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Outline

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⦿ Concept of String Theory:

From Point Particle to String
The String Spectrum
Emergence of Gravity
Scattering Amplitudes

Compactifications
T-Duality, D-branes, SM

⦿ Applications:

Gauge/Gravity Duality

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Fluxes, Moduli Stabilization

Fluxes and Geometry

⦿ Applications:

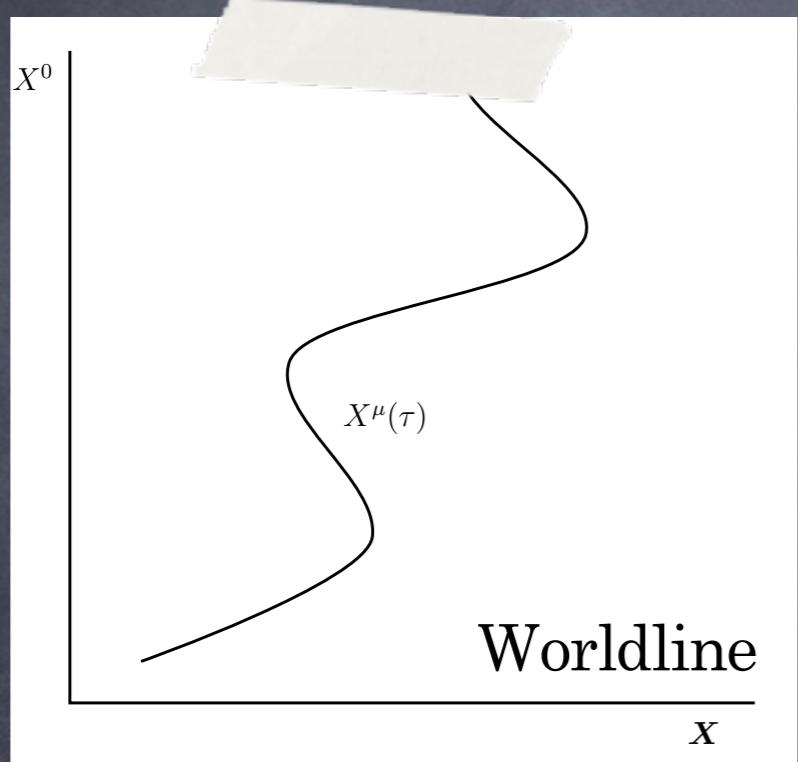
Gauge/Gravity Duality

String Cosmology

From Particles to Strings

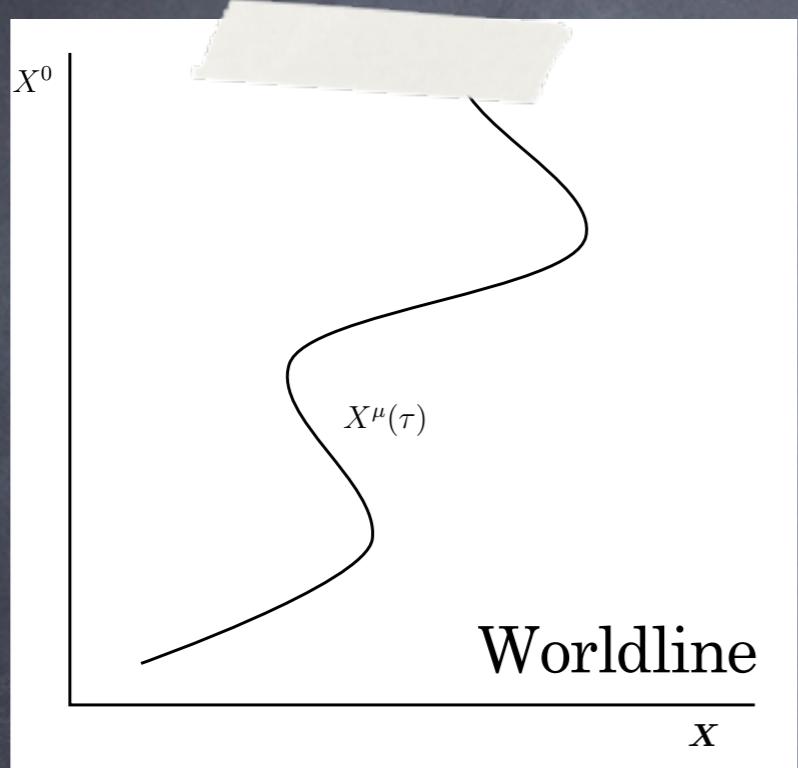
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Point Particle



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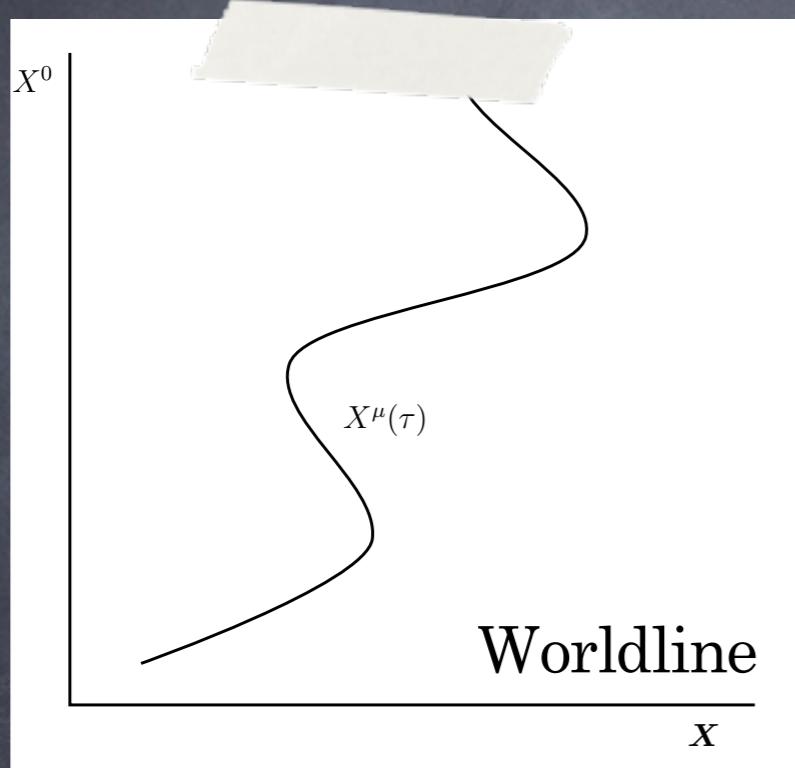


Minimal Length

$$S_{\text{pp}} = -m \int d\tau \sqrt{-\dot{X}^\mu \dot{X}_\mu}$$

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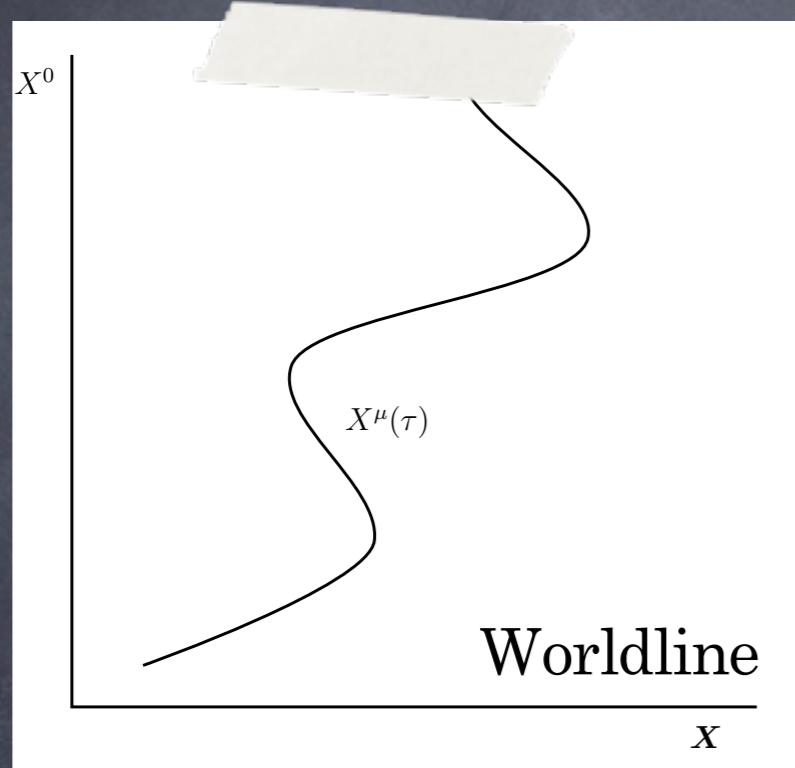
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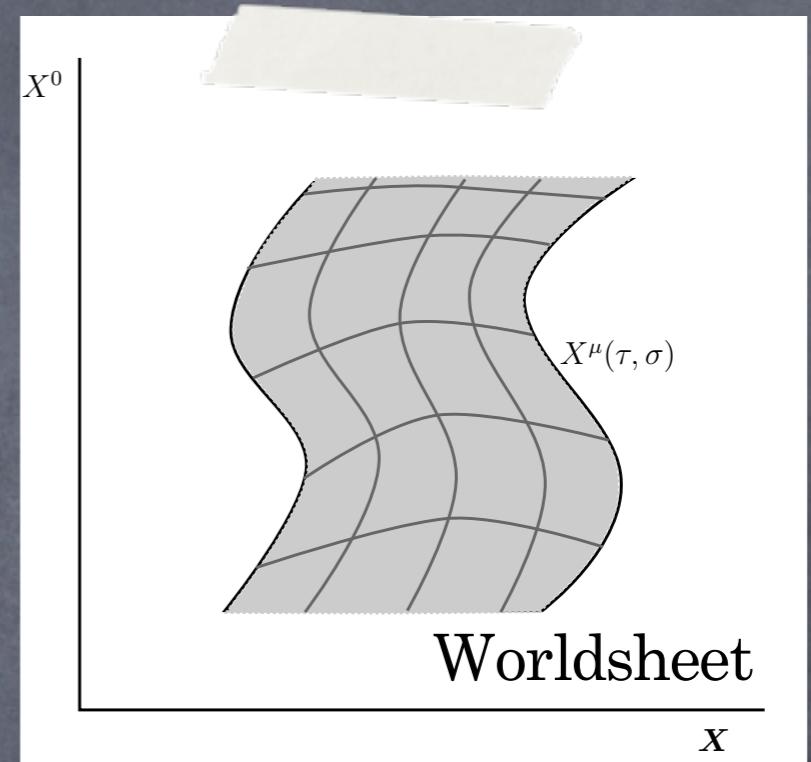
→ Free Motion

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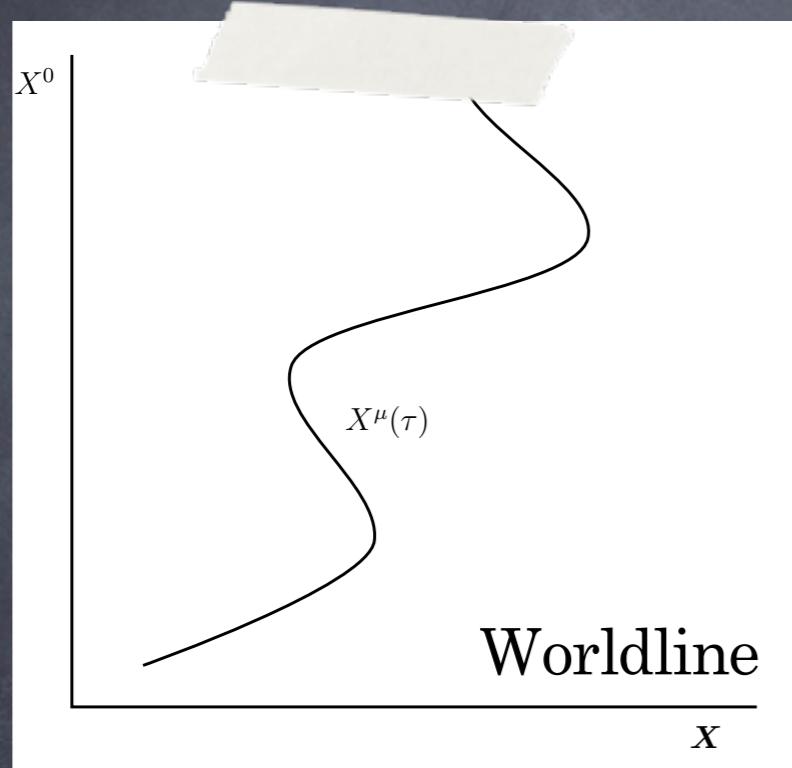
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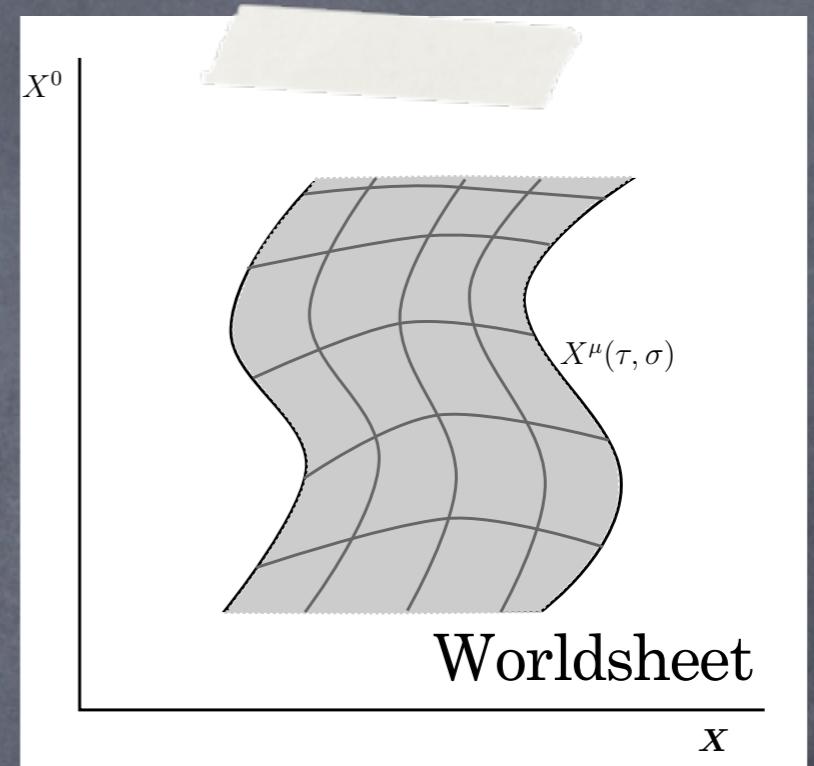
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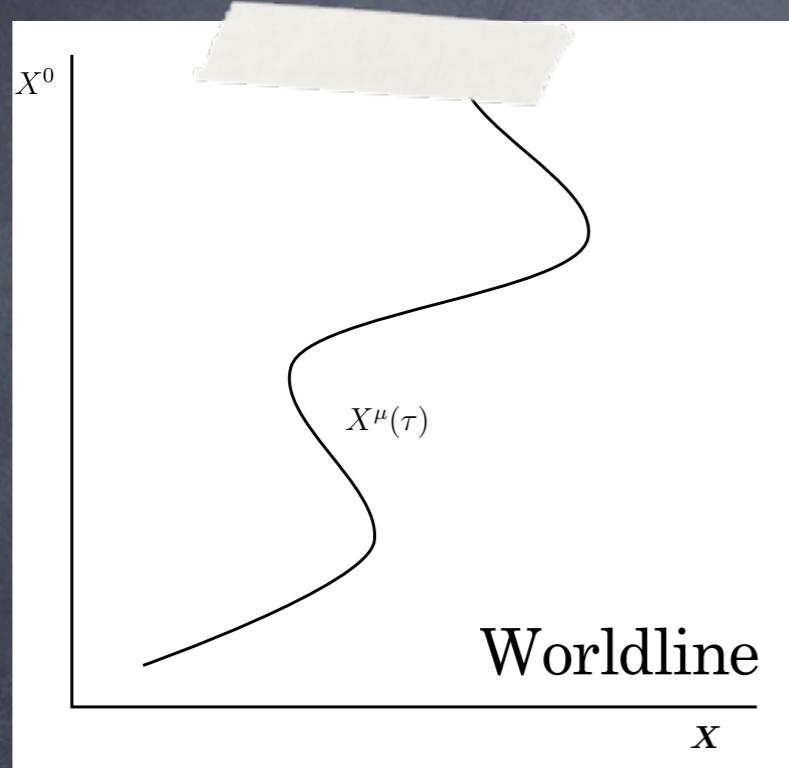
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Minimal Area

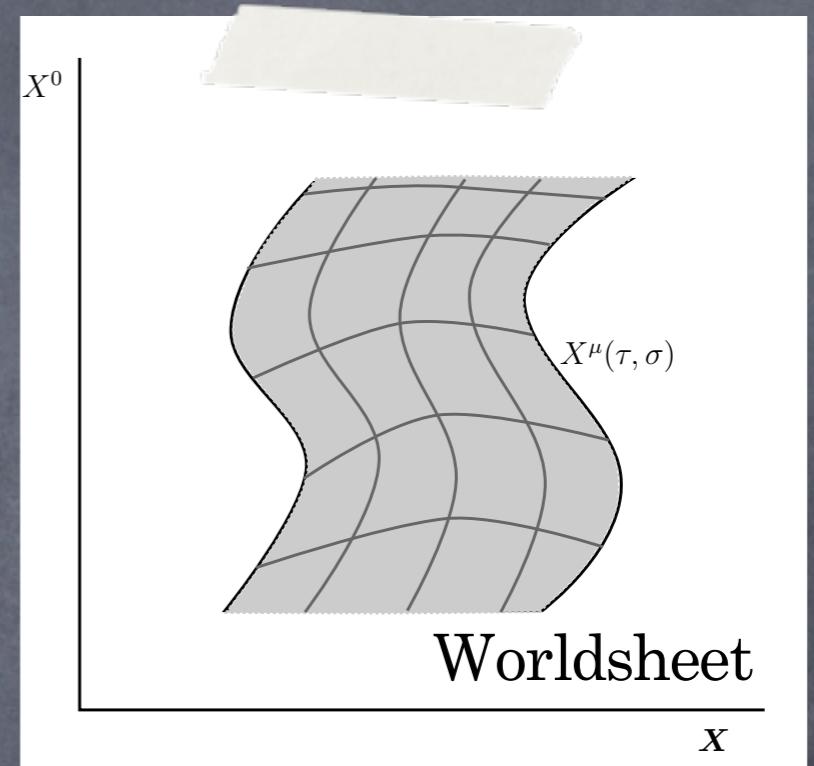
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→ More Complicated

Closer Look at the Action

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Coupling

Open String Spectrum

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$D = 26$

Gauge field

Closed String Spectrum

The closed string spectrum consists of two types of states:

- Massless states (gravitons, gauge bosons, fermions) which correspond to the massless representations of the Virasoro algebra.
- Massive states (excited states) which correspond to the massive representations of the Virasoro algebra.

The massless states are represented by the Virasoro algebra, which is a central extension of the Virasoro Lie algebra. The Virasoro algebra is generated by the Virasoro operators L_n and \bar{L}_n , where $n \in \mathbb{Z}$. The Virasoro operators satisfy the commutation relations:

$$[L_m, L_n] = m n L_{m+n} + \frac{c}{24} (m^3 - m) \delta_{m+n,0} I$$
$$[\bar{L}_m, \bar{L}_n] = \bar{n} \bar{L}_{m+n} + \frac{\bar{c}}{24} (\bar{m}^3 - \bar{m}) \delta_{m+n,0} I$$
$$[L_m, \bar{L}_n] = 0$$

where I is the identity operator and c is the central charge. The Virasoro algebra is a central extension of the Virasoro Lie algebra, which is generated by the Virasoro operators L_n and \bar{L}_n , where $n \in \mathbb{Z}$. The Virasoro Lie algebra is a Lie algebra with the same commutation relations as the Virasoro algebra, except that the central charge c is zero. The Virasoro Lie algebra is a subalgebra of the Virasoro algebra.

The massive states are represented by the massive representations of the Virasoro algebra. These representations are more complex than the massless representations, and they include additional operators such as the energy-momentum tensor and the spin operators. The massive representations are also more difficult to study than the massless representations, because they involve more complex mathematical structures such as the modular group and the Ramanujan theta functions.

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Graviton

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$D = 10$

No Tachyons

Spacetime
SUSY

Spacetime
Fermions

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Non-linear sigma model \rightarrow Interactions

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- ⦿ Weyl invariance \rightarrow Einstein Eq $R_{\mu\nu} = 0$
- ⦿ With worldsheet SUSY \rightarrow Supergravity

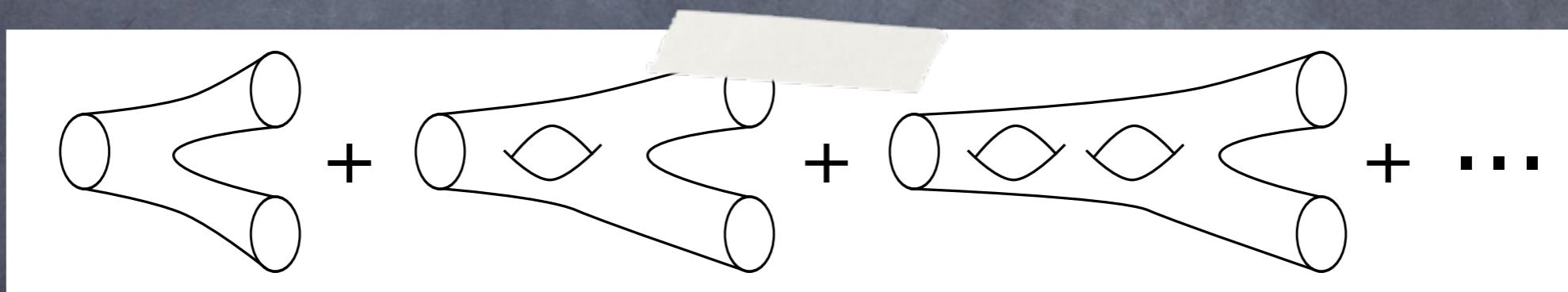
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Feynman Path Integrals \Rightarrow Feynman Diagrams

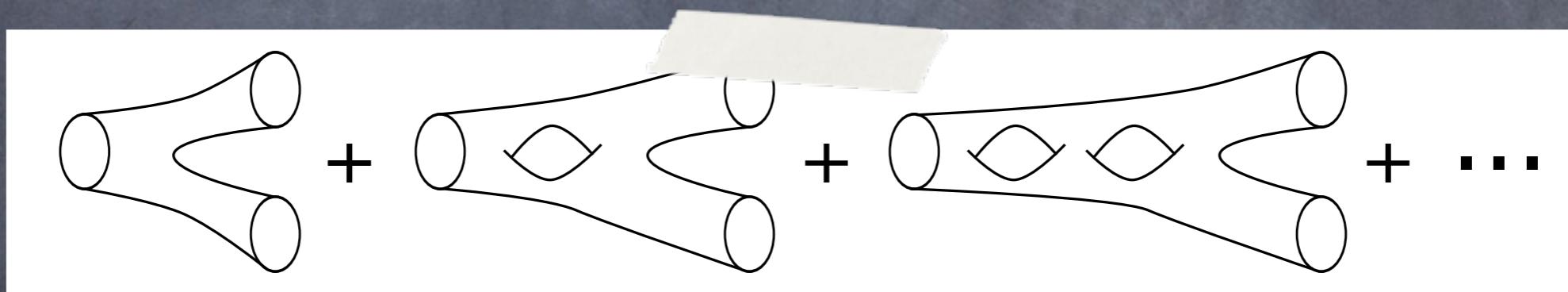
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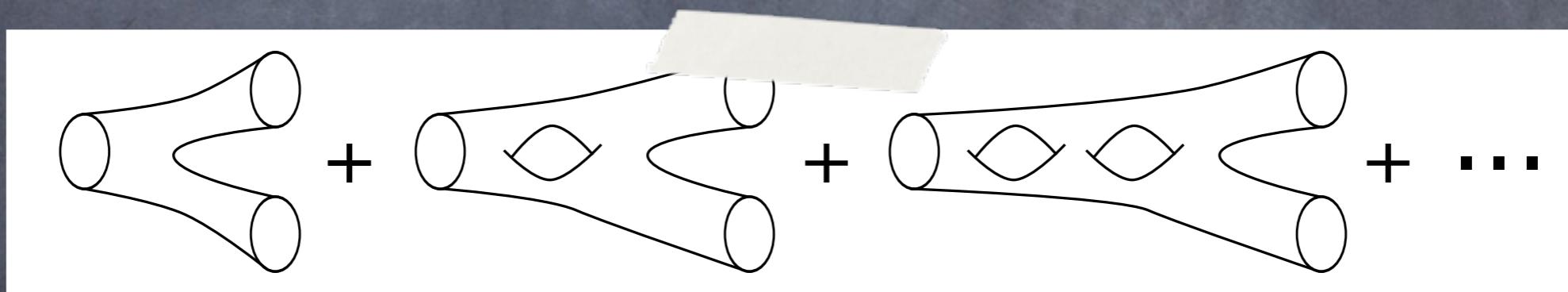


$$Z = \int_W \mathcal{D}X^\mu \mathcal{D}\gamma_{ab} e^{-I'} = \sum_g \int_{W_g} \mathcal{D}X^\mu \mathcal{D}\gamma_{ab} e^{-I'}$$

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Coupling

$$= \sum_g e^{-\lambda(2-2g)} \int_{W_g} \mathcal{D}X^\mu \mathcal{D}\gamma_{ab} e^{-I_P}$$

Take Home Message

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- Open string spectrum contains gauge field
- Closed string spectrum contains graviton
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