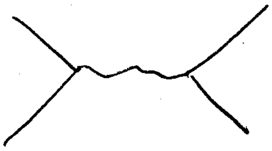
Hot field theory

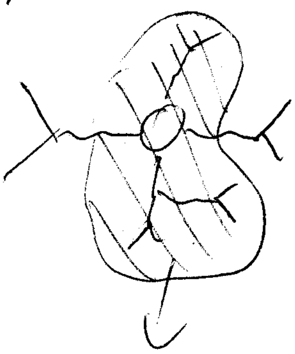
1) Motivation.

Thermal field theory: statistical, relativistic qft

$T=0$



"simple"



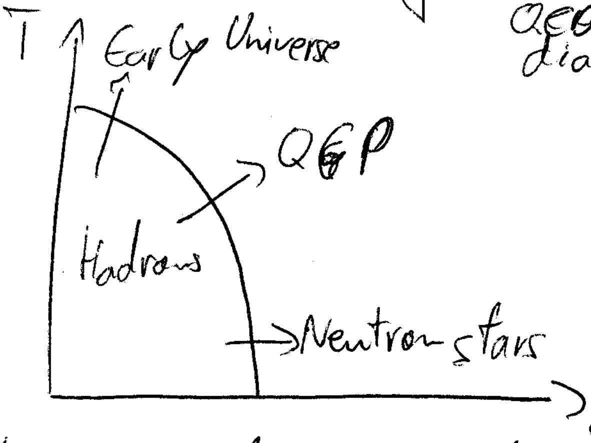
"complicated"

average: over all states between "simple" $T>0$ theory

We need for:

- High T → early universe
- High ρ → QGP → High density (SN, neutron stars)

QCD phase diagram



↳ corrections to decay rates, scattering rates

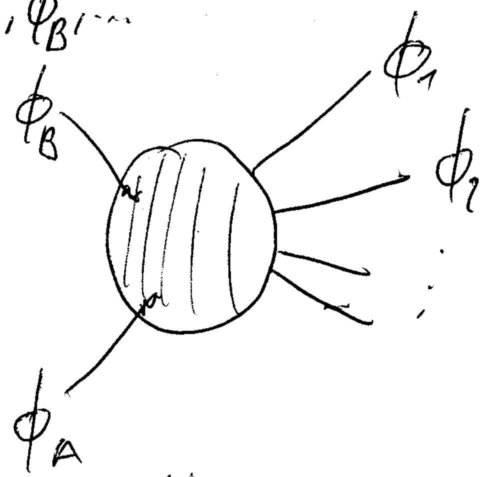
- new excited states: ↳ thermal masses
  ↳ quasiparticles (e.g. longitudinal photons, Plasminos..)

2) Thermal field theory

2.1.) Vacuum FT

- Calculate Decay Rates, Scattering cross sections

fields : $\phi_A, \phi_B, \dots$



Probability for this process:

$P = |\langle \phi_1 \phi_2 \dots | \phi_A \phi_B \rangle|^2 \to$ fourier transform

$\to {}_{out}\langle p_1 p_2 \dots | k_A k_B \rangle_{in} \equiv \langle p_1 p_2 \dots | S | k_A k_B \rangle$

$S = 1 + iT \to$ interaction part

$$\langle p_1 p_2 \dots | iT | k_A k_B \rangle = (2\pi)^4 \delta^4(k_A + k_B - p_1 - p_2 \dots)\, i\mathcal{M}$$

$i\mathcal{M}$: Feynman rules in momentum space
read them $\mathcal{L}_{int}$

ϕ^4-theory : $\mathcal{L}_{int} = -\frac{g^2}{4!}\phi^4$

Feynman rules : 1) $\circ\!\!-\!\!-$ 1

2) $\times$ $\left(-i\frac{g^2}{4!}\right)$

3) $\circ\!\!-\!\!-\!\!\circ$ $i\Delta_F(x-y) = \langle 0 | T(\phi_x \phi_y) | 0 \rangle$

4) $\int d^4 k$ loops   5) symmetry

2.2) Density Operator (von Neumann 1927)

$$\rho = \sum p_n \underbrace{|n\rangle\langle n|}_{\text{classical probabilities}}$$

Example: polarization of photons $|L\rangle, |R\rangle$

$$|\varphi\rangle = (\alpha|L\rangle + \beta|R\rangle)\, C \quad \text{(pure state)}$$

$$\rho = \frac{1}{2}|L\rangle\langle L| + \frac{1}{2}|R\rangle\langle R|$$

Canonical ensemble

$$\rho = C\sum_n e^{-\beta E_n}\,|n\rangle\langle n| \qquad \beta = \frac{1}{T}$$

$$C = \frac{1}{\sum_n e^{-\beta E_n}} = \frac{1}{Z(\beta)}$$

$$\rho = \frac{1}{Z(\beta)}\, e^{-\beta H}$$

2.3) Green's functions @ $T > 0$

$$\langle 0|A|0\rangle \to \langle 0|T(\phi_x\phi_y)|0\rangle$$

@ finite T: $\mathrm{tr}(\rho A)$

Propagator: $$\langle A\rangle_\beta = \mathrm{tr}\left(\frac{1}{Z}e^{-\beta H}A\right) = \frac{1}{Z}\sum\langle n|A|n\rangle e^{-\beta E_n}$$

$$i\Delta_F(x-y) = \frac{1}{Z}\sum\langle n|T(\phi_x\phi_y)|n\rangle e^{-\beta E_n} = \ldots =$$

$$= \int\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\left[\left[\overset{T=0}{1} + \overset{\text{induced production @ } y}{n_\beta(\omega_k)}\right]e^{-ik(x-y)} + \underbrace{n_\beta(\omega_k)\, e^{ik(x-y)}}_{\text{absorption at } x}\right]$$

(Bose enhancement)

y —— x spontaneous production ① $T=0$

$$\omega_k = \sqrt{k^2 + m^2} \qquad n_\beta(\omega_k) = \frac{1}{e^{\beta\omega_k} - 1}$$

24) Imaginary Time Formalism

imaginary time $t \to -i\tau$, τ real

CLAIM: $i\Delta_F(x) = iT \sum_{k_0} \int \frac{d^3k}{(2\pi)^3} \frac{i}{k^2-m^2} e^{-ik\cdot x}$

$k_0 = 2\pi i n T$ Matsubara modes

$$\Delta(x-y) = \frac{1}{Z} \mathrm{Tr}\left(e^{-\beta H} T\phi(x,\tau)\phi(y,0)\right) = \ldots =$$

$$= \frac{1}{Z} \mathrm{Tr}\left(e^{-\beta H} T(\phi(x,\tau)\phi(y,\beta))\right) \quad \beta > \tau$$

$\Delta = \Delta(\tau + n\beta) \quad n \in \mathbb{Z}$

$\tau \in [0,\beta[$

$\Rightarrow$ FEYNMAN RULES:

1) $i\Delta_F^{T>0}(k) = \frac{i}{k^2-m^2}$

$k_E^2 = k_0^2 + k_1^2 + k_2^2 + k_3^2$

2) $\int \frac{dk_0}{2\pi} \to iT \sum_{k_0 = 2\pi i n T}$

3) $-i4!g^2$

4) SYMMETRY

FERMIONS: $i\Delta_F^{T>0}(k) = \frac{(\not{k}+m)i}{k^2-m^2}$

$k_0 = (2n+1)\pi i T$