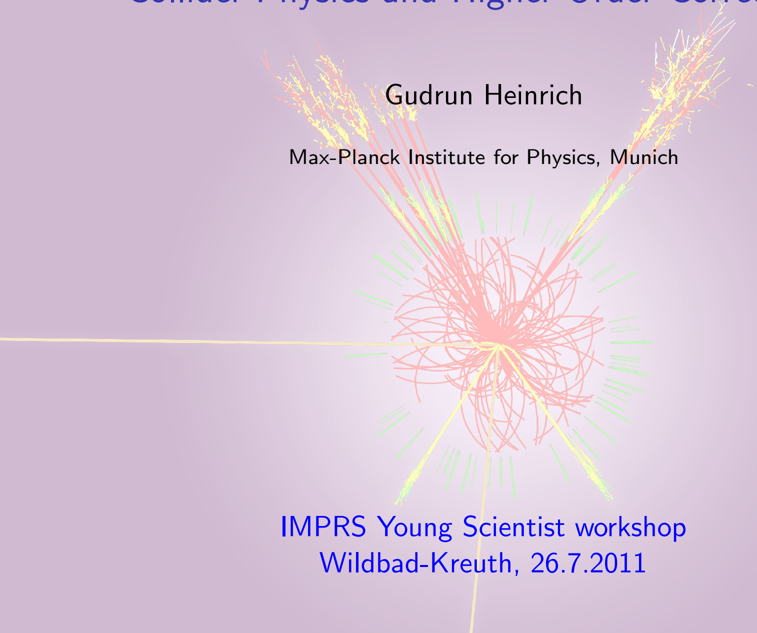
Collider Physics and Higher Order Corrections

Gudrun Heinrich

Max-Planck Institute for Physics, Munich



IMPRS Young Scientist workshop
Wildbad-Kreuth, 26.7.2011

Early Colliders

- luminosity low
- detectors poor

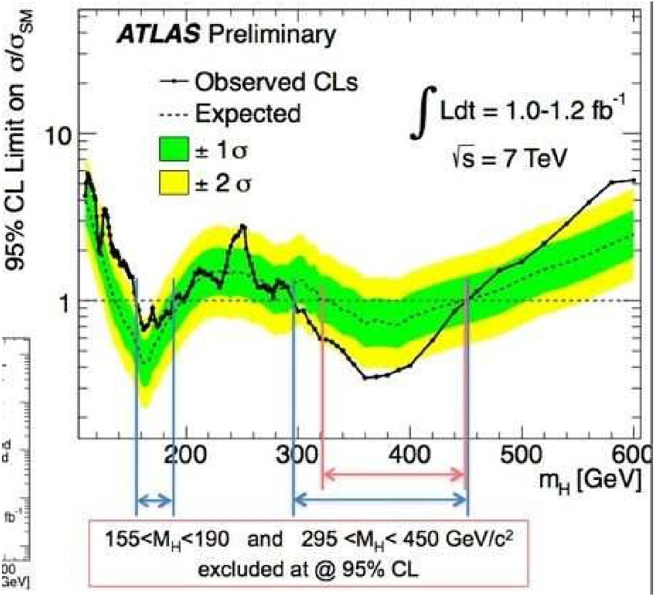
Early Theory

Modern Colliders

- luminosity higher
- detectors better

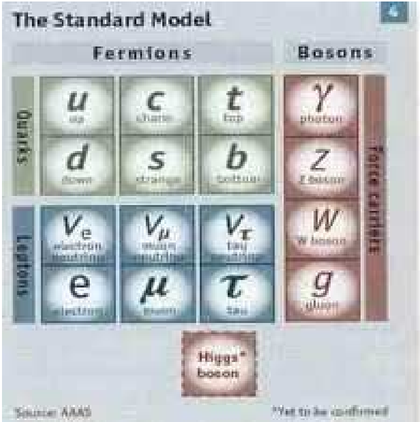
Modern Theory

Modern Phenomenology



K.Cranmer, EPS 2011

The Standard Model



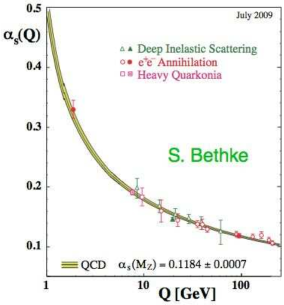
important concepts:

- local gauge theory $SU(2) \times U(1) \times SU(3)_c$
- renormalizability
- perturbation theory

Strong Interactions

basic principles of Quantum Chromo-Dynamics (QCD):

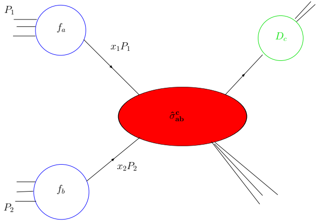
- asymptotic freedom: coupling $\alpha_s(Q^2) \to 0$ for $Q^2 \to \infty$



constituents of hadrons (quarks and gluons) can be considered as freely interacting at high energies (i.e. short distances)

- factorisation: systematic separation of long-distance effects (non-perturbative) and short-distance cross sections ("hard scattering")

factorisation



$$\sigma_{pp\to X} = \sum_{a,b,c} f_a(x_1,\mu_f^2) f_b(x_2,\mu_f^2) \otimes \hat{\sigma}_{ab}\left(p_1, p_2, \frac{Q^2}{\mu_f^2}, \frac{Q^2}{\mu_r^2}, \alpha_s(\mu_r^2)\right)$$
$$\otimes D_{c\to X}(z,\mu_f^2) + \mathcal{O}(\Lambda/Q)$$

f_a, f_b: parton distribution functions (from fits to data)

$\hat{\sigma}_{ab}$: partonic hard scattering cross section

calculable order by order in perturbation theory

$D_{c\to X}(z,\mu_f^2)$: describing the final state e.g. fragmentation function, jet observable, etc.

Perturbative expansion

expansion in strong coupling α_s:

$$\hat{\sigma} = \alpha_s^k(\mu) \left[\hat{\sigma}^{\mathrm{LO}} + \alpha_s(\mu)\, \hat{\sigma}^{\mathrm{NLO}}(\mu) + \alpha_s^2(\mu)\, \hat{\sigma}^{\mathrm{NNLO}}(\mu) + \dots \right]$$

μ-dependence comes from truncation of perturbative series

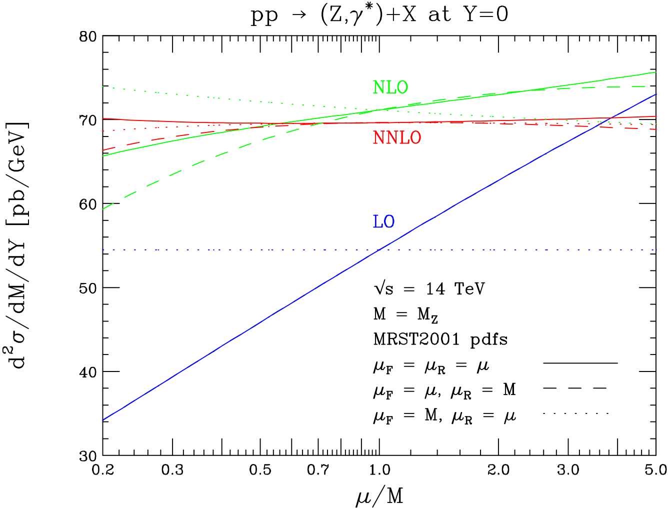
calculation at n-th order:

$$d\hat{\sigma}^{(n)}/d\ln(\mu^2) = \mathcal{O}(\alpha_s^{n+1})$$

truncation of perturbative series at LO

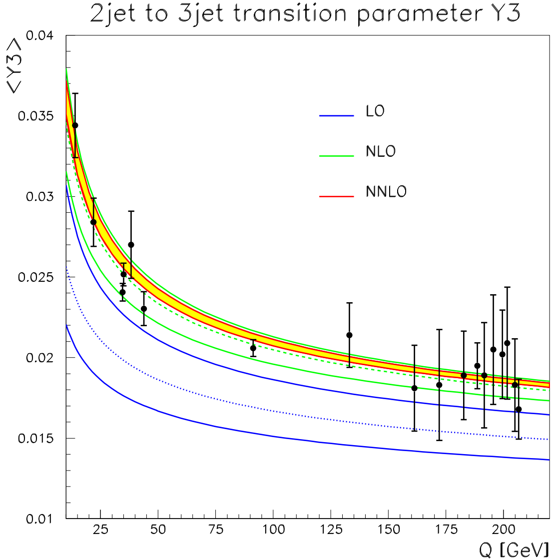
$\Rightarrow$ large renormalisation/factorisation scale dependence

Stabilisation of scale dependence at higher orders



[Anastasiou et al. 04]

shortcomings of leading order predictions



example:

3-jet observable

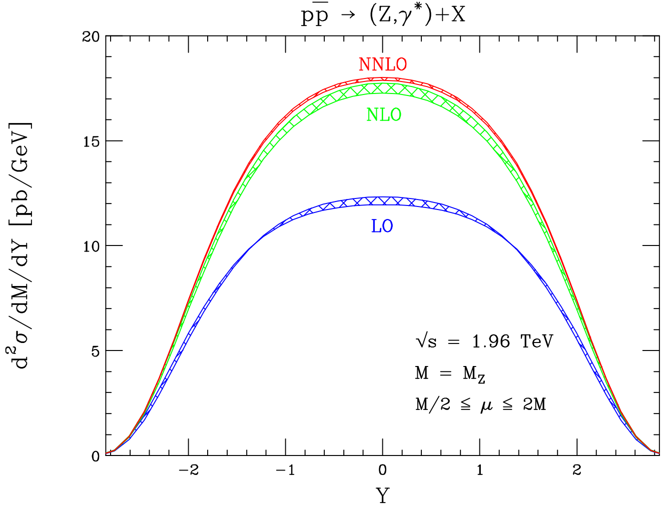
in e^+e^- annihilation

[A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, GH '09]

uncertainty bands:

$M_Z/2 < \mu < 2\,M_Z$

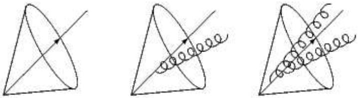
Shortcomings of Leading Order Predictions



[Anastasiou et al. 04]

Shortcomings of Leading Order Predictions

- poor jet modelling



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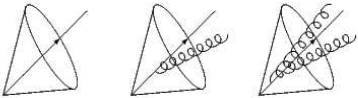

- cases where shapes of distributions are not well predicted by LO (new partonic processes become possible beyond LO)

Shortcomings of Leading Order Predictions

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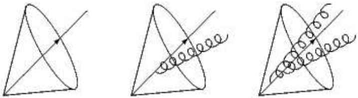
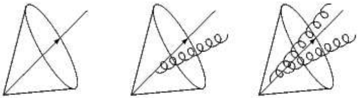


- cases where shapes of distributions are not well predicted by LO (new partonic processes become possible beyond LO)

- Minimal Supersymmetric Standard Model (MSSM):
  would be ruled out already without radiative corrections:
  mass of lightest Higgs boson at LO: $M_h \leq min(M_A, M_Z) \cdot |\cos 2\beta|$

Shortcomings of Leading Order Predictions

- poor jet modelling



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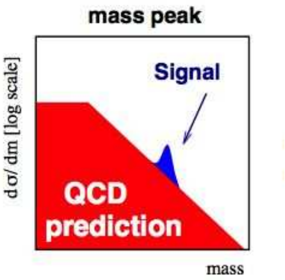
- ...

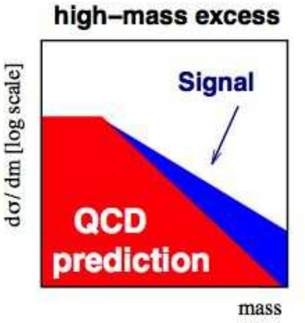
Identifying New Physics at Hadron Colliders

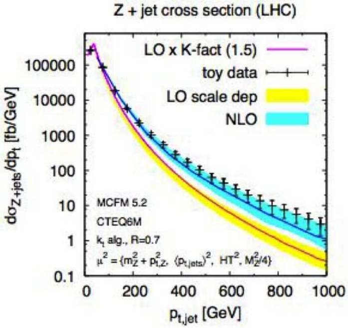


- peak: easy, backgrounds can be measured

Identifying New Physics at Hadron Colliders

- peak: easy, backgrounds can be measured
- shape: hard
  need signal/background shapes from theory

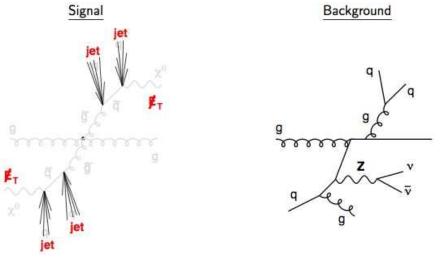




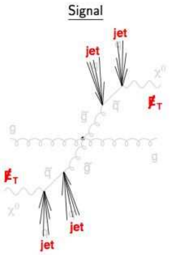
Identifying New Physics at Hadron Colliders

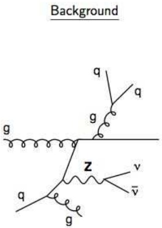
- peak: easy, backgrounds can be measured
- shape: hard
  need signal/background shapes from theory
- rate (e.g. $H \to W^+W^-$): very hard (counting experiment)
  need both shape and normalisation from theory

example heavy SUSY particles

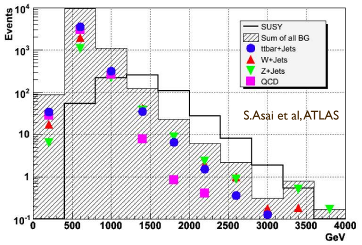


example heavy SUSY particles





Overall result, after the complete detector simulation, etc....



multi-particle final states

- to establish signals of New Physics
- to measure model parameters

  Leading Order is not sufficient !

- at LHC: typically multi-particle final states

  $\Rightarrow$ calculations of higher orders increasingly difficult

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- to establish signals of New Physics
- to measure model parameters

  Leading Order is not sufficient !

- at LHC: typically multi-particle final states

  $\Rightarrow$ calculations of higher orders increasingly difficult

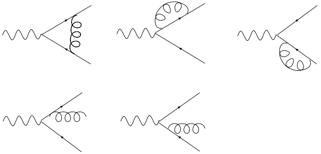
- example for time scale to add one parton:
  $pp \to 2$ jets at NLO (4-point process): Ellis/Sexton 1986
  $pp \to 3$ jets at NLO (5-point): Bern et al, Kunszt et al '93-95
  $pp \to 4$ jets at NLO (6-point): not yet available

ingredients for *m*-particle observable at NLO

virtual part (one-loop integrals):

$$\mathcal{A}^{V}_{NLO} = A_2/\epsilon^2 + A_1/\epsilon + A_0^{(v)}$$

$$d\sigma^V \sim Re\left(\mathcal{A}^{\dagger}_{LO}\,\mathcal{A}^{V}_{NLO}\right)$$



real radiation part: soft/collinear emission of massless particles

$\Rightarrow$ need subtraction terms

$$\Rightarrow \int_{\rm sing} d\sigma^S = -A_2/\epsilon^2 - A_1/\epsilon + A_0^{(r)}$$

$$\sigma^{NLO} = \underbrace{\int_{m+1}\left[d\sigma^R - d\sigma^S\right]_{\epsilon=0}}_{\text{numerically}} + \underbrace{\int_m \left[\underbrace{d\sigma^V}_{\text{cancel poles}} + \underbrace{\int_{\rm s} d\sigma^S}_{\text{analytically}}\right]_{\epsilon=0}}_{\text{numerically}}$$

NLO calculations

exploit modular structure

Tree Modules		One-Loop Module		IR Modules
$\lvert\mathcal{A}^{LO}\rvert^2$	$\oplus$	$2\,Re(\mathcal{A}^{LO\,\dagger}\mathcal{A}^{NLO,virt})$	$\oplus$	integrated IR subtraction terms
$\lvert\mathcal{A}^{NLO,real}\rvert^2$			$\ominus$	soft/collinear subtraction terms

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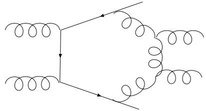
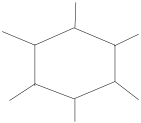
has been bottleneck so far

One-loop methods

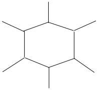
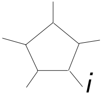
basically two categories:

- methods based on Feynman diagrams

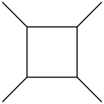
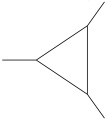
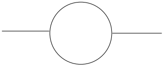
 →  + integrals with less legs

non-trivial tensor structure

scalar 6-point function

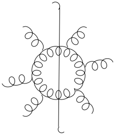
 $= \sum_{i=1}^{6} b_i$  i . . . factorial growth in complexity

reduction to set of basis integrals (4-, 3- and 2-point funcs.)

$\mathcal{A} = C_4$  $+ C_3$  $+ C_2$  $+ \mathcal{R}$

One-loop methods

- "unitarity based":

$$\mathcal{A} = \sum_{\rm cuts} \int d\, PS$$  $+ \quad \mathcal{R}$

  - use analyticity structure to compose loop amplitudes from cuts
  - efficient for coefficients of boxes, triangles, bubbles (use cuts in $D = 4$)
  - obtaining rational terms $\mathcal{R}$ needs $D \neq 4$
  - cut conditions lead to systems of equations which can be solved numerically [OPP method, Ossola, Papadopoulos, Pittau]

Progress monitor

Les Houches NLO wishlist for LHC
Status 2007

process ($V \in \{Z, W, \gamma\}$)	status
1. $pp \to V\,V$ jet	*WW jet:* Dittmaier, Kallweit, Uwer; Campbell, Ellis, Zanderighi
2. $pp \to V\,V\,V$	*ZZZ:* Lazopoulos, Melnikov, Petriello
3. $pp \to t\bar{t}\, b\bar{b}$	
4. $pp \to t\bar{t} + 2$ jets	
5. $pp \to V\,V\, b\bar{b}$	
6. $pp \to V\,V + 2$ jets	
7. $pp \to V + 3$ jets	
8. $pp \to b\bar{b}b\bar{b}$	
9. $pp \to 4$ jets	
10. EW corrections to W,Z production	

Status 2009

$pp \to W\,W$ jet	Dittmaier/Kallweit/Uwer; Campbell/Ellis/Zanderighi
	Binoth/Guillet/Karg/Kauer/Sanguinetti
$pp \to Z\,Z$ jet	Binoth/Gleisberg/Karg/Kauer/Sanguinetti; Dittmaier/Kallweit
$pp \to t\bar{t}\,b\bar{b}$	Bredenstein/Denner/Dittmaier/Pozzorini;
	Bevilacqua/Czakon/Papadopoulos/Pittau/Worek
$pp \to t\bar{t} + 2$ jets	Bevilacqua/Czakon/Papadopoulos/Worek
$pp \to Z\,Z\,Z$	Lazopoulos/Melnikov/Petriello; Hankele/Zeppenfeld
$pp \to V\,V\,V$	Binoth/Ossola/Papadopoulos/Pittau; Zeppenfeld et al.
$pp \to V\,V\,b\bar{b}$	
$pp \to W\,\gamma$ jet	Campanario/Englert/Spannowsky/Zeppenfeld
$pp \to V\,V + 2$ jets	VBF: Bozzi/Jäger/Oleari/Zeppenfeld, VBFNLO coll.
$pp \to W + 3$ jets	BlackHat coll.; Ellis/Giele/Kunszt/Melnikov/Zanderighi*
$pp \to Z + 3$ jets	BlackHat collaboration
$qq \to b\bar{b}b\bar{b}$	Binoth/Greiner/Guffanti/Guillet/Reiter/Reuter

• done • partial results * leading colour only

After 2009

- $pp \to W^+W^-b\bar{b}$ Denner,Dittmaier,Kallweit,Pozzorini '10, Bevilacqua, Czakon, van Hameren, Papadopoulos, Worek '11
- $pp \to W + 4\,\text{jets}$ (7-point functions) BlackHat collaboration '10
- $pp \to Z/\gamma + 3\,\text{jets}$ BlackHat collaboration '11
- $pp \to t\bar{t}$ +2 jets Bevilacqua, Czakon, Papadopoulos, Worek '10
- $pp \to W^+W^+jj$ Melia, Melnikov, Rontsch, Zanderighi '10
- $pp \to W^+W^-jj$ Melia, Melnikov, Rontsch, Zanderighi '11
- $gg \to b\bar{b}b\bar{b}$ Greiner, Guffanti, Reiter, Reuter '11
- EW corrections to dilepton+jet production Denner,Dittmaier, Kasprzik, Mück '11
- $e^+e^- \to 5$ jets Frederix, Frixione, Melnikov, Zanderighi '10

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- further: BIG advances in automation

Automated NLO Tools

One-loop

- FeynArts/FormCalc/LoopTools (public) Thomas Hahn
  (see next talk)
- Helac-NLO Bevilacqua, Czakon, van Hameren, Papadopoulos, Pittau, Worek
- MadLoop Hirschi,Frederix,Frixione,Garzelli,Maltoni,Pittau '11
  uses CutTools (Ossola, Papadopoulos, Pittau) and MadFKS
- Golem-Samurai (Samurai public) (see below)
  Cullen, Greiner, GH, Luisoni, Mastrolia, Ossola, Reiter, Tramontano
- dedicated programs also involving high level of automation
  Denner, Dittmaier, Pozzorini et al, VBFNLO coll., MCFM, Blackhat, Rocket, ...

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automation of subtraction for IR divergent real radiation

- MadDipole Frederix, greiner, Gehrmann 08
- Dipole subtraction in Sherpa Gleisberg, Krauss 08
- TevJet Seymour Tevlin 08
- AutoDipole Hasegawa, Moch, Uwer 08
- Helac-Phegas Czakon, Papadopoulos, Worek 09; polarized
- MadFKS Frederix, Frixione, Maltoni, Stelzer 09

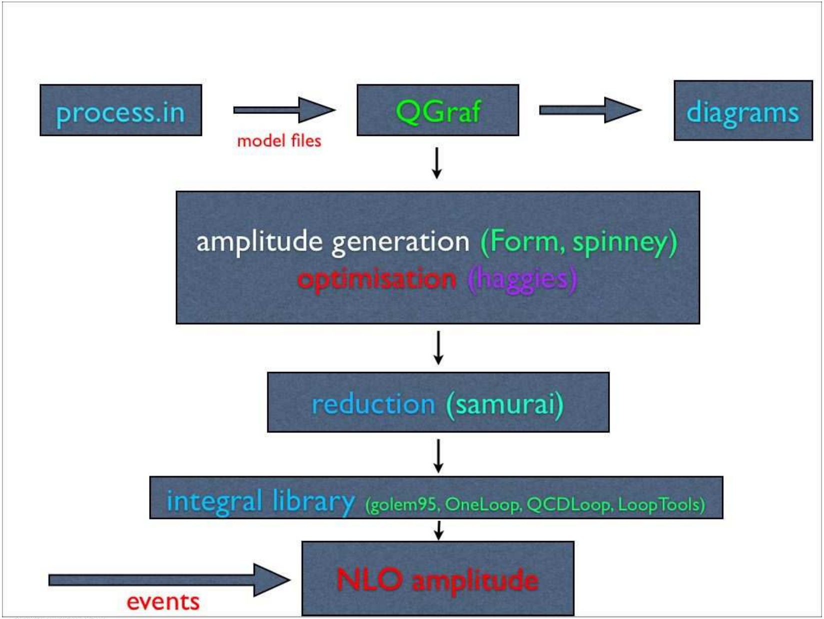
Golem-Samurai

General One-Loop Evaluator of Matrix elements &
Scattering Amplitudes from Unitarity based Reduction At
Integrand level

[Cullen, Greiner, GH, Luisoni, Mastrolia, Ossola, Reiter, Tramontano]

- algebraic generation of D-dimensional integrands based on Feynman diagrams
  - QCD, EW, BSM
  - use QGraf, FeynRules, etc. to generate integrands
- reduction by D-dimensional extension of cut-based method optionally
  - OPP-type reduction
  - traditional tensor reduction (using golem95 library)
  - tensorial reduction at integrand level GH, Ossola, Reiter, Tramontano '10
- interface with existing tools for real radiation (MadGraph/MadEvent, Sherpa, PowHeg, . . .)

Golem-Samurai structure



Monday, July 25, 2011

Golem-Samurai

public release this autumn

usage:

- edit "input card"

```
in= u,d~
out= nmu, mu+, e-, ne~, s~, c
model=smdiag
```

models can be added via **FeynRules** (Duhr) or **LanHEP** (Semenov)

```
order=gw,4,4; order=gs,2,4
zero=mB,mC,mS,mU,mD,me,mmu
one=gs,e
helicities=-+-+-++-
extensions=samurai, dred
```

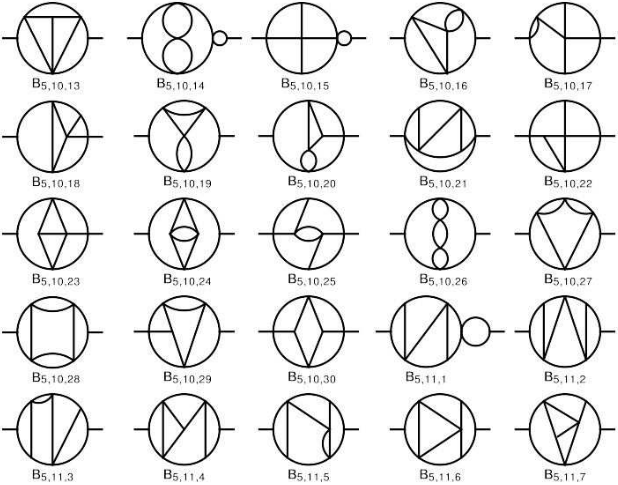
- golem-main.py process.in
- make doc $\Rightarrow$ documentation and diagram pictures
- make source $\Rightarrow$ source files
- make compile $\Rightarrow$ fully compiled code

Current Status (Comparisons)

1. $\gamma\gamma \to \gamma\gamma$
2. $\bar{u}d \to e^-\bar{\nu}_e$
3. $\bar{u}u \to \bar{d}d$
4. $dg \to dg$
5. $u\bar{d} \to e^+\nu g$
6. $u\bar{d} \to e^+\nu s\bar{s}$
7. $u\bar{d} \to e^+\nu gg$
8. $d\bar{d} \to e^+e^-g$
9. $d\bar{d} \to W^+W^-$ (1)
10. $d\bar{d} \to t\bar{t}$
11. $bg \to Hb$
12. $u\bar{u} \to g\gamma$
13. $ug \to u\gamma$
14. $gg \to g\gamma$
15. $gg \to gZ$
16. $gg \to gg$
17. $gg \to ZZ$ (1)
18. $gg \to W^+W^-$ (1)
19. $e^+e^- \to Z^* \to \bar{d}dgg$
20. $u\bar{d} \to \bar{c}se^+\nu_e\mu^+\nu_\mu$
21. $u\bar{d} \to \bar{s}ce^-\bar{\nu}_e\mu^+\nu_\mu$
22. $pp \to b\bar{b}b\bar{b}$

(1): with and without leptonic decays

Beyond One Loop



NNLO

- full NNLO cross sections:
  - e^+e^-: partonic event generator program EERAD3 for 3-jet observables in e^+e^- annihilation

    [A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, GH '07]

    [S. Weinzierl '08/'09]

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    [A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, GH '07]
    [S. Weinzierl '08/'09]
  - hadronic collisions:
    - one colour-neutral final state particle (W/Z, Higgs)
      Anastasiou, Dixon, Melnikov, Petriello; Grazzini, Catani, DeFlorian, Cieri, Ferrera
    - $t\bar{t}$, W^+W^-, $\gamma\gamma$, V+jet, dijet under construction
- different methods for double real radiation
  - antenna subtraction Gehrmann-DeRidder, Gehrmann, Glover '05
  - Dipole-like subtraction Grazzini, Catani, DeFlorian; Trocsanyi, Somogyi et al.
  - sector decomposition Binoth, GH '00, Anastasiou, Melnikov, Petriello '03, Czakon '10

Sector Decomposition

- allows to extract UV and IR singularities from (dimensionally regulated) parameter integrals in an automated way
- produces a Laurent series in ϵ
- coefficients are finite parameter integrals
  $\Rightarrow$ integrate numerically
- can be applied to multi-loop integrals and phase space integrals

Sector Decomposition

public programs:

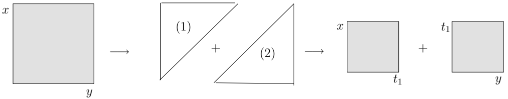
- sector_decomposition (uses `Ginac`) Bogner, Weinzierl '07
- FIESTA (uses `Mathematica`) A. Smirnov, V.Smirnov,M. Tentyukov '08
- SecDec (uses `Mathematica` and Fortran/C) Jon Carter, GH '10

  http://projects.hepforge.org/secdec

current limitation:
multi-scale integrals limited to Euclidean region
(e.g. no thresholds)

extension of SecDec to general kinematics under construction
S. Borowka, GH

basics of sector decomposition



$$I = \int_0^1 dx \int_0^1 dy\, x^{-1-\epsilon} (x+y)^{-1} [\underbrace{\Theta(x-y)}_{(1)} + \underbrace{\Theta(y-x)}_{(2)}]$$

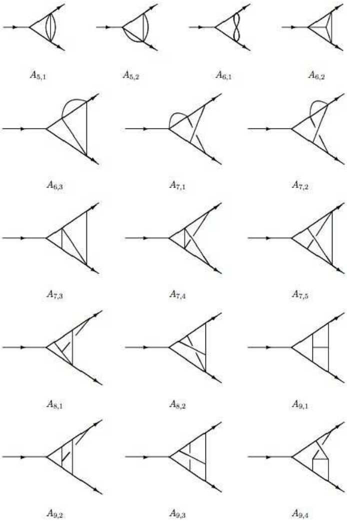
subst. (1) $y = x\,z$ (2) $x = y\,z$ to remap to unit cube

$$I = \int_0^1 dx\, x^{-1-\epsilon} \int_0^1 dz\, (1+z)^{-1} + \int_0^1 dy\, y^{-1-\epsilon} \int_0^1 dz\, z^{-1-\epsilon} (1+z)^{-1}$$

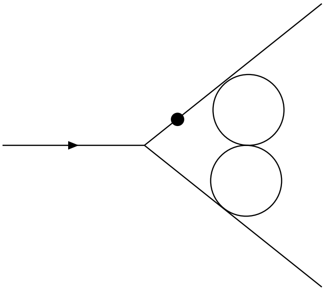
singularities are disentangled, number of integrals doubled

3-loop example

master integrals for 3-loop form factors



3-loop example



evaluation of analytic result:

$A_{6,1} = 0.166667/\epsilon^3 + 1.83333/\epsilon^2 + 18.1232/\epsilon + 125.32$

SecDec demo

Outlook

- in order to discover/understand "New Physics":
  theory predictions for signals and backgrounds
  must be well under control

  need accuracy beyond Leading Order

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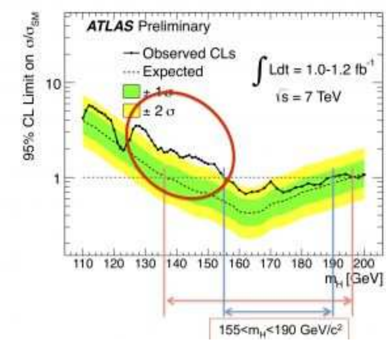
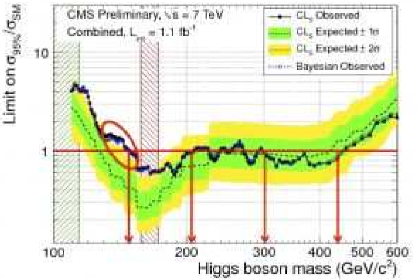
- stay tuned !





backup slides

basics of general sector decomposition

D-dim. integral of some matrix element squared typically contains overlapping infrared poles of the form

[$s_{ij} = (p_i + p_j)^2$]

$$\int \mathrm{d}\Phi^{(D)} \, |\mathrm{ME}|^2 \sim \int \mathrm{d}s_{13} \, \mathrm{d}s_{23} \; s_{13}^{-1-\epsilon} \; \frac{\mathcal{F}(s_{13}, s_{23})}{s_{13} + s_{23}}$$

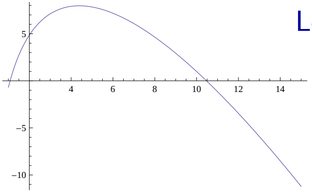
$$\sim \int_0^1 dx \, dy \; x^{-1-\epsilon} \; \frac{\mathcal{F}(x, y)}{x + y}$$

singularities for $x, y \to 0$ need to be factorised

sector decomposition is an algorithmic way to factorise this type of overlapping singularities

asymptotic complexity

- unitarity based methods:
  complexity of colour ordered amplitudes:
  $\tau_{\rm tree} \times \tau_{\rm cuts} \sim N^4 \times \begin{pmatrix} N \\ 5 \end{pmatrix} \overrightarrow{\text{N large}} \; N^9$
- Feynman diagram reduction:
  $\tau_{\rm diagrams} \times \tau_{\rm form\,factors} \sim 2^N \times \Gamma(N)$



$\text{Log}(N^9/(\Gamma(N)\,2^N))$

generic event

1. hard interaction
   $\hat{\sigma} = \alpha_s^k \, \hat{\sigma}^{\mathrm{LO}} + \alpha_s^{k+1} \, \hat{\sigma}^{\mathrm{NLO}} + \ldots$
   calculable order by order
   in perturbation theory
2. parton shower
   soft and collinear branching,
   treatment within perturbative
   QCD framework
3. hadronization
   non-perturbative models,
   fits to data
4. (underlying event)