Small circles, big circles and branes

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Outline

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1. Small circles
2. Big circles

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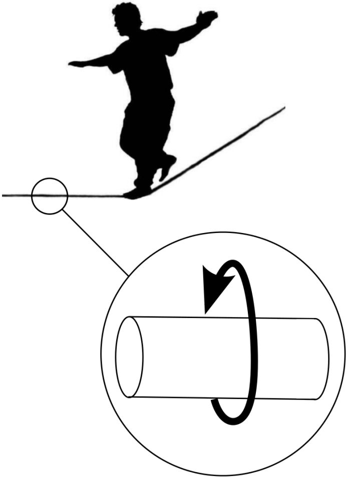
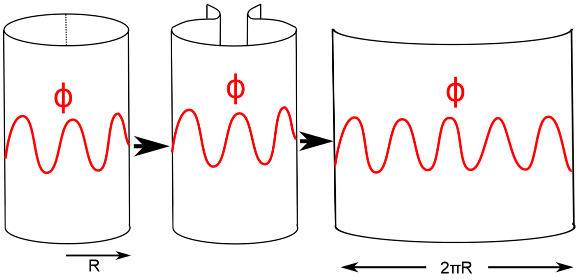
Tiny extra dimensions

- Bosonic string theory is 25+1-dimensional.
- Our universe is 3+1-dimensional.
- We resolve this discrepancy by compactifying the extra dimensions from string theory.



Scalar fields and extra dimensions



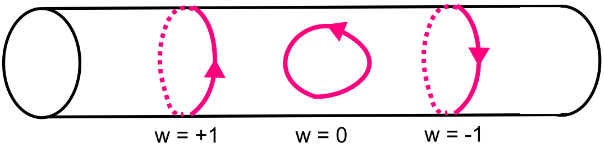
$$\phi(x^\mu, x^{25}) = \phi(x^\mu, x^{25} + 2\pi R)$$

$$\phi(x^M) = \sum_n \phi_n(x^\mu) \exp\left(\frac{inx^{25}}{R}\right)$$

$$\partial_M \partial^M \phi = m^2 \phi \qquad \Rightarrow \qquad \partial_\mu \partial^\mu \phi_n = \left(m^2 + \frac{n^2}{R^2}\right) \phi_n$$

Strings and extra dimensions

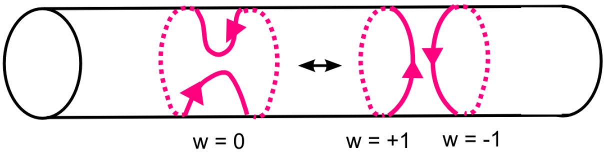
- The string mass also depends on its momentum in the compact dimension, just like a point particle.
- But strings can wind around the compact dimension as well:



- The number of times a string winds is called the *winding number*, w.
- The winding number also influences the string mass by adding extra tension.

String mass

- Winding number is conserved, just like momentum:



- Can define momentum in the compact dimension by

$$p_L = \frac{n}{R} + \frac{wR}{\alpha'} \qquad p_R = \frac{n}{R} - \frac{wR}{\alpha'} \,.$$

- The mass becomes

$$m^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{\alpha'^2} + \frac{2}{\alpha'}\left(N + \tilde{N} - 2\right)$$

Some limits

$$m^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{\alpha'^2} + \text{oscillators}$$

$R \to \infty$

- Winding states become infinitely massive.
- Compact momentum becomes a continuous spectrum.

$R \to 0$

- Winding states become a continuous spectrum.
- Compact momentum states become infinitely massive.

In both cases, the spectrum is that of a *noncompact* dimension!

A surprising duality

$$m^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{\alpha'^2} + \text{oscillators}$$

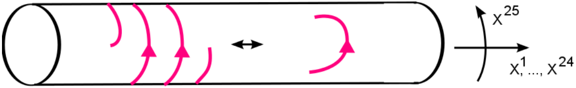
remains unchanged when

$$R \leftrightarrow \frac{\alpha'}{R}, \qquad n \leftrightarrow w$$

and in fact the situations are physically identical!
This is known as *T-duality*.
Small circles are the same as big circles.

What about open strings?

- Open strings can always be unwrapped — they have no winding number.



- Mass spectrum

$$m^2 = \frac{n^2}{R^2} + \text{oscillators}$$

is not invariant under $R \leftrightarrow \frac{\alpha'}{R}$.

- Is this the end of T-duality?

Open string end points

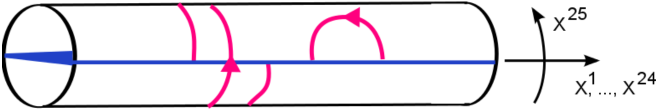
- What makes open strings different is the end points.
- There are two types of boundary conditions:

$$\partial_\sigma X = 0 \quad \text{(Neumann)} \qquad\qquad \partial_\tau X = 0 \quad \text{(Dirichlet)}$$

- For Neumann boundary conditions, the string endpoints are free to move.
- For Dirichlet boundary conditions, the string endpoints are stuck to a surface, called a *D-brane*.

When there are no Dirichlet boundary conditions, the string is on a D25-brane that fills all space.

Putting a D-brane in the compact dimension



Notice:

- The open strings are now stuck. $p^{25} = 0$.
- It is now possible to define a winding number.

We can rescue T-duality for open strings

- $m^2 = \frac{n^2}{R^2} + \text{oscillators}$
- Radius R
- D25-brane
- Momentum mode n

- $m^2 = \frac{w^2 R^2}{\alpha'^2} + \text{oscillators}$
- Radius $\frac{\alpha'}{R}$
- D24-brane
- Winding number w

We can conclude:

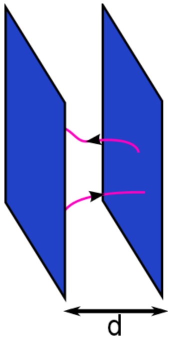
D-branes are a fundamental part of string theory.

Let's look at D-branes further

- For two D-branes separated by distance d,

$$m^2 = \frac{d^2}{(2\pi\alpha')^2} + \text{oscillator}$$

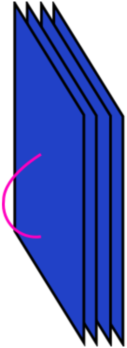
- For massless excitations, each string forms a $U(1)$ gauge field A_μ.



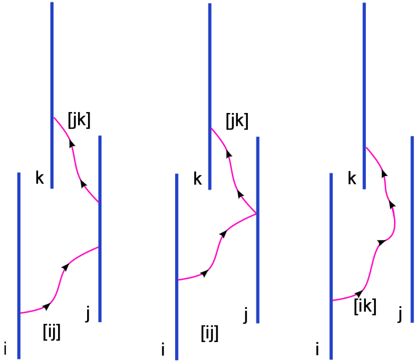
Coincident branes

- Consider a stack of N coincident branes.
- A string can start on brane i and end on brane j.
- $\Rightarrow$ there are N^2 possible strings.
- $\Rightarrow$ there are N^2 possible $U(1)$ gauge fields.

A $U(1)^{N^2}$ gauge symmetry on the branes?



Open string interactions on coincident branes

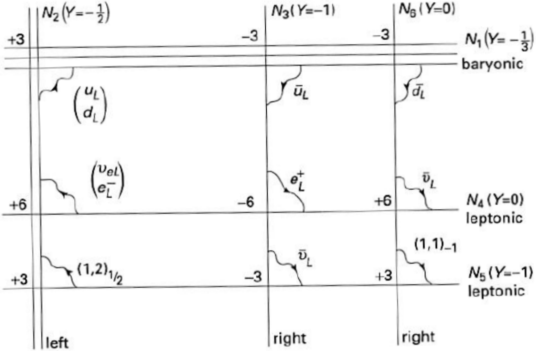


Because of interactions, the gauge group becomes $U(N)$.

Open string interactions on coincident branes

- Can label the string ground state as $|p^{\mu}; ij\rangle$.
- This label is called a *Chan-Paton factor*.
- The $U(N)$ symmetry rotates the N indistinguishable branes.

The Standard Model from D-branes



Source: Zwiebach

Conclusion

Small circles

Tiny extra dimensions, winding strings.

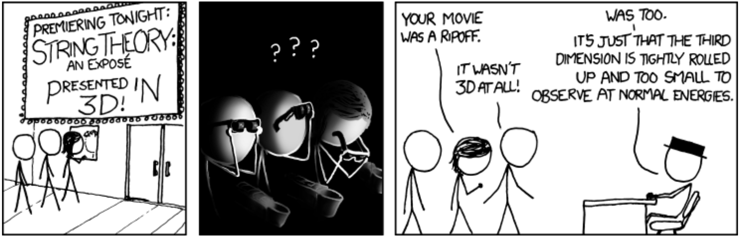
Big circles

T-duality relates small circles to big circles while keeping the physics the same.

Branes

Branes are required for T-duality of open strings. They are also useful for building gauge theories.

Thank you!



xkcd.com/848/

- B. Zwiebach, *A First Course in String Theory*
- J. Polchinski, *String Theory*