# Soft Non-Global Structure at Two Loops in SCET

Robert M. Schabinger

with Randall Kelley, Matthew D. Schwartz, and Hua Xing Zhu (arXiv:1112.3343)

Instituto de Física Teórica UAM/CSIC
Universidad Autónoma de Madrid

# Outline

# Thrust in Soft-Collinear Effective Theory

Thrust,

$$T = \max_{\mathbf{x}} \left\{ \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{x}|}{\sum_i |\mathbf{p}_i|} \right\},$$

is a well-studied event shape variable that requires resummation in the end-point region, $1 - T = \tau \to 0$.

- The framework of soft-collinear effective theory is a convenient one in which to discuss factorization and resummation.
- In the context of thrust in the end-point region, the hard scale is simply $\sqrt{s}$ and one defines the scaling behavior of a soft or collinear momentum by

$$p_{\eta \text{ collinear}} \approx \sqrt{s}\,(\tau, 1, \sqrt{\tau}) \quad p_{\text{soft}} \approx \sqrt{s}\,(\tau, \tau, \tau)$$

$$p = (p^+, p^-, p_\perp) \qquad p^2 = p^+ p^- - p_\perp^2$$

# Factorization For Thrust-Like Observables ($\tau$ or $\tau_\omega$)

Catani et. al. Nucl. Phys. **B407**, 3 (1993), Schwartz arXiv:0709.2709, Ellis et. al. **JHEP** 1011:101, 2010, Kelley et. al. arXiv:1102.0561

$$\frac{1}{\sigma_{tree}}\frac{d\sigma}{d\tau_{\text{alg}}} = H(\sqrt{s},\mu)\int dk_L dk_R dM_L^2 dM_R^2 J_{\mathbf{n}}(M_L^2-\sqrt{s}\ k_L,\mu)\times$$
$$\times J_{\bar{\mathbf{n}}}(M_R^2-\sqrt{s}\ k_R,\mu)S_{\text{alg}}(k_L,k_R,\mu)\delta\left(\tau_{\text{alg}}-\frac{M_L^2+M_R^2}{s}\right)+\cdots$$

- $H$ is a “hard function” which captures the effects associated with the short-distance hard scattering process.
- $J_{\mathbf{n}}$ and $J_{\bar{\mathbf{n}}}$ are “jet functions” which capture the effects associated with the radiation of collinear gluons off of the partons which emerge from the hard scattering.
- $S_{\text{alg}}$ is a “soft function” which captures the effects associated with the radiation of soft gluons off of the partons which emerge from the hard scattering and the exchange of soft partons between them. It depends on the algorithm used to determine how the radiated soft partons are clustered into jets.

# The Hemisphere Jet Algorithm

One defines two hemispheres (left and right) by dropping a plane perpendicular to the thrust axis at the collision point and defining

$$P^{\mu}_{L(R)} = \text{four} - \text{vector sum of all radiation in left(right) hemisphere.}$$

In soft-collinear effective theory, the $-(+)$ component of the $\eta(\bar{\eta})$ collinear momenta (of order $\sqrt{s}$) is fixed after the hard modes are integrated out but the $+(-)$ component is determined by the interaction of collinear fields with a soft background. It is therefore trivial to determine the parameters $M^2_{L(R)}$ and $k_{L(R)}$:

$$M_L^2 = P_L^2 \qquad M_R^2 = P_R^2 \qquad k_L = \eta \cdot P_L \qquad k_R = \bar{\eta} \cdot P_R$$

Clearly, this also fixes the sum of the $+(-)$ components of the soft momenta being radiated into the left(right) hemisphere to be $k_L(k_R)$.

# The Thrust Cone Jet Algorithm

- A soft parton in the left hemisphere is said to be in the **n** jet if the quantity $\frac{1}{2}(1-\cos\theta)$ is less than $R$, where $\theta$ is the direction of the soft parton as measured from the direction of the thrust axis.
- A soft parton in the right hemisphere is said to be in the $\bar{\mathbf{n}}$ jet if the quantity $\frac{1}{2}(1-\cos\theta)$ is less than $R$, where $\theta$ is again the direction of the soft parton as measured from the direction of the thrust axis.
- The algorithm is sensitive to the radiation in neither jet and for the algorithm to be well-defined an explicit veto procedure must be used to reject events with a significant amount of energy deposited out of all jets.

$$S_{T\text{cone}}(k_L,k_R,\mu) = \int_0^{\omega} d\lambda\, S_R(k_L,k_R,\lambda,\mu)$$

Here $\omega$ is called the *veto scale* and $R < \frac{1}{2}$ is called the *jet radius.*

# The Definition of the Hemisphere Soft Function

$$S_{\text{hemi}}(k_L, k_R, \mu) =$$
$$\frac{1}{N_c} \sum_{X_s} \delta\Big(k_L - \eta \cdot P^L_{X_s}\Big)\delta\Big(k_R - \bar{\eta} \cdot P^R_{X_s}\Big)\langle 0|Y_{\bar{\eta}} Y_\eta|X_s\rangle\langle X_s|Y^\dagger_\eta Y^\dagger_{\bar{\eta}}|0\rangle$$

- $P_s^{L(R)}$ is the total soft momentum of final state $X_s$ entering the left(right) hemisphere.
- The $Y's$ are Fourier transformed soft Wilson lines encapsulating the interaction of the “frozen” collinear quark and anti-quark with the soft gluon background.
- At $\mathcal{O}(\alpha_s^2)$, there are two soft partons emitted which can either travel into the same hemisphere or into opposite hemispheres.
- In earlier work, Hornig et. al. (**JHEP** 1108:054,2011) and our group (Phys.Rev.**D84**:045022,2011) calculated $S_{\text{hemi}}(k_L, k_R, \mu)$ to $\mathcal{O}(\alpha_s^2)$.

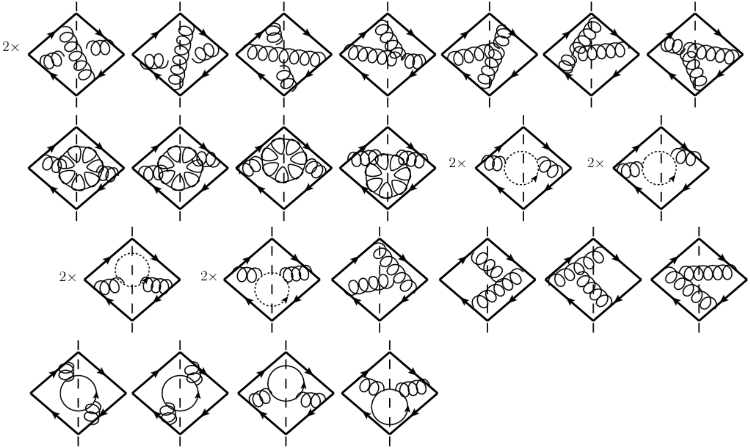
# The Definition of the Thrust Cone Soft Function

$$S_R(k_L, k_R, \lambda, \mu) =$$
$$\frac{1}{N_c} \sum_{X_s} \delta\Big(k_L - \eta \cdot P^L_{X_s}\Big) \delta\Big(k_R - \bar{\eta} \cdot P^R_{X_s}\Big) \delta\Big(\lambda - E_{X_s}\Big) \langle 0 | Y_{\bar{\eta}} Y_{\eta} | X_s \rangle \langle X_s | Y^\dagger_{\eta} Y^\dagger_{\bar{\eta}} | 0 \rangle$$

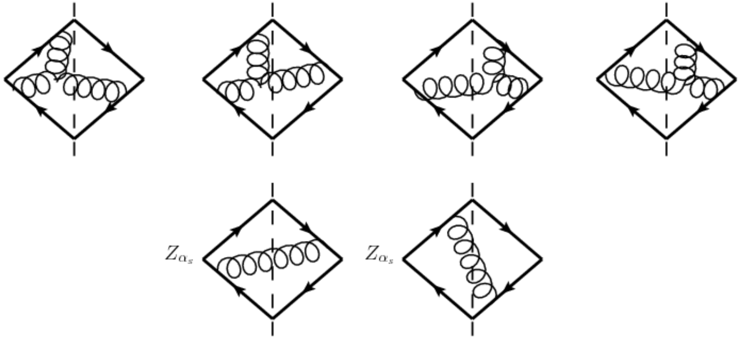
At $\mathcal{O}\left(\alpha_s^2\right)$, the phase-space of the two soft partons naturally splits up into four different contributions:

- Both soft partons clustered into the same jet.
- One soft parton clustered into the **n** jet and one soft parton clustered into the $\bar{\mathbf{n}}$ jet.
- One soft parton clustered into a jet and the other out of all jets.
- Both soft partons out of all jets.

# $\mathcal{O}(\alpha_s^2)$ Double-Cut Soft Feynman Diagrams



# $\mathcal{O}(\alpha_s^2)$ Single-Cut Soft Feynman Diagrams




$$Z_{\alpha_s} = 1 - \left(\frac{11}{3}C_A - \frac{4}{3}T_F n_f\right)\frac{\alpha_s}{4\pi\epsilon} + \mathcal{O}(\alpha_s^2)$$

# The Integrated Jet Thrust Distribution

It is difficult to work with the cone thrust soft function directly because it contains singular distributions.

The soft contribution to the integrated jet thrust distribution

$$\int_0^{\tau_\omega} d\tau'_\omega \int dk_L dk_R \int_0^{\omega} d\lambda S_R(k_L, k_R, \lambda, \mu)\delta\left(\tau'_\omega - \frac{k_L + k_R}{\sqrt{s}}\right)$$

is easier to analyze because it is just an ordinary function.

At first sight, the situation seems hopeless since the part of the answer that remains after the prediction of the factorization theorem is subtracted off is horribly complicated and, when $\tau_\omega\sqrt{s} >> 2\omega R$, it develops logarithmic singularities.

# Soft Non-Global Logarithms

We found that the logarithms that appear in the $\tau_\omega\sqrt{s} >> 2\omega R$ limit have the form (confirms partial results of Hornig et. al. **JHEP** 1201:149,2012)

$$C_F C_A\left[-\frac{8\pi^2}{3}\ln^2\left(\frac{\tau_\omega\sqrt{s}}{2R\omega}\right)+\left(-\frac{8}{3}+\frac{88\pi^2}{9}-16\zeta_3\right)\ln\left(\frac{\tau_\omega\sqrt{s}}{2R\omega}\right)\right] + C_F n_f T_F\left(\frac{16}{3}-\frac{32\pi^2}{9}\right)\ln\left(\frac{\tau_\omega\sqrt{s}}{2R\omega}\right)$$

The appearance of these logarithms implies that a novel exponentiation of the non-global structure not accounted for by the factorization theorem must occur as well or the thrust cone jet algorithm is unsatisfactory from a theoretical point of view.

In fact, the resummation of the non-global structure that appears in this and other contexts is not understood and remains an interesting open problem.

# The Small $R$ Non-Global Contribution To The Integrated Jet Thrust Distribution

Remarkably, taking the small $R$ limit before integrating allows us to capture not only the extreme small $R$ asymptotics but also the dominant power corrections to them.

Using our freedom to absorb an overall $R$-dependent constant into the two-loop Wilson coefficient, we find

$$2\,\mathcal{R}_f\left(\frac{\tau_\omega\sqrt{s}}{2R\omega}\right)$$

for the small $R$ contribution to the soft non-global structure, where $\mathcal{R}_f(z)$ is the soft non-global contribution to integrated doubly differential hemisphere mass distribution calculated by us in Phys.Rev.**D84**:045022, 2011.

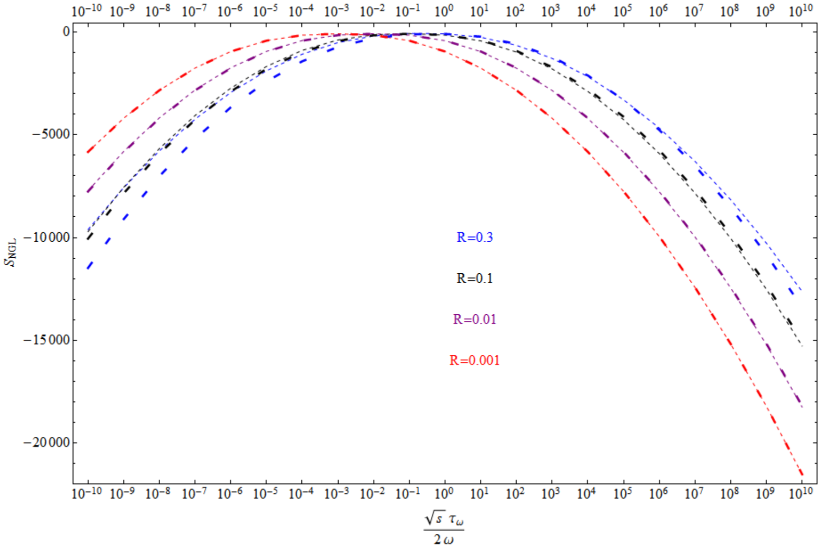
# The Integrated Two-Loop Hemisphere Soft Function

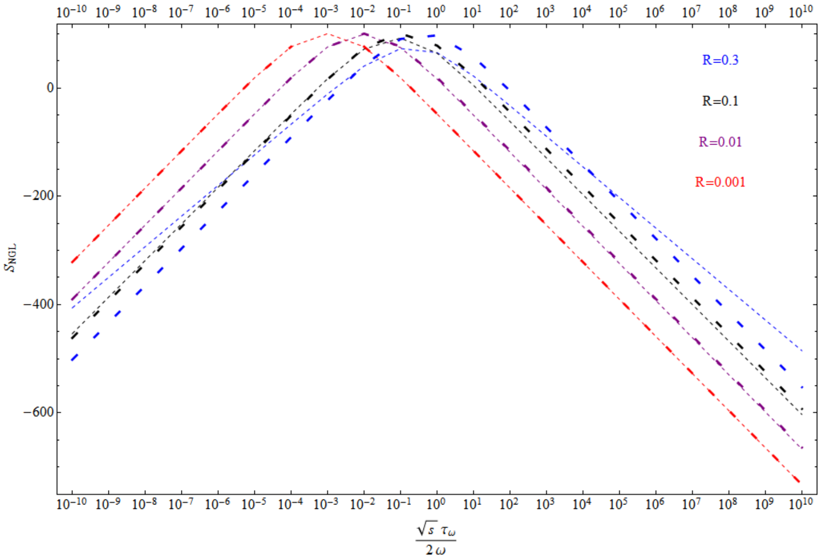
$$\mathcal{R}(M_L^2, M_R^2, \mu) \equiv \int_0^{M_L^2} dk_L \int_0^{M_R^2} dk_R \; S_{\text{hemi}}(k_L, k_R, \mu)$$
$$\equiv \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\mathcal{R}_\mu\left(\frac{M_L^2}{\mu}, \frac{M_R^2}{\mu}\right) + \mathcal{R}_f\left(\frac{M_L^2}{M_R^2}\right)\right)$$

$$\mathcal{R}_\mu\left(\frac{X}{\mu}, \frac{Y}{\mu}\right) = \left[\frac{88}{9}\ln^3\left(\frac{X}{\mu}\right) + \frac{4\pi^2}{3}\ln^2\left(\frac{X}{\mu}\right) - \frac{268}{9}\ln^2\left(\frac{X}{\mu}\right) - \frac{11\pi^2}{9}\ln\left(\frac{XY}{\mu^2}\right)\right.$$
$$\left. + \frac{404}{27}\ln\left(\frac{XY}{\mu^2}\right) - 14\zeta_3\ln\left(\frac{XY}{\mu^2}\right) + X \leftrightarrow Y\right] C_F C_A + \left[-\frac{32}{9}\ln^3\left(\frac{X}{\mu}\right)\right.$$
$$\left. + \frac{80}{9}\ln^2\left(\frac{X}{\mu}\right) + \frac{4\pi^2}{9}\ln\left(\frac{XY}{\mu^2}\right) - \frac{112}{27}\ln\left(\frac{XY}{\mu^2}\right) + X \leftrightarrow Y\right] C_F T_F n_f$$

$$\mathcal{R}_f\left(\frac{X}{Y}\right) = \Bigg[-88\mathrm{Li}_3\left(-\frac{X}{Y}\right) - 16\mathrm{Li}_4\left(\frac{1}{\frac{X}{Y}+1}\right) - 16\mathrm{Li}_4\left(\frac{\frac{X}{Y}}{\frac{X}{Y}+1}\right) + 16\times$$
$$\times\mathrm{Li}_3\left(-\frac{X}{Y}\right)\ln\left(\frac{X}{Y}+1\right) + \frac{88\mathrm{Li}_2\left(-\frac{X}{Y}\right)\ln\left(\frac{X}{Y}\right)}{3} - 8\mathrm{Li}_3\left(-\frac{X}{Y}\right)\ln\left(\frac{X}{Y}\right) - 16\zeta_3\times$$
$$\times\ln\left(\frac{X}{Y}+1\right) + 8\zeta_3\ln\left(\frac{X}{Y}\right) - \frac{4}{3}\ln^4\left(\frac{X}{Y}+1\right) + \frac{8}{3}\ln\left(\frac{X}{Y}\right)\ln^3\left(\frac{X}{Y}+1\right)$$
$$+\frac{4\pi^2}{3}\ln^2\left(\frac{X}{Y}+1\right) - \frac{4\pi^2}{3}\ln^2\left(\frac{X}{Y}\right) - \frac{4\left(3\left(\frac{X}{Y}-1\right)+11\pi^2\left(\frac{X}{Y}+1\right)\right)\ln\left(\frac{X}{Y}\right)}{9\left(\frac{X}{Y}+1\right)}$$
$$-\frac{154\zeta_3}{9} + \frac{4\pi^4}{3} - \frac{335\pi^2}{54} - \frac{2032}{81}\Bigg] C_F C_A + \Bigg[32\mathrm{Li}_3\left(-\frac{X}{Y}\right) - \frac{32}{3}\mathrm{Li}_2\left(-\frac{X}{Y}\right)\times$$
$$\times\ln\left(\frac{X}{Y}\right) + \frac{8\left(\frac{X}{Y}-1\right)\ln\left(\frac{X}{Y}\right)}{3\left(\frac{X}{Y}+1\right)} + \frac{16\pi^2}{9}\ln\left(\frac{X}{Y}\right) + \frac{56\zeta_3}{9} + \frac{74\pi^2}{27} - \frac{136}{81}\Bigg] C_F n_f T_F$$

# Robustness Of The Small $R$ Approximation




R=0.3
R=0.1
R=0.01
R=0.001
$S_{\mathrm{NGL}}$
0
−200
−400
−600
$10^{-10}$ $10^{-9}$ $10^{-8}$ $10^{-7}$ $10^{-6}$ $10^{-5}$ $10^{-4}$ $10^{-3}$ $10^{-2}$ $10^{-1}$ $10^{0}$ $10^{1}$ $10^{2}$ $10^{3}$ $10^{4}$ $10^{5}$ $10^{6}$ $10^{7}$ $10^{8}$ $10^{9}$ $10^{10}$
$\frac{\sqrt{s}\,\tau_\omega}{2\omega}$

# Outlook

There are a number of interesting questions that remain:

- Is the general NGL resummation problem any easier for the simpler case of a hemisphere jet algorithm? If so, it seems likely that our results would facilitate a resummation of the NGLs that appear in the integrated jet thrust distribution for small but experimentally accessible values of the jet radius.
- Does a resummation of the leading soft NGLs significantly affect the distribution? It might be interesting to try and reproduce the analysis of Banfi et. al. in SCET.
- Is it possible to design a jet algorithm which would, by construction, be free of NGLs?