Numerical evaluation of jet amplitudes at NLO

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HP2, 6 September 2012, München

General solutions to virtual corrections

HP2:

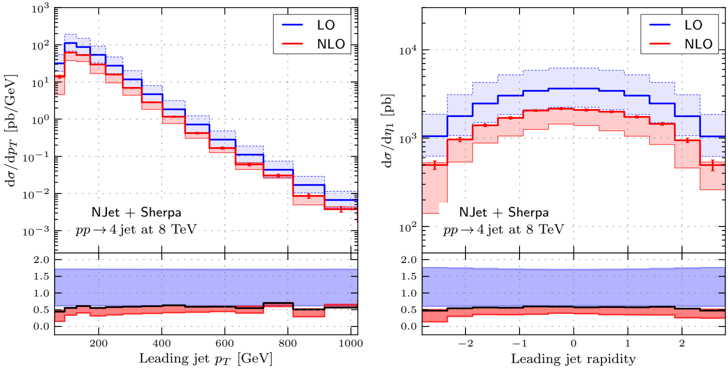
- Helac-NLO [public] [arXiv:1110.1499]
- GoSam [public] [arXiv:1111.2034]
- BlackHat [private, W/Z+jets, multi-jets] [arXiv:0803.4180+...]
- MadLoop(5) [private, SM+More] [arXiv:1103.0621]
- Open Loops [private, QCD SM] [arXiv:1111.5206]

- Rocket [private, W+jets, WW+jets, $t\bar{t}$+jet] [arXiv:0805.2152+...]
- MCFM [public, max. $2 \to 3$] [http://mcfm.fnal.gov]
- Feynman based approaches:
  VBFNLO [public], Denner et al., FeynCalc [public], Reina et al. ...

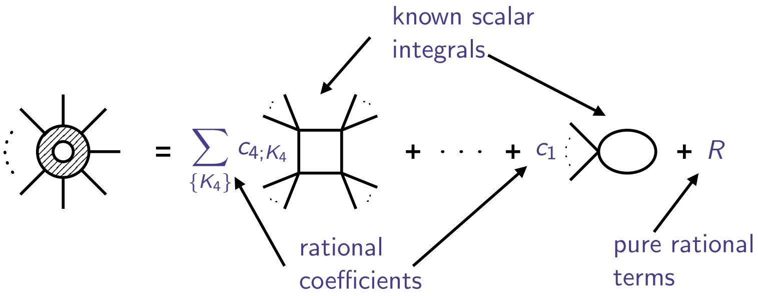
New HP2

- NJet [public, multi-jets] [arXiv:1209.0100]

NJet (virtual) + Sherpa (real+integration) = full colour jet XS



NLO QCD corrections to 3 and 4 jets at 8 TeV [arXiv:1209.0098]



- Gauge theory amplitudes reduced to box topologies or simpler

  [Passarino,Veltman;Melrose]

- Isolate logarithms with cuts and exploit on-shell simplifications
- General cutting principle: [Bern,Dixon,Kosower]
  - apply δ-functions to left and right sides
  - generate and solve the linear system for the coefficients

`NGluon`: public C++ library for evaluating multi-parton primitive amplitudes via unitarity (now part of `NJet`) [Comput.Phys.Commun. 182 (2011)]

Main features

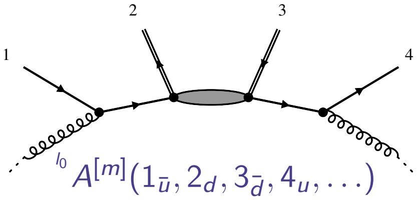
- Efficient tree amplitudes using Berends-Giele recursion.
- Rational terms from massive loop cuts.
- Extraction of integral coefficients via Fourier projections.
- Everything is in 4 dimensions (except loop integrals).

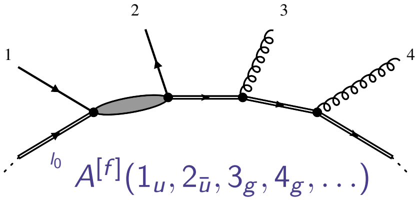
NJet: public C++ library for evaluating multi-parton matrix elements in massless QCD [`https://bitbucket.org/njet/njet`] [arXiv:1209.0100]

Additions since the first public version

- Multi-quark primitive amplitudes.
- NParton – interface to partonic primitive amplitudes.
- Full colour-summed amplitudes for up to 5 outgoing partons.
- Binoth Les Houches Accord interface for MC generators.

NParton computes arbitrary multi-fermion primitives.





All primitives are separated into two classes

- With **mixed** fermion and gluon loop content ($l_0 =$ gluon)
- With internal **fermion loops** ($l_0 =$ quark)

These two classes cover all partonic primitives in one loop QCD.

Colour decomposition of an L-loop amplitude:

$$\mathcal{A}_n^{(L)}(\{p_i\}) = \sum_c \underbrace{T_c(\{a_i\})}_{\text{colour basis}} \underbrace{\boxed{A_{n;c}^{(L)}(p_1,\ldots,p_n)}}_{\text{partial amplitudes}}$$

Knowing partial amplitudes we can get matrix elements:

$$\sum_{\text{hel}} \sum_{\text{col}} \mathcal{A}_n^{(1)} \mathcal{A}_n^{(0)\dagger} = \sum_{\text{hel}} \sum_{cc'} A_{n;c}^{(1)} \cdot \mathcal{C}_{cc'} \cdot A_{n;c'}^{(0)\dagger}$$

Partial-Primitive decomposition for pure gluons:

- Tree level: Kleiss-Kuijf basis of $(n-2)!$ primitives
- One-loop: a basis of $(n-1)!/2$ primitives.

Partial-Primitive decomposition for multi-quark case:

No analytic formula. Reconstruct partials using diagram matching.

[Ellis,Kunszt,Melnikov,Zanderighi], [Ita,Ozeren]

1. Generate all diagrams[1] D_i for a given n-parton amplitude $\mathcal{A}_n$

$$\mathcal{A}_n = \sum_{i=1}^{N_{\text{dia}}} D_i = \sum_{i=1}^{\hat{N}_{\text{dia}}} C_i K_i \qquad C_i = \sum_c T_c F_{ci}$$

2. Write all possible primitives P_i as combinations of colour-stripped diagrams K_i using matching matrix M_{ij}

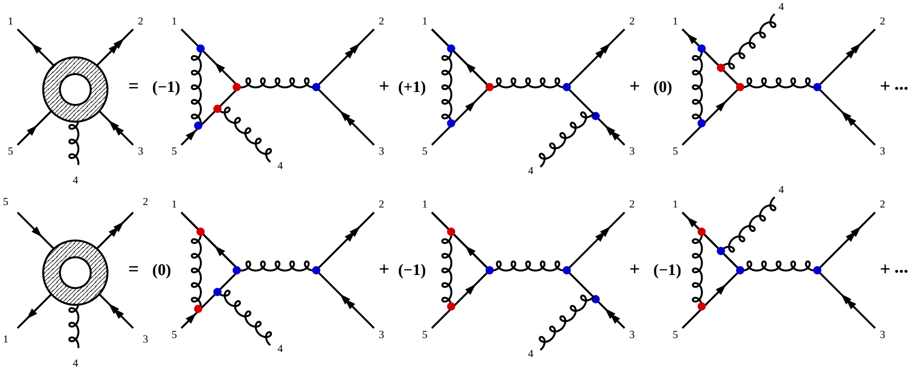
$$P_i = \sum_{j=1}^{\hat{N}_{\text{dia}}} M_{ij} K_j \qquad i \in \{1, 2, \dots, N_{\text{pri}}\} \qquad N_{\text{pri}} = N_{\text{pri}}^{[m]} + N_{\text{pri}}^{[f]}$$

$$N_{\text{pri}}^{[f]} = (n-1)! \qquad N_{\text{pri}}^{[m]} = \begin{cases} (n-1)! & n_q = 0 \\ n_q(n-1)!/2 & n_q = 2, 4, \dots \end{cases}$$

[1] only topologies are needed

Matching Matrix M_{ij}

$$\color{red}{P_i} = \sum_{j=1}^{\hat{N}_{\text{dia}}} \color{green}{M_{ij}}\color{blue}{K_j} \qquad M_{ij} \in \{0,\ 1,\ -1\}$$



Matching of $A_5^{[m]}(1_{\bar{d}}, 2_u, 3_{\bar{u}}, 4_g, 5_d)$ and $A_5^{[m]}(1_{\bar{d}}, 4_g, 3_{\bar{u}}, 2_u, 5_d)$.
Each vertex is either ordered or unordered with respect to the colour ordered Feynman rules and the primitive in question.

Partial Amplitudes and Colour Summation

$$P_i = \sum_{j=1}^{\hat{N}_{\text{dia}}} M_{ij} K_j \qquad M_{ij} \in \{0,\ 1,\ -1\}$$

Number of **independent** primitive amplitudes (denoted $\hat{P}_j$)

$$\hat{N}_{\text{pri}} = \textbf{rank}\,\mathbf{M}$$

$$\hat{N}_{\text{pri}} \leq (N_{\text{pri}} = N_{\text{rows}}) \quad \hat{N}_{\text{pri}} \leq (\hat{N}_{\text{dia}} = N_{\text{cols}})$$

Reduced row echelon form of $\hat{\mathbf{M}} = [\mathbf{M}|-\mathbb{1}]$

- upper $\hat{N}_{\text{pri}}$ rows — **solution** of K_j in terms of $\hat{P}_i$
- lower $N_{\text{pri}} - \hat{N}_{\text{pri}}$ rows — left null space of $\mathbf{M}$ (relations)

$$K_i = \sum_{j=1}^{\hat{N}_{\text{pri}}} B_{ij} \hat{P}_j \qquad \{\hat{P}_j\}_{\hat{N}_{\text{pri}}} \subset \{P_j\}_{N_{\text{pri}}}$$

Putting everything together

- Colour factors in terms of the colour "trace basis"
- Kinematic factors in terms of independent primitives

$$C_i = \sum_c T_c F_{ci} \qquad\qquad K_i = \sum_{j=1}^{\hat{N}_{\text{pri}}} B_{ij} \hat{P}_j$$

$$\mathcal{A}_n = \sum_{i=1}^{\hat{N}_{\text{dia}}} C_i K_i = \sum_c T_c \sum_{j=1}^{\hat{N}_{\text{pri}}} \underbrace{\sum_{i=1}^{\hat{N}_{\text{dia}}} F_{ci} B_{ij}}_{Q_{cj}} \hat{P}_j$$

We obtain partial amplitudes in terms of a basis of independent primitive amplitudes $\hat{P}_j$ for a given class of primitives

$$\mathcal{A}_n = \sum_c T_c \boxed{\sum_{j=1}^{\hat{N}_{\text{pri}}} Q_{cj} \hat{P}_j}$$

Number of primitives in tree, mixed and fermion loop amplitudes

Process	$N_{\text{pri}}^{[0]}$	$N_{\text{pri}}^{[m]}$	$N_{\text{pri}}^{[f]}$
$4\,g$	2	3	3
$\bar{u}u + 2\,g$	2	6	1
$\bar{u}u\bar{d}d$	1	4	1

Process	$N_{\text{pri}}^{[0]}$	$N_{\text{pri}}^{[m]}$	$N_{\text{pri}}^{[f]}$
$5\,g$	6	12	12
$\bar{u}u + 3\,g$	6	24	6
$\bar{u}u\bar{d}dg$	3	16	3

Process	$N_{\text{pri}}^{[0]}$	$N_{\text{pri}}^{[m]}$	$N_{\text{pri}}^{[f]}$
$6\,g$	24	60	60
$\bar{u}u + 4\,g$	24	120	33
$\bar{u}u\bar{d}d + 2\,g$	12	80	13
$\bar{u}u\bar{d}d\bar{s}s$	4	32	4

Process	$N_{\text{pri}}^{[0]}$	$N_{\text{pri}}^{[m]}$	$N_{\text{pri}}^{[f]}$
$7\,g$	120	360	360
$\bar{u}u + 5\,g$	120	720	230
$\bar{u}u\bar{d}d + 3\,g$	60	480	75
$\bar{u}u\bar{d}d\bar{s}sg$	20	192	20

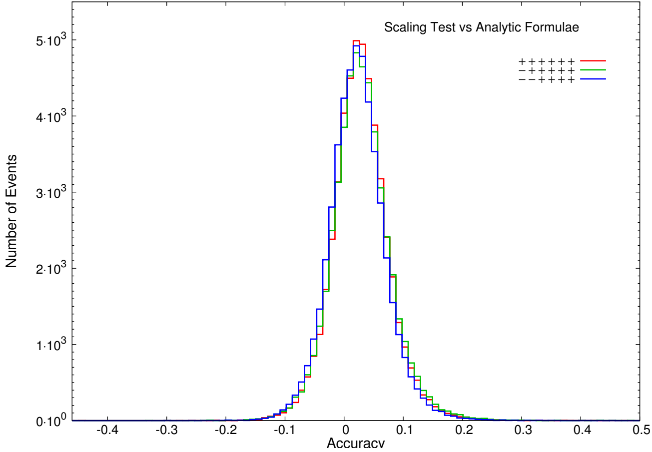
Example: fermion loop contribution to the six-quark amplitude

$$A_{6q}(1_{\bar{u}}, 2_u, 3_{\bar{d}}, 4_d, 5_{\bar{s}}, 6_s) = A_{6q}^{mixed} + n_f A_{6q}^{q\text{-}loop}$$

$$A_{6q}^{q\text{-}loop} = A^{[f]}(123456)\cdot(T_3+T_4-T_1 N_c - T_6/N_c)/N_c + A^{[f]}(124365)\cdot(T_5-T_3)/N_c$$
$$+A^{[f]}(125634)\cdot(2T_5-T_2 N_c-T_6/N_c)/N_c + A^{[f]}(126534)\cdot(T_5-T_4)/N_c$$

$$T_1 = \delta_{16}\delta_{32}\delta_{54} \qquad T_2 = \delta_{14}\delta_{36}\delta_{52} \qquad T_3 = \delta_{12}\delta_{36}\delta_{54}$$
$$T_4 = \delta_{14}\delta_{32}\delta_{56} \qquad T_5 = \delta_{16}\delta_{34}\delta_{52} \qquad T_6 = \delta_{12}\delta_{34}\delta_{56}$$

Scaling test to determine accuracy

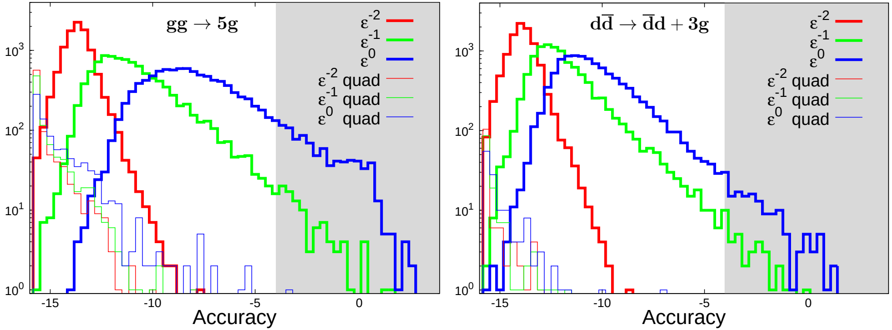


$$\log\left(\frac{A_{\mathtt{NGluon}} - A_{\text{analytic}}}{A_{\text{analytic}}}\right) - \log\left(\frac{2(A_{\mathtt{NGluon}} - A^{\text{scaled}}_{\mathtt{NGluon}})}{A_{\mathtt{NGluon}} + A^{\text{scaled}}_{\mathtt{NGluon}}}\right)$$

Reliable, but essentially statistical.

A safety margin of 2 digits is advised.

Left: 7 gluon squared amplitude. Right: 4 quark + 3 gluons.



Thick lines – double precision.

Thin lines – fixed with quadruple precision.

Full colour and helicity sum time per point [clang, Xeon 3.30 GHz].

process	T_{sd}[s]	$T_{4\text{ dig.}}$[s] (%)	process	T_{sd}[s]	$T_{4\text{ dig.}}$[s] (%)
$4\,g$	0.030	0.030 (0.00)	$5\,g$	0.22	0.22 (0.22)
$\overline{u}u{+}2g$	0.032	0.032 (0.00)	$\overline{u}u{+}3g$	0.34	0.35 (0.06)
$\overline{u}u\overline{d}d$	0.011	0.011 (0.00)	$\overline{u}u\overline{d}d{+}g$	0.11	0.11 (0.00)
$\overline{u}u\overline{u}u$	0.022	0.022 (0.00)	$\overline{u}u\overline{u}u{+}g$	0.22	0.22 (0.03)

process	T_{sd}[s]	$T_{4\text{ dig.}}$[s] (%)	process	T_{sd}[s]	$T_{4\text{ dig.}}$[s] (%)
$6\,g$	6.19	6.81 (1.37)	$7\,g$	171.3	276.7 (8.63)
$\overline{u}u{+}4g$	7.19	7.40 (0.38)	$\overline{u}u{+}5g$	195.1	241.2 (3.25)
$\overline{u}u\overline{d}d{+}2g$	2.05	2.06 (0.08)	$\overline{u}u\overline{d}d{+}3g$	45.7	48.8 (0.88)
$\overline{u}u\overline{u}u{+}2g$	4.08	4.15 (0.21)	$\overline{u}u\overline{u}u{+}3g$	92.5	101.5 (1.29)
$\overline{u}u\overline{d}d\overline{s}s$	0.38	0.38 (0.00)	$\overline{u}u\overline{d}d\overline{s}sg$	7.9	8.1 (0.23)
$\overline{u}u\overline{d}d\overline{d}d$	0.74	0.74 (0.00)	$\overline{u}u\overline{d}d\overline{d}dg$	15.8	16.2 (0.29)
$\overline{u}u\overline{u}u\overline{u}u$	2.16	2.17 (0.02)	$\overline{u}u\overline{u}u\overline{u}ug$	47.1	48.6 (0.41)

All times include two evaluations for the scaling test.

Both the phase-space and the amplitude are symmetric under permutations of the final state gluons:

$$\int \mathrm{d}PS_n(g_1,\ldots,g_n) = \int \mathrm{d}PS_n(\sigma\{g_1,\ldots,g_n\})$$

$$\mathcal{A}_{gg\to n(g)}(g,g,g_1,\ldots,g_n) = \mathcal{A}_{gg\to n(g)}(g,g,\sigma\{g_1,\ldots,g_n\})$$

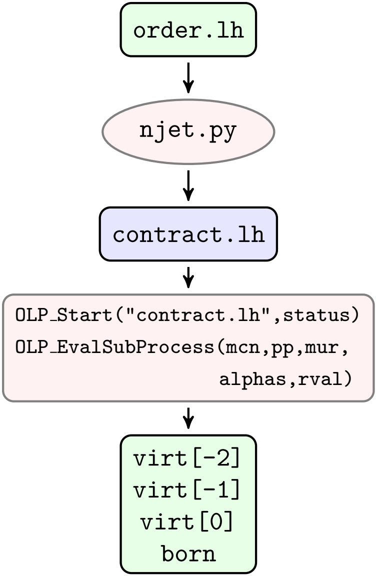
We can replace symmetric colour sum with a simpler object which gives original sum after symmetrization (or PS integration)

$$\sigma^V_{gg\to n(g)} = \int \mathrm{d}PS_n \boxed{A^{(0)\dagger}\cdot\mathcal{C}_{(n-2)!\times(n-1)!/2}\cdot A^{(1)}}$$

$$= (n-2)! \int \mathrm{d}PS_n \boxed{A^{(0)\dagger}\cdot\mathcal{C}^{\text{dsym}}_{(n-2)!\times(n-1)}\cdot A^{(1),\text{dsym}}}$$

	$gg\to 3g$	$gg\to 4g$	$gg\to 5g$
Standard sum	0.22	6.19	171.31
De-symmetrized	0.07	0.50	2.76
Speedup	$\times$ 3	$\times$ 12	$\times$ 60

Binoth Les Houches Accord Interface



Create an 'order' file

NJet takes an 'order' file
and returns a 'contract' file

Check that requested
options were accepted

Link with `libnjet.so`
Call 'Start' once to initialize
Call 'EvalSubProcess' to evaluate PS points

Use returned values to calculate XS
$1/eps^2$, $1/eps$, $finite$, $born$

BLHA example

order file

```
# OLE_order for 5jet production



MatrixElementSquareType CHsummed
CorrectionType          QCD
IRregularisation        CDR
AlphasPower             5
# process list
21 21 -> 21 21 21 21 21
1 -1 -> 21 21 21 21 21
1 -1 -> 21 -2 2 21 21
1 -1 -> 21 -1 1 21 21
1 -1 -> 21 -2 2 -3 3
1 -1 -> 21 -2 2 -2 2
1 -1 -> 21 -1 1 -1 1
```

contract file

```
# OLE_order for 5jet production
# Generated file. Do not edit by hand.
# Signed by NJet 3900867518.
# 12 1 1e-05 0.01 0 1 1 1 1 0 3 5
MatrixElementSquareType CHsummed | OK
CorrectionType QCD | OK
IRregularisation CDR | OK
AlphasPower 5 | OK
# process list
21 21 -> 21 21 21 21 21 | 1 1 # 70 120 4 64 0 (-2 -1 3 4 5 6 7)
1 -1 -> 21 21 21 21 21 | 1 2 # 71 120 4 9 0 (-1 -2 3 4 5 6 7)
1 -1 -> 21 -2 2 21 21 | 1 3 # 72 6 4 9 0 (-1 -2 4 5 3 6 7)
1 -1 -> 21 -1 1 21 21 | 1 4 # 73 6 4 9 0 (-1 -2 4 5 3 6 7)
1 -1 -> 21 -2 2 -3 3 | 1 5 # 74 1 4 9 0 (-1 -2 4 5 6 7 3)
1 -1 -> 21 -2 2 -2 2 | 1 6 # 75 4 4 9 0 (-1 -2 4 5 6 7 3)
1 -1 -> 21 -1 1 -1 1 | 1 7 # 76 4 4 9 0 (-1 -2 4 5 6 7 3)
```

Additional options

- NJetSwitchAcc — relative accuracy threshold
- NJetReturnAccuracy — return accuracy estimate
- NJetType — loop, tree or loopds
- NJetNf — number of light flavours
- NJetNc — SU(Nc) group parameter

Tools (linked together with BLHA interface)

- NJet — full colour virtual matrix elements
  scalar integrals — QCDLoop/FF [Ellis,Zanderighi,van Oldenborgh]
  extended precision — libqd [Hida,Li,Bailey]
- Sherpa/Amegic++ — trees, CS subtraction, PS integration [Gleisberg,Krauss,Kuhn,Soff,...]

Parameters

- $pp \to$ 3 and 4 jets at 8 TeV
- MSTW2008 PDF set, $\alpha_s(M_Z)$ from PDFs
- $\mu_R = \mu_F = \hat{H}_T/2$, scale variations $\hat{H}_T/4$ and $\hat{H}_T$
- anti-kt $R = 0.4$, $p_T^{\text{1st}} > 80$ GeV, $p_T^{\text{other}} > 60$ GeV, $\eta < 2.8$

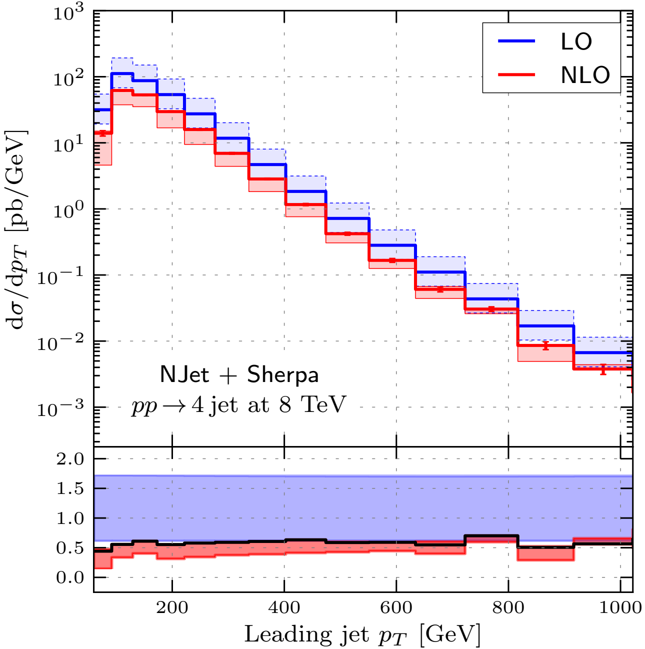
	3 jets	4 jets
σ_{LO}	$126.65(0.05)^{+66.56}_{-40.40}$ nb	$14.36(0.01)^{+10.38}_{-5.6}$ nb
σ_{NLO}	$72.57(0.16)^{+2.71}_{-28.08}$ nb	$8.15(0.09)^{+0.0}_{-3.24}$ nb
$\sigma_{8\text{ TeV}}/\sigma_{7\text{ TeV}}$	$\sim 35\%$	$\sim 46\%$

Reduced scale uncertainty

NLO scale variations are about 25% of the LO scale uncertainty for both 3 and 4 jet cross-sections

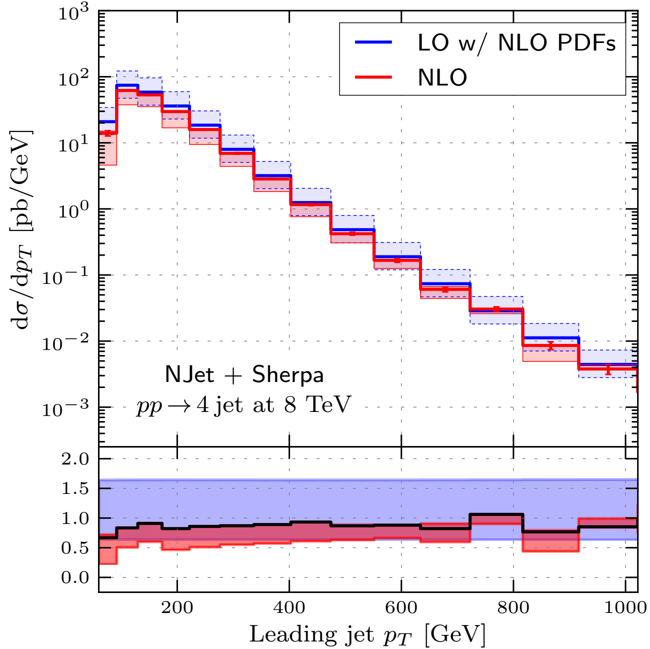
Cross-check at 7 TeV

Agreement with first 4 jet results by BlackHat collaboration [Bern,Diana,Dixon,Febres Cordero,Hoeche,Kosower,Ita,Maitre,Ozeren] **[arXiv:1112.3940]**

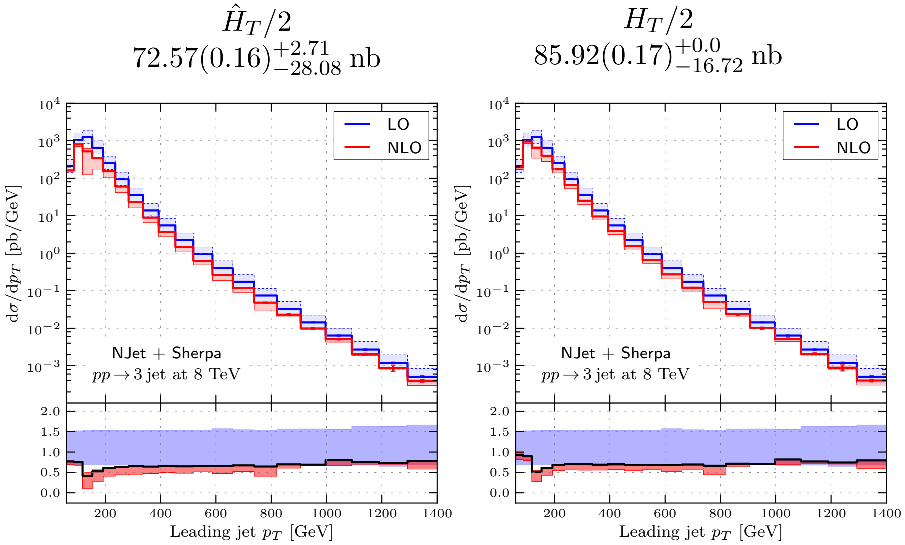


LO
NLO
NJet + Sherpa
$pp \to 4$ jet at 8 TeV
$d\sigma/dp_T$ [pb/GeV]
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
2.0
1.5
1.0
0.5
0.0
200
400
600
800
1000
Leading jet p_T [GeV]

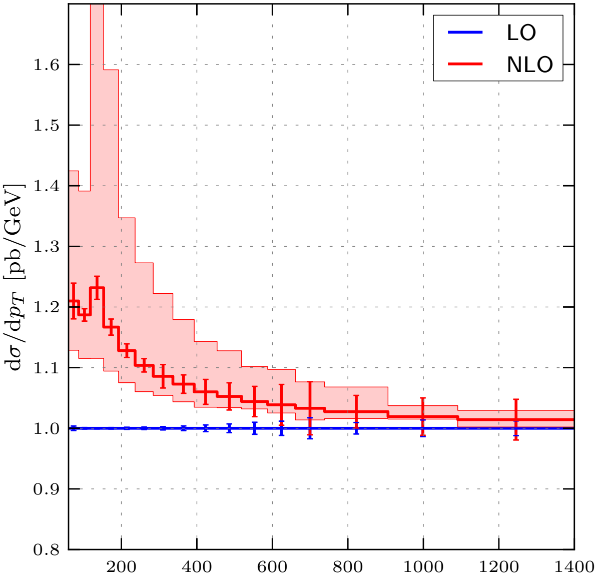
4 jets, leading jet pT at LO with NLO PDFs and alphas



3 jets, leading jet pT at NLO, $\hat{H}_T$ vs H_T



3 jets, leading jet pT at NLO, $d\sigma(H_T)/d\sigma(\hat{H}_T)$



Summary

- NJet: numerical evaluation of one-loop partonic amplitudes in massless QCD.
- General construction for primitive and partial amplitudes.
- Full colour results for ≤ 5 jets (+ fast 'desymmetrized' colour sums for gluons).
- Binoth Les Houches Accord interface.
- NJet+Sherpa: 3 and 4 jets NLO at 8 TeV.
- More applications feasible: 5 jets, parton shower matching...
- Publicly available from the NJet project page `https://bitbucket.org/njet/njet`

Bonus material

Desymmetrized 5 gluon amplitude

Desymmetrized sum contains 4 loop primitives of each kind (mixed and q-loop)

$$\mathcal{A}_{5g}^{(0)\dagger} \overset{\text{dsym}}{\otimes} \mathcal{A}_{5g}^{(1)} = 6N_c(N_c^2-1)\times$$

$$\times\Big\{N_c\Big[A^{[m]}(\mathbf{12}345)\Big(\;12\big(A^{[0]}(12453)+A^{[0]}(12534)+A^{[0]}(12543)\big)+A^{[0]}(12345)N_c^2\Big)^\dagger$$

$$+A^{[m]}(\mathbf{1}3\mathbf{2}45)\Big(\;12A^{[0]}(12534)-\big(A^{[0]}(12345)+A^{[0]}(12435)+A^{[0]}(12453)\big)N_c^2\Big)^\dagger$$

$$+A^{[m]}(\mathbf{1}34\mathbf{2}5)\Big(-12A^{[0]}(12354)+\big(A^{[0]}(12435)+A^{[0]}(12453)+A^{[0]}(12543)\big)N_c^2\Big)^\dagger$$

$$+A^{[m]}(\mathbf{1}345\mathbf{2})\Big(-12\big(A^{[0]}(12345)+A^{[0]}(12354)+A^{[0]}(12435)\big)-A^{[0]}(12543)N_c^2\Big)^\dagger\Big]$$

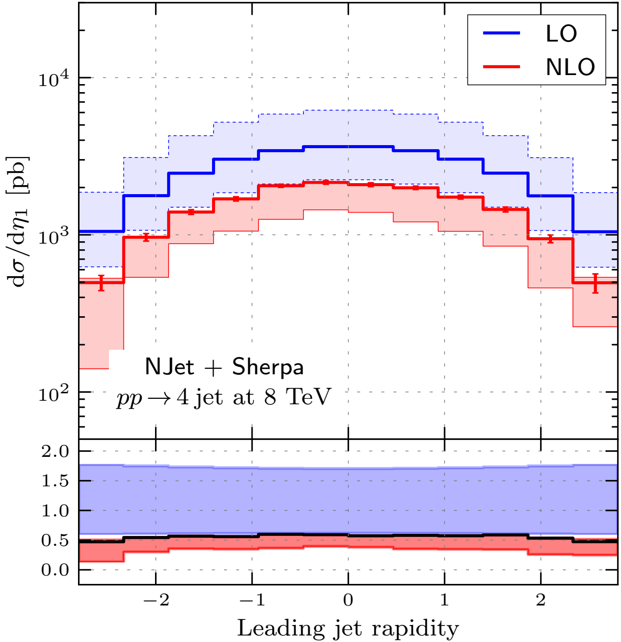
$$-N_f\Big[A^{[f]}(\mathbf{12}345)\Big(\;2\big(A^{[0]}(12453)+A^{[0]}(12534)+A^{[0]}(12543)\big)+A^{[0]}(12345)N_c^2\Big)^\dagger$$

$$+A^{[f]}(\mathbf{1}3\mathbf{2}45)\Big(\;2A^{[0]}(12534)-\big(A^{[0]}(12345)+A^{[0]}(12435)+A^{[0]}(12453)\big)N_c^2\Big)^\dagger$$

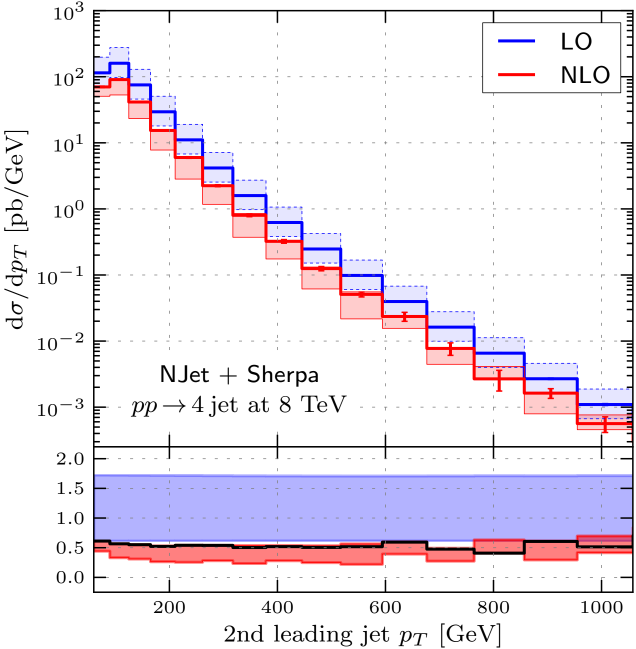
$$+A^{[f]}(\mathbf{1}34\mathbf{2}5)\Big(-2A^{[0]}(12354)+\big(A^{[0]}(12435)+A^{[0]}(12453)+A^{[0]}(12543)\big)N_c^2\Big)^\dagger$$

$$+A^{[f]}(\mathbf{1}345\mathbf{2})\Big(-2\big(A^{[0]}(12345)+A^{[0]}(12354)+A^{[0]}(12435)\big)-A^{[0]}(12543)N_c^2\Big)^\dagger\Big]\Big\}$$

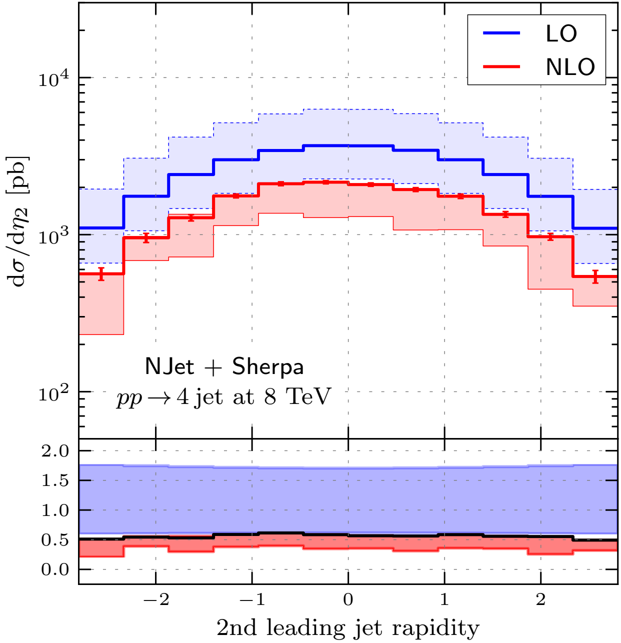
4 jets, leading jet rapidity at NLO



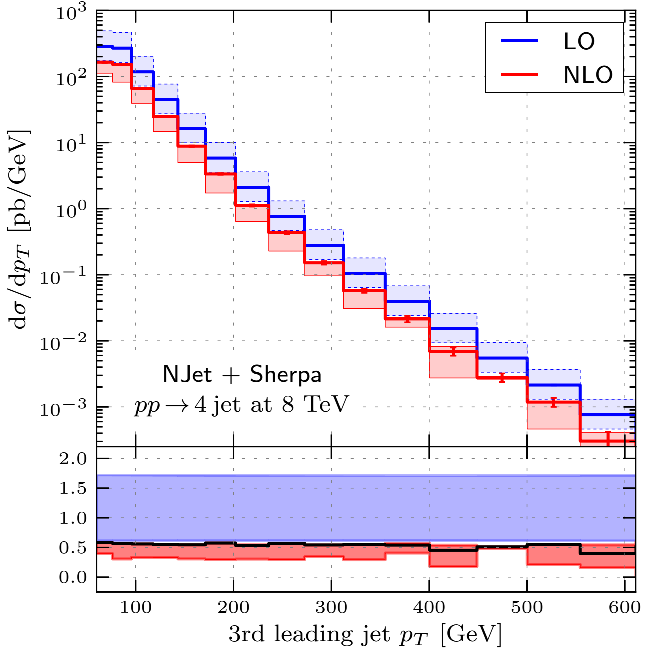
4 jets, 2nd jet pT at NLO



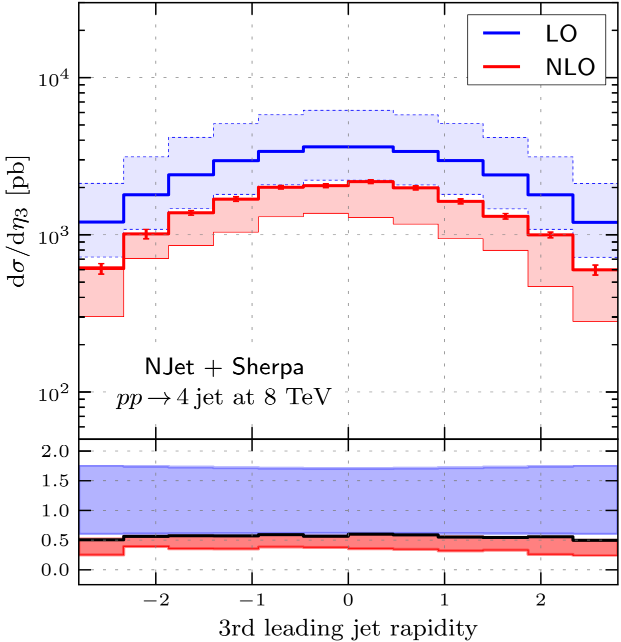
4 jets, 2nd jet rapidity at NLO



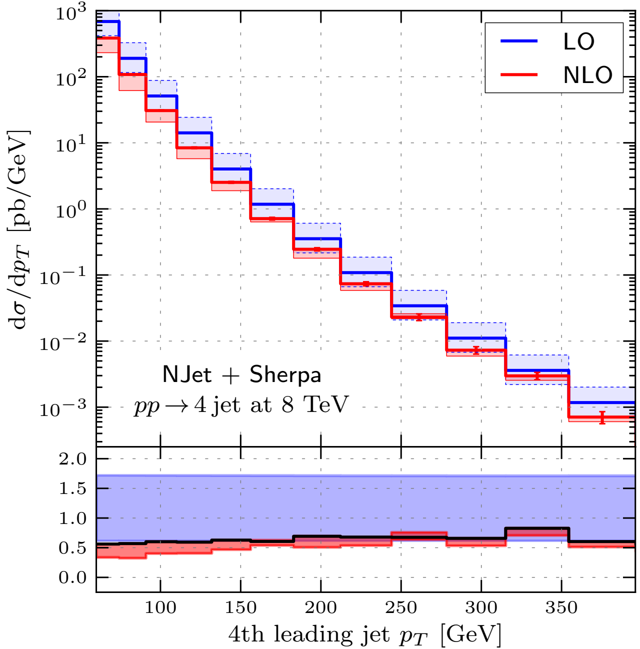
4 jets, 3d jet pT at NLO



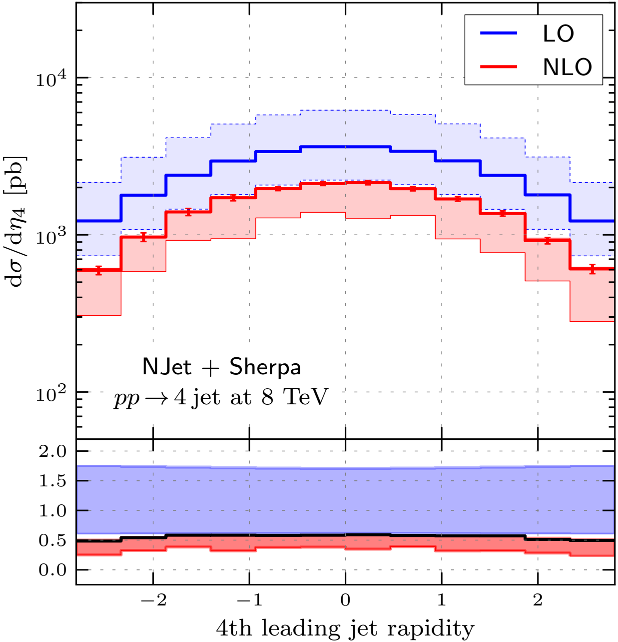
4 jets, 3rd jet rapidity at NLO



4 jets, 4th jet pT at NLO



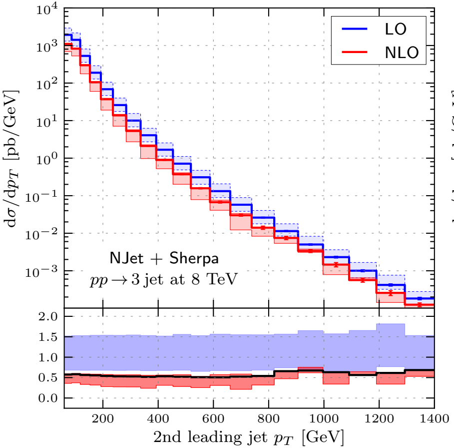
4 jets, 4th jet rapidity at NLO



3 jets, 2nd jet pT at NLO, $\hat{H}_T$ vs H_T

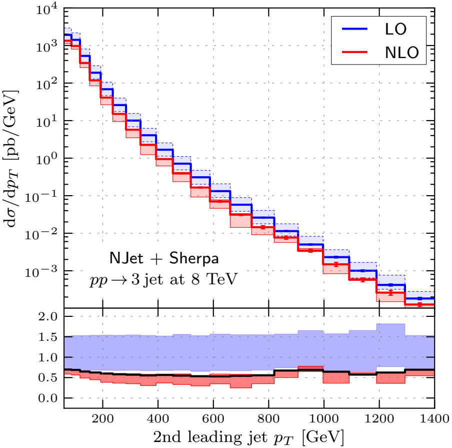
$\hat{H}_T/2$

$72.57(0.16)^{+2.71}_{-28.08}$ nb



$H_T/2$

$85.92(0.17)^{+0.0}_{-16.72}$ nb



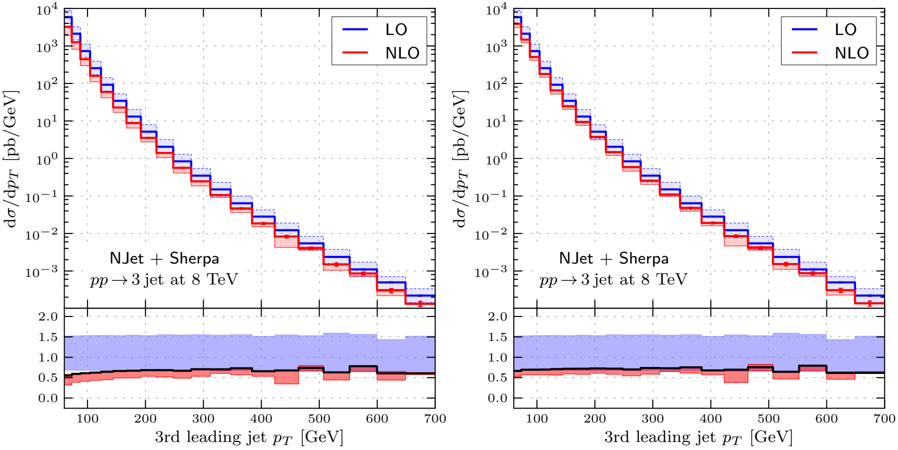
3 jets, 3rd jet pT at NLO, $\hat{H}_T$ vs H_T

$\hat{H}_T/2$

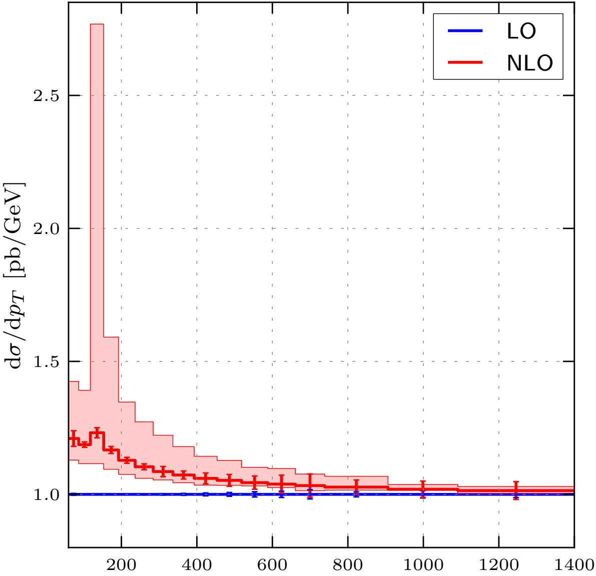
$72.57(0.16)^{+2.71}_{-28.08}$ nb

$H_T/2$

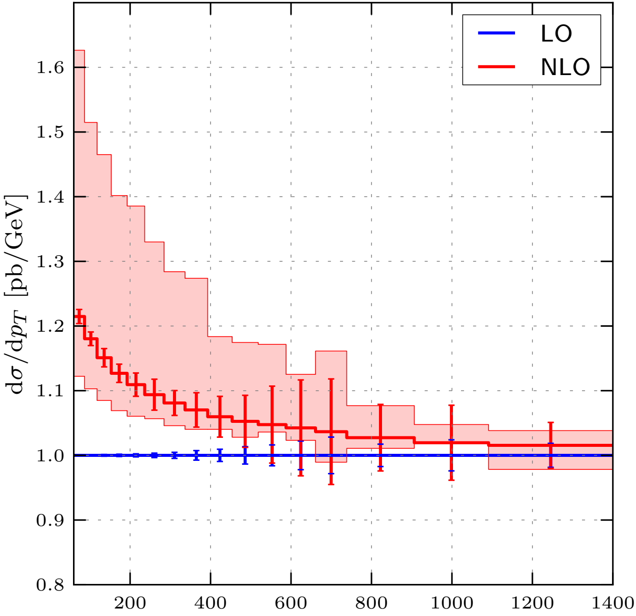
$85.92(0.17)^{+0.0}_{-16.72}$ nb



3 jets, leading jet pT at NLO, $d\sigma(H_T)/d\sigma(\hat{H}_T)$



3 jets, 2nd jet pT at NLO, $d\sigma(H_T)/d\sigma(\hat{H}_T)$



3 jets, 3rd jet pT at NLO, $d\sigma(H_T)/d\sigma(\hat{H}_T)$