Selected phenomenological implications of a 4th generation in the Standard Model

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Outline

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- There is a natural way to gather fermions into **generations**, with two leptons and two quarks in each
- Nature has chosen to repeat the same structure **three** times, the only difference being the mass scales

	I	II	III
Quarks	u , d	c , s	t , b
Leptons	e , ν_e	μ , ν_μ	τ , ν_τ

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If a fourth neutrino exists, it has to be heavier than $M_Z/2$!

- A stable heavy neutrino is **disfavoured** by cosmological considerations $\Rightarrow$ we choose an **unstable** ν_τ' which can decay into lighter leptons: **PMNS mixing** (**P**ontecorvo-**M**aki-**N**akagawa-**S**akata)

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- **Experimental lower bound** on new masses from collider physics

$m_{\tau'} > 101.9$	GeV	(L3)	$m_{\nu'_\tau} > 90.3\ (80.5)$	GeV	(L3)
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- **Radiative Corrections**: loop effects

  $\Rightarrow$ contribution to EW gauge bosons propagators (*Oblique Corrections*)

  $\Rightarrow$ induced FCNC processes (*Lepton Flavour Violating*)

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The SM *theoretical predictions* leave space to further corrections: how much?

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$\mathbf{m_H = ?}$

New Physics

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Generally:

$$X^{th} = X^{th}_{SM3}(m_H) + c^X_S \mathbf{S} + c^X_T \mathbf{T} + c^X_U \mathbf{U}$$

$$X = M_W \, , \, \Gamma_Z \, , \, R_h \, , \, A^e_{LR} \, , \, A^b_{FB} \, , \, Q_W(^{133}_{55}\mathrm{Ce}) \, , \, \ldots$$

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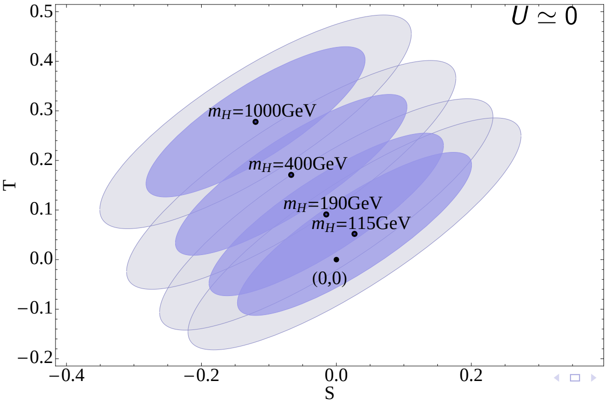


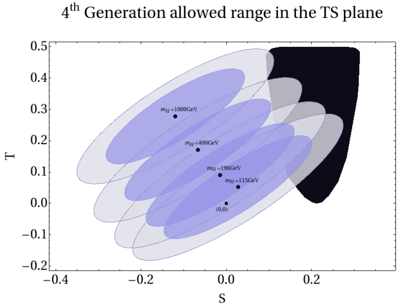
If SM with 3 generations is the "right" theory:

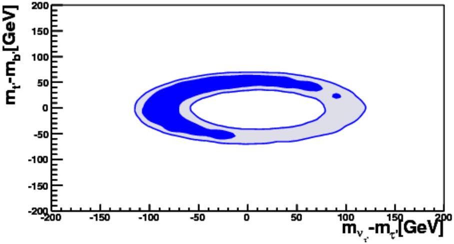
$$\mathbf{m_H < 285\ GeV}$$

(at 2σ)

Technically **S, T, U** are non-divergent combinations of the γ , **Z** , **W** gauge bosons self-energies.

It has been computed the contribution from the Fourth Generation: loop functions depend on new fermions **masses**, whose value can then be **constrained**.





$m_H = 115$ GeV

We find

- T parameter contribution is the most constraining
- T depends on mass differences in the doublet, and regions where $m_{\nu'_\tau} < m_{\tau'}$ and $m_{b'} < m_{t'}$ are preferred
- Higgs masses up to **1 TeV** are possible
- With growing m_H, mass non-degeneracy increases

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- With a fourth family, the $\mathbf{3 \times 3}$ PMNS submatrix **is not unitary**:

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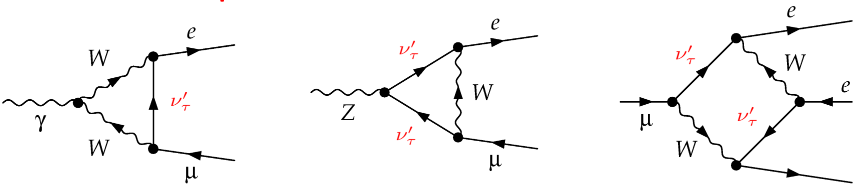
- G_F^{SM4} is the "true" Fermi's which enters in other decays
- It is actually a **new** parameter of the Fourth Generation

$$G_F^{SM3} < G_F^{SM4} < 1.002\, G_F^{SM3}$$

- PMNS mixing $\rightarrow$ off-diagonal coupling between ν'_τ and charged leptons
- The heavy neutrino modifies the $\boldsymbol{\mu}$**e**$\boldsymbol{\gamma}$, $\boldsymbol{\mu}$**eZ** and $\boldsymbol{\mu}$**eee effective vertices** which **violate lepton flavour number conservation**



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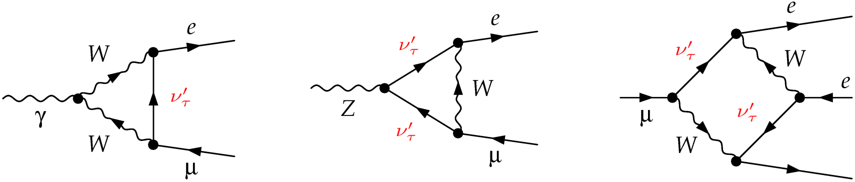

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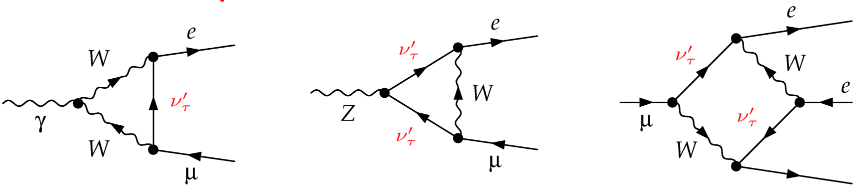

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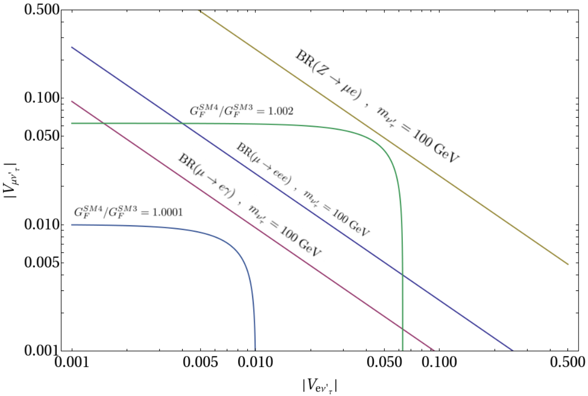
- Branching ratios depend on $m_{\nu'_\tau}$ and on the neutrino-light charged lepton mixing
- Yukawa interaction does not allow the suppression for $m_{\nu'_\tau} \rightarrow +\infty$: **non decoupling**

- LFV have **never been observed**

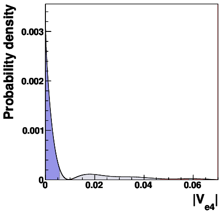
$$BR(\mu \to e\gamma) < 1.2 \times 10^{-11} \qquad BR(\mu \to eee) < 1.0 \times 10^{-12}$$
$$BR(Z \to \mu e) < 1.7 \times 10^{-6}$$



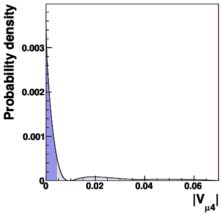
- The Z effective coupling is particularly sensitive to $m_{\nu'_\tau}$
- Included τ physics

Numerical Analysis

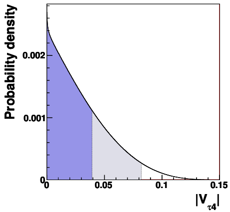
- Bayesian approach
- Model parameters: fermion masses and $V_{e\nu'_\tau}$, $V_{\mu\nu'_\tau}$ and $V_{\tau\nu'_\tau}$
- Flat priors
- Marginalization of posterior distributions



$|V_{e\nu'_\tau}| < 0.044$

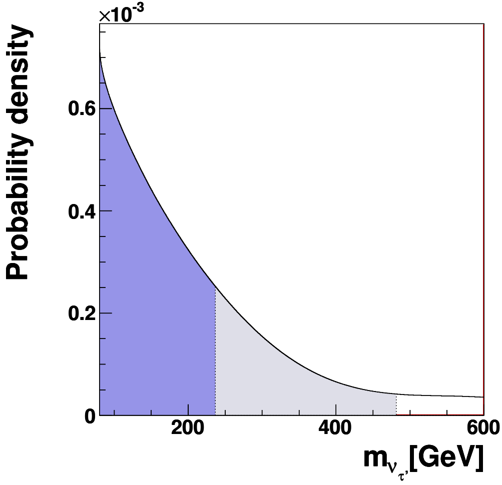


$|V_{\mu\nu'_\tau}| < 0.049$



$|V_{\tau\nu'_\tau}| < 0.083$

$(at \quad 2\sigma)$



$$m_{\nu'_\tau} < 213 \quad \text{GeV} \ (1\sigma)$$

Conclusions:

- A new generation is **compatible** with EWPO measurements (**EWPO**) and possibly enlarge the allowed mass range for the Higgs
- LFV decays receive significant contributions that can **saturate** the up-to-date upper bounds
- These same decays allow to estimate an **upper bound** on 4th neutrino mass as well on absolute values of mixing matrix elements

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Currently working on:

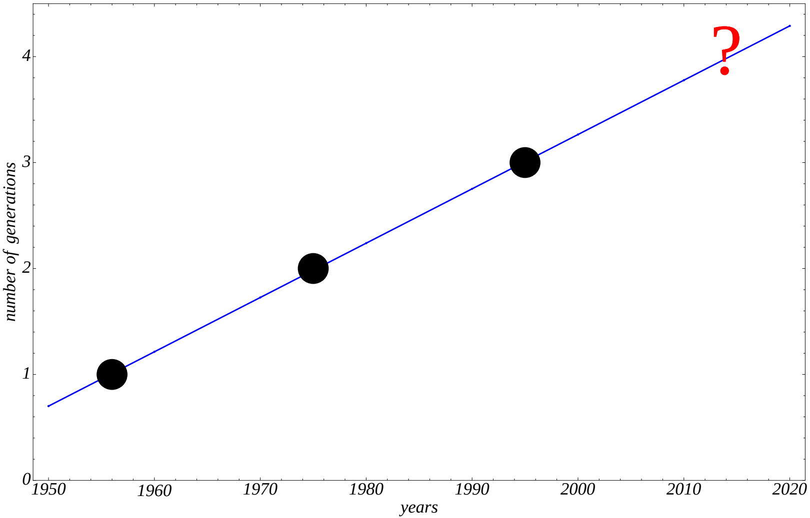
- Including the effects of heavy quarks to hadronic physics, particularly on CP violating decays and FCNC processes
- Studying the effects of the non-unitarity of the 3×3 CKM submatrix

Future developments:

- Include a Majorana neutrino (there is no see-saw)
- Study the model response to an actual measurement of $\mu \to e\gamma$

THE END

Thank you for your attention


?
4
3
2
1
0
number of generations
1950
1960
1970
1980
1990
2000
2010
2020
years