Flux compactifications and geometry in string theory

Peter Patalong

String theory group, Arnold-Sommerfeld-Center

July 26$^{\text{th}}$, 2011

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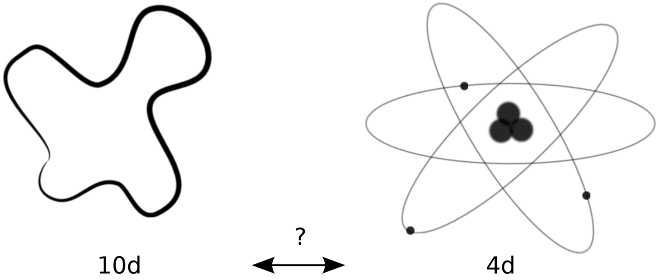
Geometry?

A problem...



… and its meaning

Structure of space-time?

- "Ordinary" space-time for the 4-dimensional part
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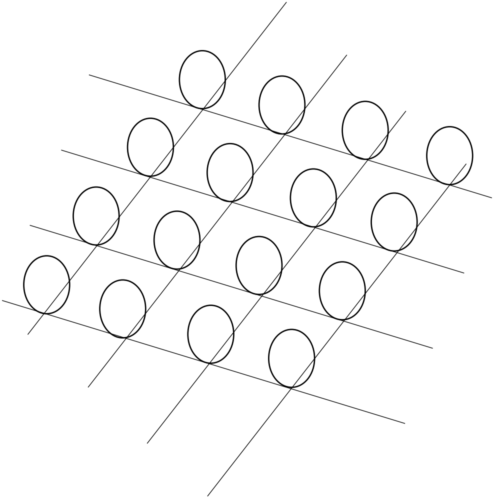
Mathematically: a manifold. What kind of manifold?

- First guess: a product

$$M_{10} = M_4 \times M_6$$

Example: $\mathbb{R}^2 \times S^1$



The geometry of M_4

- General relativity: a four-dimensional, smooth, connected Lorentzian manifold
- Cosmology: homogeneous and isotropic universe
  $\Rightarrow$ Maximally symmetric space
  $\Rightarrow$ Constant curvature
  $\Rightarrow$ M_4 is either Minkowski, AdS or dS

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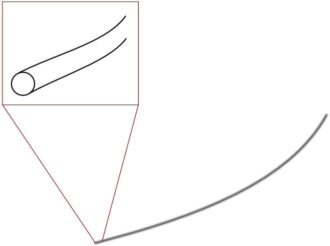

Ok, let's do that!

- Find a solution to the theory:
  - Solve the equations of motion
  - Here: type II string theory in its low-energy approximation (supergravity)
  - Take the easiest setup: a solution that gives 4d vacuum
  - Expectation value of the fermion fields: $<(\text{fermion})>= 0$
- Preserve supersymmetry:

$$\delta_\varepsilon(\text{fermion}) = (\text{boson}) \overset{!}{=} 0$$
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- Supersymmetry $\Rightarrow$ equations of motion (luckily!)

Equations, equations!

- Variation of the gravitino field:

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- Variation of the gravitino field:

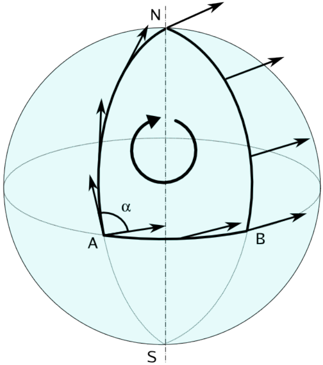
$$\delta_\varepsilon \Psi_\mu = \nabla_\mu \varepsilon \stackrel{!}{=} 0$$

- Can be decomposed:
  - On M_4: $\mathcal{R}_4 = 0$
  - On M_6: $\nabla_m \eta = 0$

    $\Rightarrow$ Internal manifold has covariantly constant spinor

    $\Rightarrow$ Strong geometrical constraint: reduced holonomy

Holonomy?



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The result: a Calabi-Yau manifold

- Holonomy reduced to $SU(3)$, Ricci-flat ($Ric_{\mu\nu} = 0$)
- Examples: Torus T^2 (in 2d), T^4 and $K3$ (in 4d)

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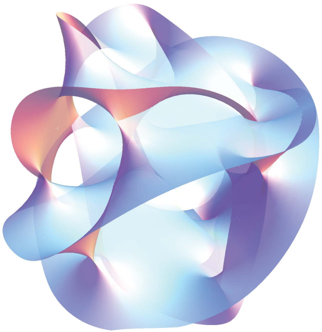

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Not good

A Calabi-Yau geometry is not satisfactory:

- M_4 is bound to be Minkowski.
- There are moduli.
- There is too much supersymmetry.

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- Can we allow for more?
  Yes, a warped product!

  $$\mathrm{d}s^2 = e^{2A(y)} g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + g_{mn} \mathrm{d}y^m \mathrm{d}y^n$$

  Size of M_4 depends on the position in M_6.

Susy does not like warping

- The internal components of $\nabla_\mu \varepsilon \stackrel{!}{=} 0$ become

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- M_6 compact $\Rightarrow \nabla A = 0$
  $\Rightarrow$ no warping, M_4 again Minkowski.

Flux?

Fluxes in the closed string spectrum

- Type II closed string spectrum:

NS-NS	$\mathbf{8}_v \otimes \mathbf{8}_v = \mathbf{35} \oplus \mathbf{28} \oplus \mathbf{1}$	$g_{\mu\nu},\ B_{\mu\nu},\ \Phi$
R-R	$\mathbf{8}_s \otimes \mathbf{8}_{s/c} =$ p-forms	RR-fields
NS-R	$\mathbf{8}_v \otimes \mathbf{8}_{s/c} = \mathbf{8}_{s/c} \oplus \mathbf{56}_{s/c}$	$\Psi_\mu,\ \lambda$
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- RR-fluxes:
  - Tensor decomposition:

$$\mathbf{8}_s \otimes \mathbf{8}_c = C^{(1)} \oplus C^{(3)} \qquad \text{IIA}$$

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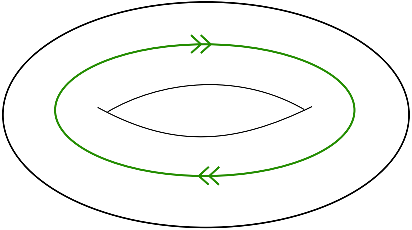
  - Take RR-fields as potentials: $F^{(n+1)} = \mathrm{d}C^{(n)}$
- H-flux: $H = \mathrm{d}B$

Flux in M_6

- Take electrodynamics as inspiration: field strength $F^{(2)}$
- Fluxes can wrap cycles in the internal manifold.



Fluxes and geometry I

- Flux lines $\rightarrow$ energy density
- Decreasing volume $\rightarrow$ increasing density $\rightarrow$ "pressure"
- Mathematically: Derive equations of motion from action and find for example

$$\mathcal{R}_6 + \frac{1}{2} g_s^2 |F_2|^2 + \frac{3}{2} \left(g_s^2 |F_0|^2 - |H|^2 \right) = 0$$

Fluxes and geometry II

- Supersymmetry variation reloaded:

$$\delta_\varepsilon \Psi_\mu = \left(\nabla_\mu + \frac{1}{8} \mathcal{P} \not{H}_\mu \right) \varepsilon + \frac{1}{16} e^\Phi \sum_n \frac{1}{(2n)!} \not{F}^{(2n)} \Gamma_\mu \mathcal{P}_n \varepsilon$$

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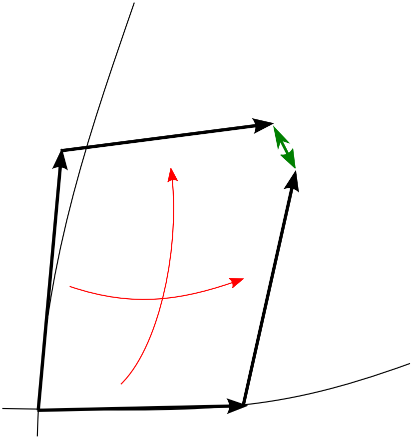
- Decompose derivative of the spinor,

$$\nabla_\mu \varepsilon = (\text{torsion classes}) \neq 0$$

$\Rightarrow$ Fluxes induce torsion!

Torsion?



Torsion!

Classification of M_6 with respect to its torsion:

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Classification of M_6 with respect to its torsion:

- Complex
- Symplectic
- Half-flat
- Special Hermitean
- Nearly Kähler
- Almost Kähler
- Kähler
- Conformal Calabi-Yau
- Calabi-Yau

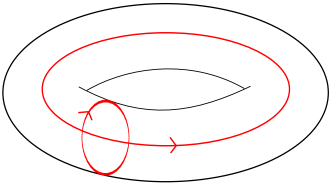
Fluxes and geometry III

Geometry also affects the choice of the fluxes:

- Dirac quantisation condition

$$\int_{\Sigma_p} F^{(p)} = N_p \in \mathbb{Z}$$

- Values are determined by integrals over cycles
- Number of different cycles is determined by topological features of M_6



Compactification?

$$\begin{aligned} \int \mathrm{d}^{10}x \; (10\mathrm{d}) \quad &\rightarrow \quad \int \mathrm{d}^4 x \int \mathrm{d}^6 x \; (4\mathrm{d}) \cdot (6\mathrm{d}) \\ &= \quad (\text{number}) \cdot \int \mathrm{d}^4 x \; (4\mathrm{d}) \end{aligned}$$

Geometry and compactification I

- A type II example:

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^{10}x \; e^{-2\phi} \sqrt{|g|} \left(\mathcal{R} + 4|\mathrm{d}\phi|^2 - \frac{1}{2}|H|^2 \right)$$

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- Integrating out the internal dimensions:

$$S = M_4^2 \int \mathrm{d}^4x \sqrt{|g_{\mu\nu}|} \left(\mathcal{R}_4 + \mathrm{kin} + \sigma^{-2}\rho^{-1}\mathcal{R}_6 - \frac{1}{2}\sigma^{-2}\rho^{-3}|H|^2 \right)$$

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  - give rise to masses for moduli.

Take-away message

Fluxes $\rightarrow$ Internal geometry $\rightarrow$ 4-dimensional theory