The Color Dipole Picture of low-x DIS

Dieter Schildknecht

Universität Bielefeld & Max Planck Institut für Physik, München

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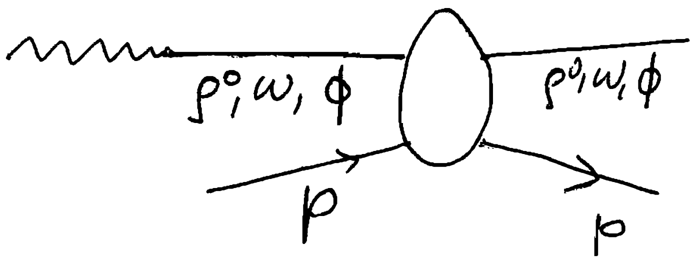
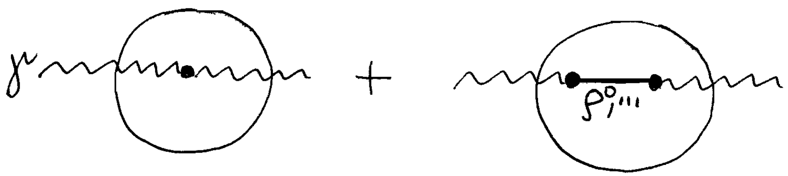
1. Introduction

1960's Vector Meson Dominance



J.J. Sakurai (1960, ...)

Shadowing in γA interactions



Leo Stodolsky (1967)

1969 DIS SLAC-MIT Collaboration

Bjorken scaling,
parton model

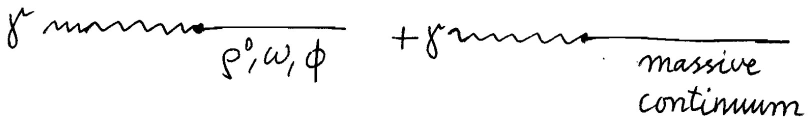
Volume 40B, number 1 PHYSICS LETTERS 12 June 1972

GENERALIZED VECTOR DOMINANCE
AND INELASTIC ELECTRON-PROTON SCATTERING *

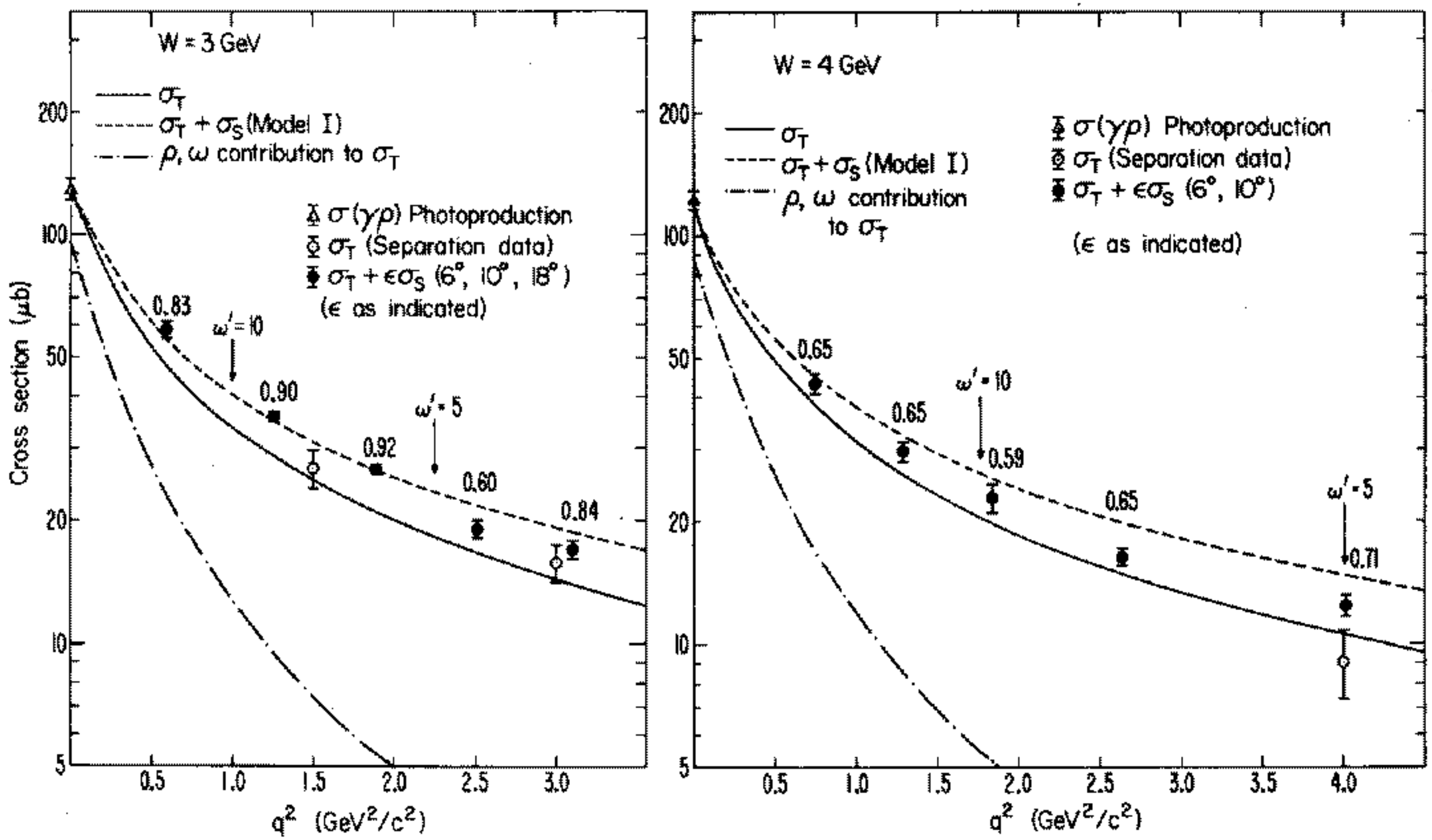
J. J. SAKURAI and D. SCHILDKNECHT **
Department of Physics, University of California,
Los Angeles, USA

Received 30 March 1972

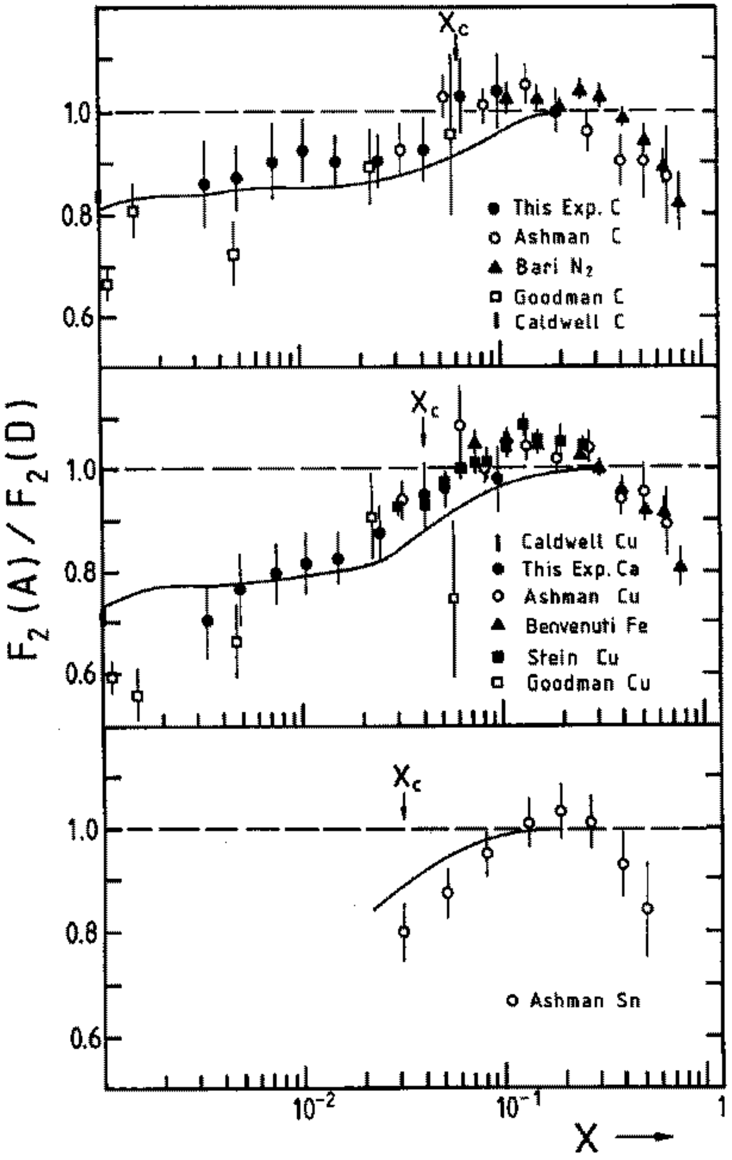
We propose a model of inelastic electron-proton scattering which takes into account the coupling of the photon to higher-mass vector states. Both the virtual photon-proton cross section σ_T (predicted with essentially no adjustable parameters) and the q^2 dependence of R are in exceedingly good agreement with the SLAC-MIT data in the diffraction region.



(1972)


W = 3 GeV
σ_T
$\sigma_T + \sigma_S$ (Model I)
ρ, ω contribution to σ_T
$\sigma(\gamma p)$ Photoproduction
σ_T (Separation data)
$\sigma_T + \epsilon\sigma_S$ (6°, 10°, 18°)
(ϵ as indicated)
0.83
$\omega' = 10$
0.90
0.92
$\omega' = 5$
0.60
0.84
Cross section (μb)
q^2 (GeV^2/c^2)
W = 4 GeV
σ_T
$\sigma_T + \sigma_S$ (Model I)
ρ, ω contribution to σ_T
$\sigma(\gamma p)$ Photoproduction
σ_T (Separation data)
$\sigma_T + \epsilon\sigma_S$ (6°, 10°)
(ϵ as indicated)
0.65
0.65
$\omega' = 10$
0.59
0.65
$\omega' = 5$
0.71
q^2 (GeV^2/c^2)

1989 Shadowing EMC Collaboration



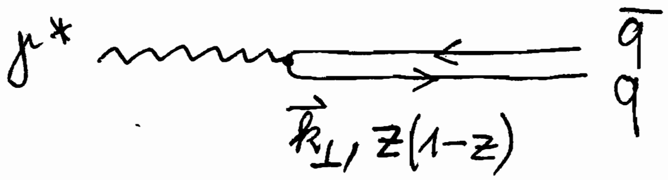
D. Schildknecht (1973)
C. Bilchak and D. Schildknecht (1989)

1994 HERA

DIS for $x_{bj} \ll 0.1$, High-mass diffractive production ("rap-gap" events) at HERA

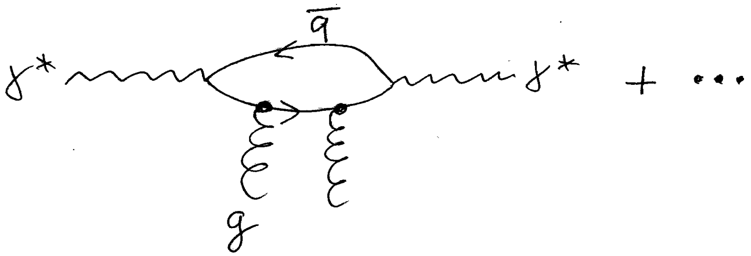
Modern picture of low-x DIS:

i) $\boldsymbol{q\bar{q}}$ internal structure



Nikolaev, Zakharov (1991)

ii) $\boldsymbol{q\bar{q}}$-dipole interaction

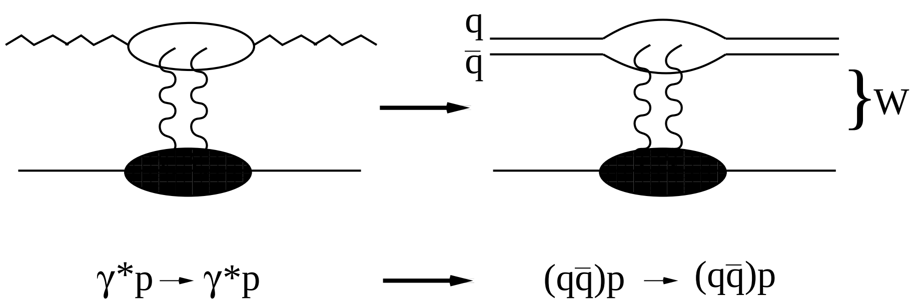


Low (1975)

Nussinov (1975)

2. The CDP: Model-independent Results.

The longitudinal and the transverse photoabsorption cross section



$$\text{A)} \qquad \sigma_{\gamma^*_{L,T}}(W^2, Q^2) = \int dz \int d^2\vec{r}_\perp |\psi_{L,T}(\vec{r}_\perp, z(1-z), Q^2)|^2 \;\; \sigma_{(q\bar{q})p}(\vec{r}_\perp, z(1-z), W^2)$$

Remarks: i) $|\psi_{L,T}(\vec{r}_\perp, z(1-z), Q^2)|$: Probability for $\gamma^*_{L,T} \to q\bar{q}$ fluctuation

ii) $\sigma_{(q\bar{q})p}(\vec{r}_\perp, z(1-z), W^2)$: $(q\bar{q})p$ cross section dependent on W^2 (not on $x \equiv \frac{Q^2}{W^2}$)

Lifetime of $q\bar{q}$ fluctuation:

$$\frac{1}{\Delta E} = \frac{2\nu}{Q^2+M^2_{q\bar{q}}} = \frac{1}{x+\frac{M^2_{q\bar{q}}}{W^2}}\frac{1}{M_p} \gg \frac{1}{M_p}, \qquad \begin{matrix} Q^2 \equiv -q^2 \geq 0 \\ x < 0.1 \end{matrix}$$

B) Gauge-invariant two-gluon coupling:

$$\begin{aligned}
\sigma_{(q\bar{q})p}(\vec{r}_\perp, z(1-z), W^2) &= \int d^2\vec{l}_\perp \tilde{\sigma}(\vec{l}_\perp^{\,2}, z(1-z), W^2)\left(1 - e^{-i\,\vec{l}_\perp\cdot\vec{r}_\perp}\right) \\
&\cong \frac{\pi}{4}\vec{r}_\perp^{\,2} \int d\vec{l}_\perp^{\,2}\vec{l}_\perp^{\,2}\tilde{\sigma}(\vec{l}_\perp^{\,2}, z(1-z), W^2).
\end{aligned}$$

Nikolaev, Zakharov (1991)

"color transparency" for

$$\vec{r}_\perp^{\,2}\vec{l}_\perp^{\,2} < \vec{r}_\perp^{\,2}\vec{l}_\perp^{\,2}{}_{Max}(W^2) < 1$$

$\sigma_{\gamma^*_{L,T}p}(W^2, Q^2)$ for large Q^2

$$|\psi_{L,T}(r_\perp, z(1-z), Q^2)|^2 \sim K^2_{0,1}(\underbrace{r_\perp\sqrt{z(1-z)}\sqrt{Q^2}}_{\equiv r'_\perp Q}) \sim \frac{1}{r'_\perp Q} e^{-2r'_\perp Q} \quad \text{for} \quad r'_\perp Q \gg 1.$$

Dominant contribution from $r'^2_\perp Q^2 < 1$.

For $l'^2_{\perp\ \mathrm{Max}}(W^2) < Q^2$,

$$r'^2_\perp l'^2_{\perp\ \mathrm{Max}}(W^2) = r^2_\perp l^2_{\perp\ \mathrm{Max}}(W^2) < 1. \qquad \left(\vec{l}'^2_\perp = \frac{l^2_\perp}{z(1-z)}\right)$$

Color transparency, $\sigma_{(q\bar{q})p} \sim r^2_\perp$.

Using $\int dy y^3 K_0^2(y) = \frac{1}{3}$,

$\int dy y^3 K_1^2(y) = \frac{2}{3}$,

$$\sigma_{\gamma^*_{L,T}}(W^2, Q^2) = \alpha \sum Q_q^2 \frac{1}{Q^2} \int dz \int d\vec{l}'^2_\perp \vec{l}'^2_\perp \tilde{\sigma}(\vec{l}'^2_\perp, z(1-z), W^2) \begin{cases} 1, \\ 2\rho_W. \end{cases}$$

Substitution rule: $\sigma_{\gamma_L^* p}(W^2, Q^2) \to \sigma_{\gamma_T^* p}(W^2, Q^2)$

via: $K_0^2(r'_\perp Q) \to K_1^2(r'_\perp Q)$,

$$\sigma_{(q\bar{q})p}(\vec{r}_\perp^{\,2}, \ldots) \to \sigma_{(q\bar{q})p}(\rho_W \vec{r}_\perp^{\,2}, \ldots)$$

Transverse-size enhancement (for $\rho_{\mathrm{W}} > 1$):

$$(\gamma_L^* \to q\bar{q}) \to (\gamma_T^* \to q\bar{q})$$

$$R(W^2, Q^2) \equiv \frac{\sigma_{\gamma_L^* p}(W^2, Q^2)}{\sigma_{\gamma_T^* p}(W^2, Q^2)} = \frac{1}{2\rho_W}$$

$(q\bar q)^{J=1}_{L,T}$ states : $\gamma^*_{L,T} \to (q\bar q)^{J=1}_{L,T}$

$$\sigma_{\gamma^*_{L,T}p}(W^2,Q^2) = \alpha \sum_q Q_q^2 \frac{1}{Q^2}\frac{1}{6} \begin{cases} \int d\vec l_\perp'^2 \vec l_\perp'^2 \bar\sigma_{(q\bar q)^{J=1}_L p}(\vec l_\perp'^2, W^2), \\ 2\int d\vec l_\perp'^2 \vec l_\perp'^2 \bar\sigma_{(q\bar q)^{J=1}_T p}(\vec l_\perp'^2, W^2). \end{cases}$$

$$\rho_W = \frac{\int d\vec l_\perp'^2 \vec l_\perp'^2 \bar\sigma_{(q\bar q)^{J=1}_T p}(\vec l_\perp'^2, W^2)}{\int d\vec l_\perp'^2 \vec l_\perp'^2 \bar\sigma_{(q\bar q)^{J=1}_L p}(\vec l_\perp'^2, W^2)}.$$

Numerical value of $\rho_W = \rho$:

$$\vec l_\perp^2 = z(1-z)\vec l_\perp'^2$$

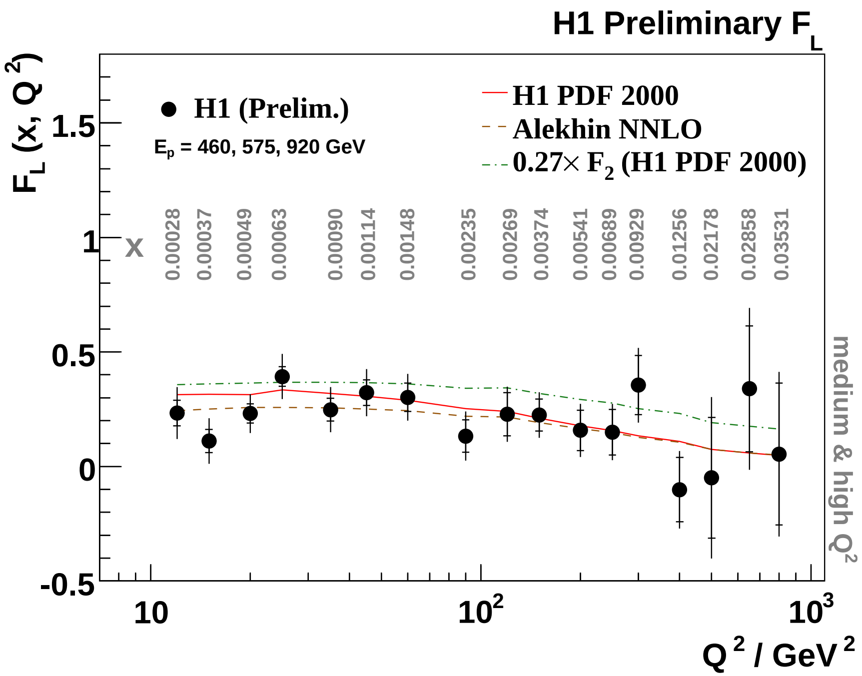
$$\langle \vec l_\perp^2 \rangle^{\vec l_\perp'^2 = const}_{L,T} = \vec l_\perp'^2 \begin{cases} 6\int dz z^2(1-z)^2 = \frac{4}{20}\vec l_\perp'^2, \\ \frac{3}{2}\int dz\ z(1-z)(1-2z(1-z)) = \frac{3}{20}\vec l_\perp'^2. \end{cases}$$

Uncertainty principle:

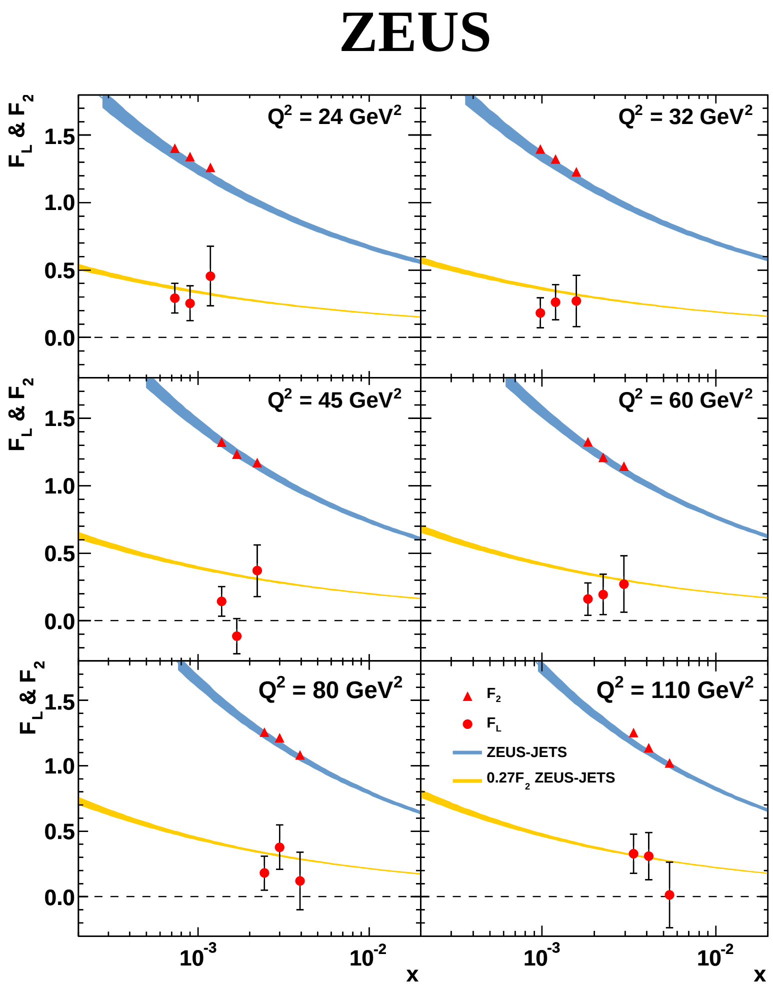
$$\rho_W = \frac{\langle r_\perp^2 \rangle_T}{\langle \vec r_\perp^2 \rangle_L} = \frac{\langle \vec l_\perp^2 \rangle_L}{\langle \vec l_\perp^2 \rangle_T} = \frac{4}{3} \equiv \rho.$$

$$R = \frac{1}{2\rho} = \begin{cases} 0.5 \text{ for } \rho = 1, & \text{ad hoc, helicity independence} \\ \frac{3}{8} = 0.375 \text{ for } \rho = \frac{4}{3}. \end{cases}$$

Kuroda, Schildknecht (2008)



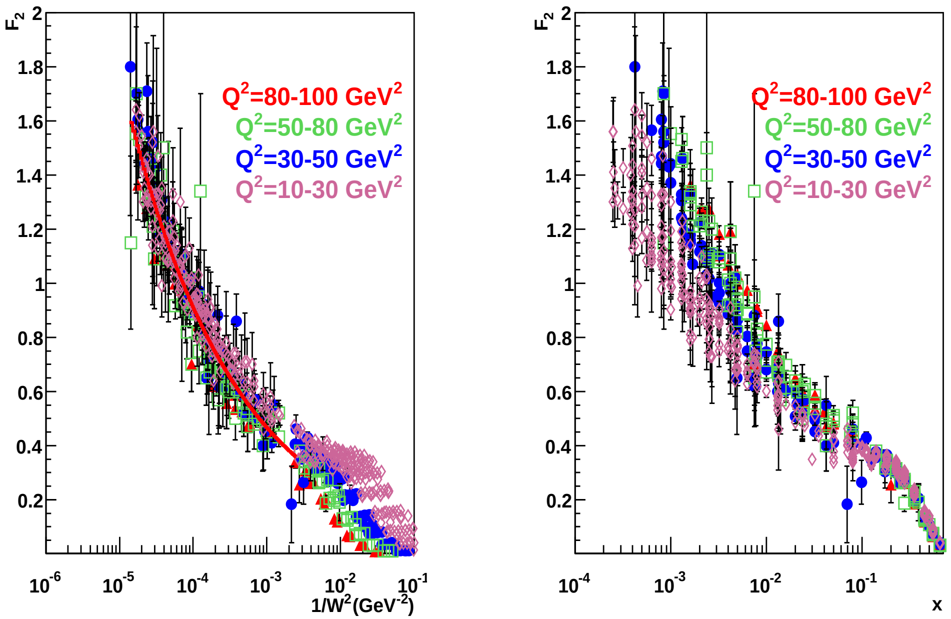
H1 Preliminary F_L
H1 (Prelim.)
E_p = 460, 575, 920 GeV
H1 PDF 2000
Alekhin NNLO
0.27× F_2 (H1 PDF 2000)
$F_L(x, Q^2)$
Q^2 / GeV^2
medium & high Q^2
x
0.00028 0.00037 0.00049 0.00063 0.00090 0.00114 0.00148 0.00235 0.00269 0.00374 0.00541 0.00689 0.00929 0.01256 0.02178 0.02858 0.03531



ZEUS
Q^2 = 24 GeV^2
Q^2 = 32 GeV^2
Q^2 = 45 GeV^2
Q^2 = 60 GeV^2
Q^2 = 80 GeV^2
Q^2 = 110 GeV^2
F_L & F_2
F_2
F_L
ZEUS-JETS
0.27F_2 ZEUS-JETS
x

The W-dependence

$$F_2(x,Q^2) \;\cong\; \frac{Q^2}{4\pi^2\alpha}\Big(\sigma_{\gamma_L^* p}(W^2,Q^2)+\sigma_{\gamma_T^* p}(W^2,Q^2)\Big)$$
$$= \frac{\sum_q Q_q^2}{4\pi^2}\int dz \int d\vec{l}_\perp^{\,2}\vec{l}_\perp^{\,2}\tilde{\sigma}(\vec{l}_\perp^{\,2}, z(1-z), W^2)(1+2\rho).$$

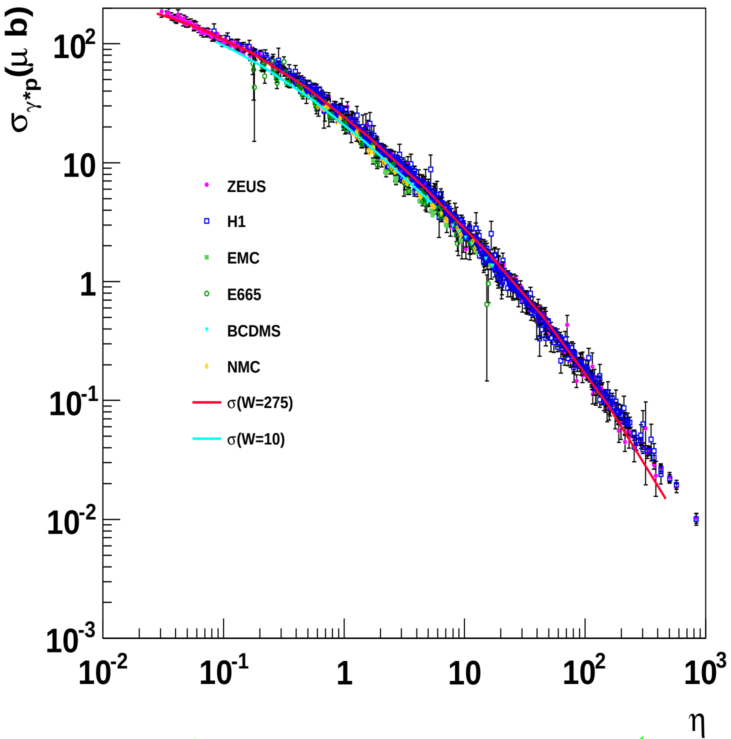


Prabhdeep Kaur (2010)

Low-x Scaling

$$\text{Empirically :} \qquad \eta \equiv \frac{Q^2 + m_0^2}{\Lambda_{sat}^2(W^2)},$$

$$\Lambda_{sat}^2(W^2) \sim (W^2)^{C_2}$$



Schildknecht, Surrow, Tentyukov (2000)

$$\begin{aligned} \sigma_{\gamma^* p}(W^2, Q^2) &= \sigma_{\gamma^* p}(\eta(W^2, Q^2)) \\ &\sim \sigma^{(\infty)} \begin{cases} ln \frac{1}{\eta(W^2,Q^2)} & , \quad \text{for } \eta(W^2, Q^2) \ll 1 \\ \frac{1}{\eta(W^2,Q^2)} & , \quad \text{for } \eta(W^2, Q^2) \gg 1 \end{cases} \end{aligned}$$

Low-x scaling: Direct consequence of CDP

$$\sigma_{(q\bar{q})^{J=1}_{L,T}p}(\vec{r}_\perp{}', W^2) = \int d^2\vec{l}_\perp{}' \bar{\sigma}_{(q\bar{q})^{J=1}_{L,T}p}(\vec{l}_\perp{}'^2, W^2)(1 - e^{-i\vec{l}_\perp{}'\cdot\vec{r}_\perp{}'})$$

$$= \pi \int d\vec{l}_\perp{}'^2 \bar{\sigma}_{(q\bar{q})^{J=1}_{L,T}p}(\vec{l}_\perp{}'^2, W^2) \cdot \underbrace{\left(1 - \frac{\int d\vec{l}_\perp{}'^2 \bar{\sigma}_{(q\bar{q})^{J=1}_{L,T}p}(\vec{l}_\perp{}'^2, W^2) J_0(l'_\perp r'_\perp)}{\int d\vec{l}_\perp{}'^2 \bar{\sigma}_{(q\bar{q})^{J=1}_{L,T}p}(\vec{l}_\perp{}'^2, W^2)}\right)}$$

i) “1 - 1” destructive interference

color transparency

$$\vec{r}_\perp{}'^2 < \frac{1}{\vec{l}_\perp{}'_{Max}(W^2)}$$

$$J_0(l'_\perp r'_\perp) \cong 1 - \frac{1}{4}(l'_\perp r'_\perp)^2 + \cdots$$

ii) “1-0=1” hadronlike “saturation”

$$\frac{1}{l'^2_{\perp\, Max}(W^2)} < r'^2_\perp$$

$$\sigma_{(q\bar{q})^{J=1}_{L,T}p}(r_\perp{}'^2, W^2) \cong \pi \int d\vec{l}_\perp{}'^2 \bar{\sigma}_{(q\bar{q})^{J=1}_{L,T}p}(\vec{l}_\perp{}'^2, W^2)$$
$$\equiv \sigma^{(\infty)}_{L,T}(W^2)$$

Note: a) $r'^2_\perp$ fixed, $W^2 \longrightarrow \infty$,

b) $r'^2_\perp \longrightarrow \infty$, W^2 fixed

$$\sigma_{\gamma^* p}(W^2, Q^2) \sim \begin{cases} \sigma^{(\infty)} \frac{\Lambda^2_{sat}(W^2)}{Q^2} \sim \frac{\sigma^{(\infty)}}{\eta(W^2,Q^2)} & , \quad \eta(W^2, Q^2) \gg 1 \qquad (i) \\ \sigma^{(\infty)} ln \frac{1}{\eta(W^2,Q^2)} & , \quad \eta(W^2, Q^2) \ll 1 \qquad (ii) \end{cases}$$

Direct consequence of CDP, NOT dependent on a specific parameterization dipole cross section.

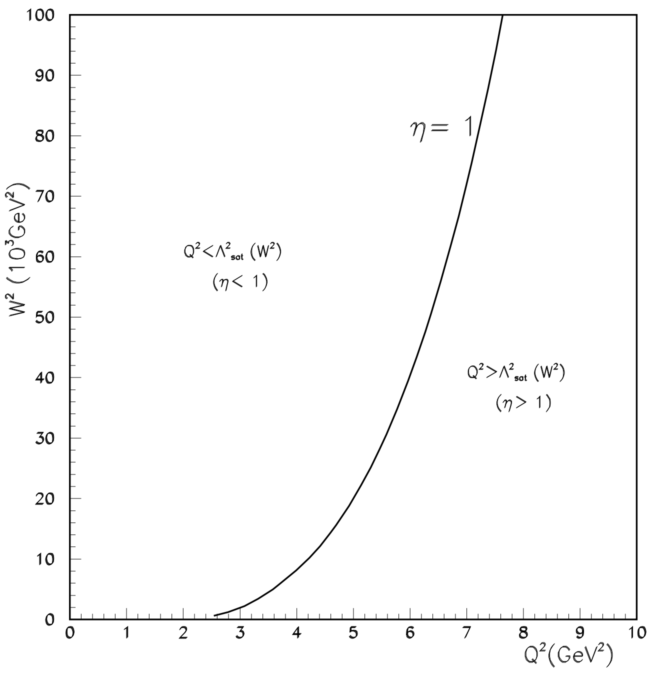
$$\begin{aligned} \Lambda^2_{sat}(W^2) &\equiv \frac{\int d\vec{l}_\perp^{\prime 2} \vec{l}_\perp^{\prime 2} \bar{\sigma}_{(q\bar{q})^{J=1}_L p}(\vec{l}_\perp^{\prime 2}, W^2)}{\int d\vec{l}_\perp^{\prime 2} \bar{\sigma}_{(q\bar{q})^{J=1}_L p}(\vec{l}_\perp^{\prime 2}, W^2)} \\ &= \frac{1}{\sigma_L^{(\infty)}(W^2)} \pi \cdot \int d\vec{l}_\perp^{\prime 2} \vec{l}_\perp^{\prime 2} \bar{\sigma}_{(q\bar{q})^{J=1}_L p}(\vec{l}_\perp^{\prime 2}, W^2) \end{aligned}$$

The limit of $\eta(W^2, Q^2) \to 0$, or $W^2 \to \infty$ at Q^2 fixed

$$\lim_{\substack{W^2\to\infty \\ Q^2 \ \text{fixed}}} \frac{\sigma_{\gamma^* p}(\eta(W^2,Q^2))}{\sigma_{\gamma^* p}(\eta(W^2,Q^2=0))} = \lim_{\substack{W^2\to\infty \\ Q^2 \ \text{fixed}}} \frac{\ln\left(\frac{\Lambda^2_{sat}(W^2)}{m_0^2}\frac{m_0^2}{(Q^2+m_0^2)}\right)}{\ln\frac{\Lambda^2_{sat}(W^2)}{m_0^2}} = 1 + \lim_{\substack{W^2\to\infty \\ Q^2 \ \text{fixed}}} \frac{\ln\frac{m_0^2}{Q^2+m_0^2}}{\ln\frac{\Lambda^2_{sat}(W^2)}{m_0^2}} = 1.$$

$$\sigma_{\gamma^* p}\left(\eta(W^2,Q^2=0)\right) = \sigma_{\gamma p}(W^2)$$

D. Schildknecht, DIS 2001 (Bologna)



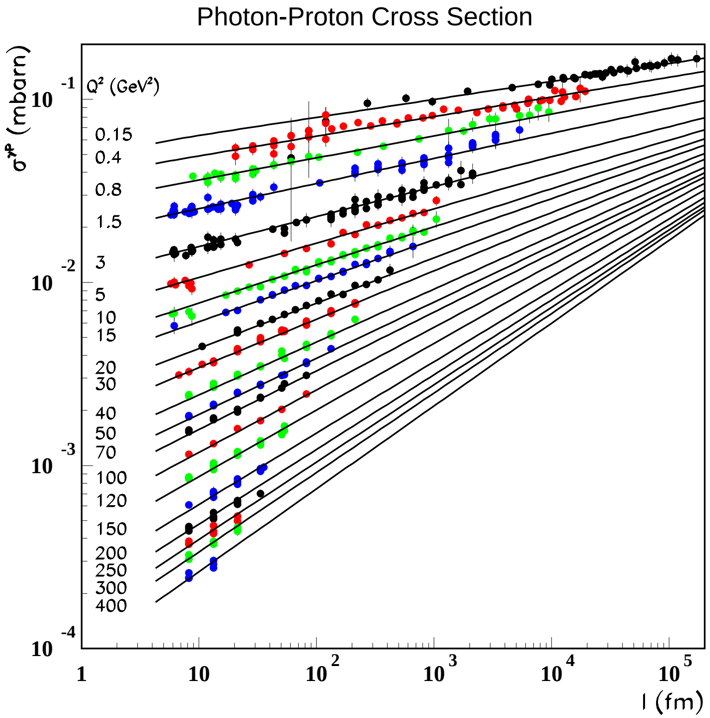
$Q^2[GeV^2]$	$W^2[GeV^2]$	$\frac{\sigma_{\gamma^* p}(\eta(W^2,Q^2))}{\sigma_{\gamma p}(W^2)}$
1.5	2.5×10^7	0.5
	1.26×10^{11}	0.63

Observation by Caldwell

$$\sigma_{\gamma^* p}(W^2, Q^2) = \sigma_0(Q^2) \left(\frac{1}{2Mp} \frac{W^2}{Q^2} \right)^{\lambda_{eff}(Q^2)}$$ A. Caldwell (2008)

Q^2-independent limit at approximately

$$W^2 \simeq 10^9 Q^2.$$



Summarizing this Section on model-independent results:

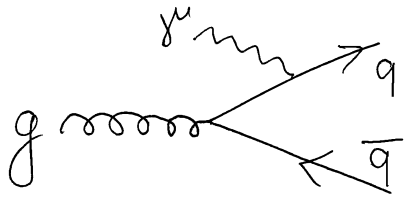
$$R = \frac{1}{2\rho} = \frac{3}{8};$$

$$\textbf{Low} - \textbf{x scaling}: \qquad \sigma_{\gamma^* p} \sim \sigma^{(\infty)} \begin{cases} \frac{1}{\eta(W^2,Q^2)} & , \quad \eta(W^2,Q^2) > 1, \\ ln \frac{1}{\eta(W^2,Q^2)} & , \quad \eta(W^2,Q^2) < 1. \end{cases}$$

$W^2 \to \infty,$

Q^2 **fixed :** $Q^2 -$ **independent limit coinciding with** $Q^2 = 0$ **photoproduction,** $\sigma_{\gamma \mathrm{p}}(\mathrm{W}^2)$**.**

3. The CDP, the Gluon Distribution Function and Evolution.



CDP $\leftrightarrow$ Photon-Gluon Fusion of pQCD

$$F_L(x,Q^2) = \frac{\alpha_s(Q^2)}{3\pi}\sum_q Q_q^2 \cdot 6 I_g(x,Q^2),$$

where $I_g(x,Q^2) \equiv \int_x^1 \frac{dy}{y}\left(\frac{x}{y}\right)^2\left(1-\frac{x}{y}\right) y g(y,Q^2)$.

$$F_L(\xi_L x, Q^2) = \frac{\alpha_s(Q^2)}{3\pi}\sum_q Q_q^2 G(x,Q^2).$$

Cooper-Sarkar et al. (1988)

$$F_2(x,Q^2) = \frac{5}{18} x \sum(x,Q^2).$$

$$\frac{\partial F_2(\xi_2 x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{3\pi}\sum_q Q_q^2 G(x,Q^2).$$

rescaling factors:

$(\xi_L,\xi_2) \simeq (0.40, 0.50)$

$(\xi_L,\xi_2) = (0.45, 0.40)$ for specific gluon distribution.

Accuracy $\lesssim 0.5$ %.

Prytz (1993)

Using $F_L(x,Q^2) = \frac{1}{2\rho+1} F_2(x,Q^2)$:

$$(2\rho+1)\frac{\partial}{\partial \ln Q^2} F_2\left(\frac{\xi_2}{\xi_L}x, Q^2\right) = F_2(x,Q^2)$$

i) CDP: $F_2(x,Q^2) = F_2(W^2)$:

$$(2\rho_W+1)\frac{\partial}{\partial \ln W^2} F_2\left(\frac{\xi_L}{\xi_2}W^2\right) = F_2(W^2)$$

ii) Power law

$$F_2(W^2) \sim (W^2)^{C_2} = \left(\frac{Q^2}{x}\right)^{C_2}$$

Compare: "hard Pomeron" solution of DGLAP evolution: $\left(\frac{1}{x}\right)^{\lambda = \text{ fixed}}$.

"hard Pomeron" Regge: $\left(\frac{1}{x}\right)^{\epsilon_0 \simeq 0.43}$

$$(2\rho_W+1)C_2\left(\frac{\xi_L}{\xi_2}\right)^{C_2} = 1$$

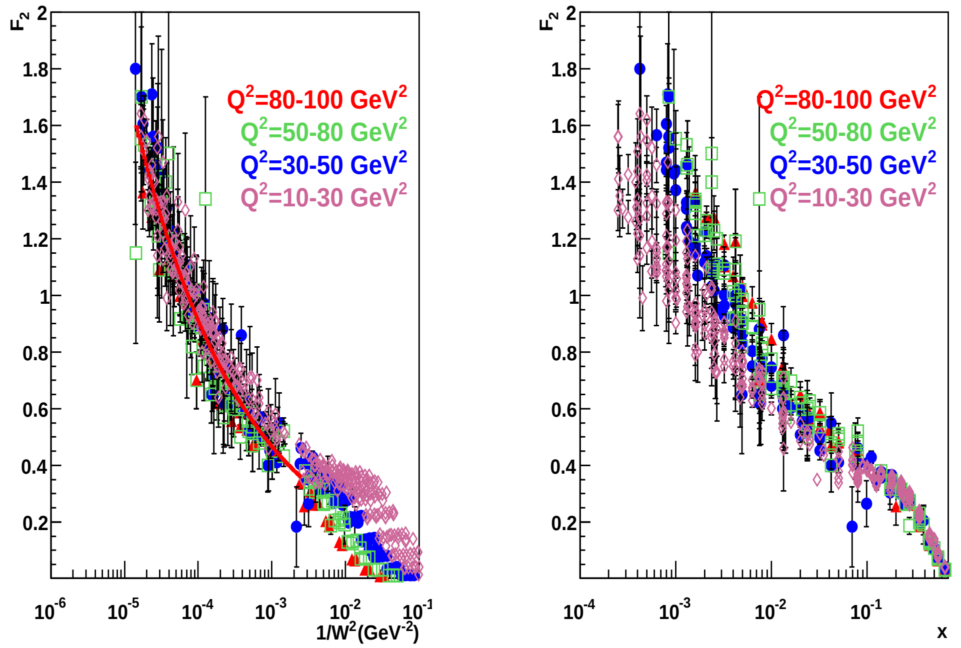
with $\rho = \frac{4}{3}$,

$$C_2 = \frac{1}{2\rho+1}\left(\frac{\xi_2}{\xi_L}\right)^{C_2} = 0.29$$

Kuroda, Schildknecht (2011)

$$F_2(W^2) = f_2 \cdot \left(\frac{W^2}{1\mathrm{GeV}^2}\right)^{C_2=0.29}$$

$$f_2 = 0.063 \quad \text{(fitted parameter)}$$



Experimental evidence for $F_2(x, Q^2) = F_2(W^2 \cong Q^2/x)$

and for the prediction of $C_2 = 0.29$.

The Gluon Distribution Function

$$
\begin{aligned}
\alpha_s(Q^2)G(x,Q^2) &= \frac{3\pi}{\sum_q Q_q^2} F_L(\xi_L x, Q^2) \\
&= \frac{3\pi}{\sum_q Q_q^2} \frac{1}{(2\rho+1)} F_2(\xi_L x, Q^2) \\
&= \frac{3\pi}{\sum_q Q_q^2 (2\rho+1)} \frac{f_2}{\xi_L^{C_2=0.29}} \left(\frac{W^2}{1\mathrm{GeV}^2}\right)^{C_2=0.29}
\end{aligned}
$$

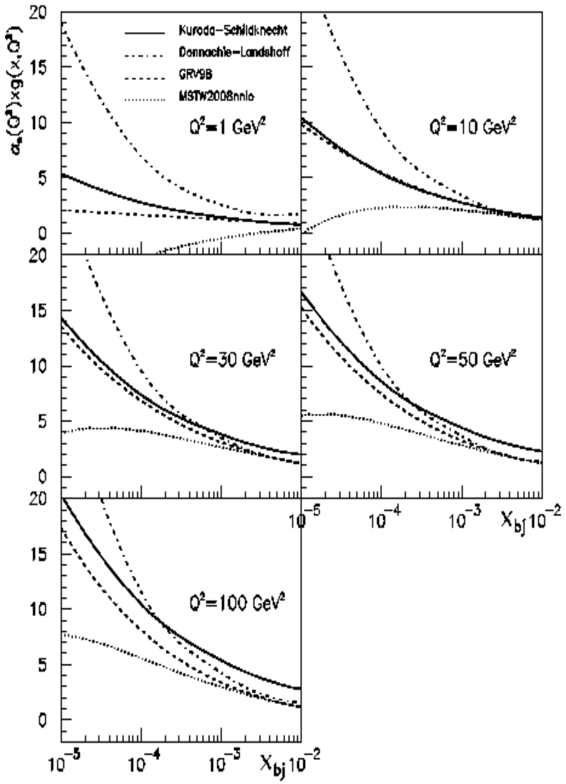
Comments:

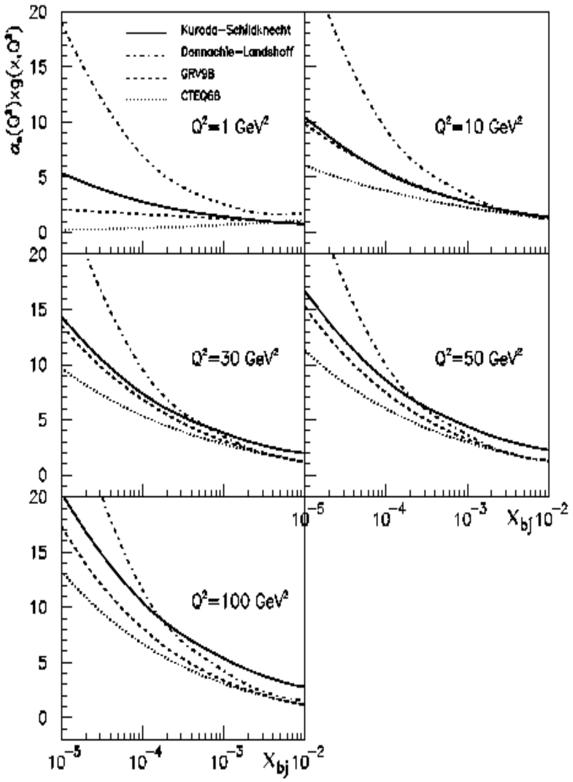
CDP: $F_{L,2} = F_{L,2}\left(W^2 = \frac{Q^2}{x}\right)$,

$\rho =$ const. $= \frac{4}{3}$,

$C_2 = 0.29$ from evolution

$f_2 = 0.063$ fit parameter





Comparison with gluon distributions from Durham data file using $\alpha_s(Q^2) = \alpha_s(Q^2)^{NLO}$

4. Models for the Dipole Cross Section

Cvetic, Schildknecht, Surrow, Tentyukov (2001)

Model-independently:

$$\sigma_{\gamma^* p} \sim \begin{cases} ln\frac{1}{\eta(W^2,Q^2)} & , \quad \eta(W^2,Q^2) \ll 1 \\ \frac{1}{\eta(W^2,Q^2)} & , \quad \eta(W^2,Q^2) \gg 1 \end{cases}$$

Detailed ansatz for dipole cross section: Interpolation between $\eta(W^2,Q^2) < 1$ and $\eta(W^2,Q^2) > 1$.

Simple ansatz with $\rho = 1$, $\left(R = \frac{1}{2\rho} = \frac{1}{2}\right)$:

$$\sigma_{(q\bar{q})p}(\vec{r}_\perp, z(1-z), W^2) = \sigma^{(\infty)}(W^2)\left(1 - J_0\left(r_\perp\sqrt{z(1-z)}\Lambda_{sat}(W^2)\right)\right)$$

$$\begin{aligned} \sigma_{\gamma^* p}(W^2,Q^2) &= \sigma_{\gamma^* p}(\eta(W^2,Q^2)) + O\left(\frac{m_0^2}{\Lambda^2_{\rm sat}(W^2)}\right) = \\ &= \frac{\alpha R_{e^+e^-}}{3\pi}\sigma^{(\infty)}(W^2)I_0(\eta) + O\left(\frac{m_0^2}{\Lambda^2_{\rm sat}(W^2)}\right), \qquad R_{e^+e^-} = 3\sum_q Q_q^2. \end{aligned}$$

$$\begin{aligned} I_0(\eta(W^2,Q^2)) &= \frac{1}{\sqrt{1+4\eta(W^2,Q^2)}} \ln \frac{\sqrt{1+4\eta(W^2,Q^2)}+1}{\sqrt{1+4\eta(W^2,Q^2)}-1} \cong \\ &\cong \begin{cases} \ln\frac{1}{\eta(W^2,Q^2)} + O(\eta \ln \eta), & \text{for } \eta(W^2,Q^2) \to \frac{m_0^2}{\Lambda^2_{\rm sat}(W^2)}, \\ \frac{1}{2\eta(W^2,Q^2)} + O\left(\frac{1}{\eta^2}\right), & \text{for } \eta(W^2,Q^2) \to \infty, \end{cases} \end{aligned}$$

Generalization to $\rho = \frac{4}{3}$.

Constraint: $m_0^2 \leq M_{q\bar{q}}^2, M_{q\bar{q}}'^2 \leq m_1^2(W^2)$; Kuroda, Schildknecht (2011)

$$\sigma_{\gamma^* p} = \sigma_{\gamma^* p}\left(\eta(W^2, Q^2), \frac{m_0^2}{\Lambda_{sat}^2(W^2)}, \xi \equiv \frac{m_1^2(W^2)}{\Lambda_{sat}^2(W^2)}\right),$$

$$\eta(W^2, Q^2) = \frac{Q^2 + m_0^2}{\Lambda_{sat}^2(W^2)},$$

$$\Lambda_{sat}^2(W^2) = C_1\left(\frac{W^2}{W_0^2} + 1\right)^{C_2} \cong \text{ const } \left(\frac{W^2}{1GeV^2}\right)^{C_2}$$

$$C_1 = 1.95 GeV^2$$

$$W_0^2 = 1081 GeV^2$$

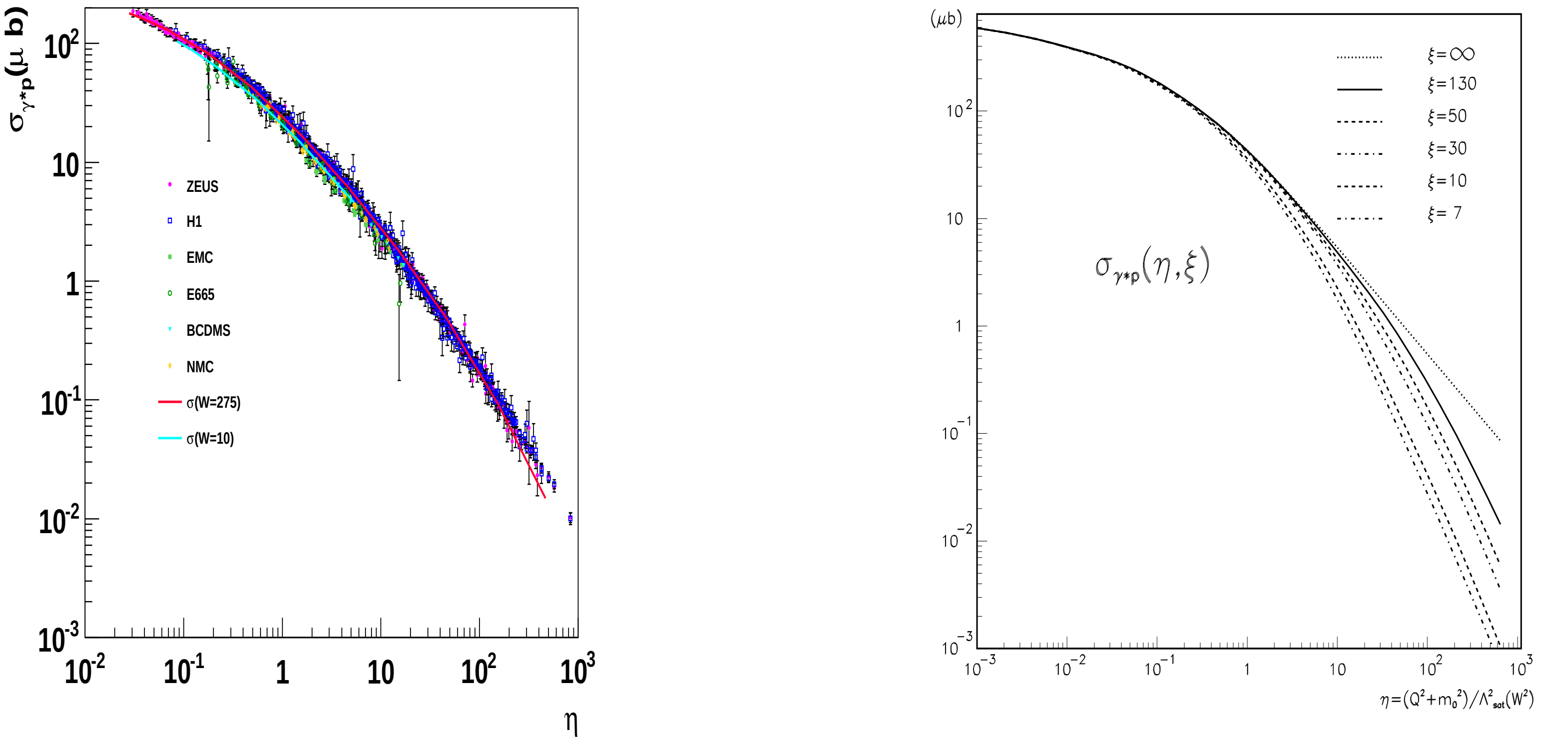
$$C_2 = 0.27(0.29)$$

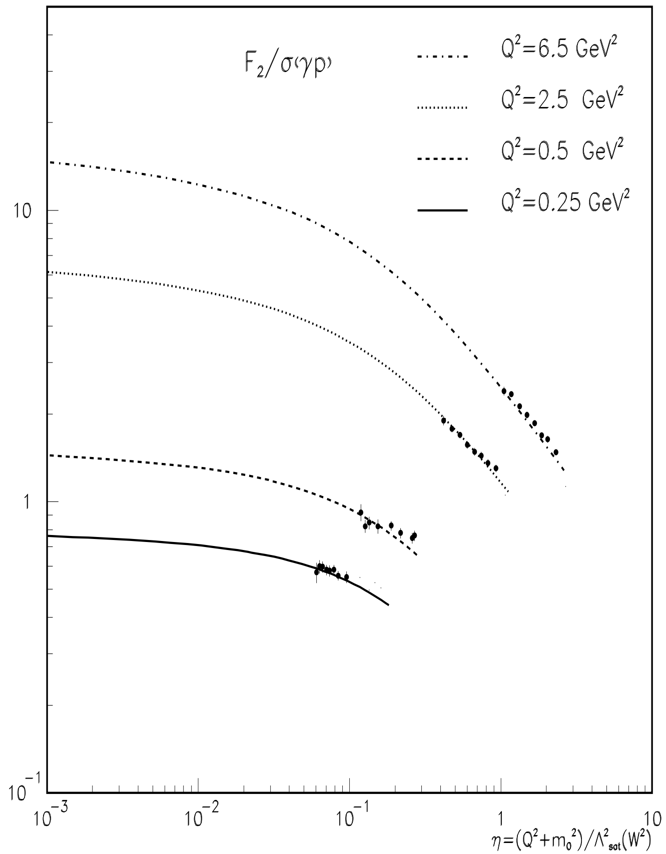
$$m_0^2 = 0.15 GeV^2$$

$$m_1^2(W^2) = \xi \Lambda_{sat}^2(W^2) = 130 \Lambda_{sat}^2(W^2)$$

Normalization by $Q^2 = 0$ photoproduction (Regge fit):

$$\sigma^{(\infty)}(W^2) \cong \begin{cases} 30mb, & (\text{for 3 active flavors}, \quad R_{e^+e^-} = 2) \\ 18mb, & (\text{for 4 active flavors}, \quad R_{e^+e^-} = \frac{10}{3}) \end{cases}$$

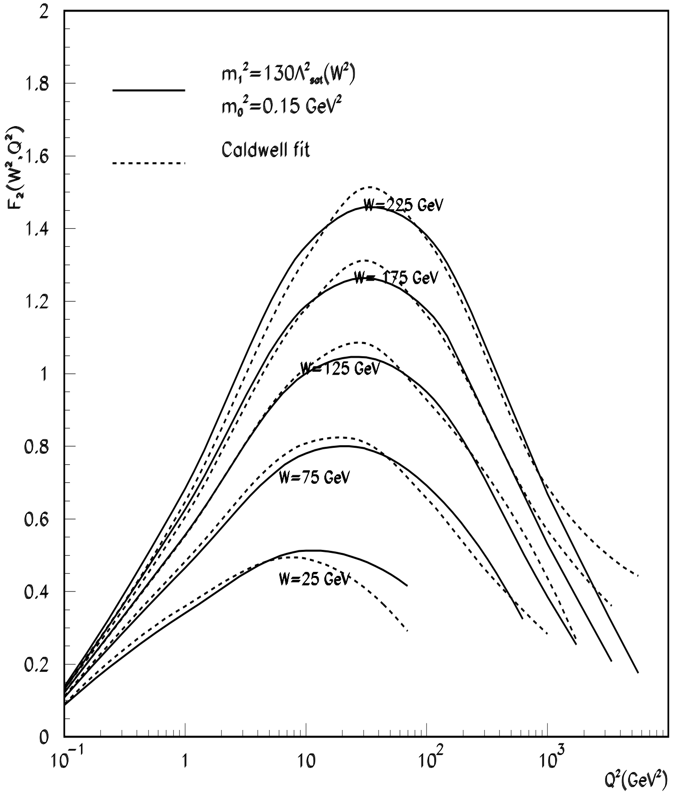

$\sigma_{\gamma^*p}(\mu\, b)$
ZEUS
H1
EMC
E665
BCDMS
NMC
σ(W=275)
σ(W=10)
η
(μb)
ξ=∞
ξ=130
ξ=50
ξ=30
ξ=10
ξ= 7
$\sigma_{\gamma*p}(\eta,\xi)$
$\eta=(Q^2+m_0^2)/\Lambda^2_{sat}(W^2)$

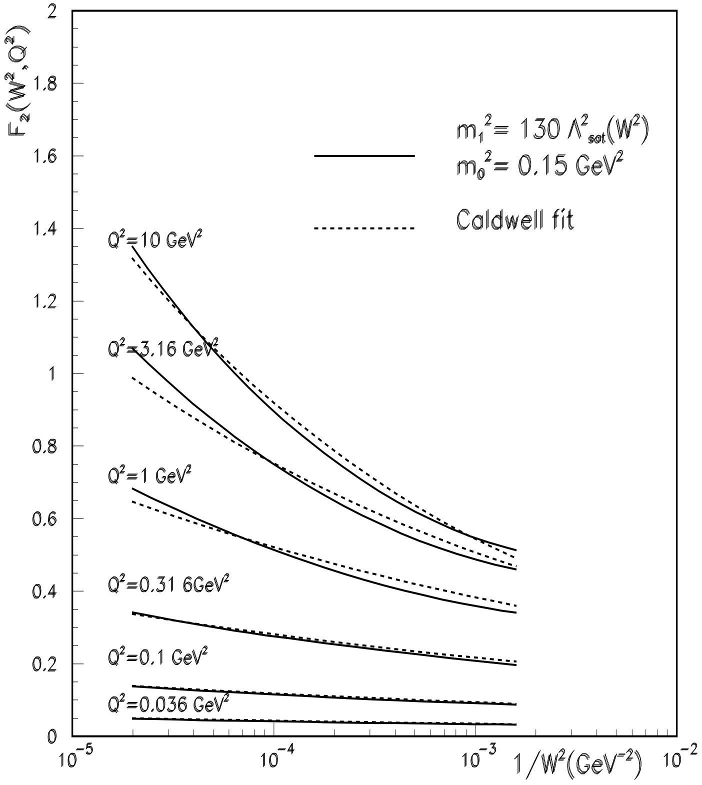


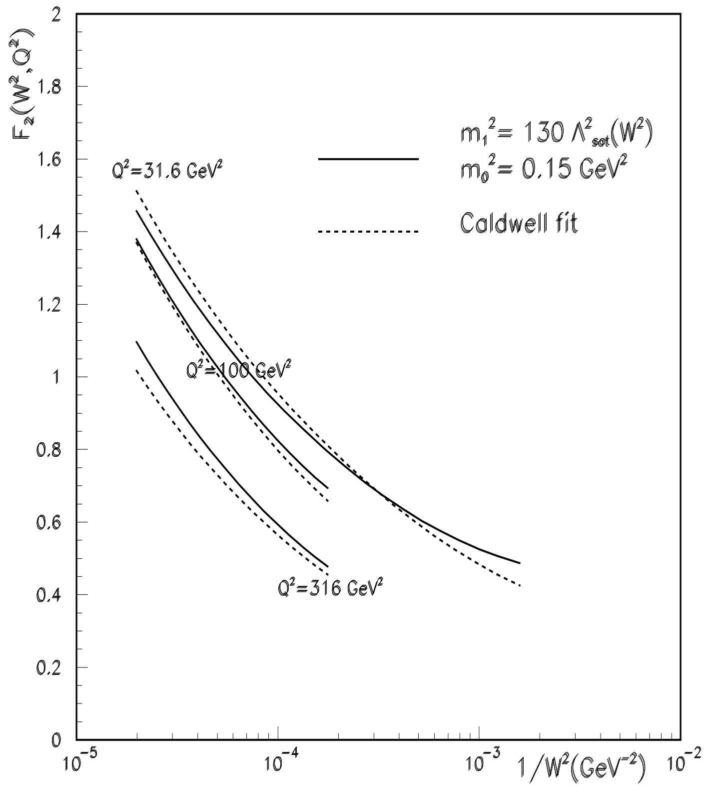
The approach to saturation.

Comparison with Caldwell 6-parameter 2 P-fit: $\sigma_{\gamma^* p} = \sigma_0 \frac{M^2}{Q^2+M^2} \left(\frac{l}{l_0}\right)^{\epsilon_0 + (\epsilon_1 - \epsilon_0)\sqrt{\frac{Q^2}{Q^2+\Lambda^2}}}$

where $$l = \frac{1}{2x_{bj}M_p}$$



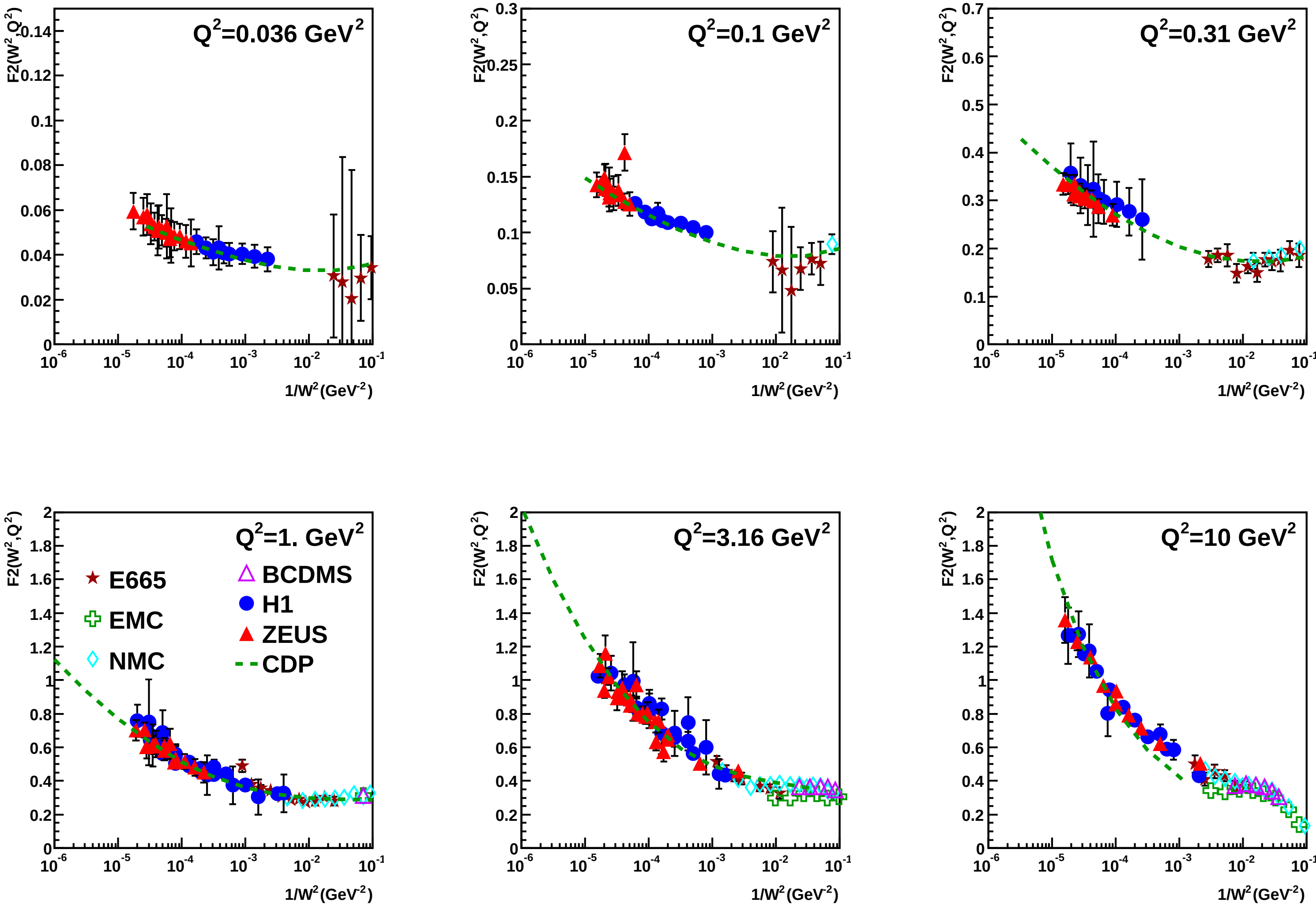


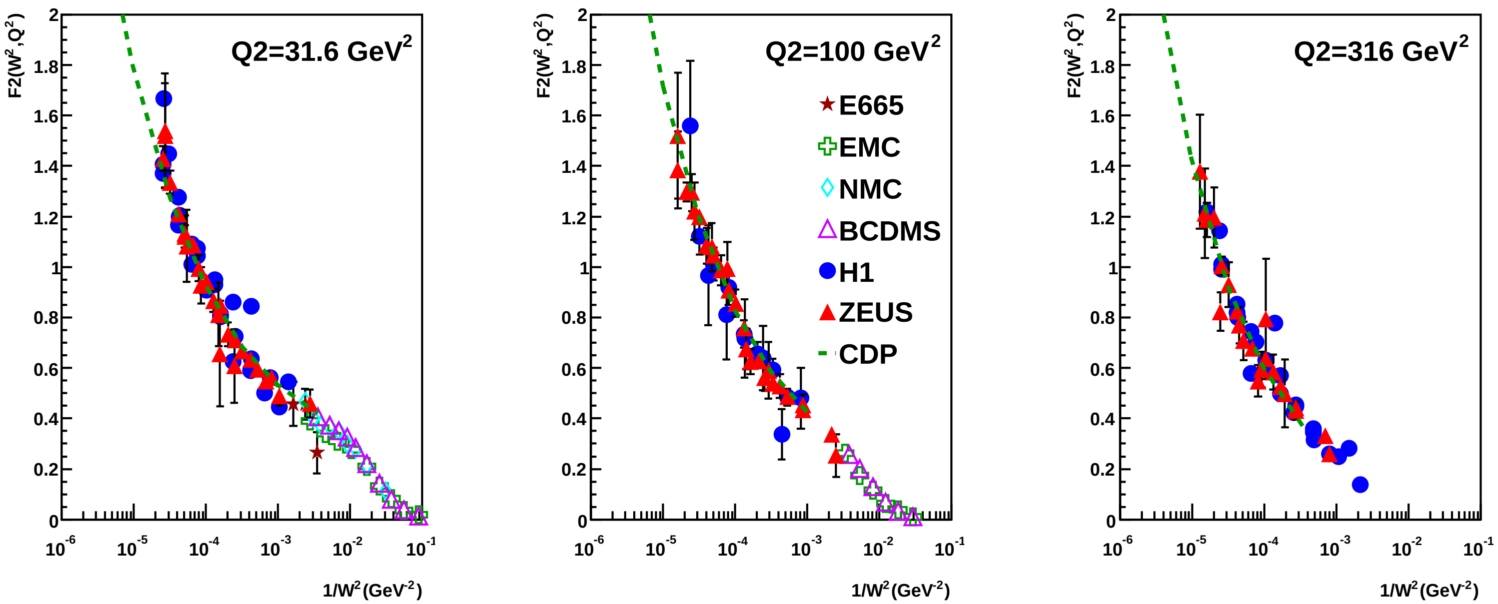


F2(W2,Q2)
m1^2= 130 Λ^2sat(W^2)
m0^2= 0.15 GeV^2
Caldwell fit
Q^2=31.6 GeV^2
Q^2=100 GeV^2
Q^2=316 GeV^2
1/W^2(GeV^-2)

Comparison with the experimental data directly

Prabhdeep Kaur (2010)





F2(W², Q²)
Q2=31.6 GeV²
Q2=100 GeV²
Q2=316 GeV²
E665
EMC
NMC
BCDMS
H1
ZEUS
CDP
1/W²(GeV⁻²)

Saturation limit: $\lim\limits_{\substack{W^2\to\infty \\ Q^2\text{fixed}}} \frac{F_2(x\cong Q^2/W^2, Q^2)}{\sigma_{\gamma p}(W^2)} = \frac{Q^2}{4\pi^2\alpha}$

Consider $Q_1^2 = 0.036\ GeV^2$

and $Q_2^2 = 0.1 GeV^2$

$$
\begin{aligned}
F_2(W^2, Q_2^2 = 0.1\mathrm{GeV}^2) &= \frac{Q_2^2}{Q_1^2} F_2(W^2, Q_1^2 = 0.036\mathrm{GeV}^2) \\
&= 2.78 F_2(W^2, Q_1^2 = 0.036\mathrm{GeV}^2).
\end{aligned}
$$

$\frac{1}{W^2}[\mathrm{GeV}^{-2}]$	$F_2(W^2, Q_1^2 = 0.036\mathrm{GeV}^2)$	$\frac{Q_2^2}{Q_1^2} F_2(W^2, Q_1^2 = 0.036\mathrm{GeV}^2)$
$2 \cdot 10^{-5}$	$\cong 0.055$	0.15
10^{-4}	$\cong 0.04$	0.11

$F_2(W^2)$ and gluon distribution.

$$F_2(W^2) = f_2 \left(\frac{W^2}{1\ GeV^2}\right)^{0.29} \quad , \quad f_2 = 0.063$$

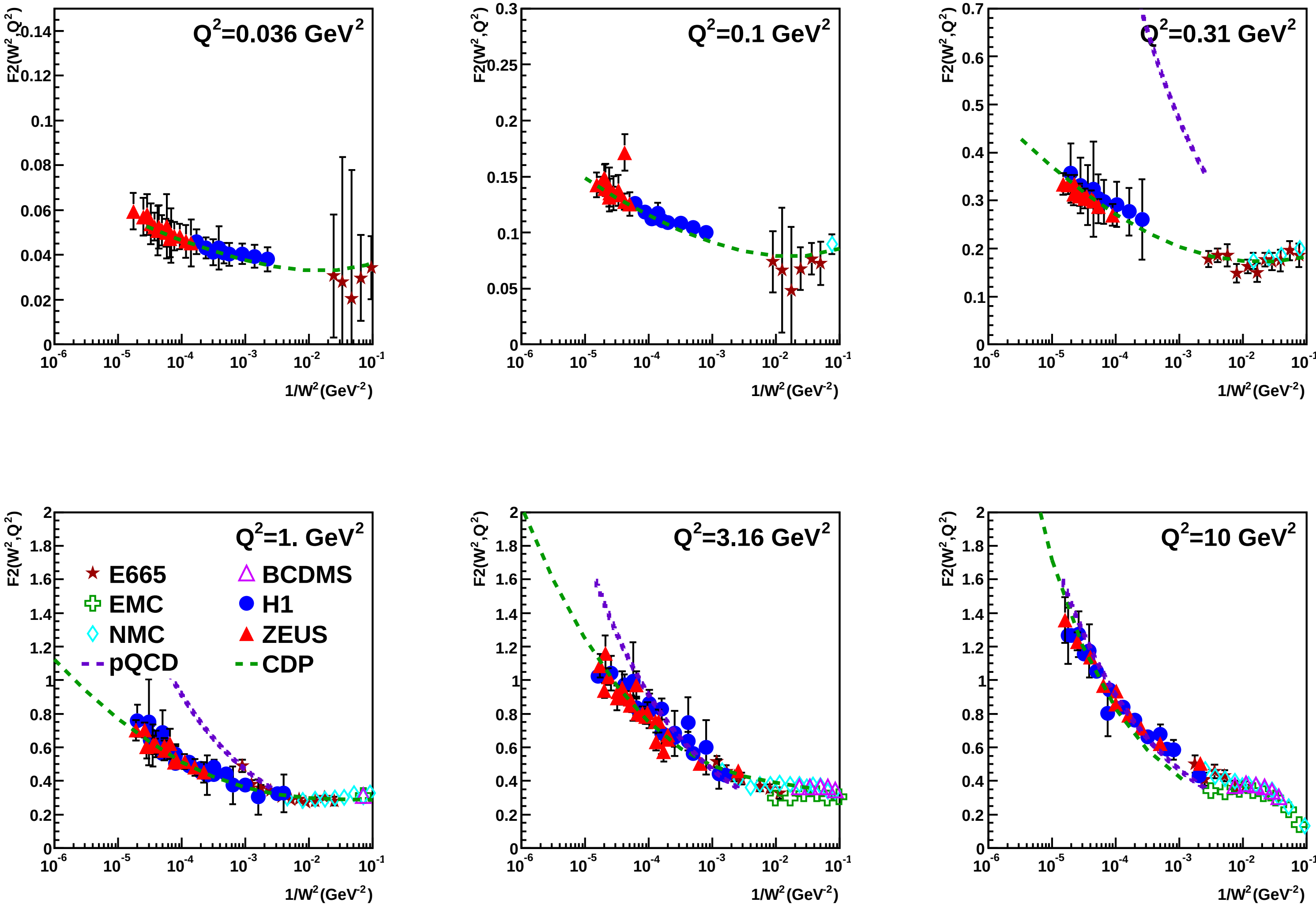
$$10GeV^2 \leq Q^2 \leq 100GeV^2$$

In terms of gluon distribution:

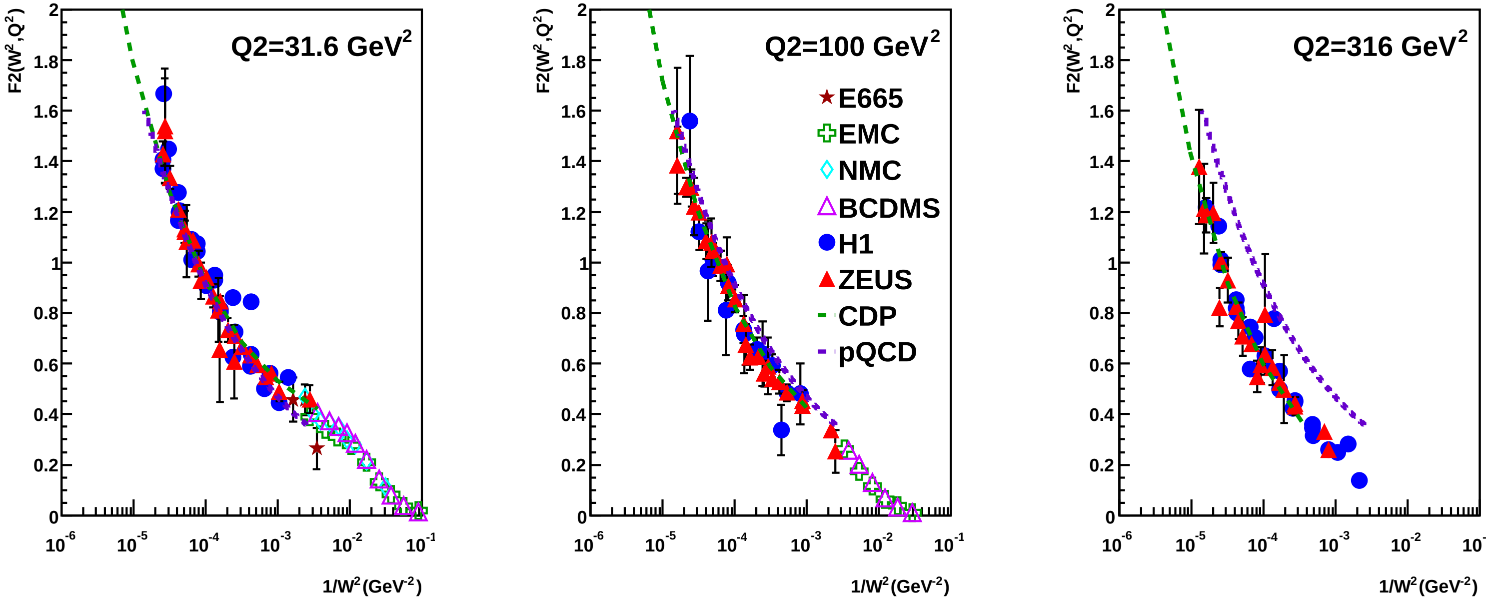
$$F_2(W^2 = \tfrac{Q^2}{x}) = \frac{(2\rho+1)\sum Q_q^2}{3\pi} \xi_L^{C_2} \alpha_s(Q^2) G(x, Q^2), \qquad \eta(W^2, Q^2) \gg 1.$$

Saturation behavior:

$$\begin{aligned} F_2(W^2, Q^2) \ &\sim \ Q^2 \sigma_L^{(\infty)} \ln \frac{\Lambda^2_{\text{sat}}(W^2)}{Q^2 + m_0^2} \\ &\sim \ Q^2 \sigma_L^{(\infty)} \ln \left(\frac{\alpha_s(Q^2) G(x, Q^2)}{\sigma_L^{(\infty)}(Q^2 + m_0^2)} \right), \qquad \eta(W^2, Q^2) \ll 1. \end{aligned}$$

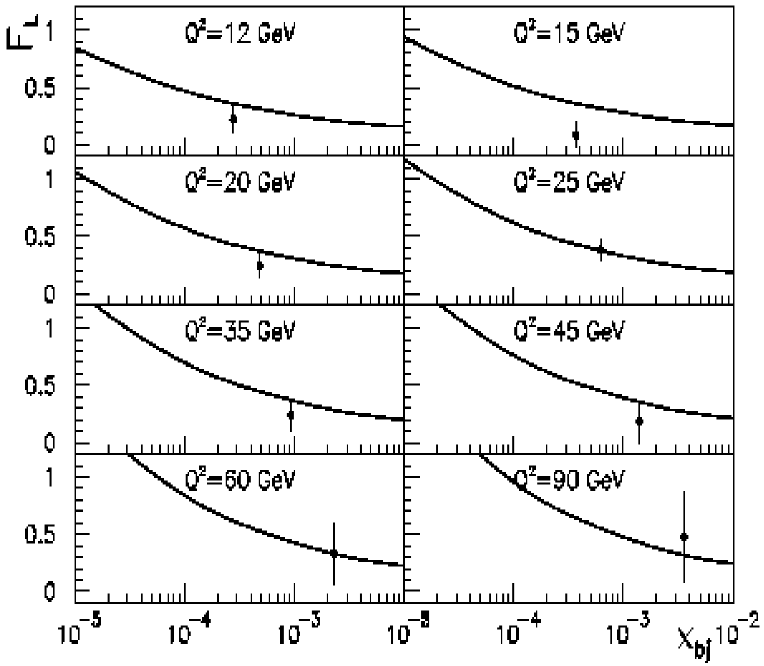


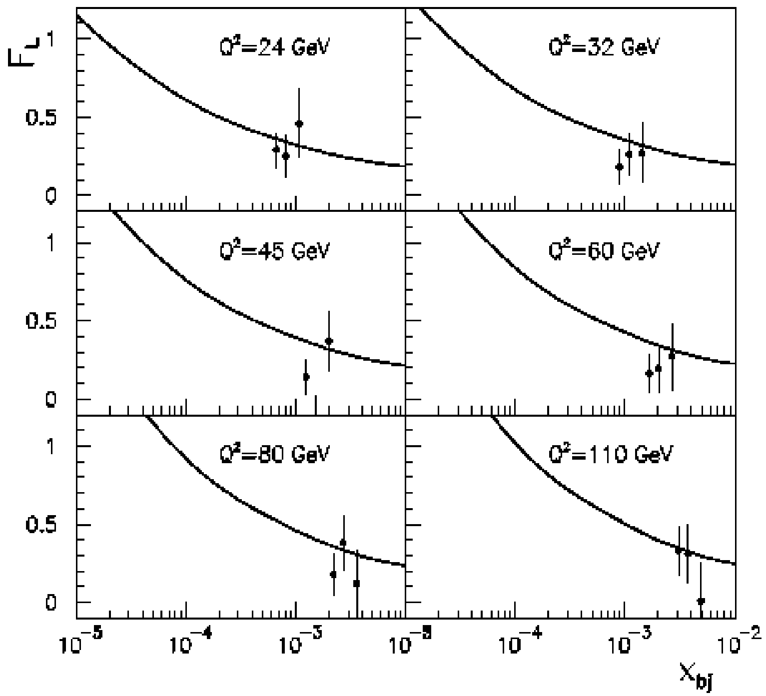
CDP and pQCD-improved parton model



CDP and pQCD-improved parton model

The longitudinal structure function, $F_L(x, Q^2)$





5. Conclusions

Gauge-invariant (two-gluon) interaction of color dipole:

i) Color transparency

$\sigma_{(q\bar{q})p}(\vec{r}_\perp^{\,2}, W^2) \sim \vec{r}_\perp^{\,2}$, **destructive interference**

relevant for $\eta(W^2, Q^2) = \frac{Q^2+m_0^2}{\Lambda^2_{sat}(W^2)} > 1, \quad \Lambda^2_{sat}(W^2) \sim (W^2)^{C_2=0.29}$

$$
\begin{aligned}
F_2(x, Q^2) &= F_2(W^2 = Q^2/x) \\
&\sim \Lambda^2_{sat}(W^2)\ , \qquad (10GeV^2, Q^2 < 100GeV^2) \\
&\sim \alpha_s(Q^2) G(x, Q^2).
\end{aligned}
$$

Peaceful coexistence between CDP and pQCD-improved parton model

ii) Saturation

$\sigma_{(q\bar{q})p}(\vec{r}_\perp{}', W^2) \sim \sigma^{(\infty)}$, destructive interference has died out,

relevant for $\eta(W^2, Q^2) < 1$,

$F_2(x, Q^2) \sim Q^2 \sigma^{(\infty)} ln \frac{\alpha_s(Q^2) G(x,Q^2)}{\sigma^{(\infty)}(Q^2+m_0^2)}$.

Smooth transition from $\eta(W^2, Q^2) \gg 1$ to $\eta(W^2, Q^2) \ll 1$, including $Q^2 = 0$.

There is only a single Pomeron.

Concrete model, interpolating the regions of $\eta(W^2, Q^2) > 1$ and $\eta(W^2, Q^2) < 1$, describes experimental data for $x \lesssim 0.1$, including $Q^2 = 0$ photoproduction.