IMPRS Young Scientist Workshop 2011

Applications of String Theory: Gauge / Gravity Duality

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Motivation

Strongly Coupled Theories:

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- Examples: QCD, Quark-Gluon Plasma, High Tc Superconductors

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- lattice methods => good for computation of, e.g., hadron masses; however dynamics hard to determine

- Gauge/Gravity Duality => relatively easy computation of the dynamics

Content

- Towards Gauge/Gravity Duality
- Maldacena's conjecture
- Thermodynamics
- Hydrodynamics
- Conclusion

Towards G/G Duality

Two possible interpretations of D-Branes:

Towards G/G Duality

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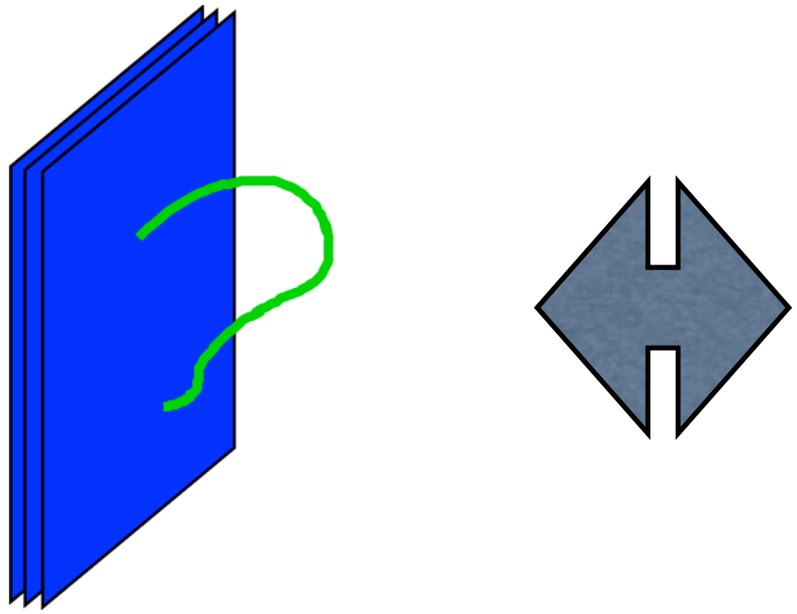

On N D-Branes open strings can end, i.e. $U(N)$ field theories live on their worldvolume

Towards G/G Duality

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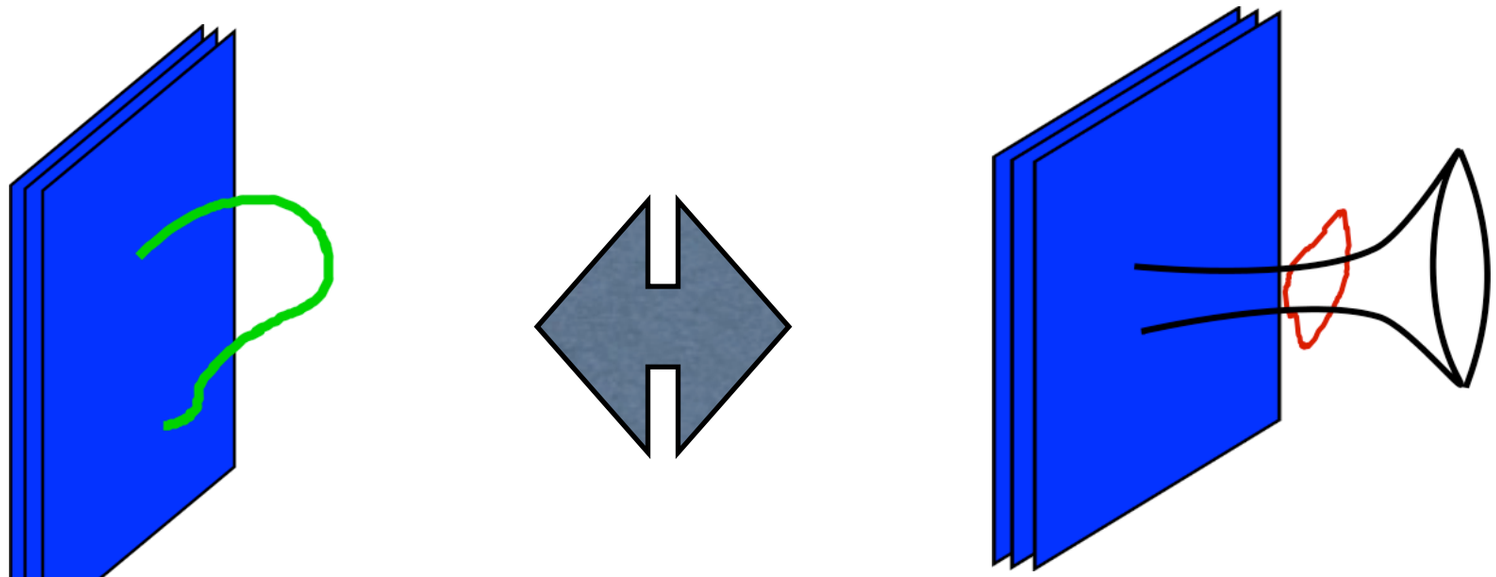


On N D-Branes open strings can end, i.e. $U(N)$ field theories live on their worldvolume

D-Branes are massive objects, i.e. they source closed strings (graviton) and therefore curve spacetime

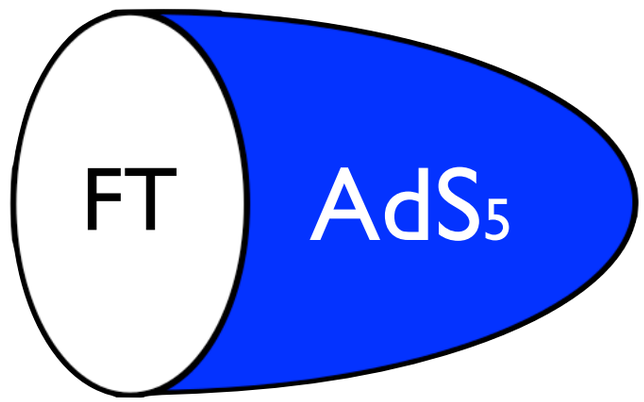
Maldacena's Conjecture

Maldacena in 1997:

for a stack of N coincident D3-branes, the

=> symmetries and the

=> dynamics (correlators)



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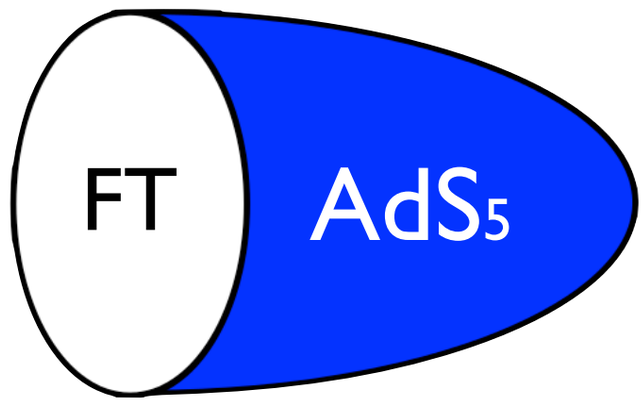


of the effective low energy theories agree

Supergravity Theory in D=5 AdS Space	DUAL	Gauge Theory in D=4 Mink. Space

weakly coupled strongly coupled

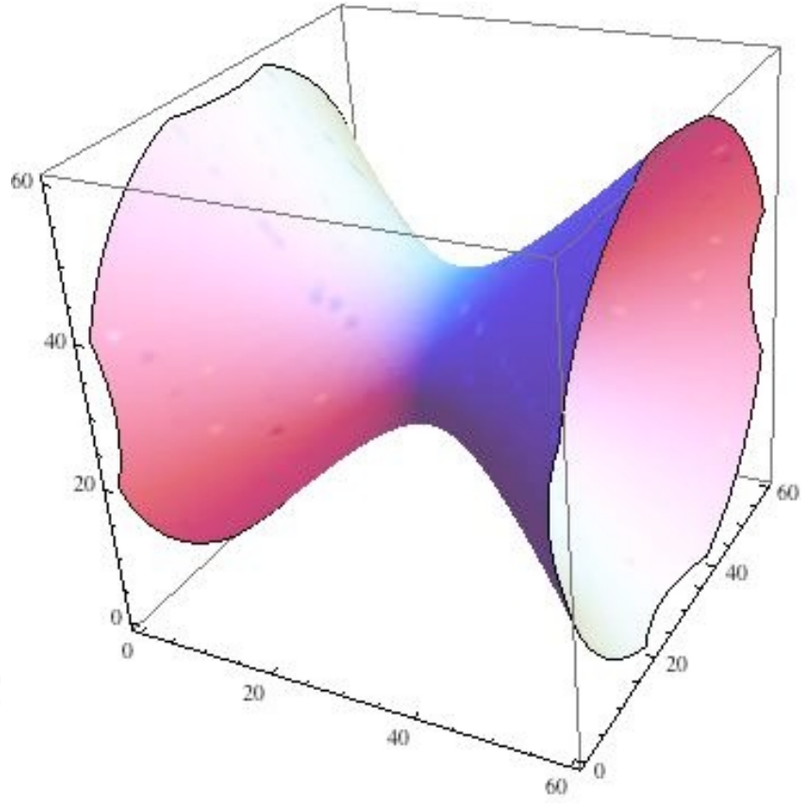
AdS Space

- constant negative curvature
- homogenous and isotropic
- conformal flat boundary

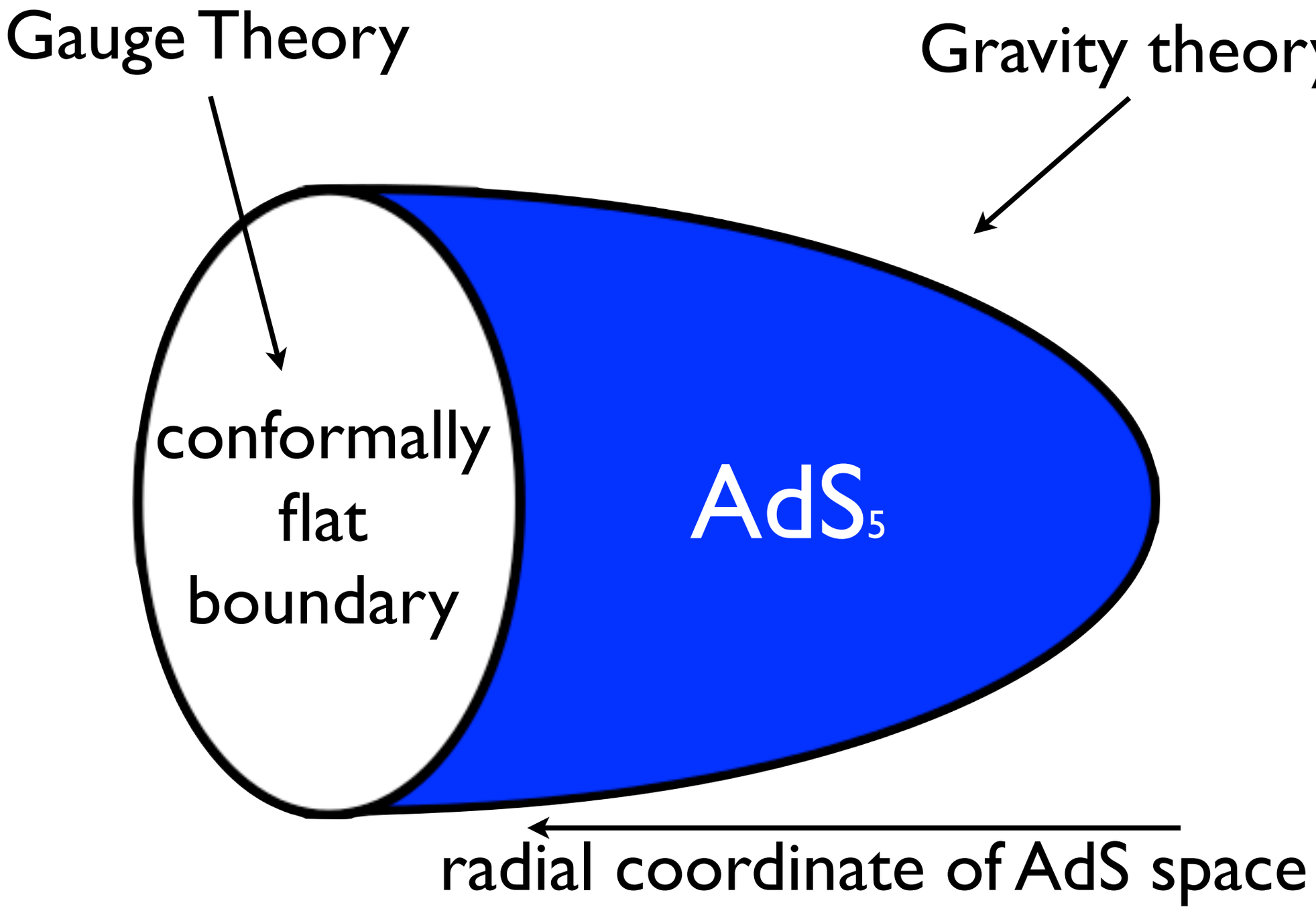
AdS_{D+1} **space:**

$$x_0^2 + x_{D+1}^2 - \sum_{i=1}^{D} x_i^2 = R^2$$

$$\mathrm{d}s^2 = -\mathrm{d}x_0^2 - \mathrm{d}x_{D+1}^2 + \sum_{i=1}^{D} \mathrm{d}x_i^2$$



AdS Space


Gauge Theory
Gravity theory
conformally
flat
boundary
AdS5
radial coordinate of AdS space

... more on the duality

Duality in a mathematical prescription:

partition function of string theory

generating functional of the FT

$$\mathcal{Z}_{\text{sugra}}[\Phi; \Phi(r_{\text{bdry}}) = \phi_{\text{bdry}}] = \left\langle e^{\int \mathrm{d}^D x \phi_{\text{bdry}} \mathcal{O}} \right\rangle_{\text{FT}}$$

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Many computations confirm the duality, however no strict mathematical proof has been found yet!!

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in general: value of supergravity field on AdS boundary sources gauge invariant operator on the field theory side

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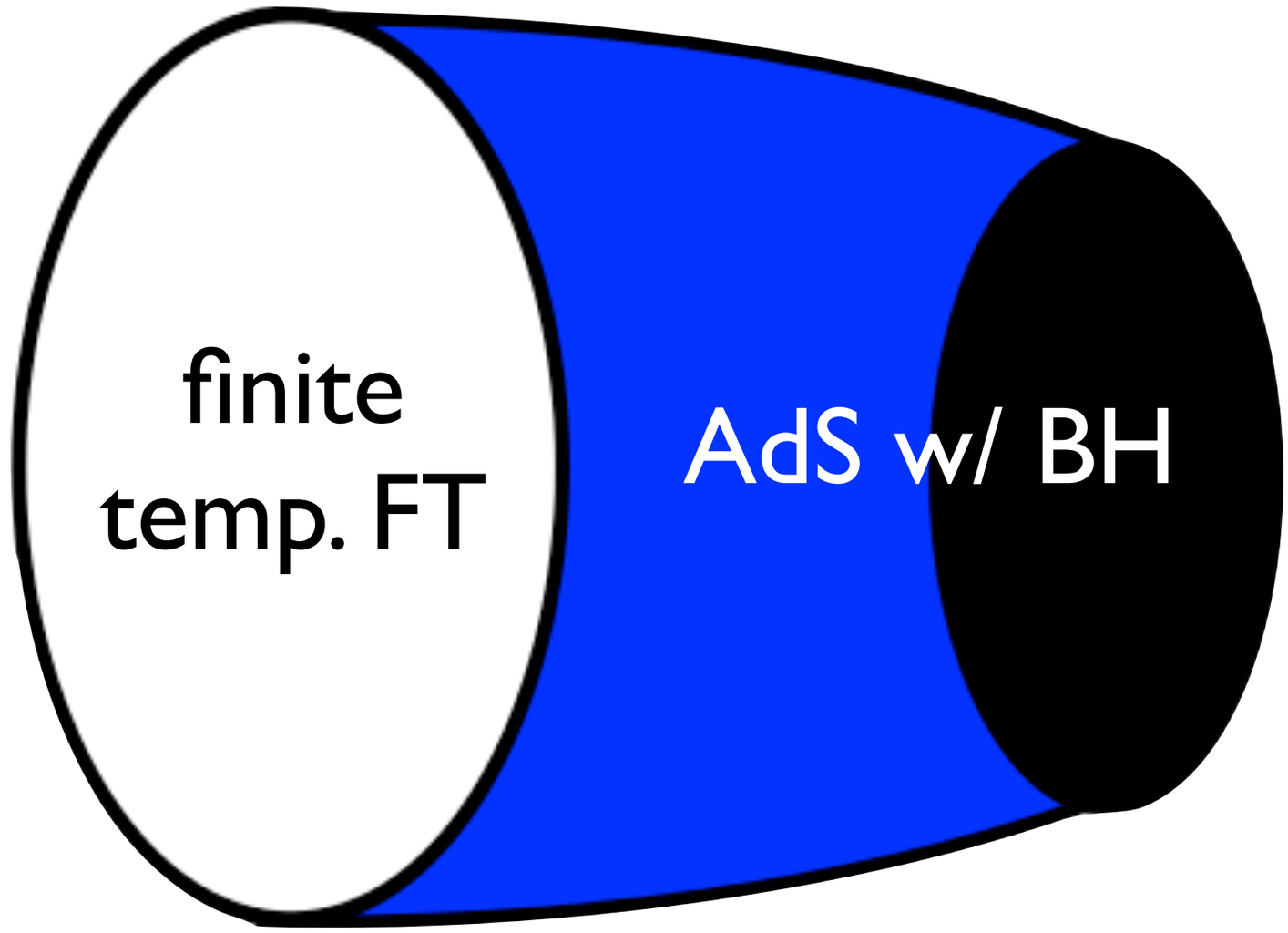
Gravity	Field Theory
• metric	• energy-momentum tensor
• $\langle A_t \rangle \neq 0$	• finite chemical pot.
• black hole	• finite temperature
• hairy black holes	• spontaneous breaking of global symmetries

Thermodynamics

Thermodynamics



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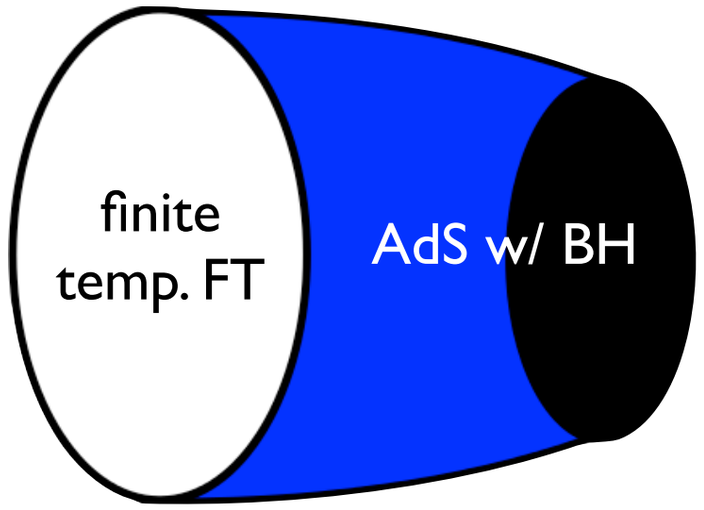

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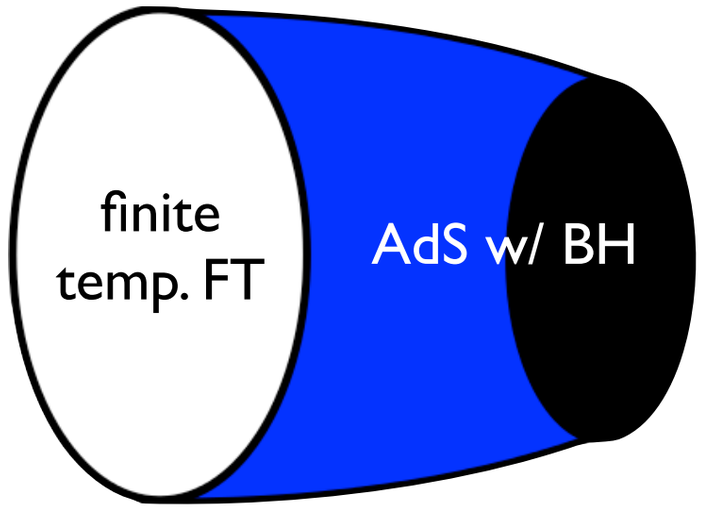
- Aim: computation of thermal properties of strongly interacting theories



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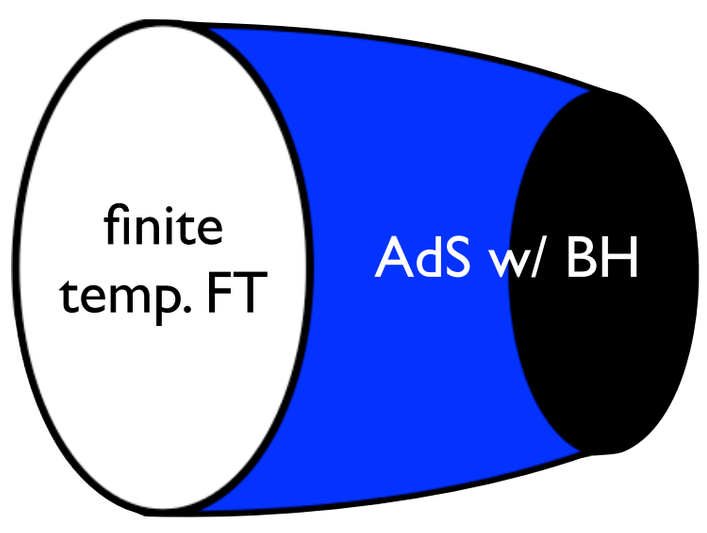

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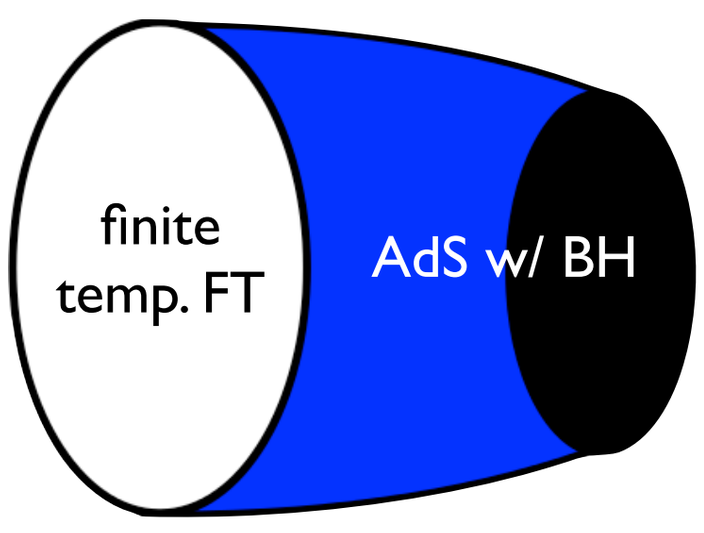

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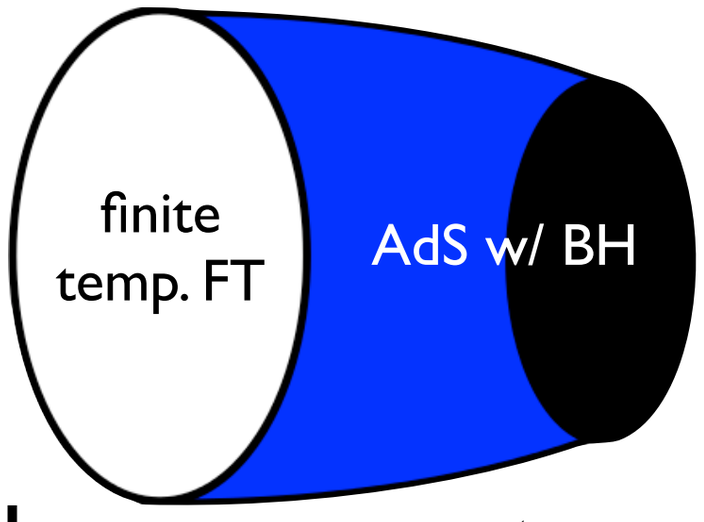
- finite density in field theory
  <=> non-vanishing gauge field component on gravity side



Computations

saddle point approximation: $\mathcal{Z}_{\rm Sugra} \simeq e^{-S_E[\phi^\star]}$

- grand canonical potential:

$$\Omega = -T \ln \mathcal{Z} \simeq T \ S_E$$

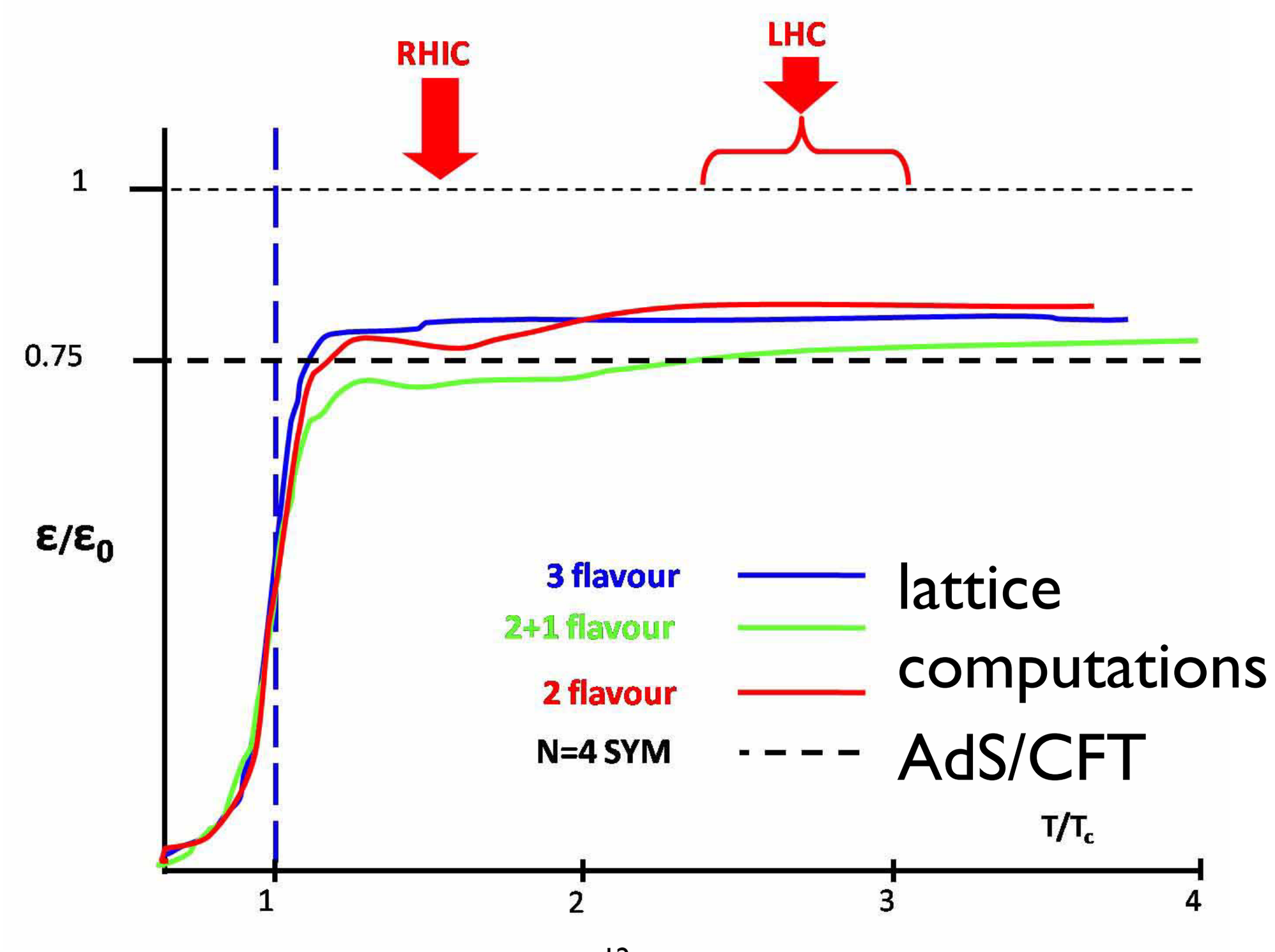
- entropy:

$$S = \frac{\partial (T \ln \mathcal{Z})}{\partial T} \simeq -\frac{\partial (T \ S_E)}{\partial T}$$

- particle number:

$$N = \frac{\partial (T \ln \mathcal{Z})}{\partial \mu} \simeq -\frac{\partial (T \ S_E)}{\partial \mu}$$

Energy Density



Myers

Hydrodynamics

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$$T^{\mu\nu} = T^{\mu\nu}_{\text{eq.}} + \Pi^{\mu\nu} \qquad J^{\mu} = J^{\mu}_{\text{eq.}} + \Upsilon^{\mu}$$

$$\Pi_{ij} \sim \eta \left(\partial_i u_j + \partial_j u_i \right)$$

The Shear Viscosity

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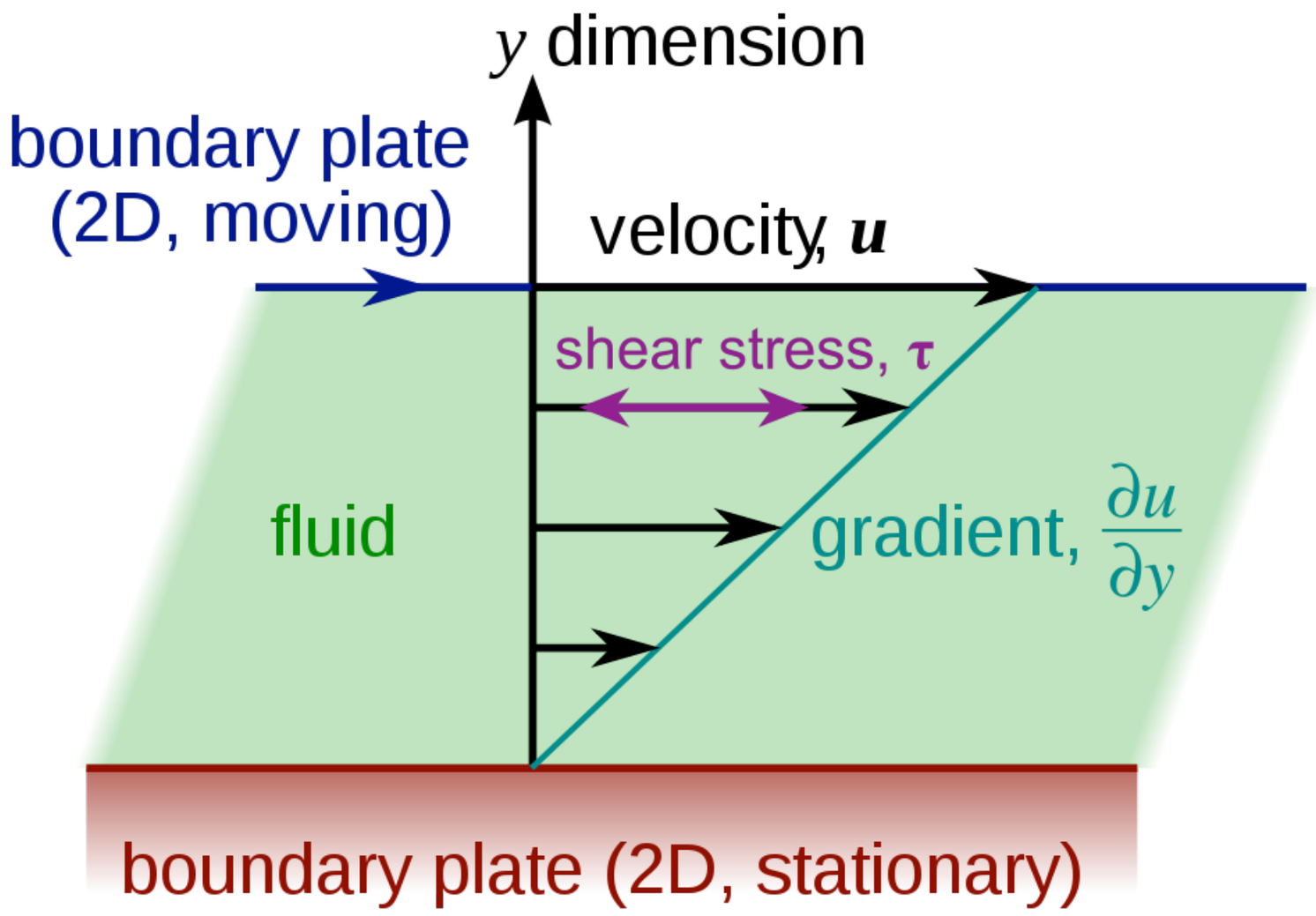

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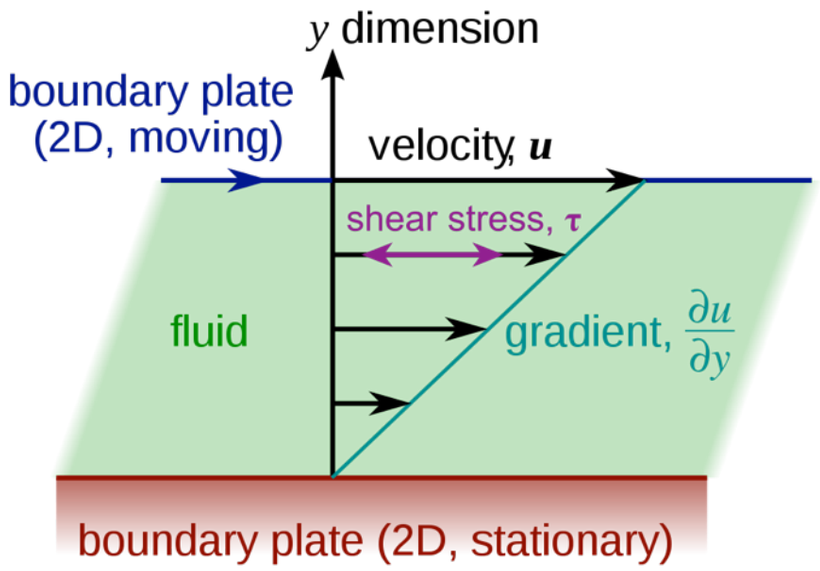
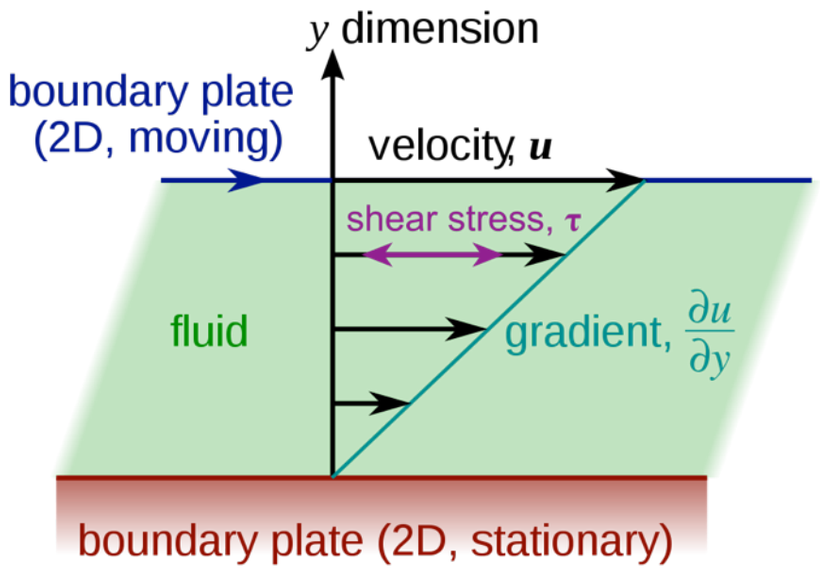


The Shear Viscosity

- measure of the momentum diffusion transverse to the momentum



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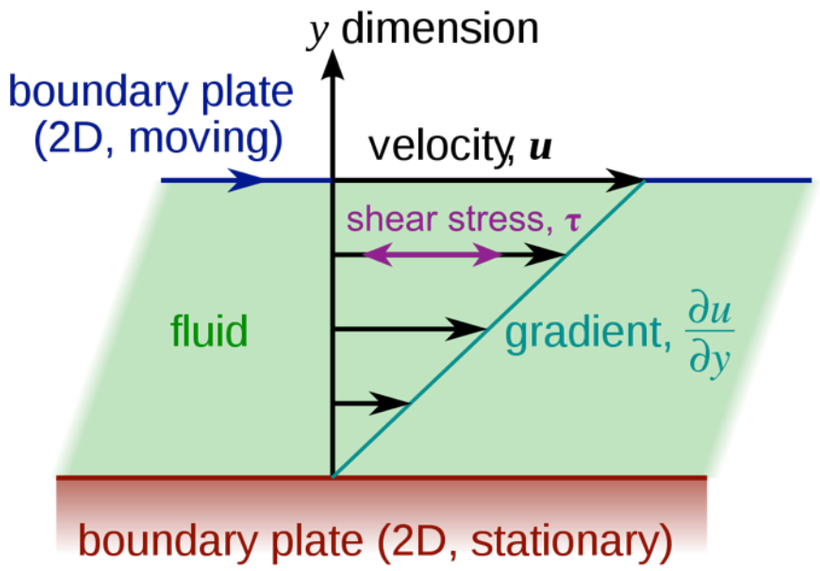

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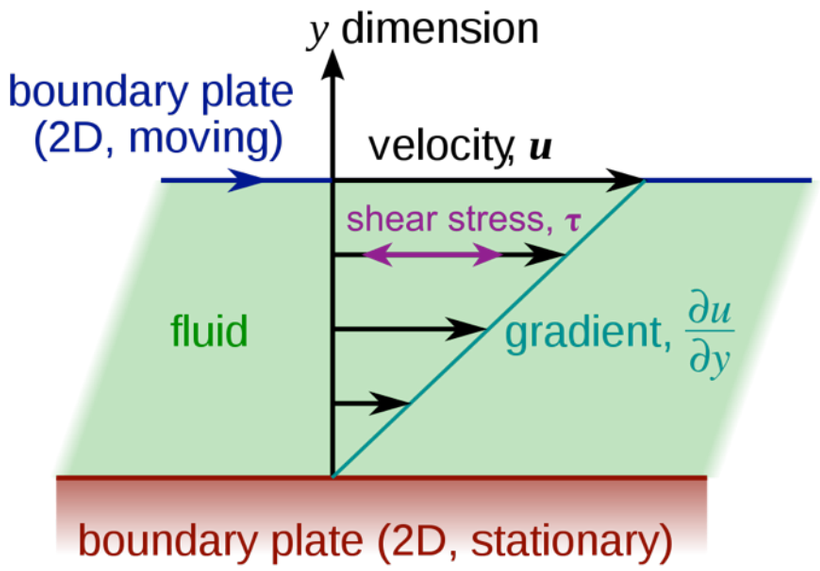
- anisotropic viscosity tensor has 21 independent components
- isotropic case: 1 independent shear viscosity η



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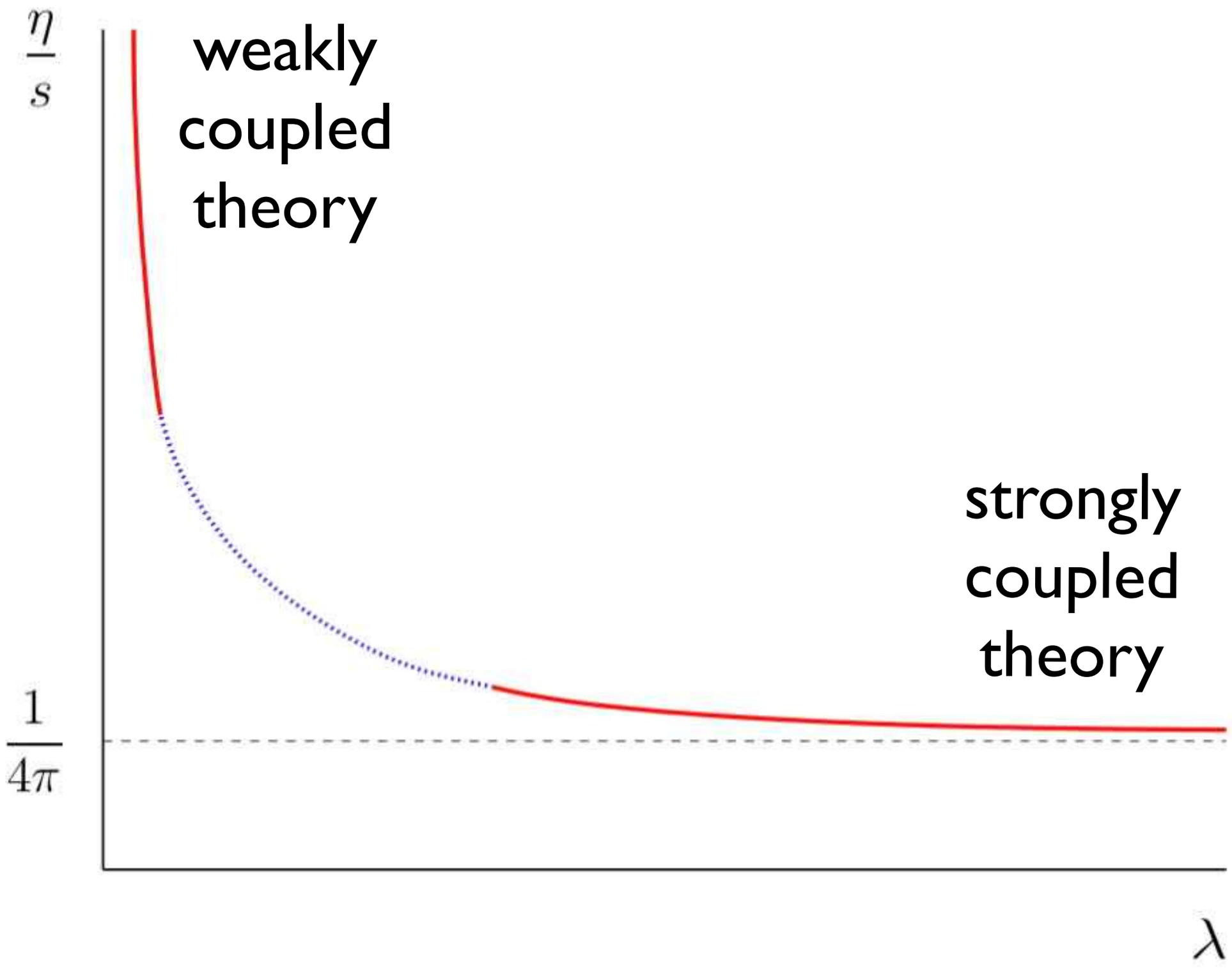
$\frac{\eta}{s}$
weakly coupled theory
strongly coupled theory
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λ

Mateos

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- Kovtun, Son and Starinets: lower bound for all fluids

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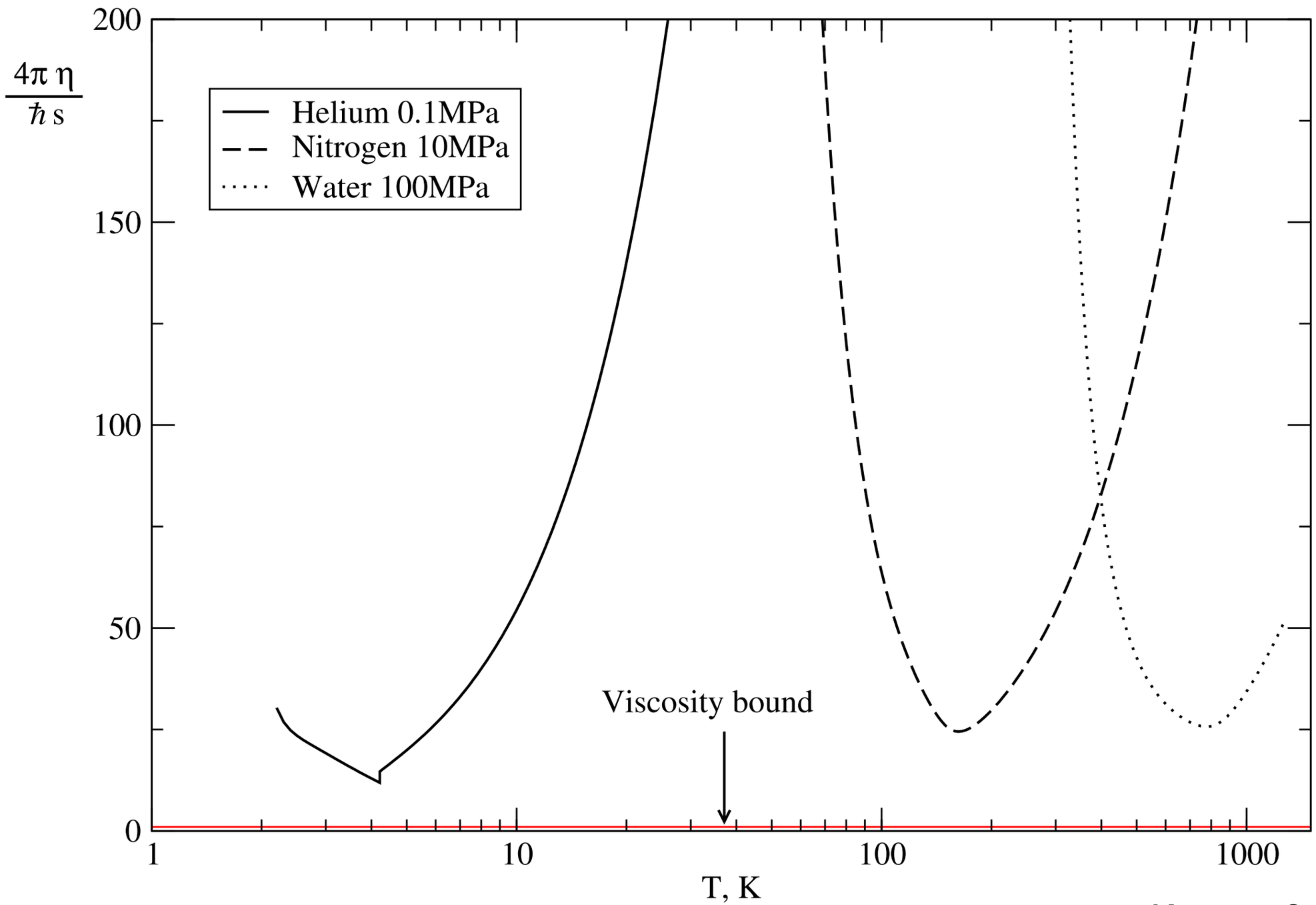

Kovtun, Son, Starinets

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- tool to determine the dynamics of strongly interacting theories with gravity dual
- predicts right order of magnitude for shear viscosity over entropy for Quark-Gluon plasma
- Outlook: using the duality to learn about Quantum Gravity!

Thank you!