Building a minimal dark matter model with kinetic mixing

- a small cookbook -

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The AIM

Extending the Standard Model gauge group

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \xrightarrow[\text{breaking}]{\text{sp. symmetry}} SU(3)_C \otimes U(1)_{\mathrm{em}}$$

by an extra $\mathrm{U}(1)_X$ via kinetic mixing

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X \xrightarrow[\text{breaking}]{\text{sp. symmetry}} SU(3)_C \otimes U(1)_{\mathrm{em}}$$

Why such a model?

- few additional parameters
  - tight constraints possible
  - lesser assumptions can be made by model builder
- operators are of dimensions < 4
  - renormalizability garanteed

The first minimal dark matter models proposed by C.Boehm and P. Fayet '03.

How to start

Let us first examine the SM and the dark gauge group independently

$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ $\qquad$ $U(1)_X$

and build the Lagrangian for the dark U$(1)_X$

Constructing the Lagrangian for the $U(1)_X$

The Lagrangian for a new group structure has three basic ingredients

$$\mathcal{L}_{\mathcal{X}} = \text{kinetic field term} + \text{particle field term} + \text{source term}$$

The field Lagrangians

Each field has its own Lagrangian

- Dirac field → Dirac particles (spin 1/2)

$$\mathcal{L}_{\mathcal{D}} = \bar{\psi}(i\partial\!\!\!/_{\mu} - m_{\psi})\psi$$

- Klein-Gordon field → scalar particles (spin 0)

$$\mathcal{L}_{\mathcal{KG}} = (\partial_{\mu}\phi)^{\dagger}(\partial^{\mu}\phi) - m_{\phi}^{2}\phi^{\dagger}\phi$$

- Vector field → vector particles (spin 1)

$$\mathcal{L}_{\mathcal{V}} = -\frac{1}{4}(\partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu})(\partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}) + \frac{1}{2}m_{A}^{2}A_{\mu}A^{\mu}$$

Constructing the Lagrangian for the U(1)$_X$

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$$\mathcal{L}_{\mathcal{X}} = \text{kinetic field term} + \text{particle field term} + \text{source term}$$

The kinetic field term

The field term consists of two contracted field strength tensors and a mass term in the massive case

$$-\frac{1}{4}C_{\mu\nu}C^{\mu\nu}+\frac{1}{2}m_X^2 C_\mu C^\mu$$

where the field strength tensor in the Abelian case is

$$C_{\mu\nu}=\partial_\mu C_\nu-\partial_\nu C_\mu \ .$$

In a non-Abelian case

$$C^a_{\mu\nu}=\partial_\mu C^a_\nu-\partial_\nu C^a_\mu+gf^{abc}C^b_\mu C^c_\nu$$

Fun fact : $C^a_{\mu\nu}C^{a\mu\nu}=(\partial_\mu C^a_\nu-\partial_\nu C^a_\mu)^2+2(\partial_\mu C^a_\nu-\partial_\nu C^a_\mu)gf^{abc}C^{b\mu}C^{c\nu}$
$+g^2 f^{eab}f^{ecd}C^a_\mu C^b_\nu C^{c\mu}C^{d\nu}$

Constructing the Lagrangian for the U(1)$_X$

The Lagrangian for a new group structure has three basic ingredients

$$\mathcal{L}_{\mathcal{X}} = -\frac{1}{4} C_{\mu\nu} C^{\mu\nu} + \frac{1}{2} m_X^2 C_\mu C^\mu$$
$$+ \text{ particle field term}$$
$$+ \text{ source term}$$

The particle field term

What type of particles are interacting with the new vector boson C_μ?

- Dirac field $\rightarrow$ Dirac particles (spin 1/2)

$$\bar{\chi}(i\partial\!\!\!/_\mu - m_\chi)\chi$$

- Klein-Gordon field $\rightarrow$ scalar particles (spin 0)

$$(\partial_\mu \chi)^\dagger (\partial^\mu \chi) - m_\chi^2 \chi^\dagger \chi$$

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$$\mathcal{L}_{\mathcal{X}} = -\frac{1}{4} C_{\mu\nu} C^{\mu\nu} + \frac{1}{2} m_X^2 C_\mu C^\mu$$
$$+ \bar{\chi}(i\partial\!\!\!/_\mu - m_\chi)\chi$$
$$+ \text{ source term}$$

The source

The source is a current resulting from local gauge invariance

$$g_X J_X^{\mu} C_{\mu} \quad .$$

- Dirac particles $\rightarrow$ Dirac current

$$J_X^{\mu} = \epsilon_{Z'}^{\chi} \bar{\chi} \gamma^{\mu} \chi$$

- Scalar particles $\rightarrow$ Klein-Gordon current

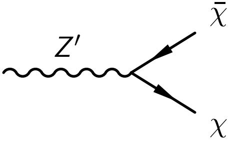
$$J_X^{\mu} = \epsilon_{Z'}^{\chi} \chi^{\dagger} (i \overleftarrow{\partial_{\mu}} - i \overrightarrow{\partial^{\mu}}) \chi$$

Where are the couplings?

The couplings are "hidden" in the source term

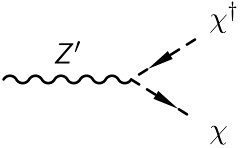
- Dirac current: $\bar{\chi}\gamma^{\mu} g_X \epsilon^{\chi}_{Z'} \chi C_{\mu}$

$$g_X \epsilon^{\chi}_{Z'} \gamma^{\mu}$$



- Klein-Gordon current: $\chi^{\dagger} g_X \epsilon^{\chi}_{Z'} (i\overleftarrow{\partial_{\mu}} - i\overrightarrow{\partial^{\mu}})\chi C_{\mu}$

$$g_X \epsilon^{\chi}_{Z'} (p_2 - p_1)^{\mu}$$



Constructing the Lagrangian for the U(1)$_X$

The Lagrangian for a new group structure has three basic ingredients

$$\mathcal{L}_{\mathcal{X}} = -\frac{1}{4} C_{\mu\nu} C^{\mu\nu} + \frac{1}{2} m_X^2 C_\mu C^\mu$$
$$+ \bar{\chi}(i\partial\!\!\!/_\mu - m_\chi)\chi$$
$$+ \bar{\chi}\gamma^\mu g_X \epsilon^\chi_{Z'} \chi C_\mu$$

Kinetic mixing - introducing a link

B. Holdom '86

Kinetic mixing constitutes a possibility to link the Standard Model with the dark sector:

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_X - \frac{\epsilon}{2} C_{\mu\nu} B^{\mu\nu} \quad (1)$$

Kinetic mixing matrix

- bring the field terms into bilinear form

$$\mathcal{L}_{\text{field}} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} F^{a}_{\mu\nu} F^{a\mu\nu} - \frac{1}{4} C_{\mu\nu} C^{\mu\nu} - \frac{\epsilon}{2} C_{\mu\nu} B^{\mu\nu} + \ldots$$

$$= -\frac{1}{4} \vec{F}^T \begin{pmatrix} 1 & \epsilon & 0 \\ \epsilon & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \vec{F} + \ldots$$

with $\vec{F} = (C_{\mu\nu}, B_{\mu\nu}, F^3_{\mu\nu})^T$

Mass matrix

- bring the mass terms into bilinear form

$$\mathcal{L}_{\text{mass}} = \frac{1}{2}(0, v)\left[\frac{g_1^2}{4}(B_\mu)^2 + \frac{g_2^2}{4}(A_\mu^3)^2 + \frac{g_1 g_2}{4}A_\mu^3 B^\mu\right]\begin{pmatrix}0\\ v\end{pmatrix} + \frac{1}{2}m_X^2 (C_\mu)^2 + \ldots$$

$$= \frac{1}{2}\vec{V}^T \begin{pmatrix} m_X^2 & 0 & 0 \\ 0 & \frac{1}{4}v^2 g_1^2 & -\frac{1}{4}v^2 g_1 g_2 \\ 0 & -\frac{1}{4}v^2 g_1 g_2 & \frac{1}{4}v^2 g_2^2 \end{pmatrix} \vec{V} + \ldots$$

with $\vec{V} = (C_\mu, B_\mu, A_\mu^3)^T$

Kinetic Mixing - Gauge Boson Masses

- Eigenvalues of the rotated mass matrix $\tilde{M}$

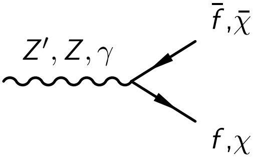
$$m_\gamma^2 = 0,\ m_Z^2 = \frac{(q-p)}{2},\ m_{Z'}^2 = \frac{(q+p)}{2}$$

with

$$q = m_X^2 \beta + \frac{g_1^2 \beta + g_2^2}{4} v^2,\ p = \sqrt{q^2 - 4 m_X^2 (m_Z^{SM})^2 \beta}$$

and $\beta = \frac{1}{1-\epsilon^2}$.

Kinetic Mixing - Neutral Current Couplings



- In canonical normalization

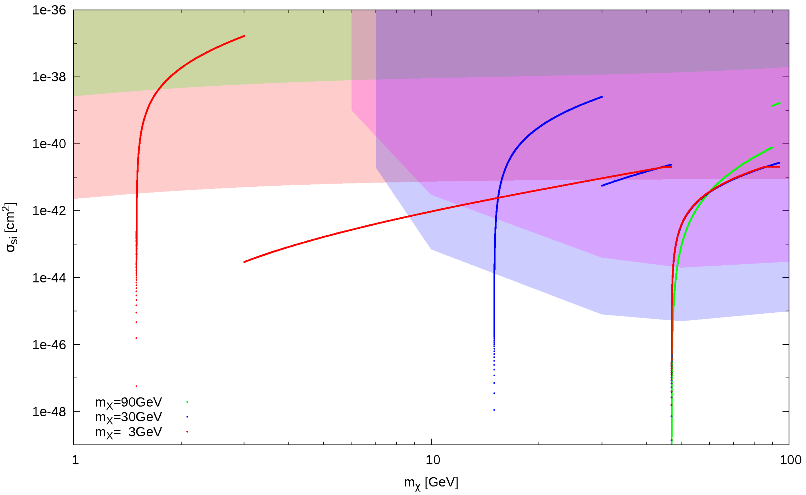
$$\mathcal{L}_{\mathcal{NC}} = \frac{g_2}{2\cos\theta_W} \bar{f}\gamma^\mu[(v'_f - \gamma_5 a'_f)Z'_\mu + (v_f - \gamma_5 a_f)Z_\mu]f$$
$$+ e\bar{f}\gamma^\mu Q_f A_\mu f + \bar{\chi}\gamma^\mu[\epsilon^\chi_{Z'} Z'_\mu + \epsilon^\chi_Z Z_\mu + \epsilon^\chi_\gamma A^\gamma_\mu]\chi$$

- For small kinetic mixing and arbitrary $m_{Z'}$ the coupling factors to SM fermions f and dark fermions χ are

$$v_f, a_f = v_f^{SM}, a_f^{SM} + \mathcal{O}(\epsilon^2)$$
$$\epsilon^\chi_Z \propto \epsilon + \mathcal{O}(\epsilon^3)$$
$$\epsilon^\chi_\gamma = 0 \text{ (no millicharge).}$$

$$v'_f \propto \epsilon + \mathcal{O}(\epsilon^3)$$
$$a'_f \propto \epsilon + \mathcal{O}(\epsilon^3)$$
$$\epsilon^\chi_{Z'} \propto g_X Q_X + \mathcal{O}(\epsilon^2)$$

A dark matter cross section prediction



preliminary

Summary

Summary

- learned how to construct a dark matter minimal Lagrangian
- used kinetic mixing to link the Standard Model with the dark sector
- discovery potential in collider and direct dark matter detection experiments

Outlook

- update limits

References

Literature

- Scalar Dark Matter candidates (C. Boehm, P. Fayet '03): `arxiv:hep-ph/0305261`
- Two U(1)'s and ϵ charge shifts (B. Holdom '86): `Phys. Lett. B, 166, 196`
- First Dark Matter Results from the XENON100 Experiment (XENON100 Collaboration '10): `arxiv:1005.0380 [astro-ph]`
- Status and Sensitivity Projections for the XENON100 Dark Matter Experiment (E. Aprile, L. Baudis '09): `arxiv:0902.4253 [astro-ph.IM]`