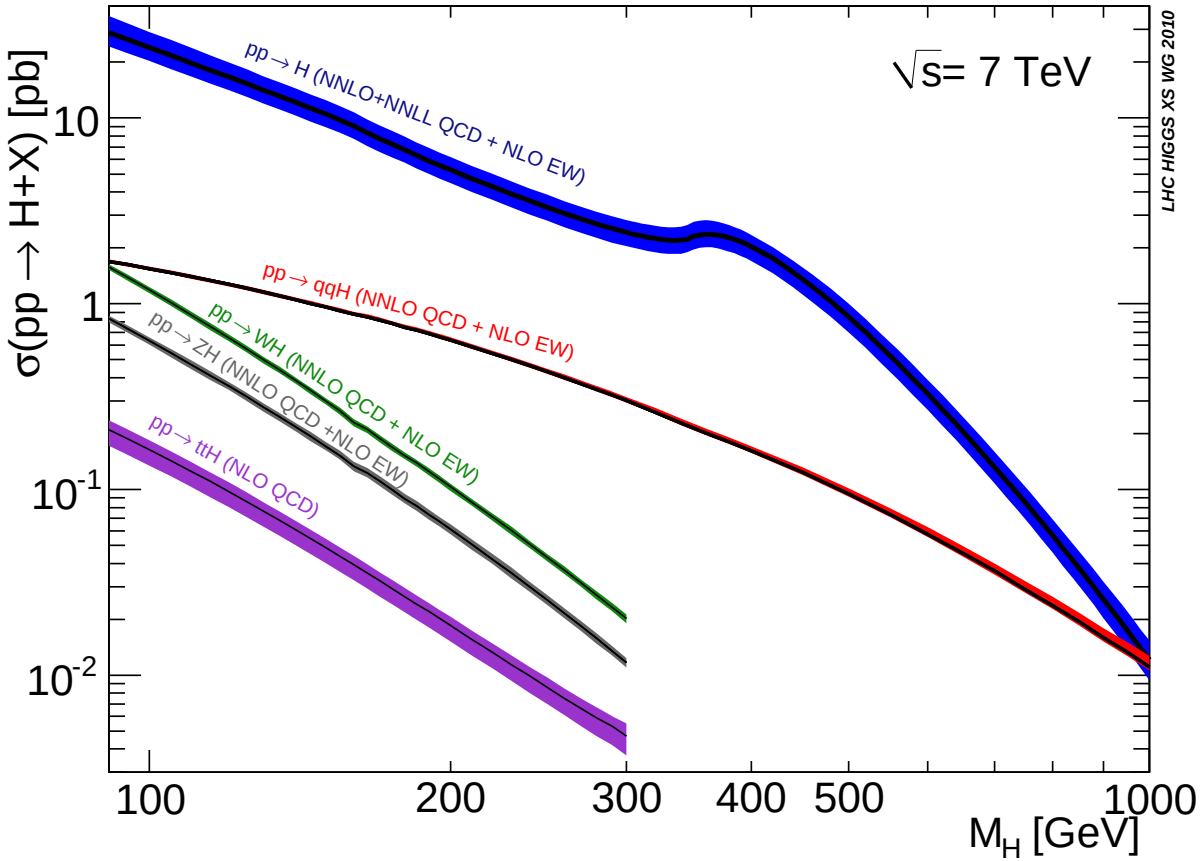
  NLO
  1.26 ± 0.09 (exp) $\pm$ 0.11 (th) GeV

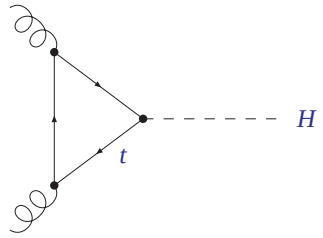
  $\text{NNLO}_{\text{approx}}$ (fit + PDG constraint)
  1.18 ± 0.09 (exp) $\pm$ 0.03 (th) GeV
- PDG quotes running masses:
  $m_c(m_c) = 1.27^{+0.07}_{-0.11}$ GeV
- Implicit $\alpha_s(M_Z)$ dependence in $m_c(m_c)$ determination from QCD sum rules
  Dehnadi, Hoang, Mateu, Zebarjad '11

Higgs cross section

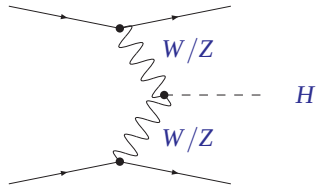
Cross section for Higgs production at the LHC

- Dominant channels for Higgs boson production LHC Higgs XS WG '10

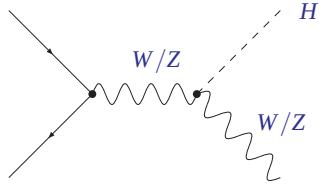




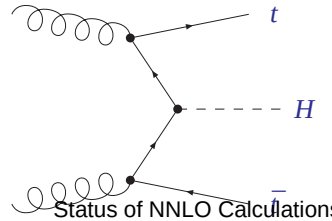
gg-fusion



weak boson fusion

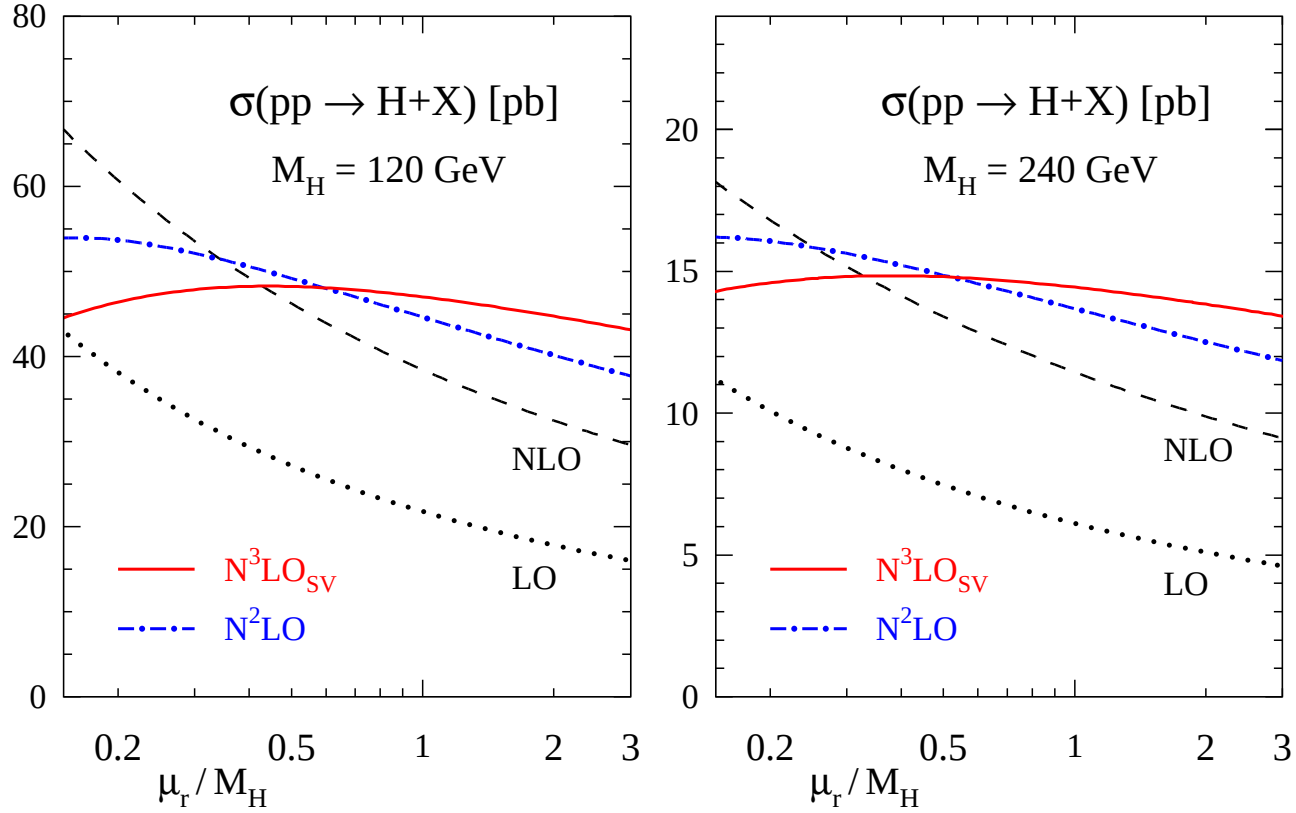


Higgs strahlung



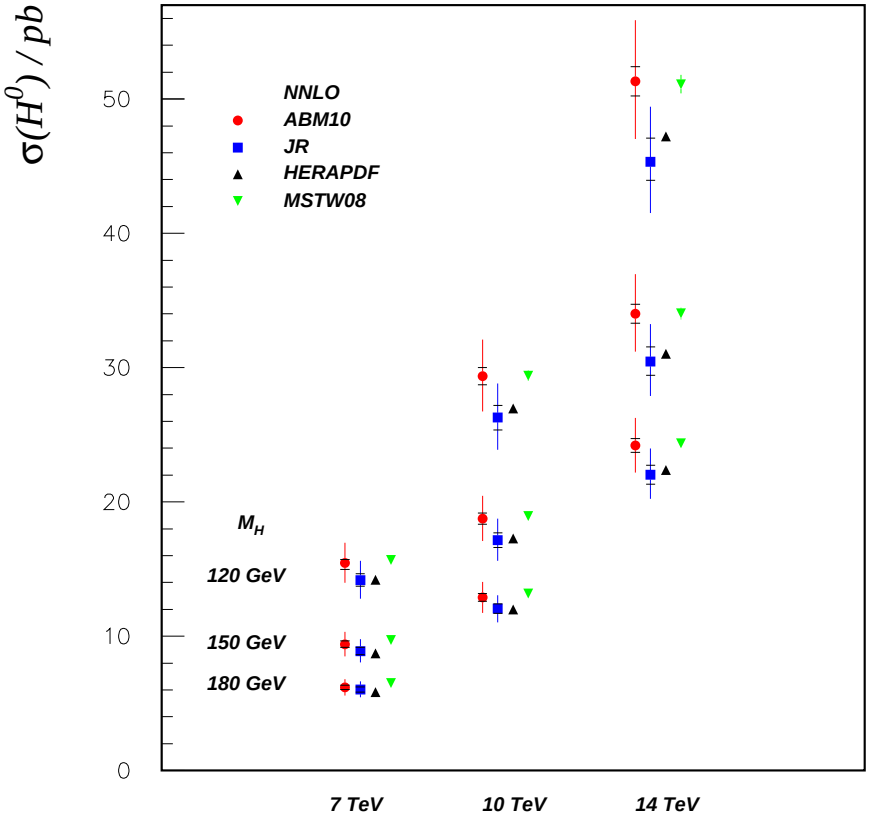
$t\bar{t}H$-channel

gg-fusion



- Apparent convergence of perturbative expansion
  - NNLO corrections still large
    Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran, Smith, van Neerven '03
  - improvement through complete soft N^3LO corrections S.M., Vogt '05
    or NNLL resummtion Catani, de Florian, Grazzini, Nason '03, Ahrens et al. '10
- Perturbative stability under renormalizaion scale variation

PDF dependence of gg-fusion cross section at LHC

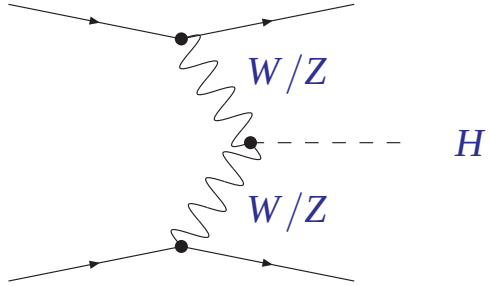


Alekhin, Blümlein, Jimenez-Delgado, S.M., E. Reya '10

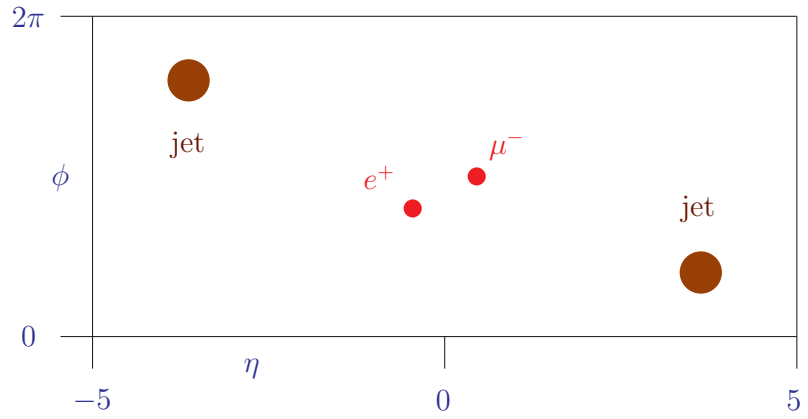
- PDF uncertainties (inner error bars) at 1σ
- Scale uncertainty (outer error bars) in range $M/2 \leq \mu_F = \mu_R \leq 2M$
- PDF uncertainty estimates by LHC Higgs XS WG too optimistic

Vector-boson fusion

- Second largest rate at LHC (WWH coupling)



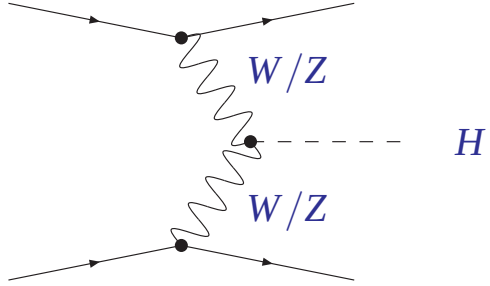
Signatures



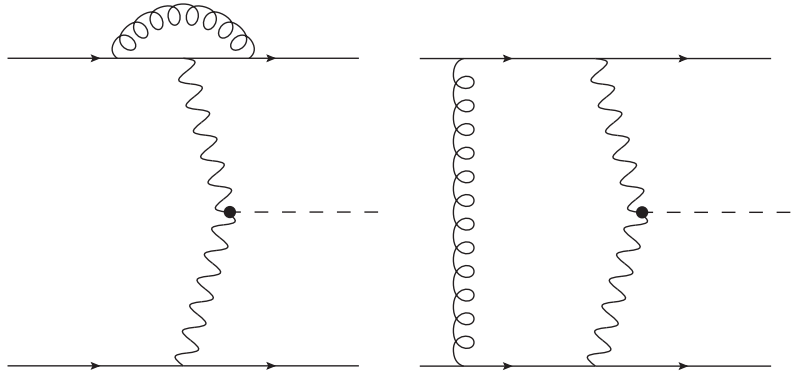
- WW, ZZ fusion $\longrightarrow$ Higgs is color singlet
  - two hard (forward) tagging jets (visible in detector)
  - no (or small) hadronic activity between tagging jets
  - color connection between forward jet and proton remnant
  - Higgs decay in the central rapidity region

Vector-boson fusion

- Second largest rate at LHC (WWH coupling)



- NLO QCD radiative corrections



Vector-boson fusion

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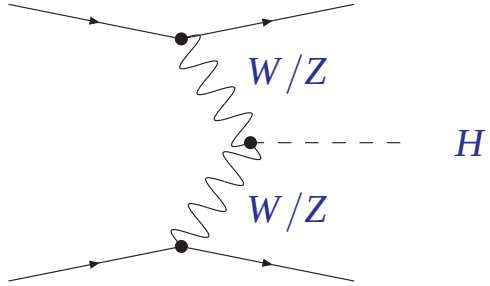
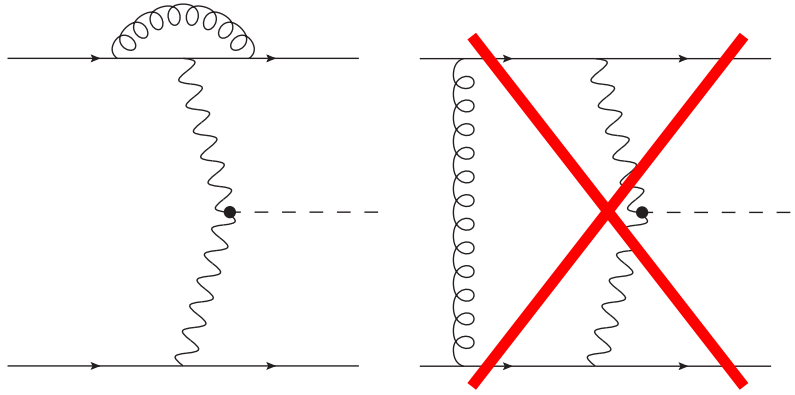
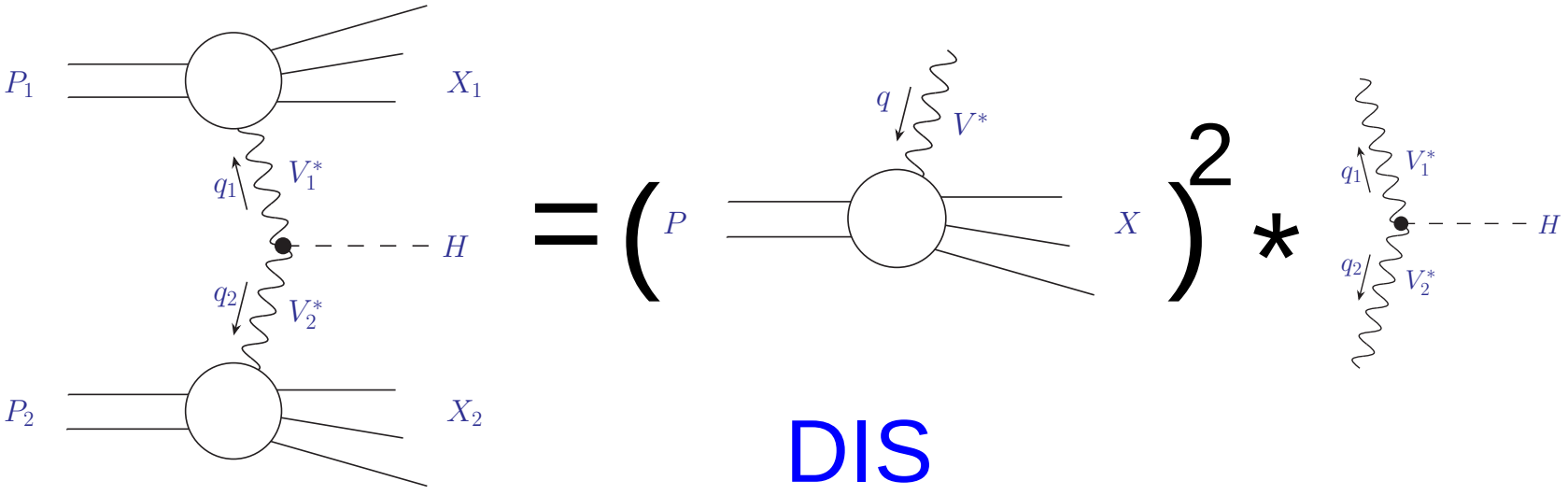


- NLO QCD corrections factorize
  (color conservation eliminates t-channel gluon in squared ME)



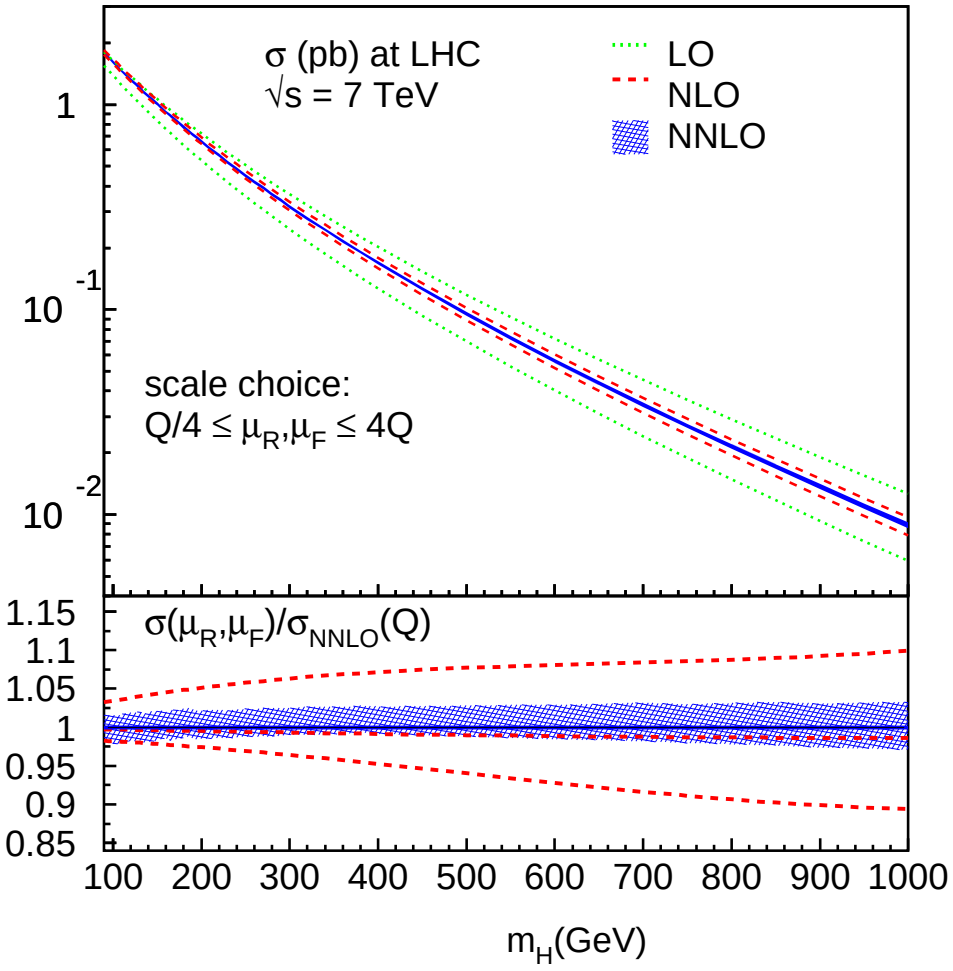
Exact factorization



DIS

- Deep-inelastic scattering building block of cross section with structure functions F_1, F_2 and F_3
- Exact factorization at NLO: so-called strucure function approach
  Han, Valencia, Willenbrock '92
- Structure function approach is NOT exact at NNLO in QCD
  - but can be still considered a good approximation, holds to $\mathcal{O}(1\%)$
  - NNLO QCD corrections to F_1, F_2 and F_3 long known
    Kazakov, Kotikov '88; Zijlstra, van Neerven '92; S.M., Vermaseren '99

Cross section for VBF at LHC

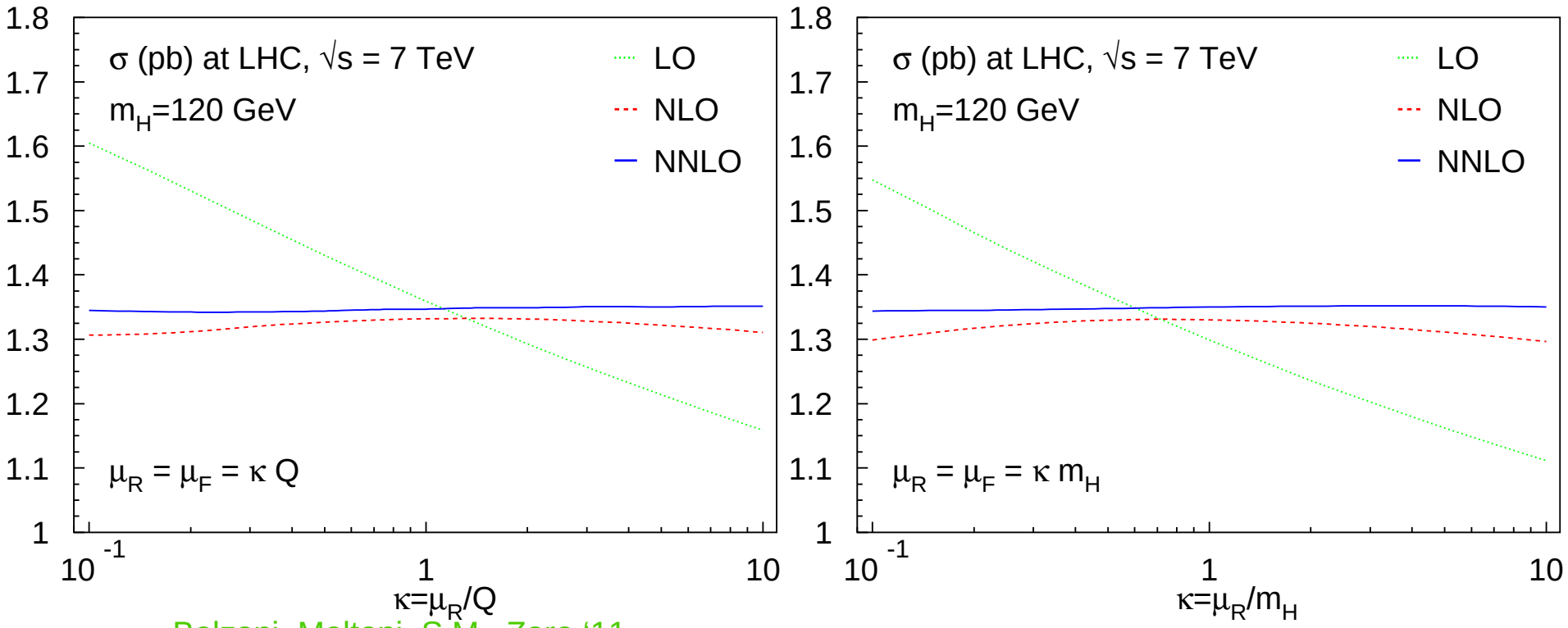


Bolzoni, Maltoni, S.M., Zaro '11

VBF at NNLO

- QCD corrections at second order small
  - apparent convergence
- NNLO results very stable at 2% against QCD scales variation (uniformly over the full mass range)
- Significant reduction of theoretical uncertainty

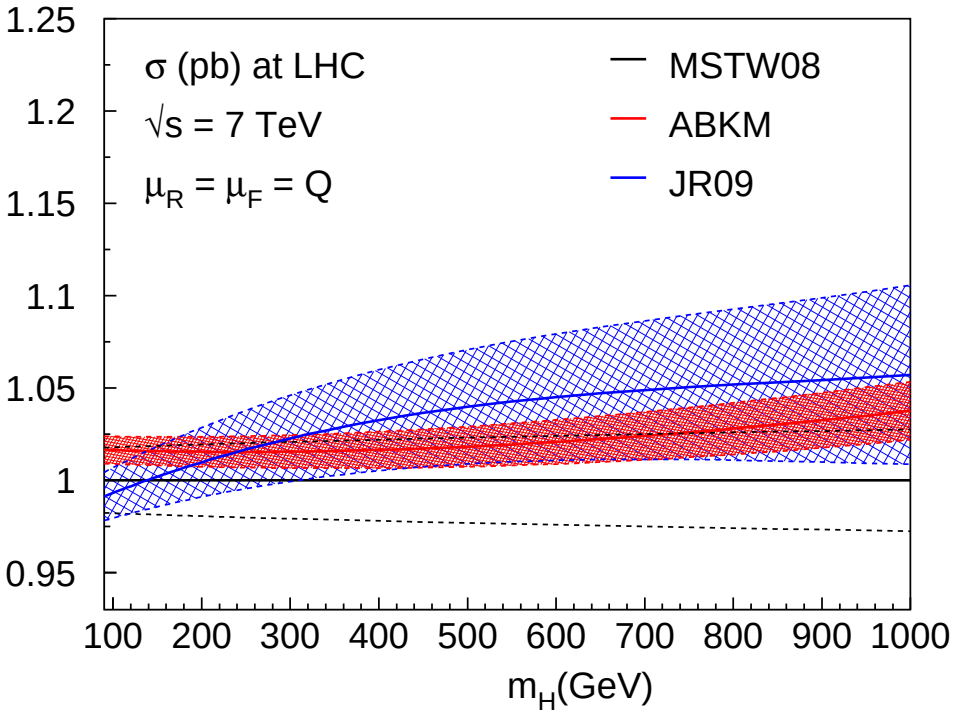
Scale stability at NNLO

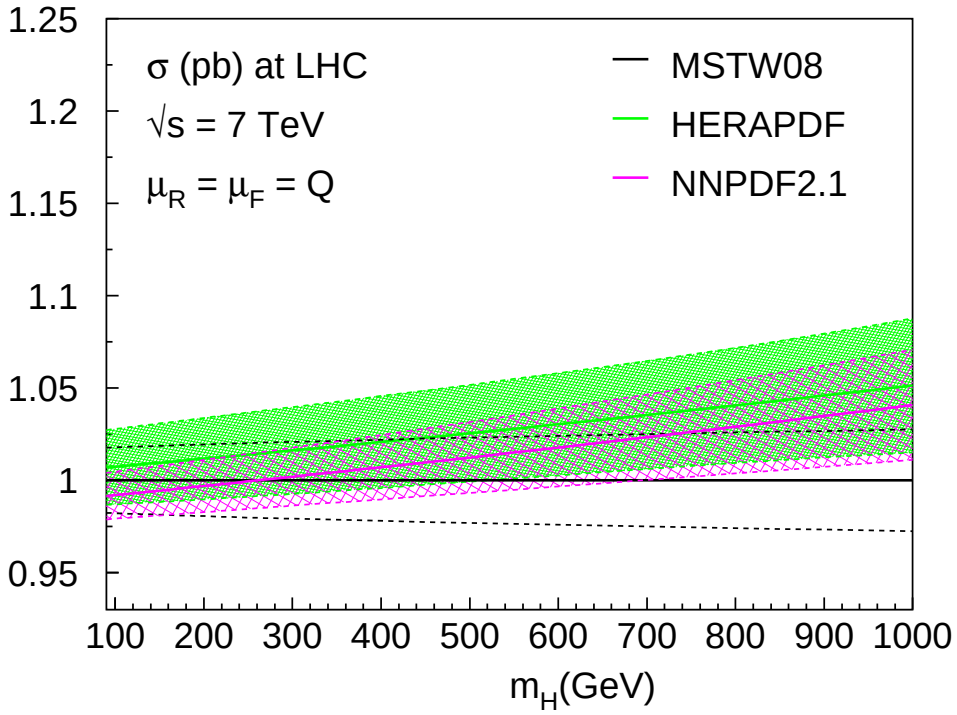


Bolzoni, Maltoni, S.M., Zaro '11

- VBF cross sections displays very good scale stability at NNLO over large range for $\mu_R = \mu_F$ preferred (minimal sensitivity)
- Scale choice $\mu_R = \mu_F \simeq Q$ preferred (minimal sensitivity)

PDF dependence of VBF cross section at LHC



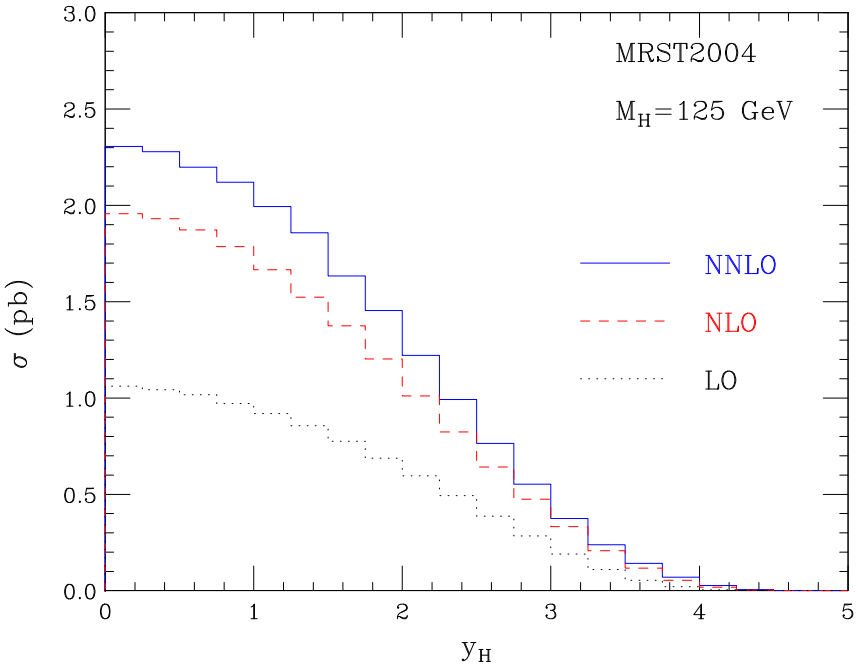


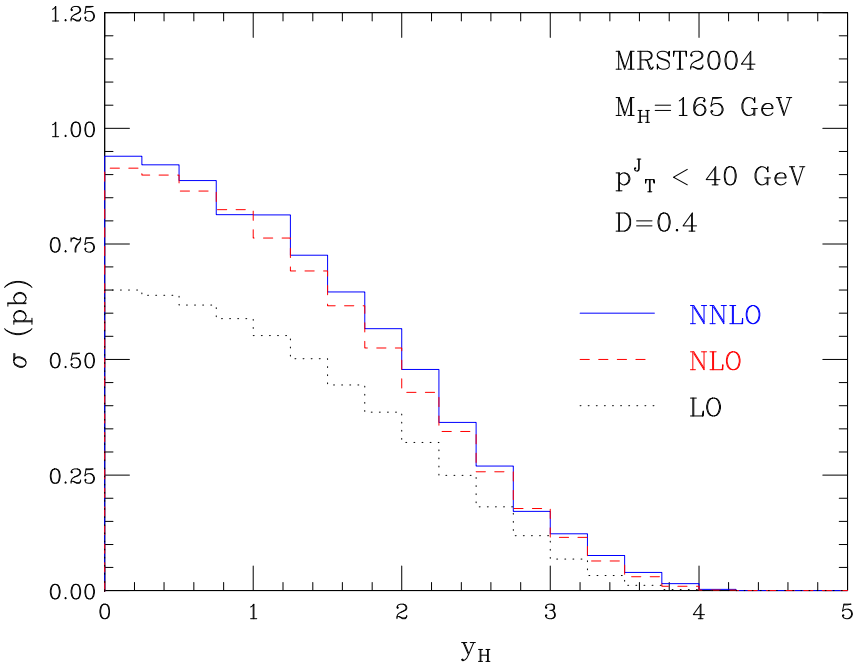
Bolzoni, Maltoni, S.M., Zaro '11

- PDF uncertainty
  - moderate for small Higgs masses $\mathcal{O}(\pm 2\%)$
  - increasingly larger for heavy Higgs bosons up to $\mathcal{O}(\pm 10\%)$

gg fusion (fully exclusive)

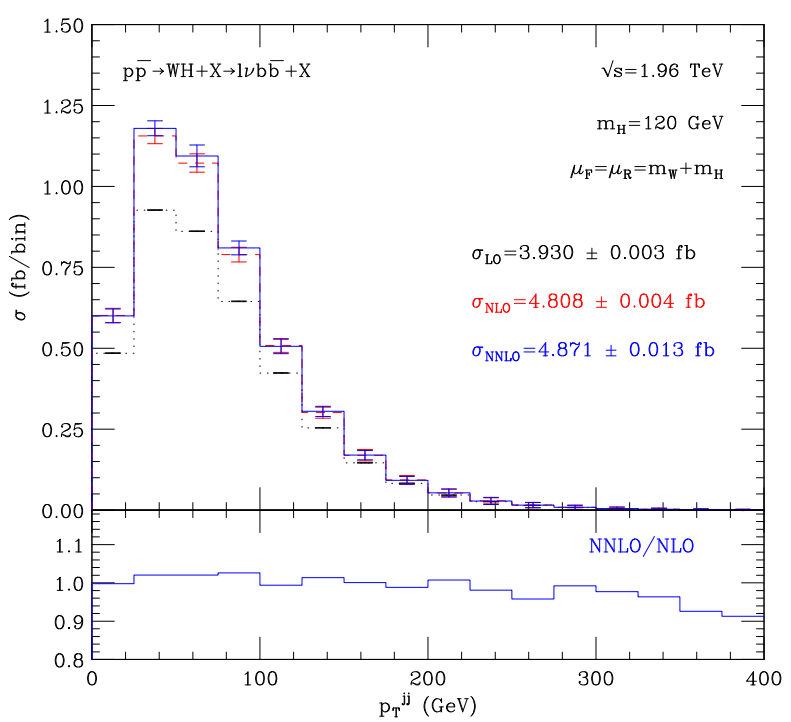
- Bin-integrated Higgs rapidity distribution including decay $H \to \gamma\gamma$
  - QCD corrections up to NNLO Anastasiou, Melnikov, Petriello '05
  - fast parton level Monte Carlo HNNLO Catani, Grazzini '07

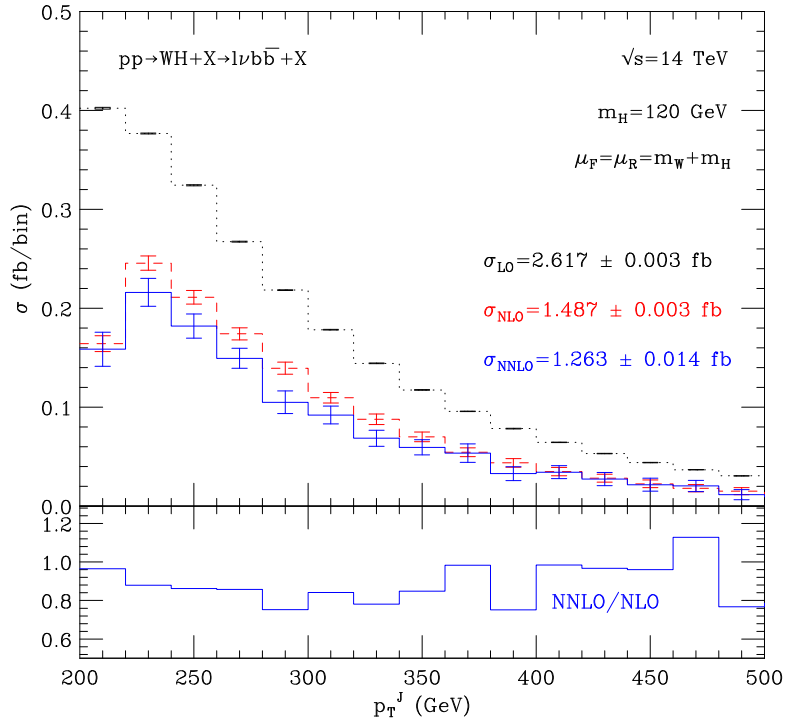




- Impact of kinematical cuts on higher order corrections (LHC $\sqrt{s} = 14$ TeV)
  - left: Higgs mass $M_h = 125$ GeV, no cuts on p_t of jets
  - right: Higgs mass $M_h = 165$ GeV and veto on jets with $p_t > 40$ GeV (k_t algorithm for jet reconstruction with jet size $D = 0.4$)

WH production (fully exclusive) Ferrara, Tramontana, Grazzini '11





- Scale dependence at the 1% level both at NLO and NNLO
- Tevatron (left): lepton $p_t > 20$ GeV, $|y| < 2$ and $p_t^{miss} > 20$ GeV
  - require two jets with $p_t > 20$ GeV, $|y| < 2$ (k_t alg. with $R = 0.4$)
- LHC $\sqrt{s} = 14$ TeV (right): lepton $p_t > 30$ GeV, $|y| < 2.5$ and $p_t^{miss} > 30$ GeV; require $p_t^W > 200$ GeV; (cone alg. with $R = 1.2$)
  - one fat jet with $p_t > 200$ GeV (and $b\bar{b}$-pair), $|y| < 2.5$; no other jet with $p_t > 20$ GeV and $|y| < 5$

Fully exclusive cross sections at NNLO

Higgs production

- Method relies on colorless final state $F = H, W^{\pm}, HW^{\pm}, \gamma\gamma, ZZ, \ldots$
  Catani, Grazzini '07

$$ij \to F + X\,, \qquad \text{where} \quad i,j = q, \bar{q}, g$$

  - ingredients:
    - two-loop amplitude for $ij \to F$
    - cross section for F+jets to NLO

- Alternative for colored final states: full fledged subtraction schemes
  - successfully applied to NNLO cross section for $e^+e^- \to 3$ jets
    Gehrmann, Gehrmann-De Ridder, Glover, Heinrich '07; Weinzierl '08
  - antenna subtraction scheme for DIS
    Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni '09
- NNLO cross sections $pp \to 2$ jets, $ep \to 1$ jet $+X$: still work in progress

Conclusions

Higgs cross section

- Theory predictions for hard parton cross section under good control
  - greatly reduced scale dependence at higher orders and convergence of perturbation theory
  - fully exclusive cross sections for gg-fusion and WH available

Deep-inelastic scattering

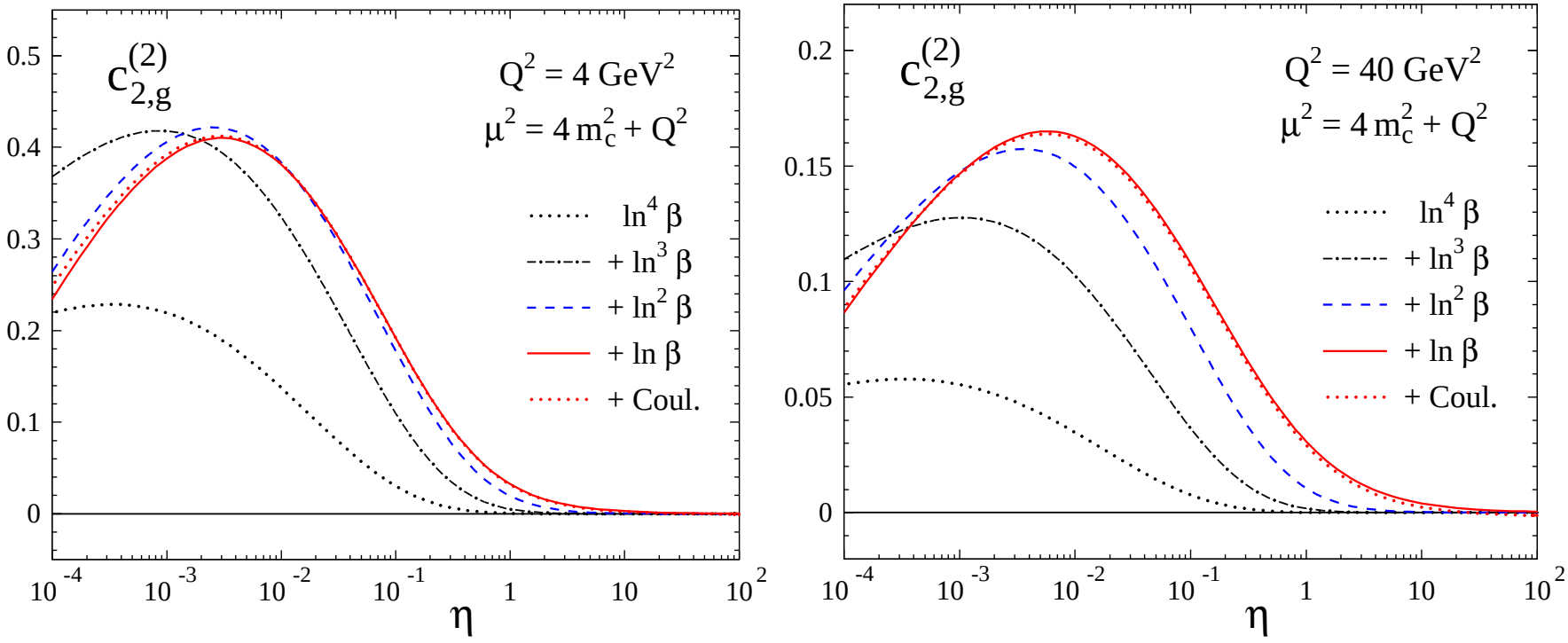
- Theory predictions for hard scattering under even better good control
  - testing ground for theory developments (thanks to OPE)
- Precision determinations of $\alpha_s(M_Z)$

Phenomenology

- DIS structure functions will remain backbone of PDF determinations
- Carry loop technology from DIS over to LHC
- ...

Extra Slides

Approximate coefficient functions at NNLO



- Distance from threshold $\eta = s/(4m^2) - 1$
  - Sudakov logarithms dominant for $\eta \ll 1$ now known to NNLL
    Lo Presti, Kawamura, S.M., Vogt '10
- Combine Sudakov logarithms with exact scale dependence at two loops $\longrightarrow$ NNLO$_{\mathrm{approx}}$

Convolution with gluon PDF

- Recall QCD factorization

$$x^{-1} F_2^c(x, Q^2, m^2) = \frac{\alpha_s \, e_q^2}{\pi^2} \sum_{i=q,\bar{q},g} \int_0^{\eta_{max}} d\eta \; f_i\left(z(\eta), \mu^2\right) \; c_{i,k}\left(\eta, \xi, \mu^2\right)$$

  - gluon PDF gives large weight to parton dynamics near threshold for $Q^2 \lesssim 10 - 30$ GeV^2