Hot Field Theory

Outline

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Previously

- TFT : average over interactions w/ bath
- vacuum : σ, Γ -> via $\mathcal{M}$ -> Feynman Rules
  -> Propagator $\langle 0 | T \phi_x \phi_y | 0 \rangle$

At $\langle 0 | A | 0 \rangle \xrightarrow{T>0} \mathrm{tr}(\rho A)$

- Thermal propagator (scalar)

induced creation   absorption @ x

$$i \Delta_F^{T>0}(x-y) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} \left\{ \left[1 + n_B(\omega_k) \right] e^{-ik(x-y)} + n_B(\omega_k) e^{ik(x-y)} \right\}$$

$T=0$ $\beta = \frac{1}{T}$

- Imaginary Time Formalism :

$t \to t = -i\tau \quad (\tau \in \mathbb{R})$ (for $\tau = \beta$ $\Delta(\tau) = \Delta(\tau + n\beta)$)

$$i\Delta_F^{T>0} = iT \sum_{k_0 = 2\pi i n T} \int \frac{d^3k}{(2\pi)^3} \frac{i}{k^2-m^2} e^{-ik\cdot x} \qquad e^{-\beta H} = e^{-itH}$$

$$\beta = \tau = it$$

Feynman Rules: • As in vacuum

$$\bullet \int \frac{dk_0}{2\pi} \to iT \sum_{k_0 = 2\pi i n T}$$

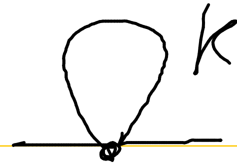
Fermions: $i S_F^{T>0}(k) = i \frac{\not{k}+m}{k^2-m^2} \qquad k_0 = (2n+1)\pi i T$

Mixed Rep.: $\Delta(\tau, \omega_k) = -T \sum_{k_0} e^{-k_0\tau} \Delta(k)$

$$\Delta(k) = -\int_0^\beta d\tau \, e^{k_0\tau} \Delta(\tau, \omega_k) = \frac{1}{k^2-m^2}$$

$$\Delta(\tau, \omega_k) = \frac{1}{2\omega_k}\left\{[1+n_B(\omega_k)]e^{-i\omega_k\tau} + n_B(\omega_k)e^{i\omega_k\tau}\right\}$$

Our first diagram :



$\mathcal{L}_{int} = -g^2 \phi^4$

$$-i\Pi = \frac{1}{2}\left(-i\,4!\,g^2\right) i\, T \sum_{k_0} \int \frac{d^3k}{(2\pi)^3}\, i\Delta(k)$$

Use $T\sum_{k_0} e^{k_0\tau} = T\sum_{n=-\infty}^{\infty} e^{2\pi i n T\tau} = T\,\delta(T\tau) = \delta(\tau)$

$$\Rightarrow T\sum_{k_0} \Delta(k) = -\int_0^\beta d\tau\, \underbrace{\left(T\sum_{k_0} e^{k_0\tau}\right)}_{\delta(\tau)} \Delta(\tau,\omega_k) = \Delta(\tau=0,\omega_k)$$

$$\Rightarrow \Pi = 12\, g^2 \int \frac{d^3k}{(2\pi)^3}\, \Delta(0,\omega_k)$$

$$= 6\, g^2 \int \frac{d^3k}{(2\pi)^3}\, \frac{1}{\omega_k}\left[\underbrace{1}_{T=0} + \underbrace{2n_B(\omega_k)}_{T>0}\right]$$

$\Pi = \Pi_{T=0} + \Pi_{T>0} \longrightarrow$ finite because n_B damps the Integral

$\Pi_{T=0}$: UV-divergent $\Rightarrow$ Renormalisation

$\Rightarrow$ Thermal does not introduce new UV-divergences

Massless case : $m = 0$

$$\boxed{\Pi = 6g^2 \int_0^\infty \frac{dk\, 4\pi k^2}{(2\pi)^3} \frac{1}{k} 2 \frac{1}{e^{k/T} - 1} \overset{x = \frac{k}{T}}{=} \frac{6}{\pi^2} g^2 T^2 \underbrace{\int_0^\infty \frac{x}{e^x - 1}}_{\zeta(2) = \frac{\pi^2}{6}} = g^2 T^2}$$

3) Hard Thermal Loop Resummation

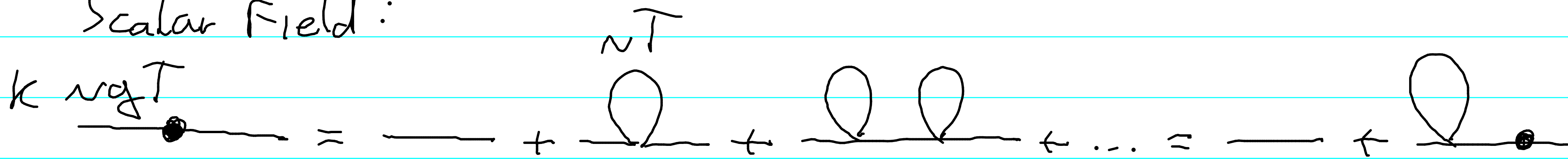
Bare thermal propagators :
- IR divergences
- gauge dependent result
- miss contributions to lower order from higher order diagrams

CURE : HTL Resummation (Braaten, Pisarski 90s)

$g \ll 1$

Separate :
- $K \sim T$ hard — bare propagators
- $K \sim gT$ soft — HTL resummed propagators

Scalar Field :



$$i\Delta^* = i\Delta + i\Delta(-i\Pi)i\Delta + \dots$$

$$\Delta^* = \Delta + \Delta\Pi\Delta + \Delta\Pi\Delta\Pi\Delta + \dots = \Delta\sum_{n=0}^{\infty}(\Pi\Delta)^n$$

$$= \Delta\frac{1}{1-\Pi\Delta} = \frac{1}{\Delta^{-1}-\Pi}$$

convergent $\Pi\Delta < 1$

$$= \frac{1}{k^2-m^2-\Pi}$$

Recall from vacuum QFT: Setting $\Delta^{-1} = 0$ gives energy-momentum relation of a „free" particle (dispersion relation, on-shell condition)

$$\Delta^{-1} = k^2 - m^2 \overset{!}{=} 0 \Rightarrow k_0 = (\pm)\omega_k \qquad \omega_k = \sqrt{k^2-m^2}$$

$$(\Delta^*)^{-1} = k^2 - m^2 - \Pi \overset{!}{=} 0 \Rightarrow k_0 = \sqrt{k^2 + \underbrace{m^2 + \Pi}_{m_{eff}^2}}$$

$m_{eff}^2 = m^2 + \Pi \approx \Pi$ if $\Pi \sim T^2 \gg m^2$

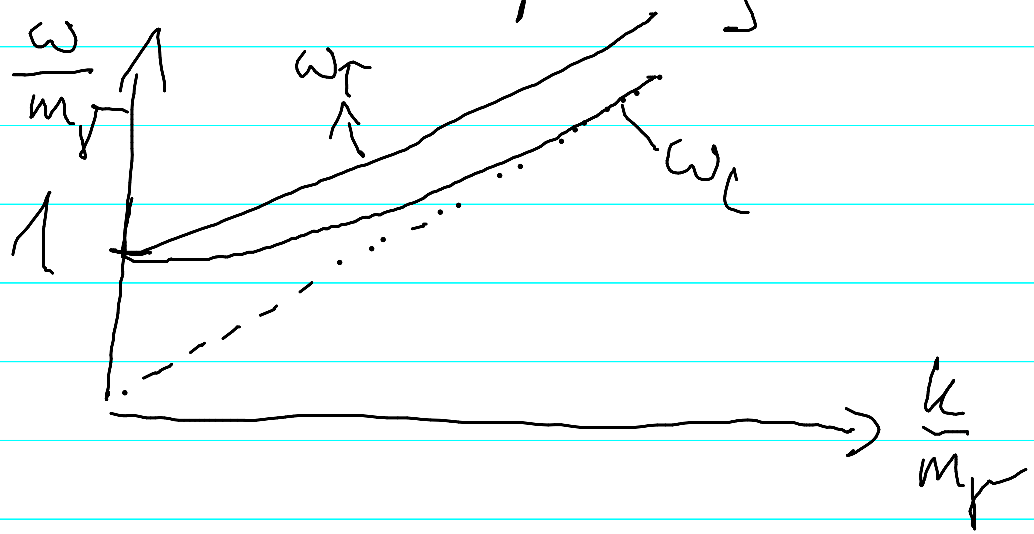
- Scalar field for $m \approx 0$: $m_{th}^2 = \Pi \approx g^2 T^2$

- Photon in QED: Propagator gets a physical Longitudinal mode

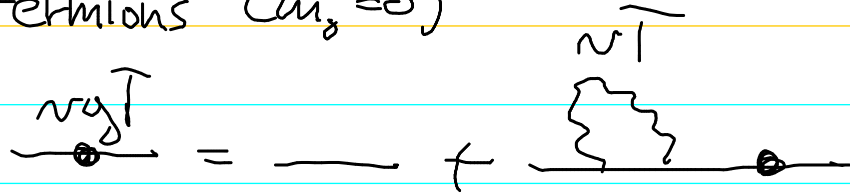
$$\sim\!\bullet\!\sim \;=\; \sim\sim \;+\; \sim\!\overset{\sim T}{\bigcirc}\!\sim \;+\; \sim\!\overset{\sim T}{\bigcirc}\!\sim\!\bigcirc\!\sim \;+\; \dots$$

High T (HTL)-limit $\quad k \sim T$

Π_L Plasmon $m_\gamma^2 = \frac{e^2 T^2}{9}$



• Fermions ($m_0 = 0$)



$$S^* = -\frac{1}{2D_+}\left(\gamma_0 - \vec{\gamma}\cdot\hat{k}\right) - \frac{1}{2D_-}\left(\gamma_0 + \vec{\gamma}\cdot\hat{k}\right)$$

ω_+ $\qquad$ $\frac{\text{Helicity}}{\text{Chirality}} = \chi = 1$ $\qquad$ ω_- $\qquad$ $\chi = -1$ $\qquad$ NEW@T>0!

$m_F^2 = \frac{e^2 T^2}{8}$ (QED)

$\omega_+ \xrightarrow{k\to\infty} \sqrt{k^2 + 2m_F^2}$

$\omega_- \xrightarrow{k\to\infty} k$

S^* conserves chirality!

SUMMARY

- TFT: Average interactions
- $\langle 0|A|0\rangle \to \mathrm{tr}(\rho A)$
- th. Propagator
- ITF $\Rightarrow$ Feynman rules
- Bath: Creation + absorption
- n_B, n_F occur naturally
- HTL res. gives improved pert. th.
- th. masses
- Modified dispersion relations
- New quasiparticles (Plasmon, Plasmino)