Higher order QCD corrections for associated VH production at hadron colliders

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Outline

1. Associated $VH$ production at hadron colliders
2. $q_T$-subtraction formalism at NNLO
3. Associated $VH$ production at NNLO: numerical results
4. Conclusions
Motivations

Associated vector boson Higgs ($VH$) production (with $H \rightarrow b \bar{b}$ and $V \rightarrow l_1 l_2$ decay) is an important mechanism for discovery and study the properties of the Higgs boson.

- At the LHC it is important channel through boosted analysis with jet reconstruction and decomposition techniques [Butterworth et al. (’08)].
- At the Tevatron is the main search channel in the low Higgs mass region.

To get closer to SM $VH$ sensitivity with the LHC 2012 data, precise theoretical predictions needed $\implies$ computation of higher-order QCD corrections.
Associated \( VH \) production

\[
\begin{align*}
    h_1(p_1) + h_2(p_2) & \rightarrow V + H + X \rightarrow \ell_1 \ell_2 + b \bar{b} + X \\
    \text{where} & \quad V = Z^0, W^\pm \quad \text{and} \quad \ell_1 \ell_2 = \ell^+ \ell^-, \ell \nu_{\ell}
\end{align*}
\]

According to the QCD factorization theorem:

\[
d\sigma(p_1, p_2) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 \ f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) \ d\hat{\sigma}_{ab}(x_1 p_1, x_2 p_2; \mu_F^2).
\]

\[
d\hat{\sigma}_{ab}(\hat{p}_1, \hat{p}_2; \mu_F^2) = d\hat{\sigma}_{ab}^{(0)}(\hat{p}_1, \hat{p}_2; \mu_F^2) + \alpha_S(\mu_R^2) d\hat{\sigma}_{ab}^{(1)}(\hat{p}_1, \hat{p}_2; \mu_F^2) + \alpha_S^2(\mu_R^2) d\hat{\sigma}_{ab}^{(2)}(\hat{p}_1, \hat{p}_2; \mu_F^2, \mu_R^2) + \mathcal{O}(\alpha_S^3).
\]

In the following we do not consider QCD corrections to \( H \rightarrow b \bar{b} \) decay.
Associated $VH$ production

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\]

\[
    + \alpha_S^2(\mu_R^2) \ d\hat{\sigma}_{ab}^{(2)}(\hat{p}_1, \hat{p}_2; \mu_F^2, \mu_R^2) + O(\alpha_S^3).
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In the following we do not consider QCD corrections to \( H \rightarrow b\bar{b} \) decay.
Associated $VH$ production: total cross section

- **NNLO QCD corrections for $WH$ are basically the same of DY ($\sim 1\text{-}3\%$ at the LHC) [Van Neerven et al. (’91), Brein, Harlander, Djouadi (’00)] → $vh@nnlo$.**

- For $ZH$, $gg \to HZ$ top-loop $\sim g^2\lambda_t^2\alpha_s^2$ (non DY-like) corrections ($+5\%$ at the LHC) [Kniehl (’90)] [Brein, Harlander, Djouadi (’00)] → $vh@nnlo$.

- **NNLO top-mediated contributions $\sim g^3\lambda_t\alpha_s^2$ to $WH$ and $ZH$ ($\sim 1\text{-}2\%$ at the LHC) recently computed:** [Brein, Harlander, Wiesemann, Zirke (’11)].

- **NLO EW corrections ($\sim 5\text{-}10\%$) [Ciccolini, Dittmaier, Krämer (’03)] [Denner, Dittmaier, Kallweit, Mück (’11)]**
Associated $VH$ production:

- **total cross section**

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Associated $VH$ production: differential distributions

- Fully differential NNLO QCD corrections for $WH$ (Drell-Yan like contributions), including tree-level $H \rightarrow b\bar{b}$ and $W \rightarrow l\nu$ decays with spin correlations [G.F,Grazzini,Tramontano(’11)].

- NNLO fully-differential decay rate $H \rightarrow b\bar{b}$ (in the $m_b = 0$ approx.) computed through new non-linear mapping method: [Anastasiou,Herzog,Lazopoulos(’12)].

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Fully-Exclusive Cross Sections at NNLO in hadron-collisions

- Experiments have finite acceptance, in particular $VH$ experimental analyses performed in extreme kinematical regimes (e.g., boosted analysis with jet veto): important to provide exclusive theoretical predictions.

- At NLO general algorithms (e.g., Dipole formalism [Catani,Seymour(’98)]) allow (relative) straightforward fully-exclusive calculations.

- At NNLO in hadronic collisions only few fully exclusive calculations exist:
  
  - **Sector decomposition**: [Binoth,Heinrich(’00)]
    
    \[ gg \rightarrow H \] [Anastasiou,Melnikov,Petriello(’04)]→FEHIP
    
    Drell-Yan [Melnikov,Petriello(’06)]→FEWZ
  
  - **$q_T$-subtraction**:
    
    \[ gg \rightarrow H \] [Catani,Grazzini(’07)]→HNNLO
    
    Drell-Yan [Catani,Cieri,de Florian,G.F.,Grazzini(’09)]→DYNLO
    
    Associated $WH$ production [G.F.,Grazzini,Tramontano(’11)]→WNNO
    
    Diphoton prod.[Catani,Cieri,de Florian,G.F.,Grazzini(’11)]→2$\gamma$NNLO
    
    (see L. Cieri talk)
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  - **Sector decomposition:** \[ \text{Binoth, Heinrich ('00)} \]
    \[ gg \to H \quad \text{Anastasiou, Melnikov, Petriello ('04)} \to \text{FEHIP} \]
    Drell-Yan \[ \text{Melnikov, Petriello ('06)} \to \text{FEWZ} \]
  - **\( q_T \)-subtraction:**
    \[ gg \to H \quad \text{Catani, Grazzini ('07)} \to \text{HNNLO} \]
    Drell-Yan \[ \text{Catani, Cieri, de Florian, G.F., Grazzini ('09)} \to \text{DYNNLO} \]
    Associated \( WH \) production \[ \text{G.F., Grazzini, Tramontano ('11)} \to \text{WNNLO} \]
    Diphoton prod. \[ \text{Catani, Cieri, de Florian, G.F., Grazzini ('11)} \to 2\gamma \text{NNLO} \]
    (see L. Cieri talk)
The $q_T$-subtraction formalism at NNLO

\[ h_1(p_1) + h_2(p_2) \to V(M, q_T) + X \]

$V$ is one or more colourless particles (vector bosons, leptons, photons, Higgs bosons,…) [Catani,Grazzini(’07)].

- **Key point I**: at LO the $q_T$ of the $V$ is exactly zero.

\[
\left. d\sigma^V_{(N)NLO} \right|_{q_T \neq 0} = d\sigma^{V+jets}_{(N)LO},
\]

for $q_T \neq 0$ the NNLO IR divergences cancelled with the NLO subtraction method.

- **Key point II**: treat the NNLO singularities at $q_T = 0$ by an additional subtraction using the universality of logarithmically-enhanced contributions from $q_T$ resummation formalism [Catani,de Florian,Grazzini(’00)].

\[
d\sigma_{nLO}^{V} \underset{q_T \to 0}{\longrightarrow} d\sigma_{LO}^{V} \otimes \Sigma(q_T/M)dq_T^2 = d\sigma_{LO}^{V} \otimes \sum_{n=1}^{\infty} \sum_{k=1}^{2n} \left( \frac{\alpha_s}{\pi} \right)^n \Sigma^{(n,k)} \frac{M^2}{q_T^2} \ln^{k-1} \frac{M^2}{q_T^2} d^2 q_T
\]

\[
d\sigma^{CT} \underset{q_T \to 0}{\longrightarrow} d\sigma_{LO}^{V} \otimes \Sigma(q_T/M)dq_T^2
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The $q_T$-subtraction formalism at NNLO

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- **Key point I:** at LO the $q_T$ of the $V$ is exactly zero.

  $$d\sigma^V_{(N)\text{NLO}}|_{q_T \neq 0} = d\sigma^{V+\text{jets}}_{(N)\text{LO}},$$

  for $q_T \neq 0$ the NNLO IR divergences cancelled with the NLO subtraction method.

- **Key point II:** treat the NNLO singularities at $q_T = 0$ by an additional subtraction using the universality of logarithmically-enhanced contributions from $q_T$ resummation formalism [Catani, de Florian, Grazzini('00)].

  $$d\sigma^V_{N^n\text{LO}} \xrightarrow{q_T \to 0} d\sigma^V_{\text{LO}} \otimes \sum(q_T/M)dq_T^2 = d\sigma^V_{\text{LO}} \otimes \sum_{n=1}^{\infty} \sum_{k=1}^{2n} \left( \frac{\alpha_S}{\pi} \right)^n \sum^{(n,k)} \frac{M^2}{q_T^2} \ln^{k-1} \frac{M^2}{q_T^2} d^2q_T$$

  $$d\sigma^C_T \xrightarrow{q_T \to 0} d\sigma^V_{\text{LO}} \otimes \sum(q_T/M)dq_T^2$$
The $q_T$-subtraction formalism at NNLO

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h_1(p_1) + h_2(p_2) \rightarrow V(M, q_T) + X
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- **Key point I**: at LO the $q_T$ of the $V$ is exactly zero.

\[
d\sigma_V^{(N)\text{NLO}}|_{q_T \neq 0} = d\sigma_{(N)\text{LO}}^{V+\text{jets}},
\]

for $q_T \neq 0$ the NNLO IR divergences cancelled with the NLO subtraction method.

- The only remaining NNLO singularities are associated with the $q_T \rightarrow 0$ limit.

- **Key point II**: treat the NNLO singularities at $q_T = 0$ by an additional subtraction using the universality of logarithmically-enhanced contributions from $q_T$ resummation formalism [Catani,de Florian,Grazzini(‘00)].

\[
d\sigma_{V_n\text{LO}}^{V} \xrightarrow{q_T \rightarrow 0} d\sigma_{LO}^{V} \otimes \Sigma(q_T/M)dq_T^2 = d\sigma_{LO}^{V} \sum_{n=1}^{\infty} \sum_{k=1}^{2n} \left(\frac{\alpha_s}{\pi}\right)^n \Sigma^{(n,k)} \frac{M^2}{q_T^2} \ln^{k-1} \frac{M^2}{q_T^2} d^2q_T
\]

\[
d\sigma_{CT}^{q_T \rightarrow 0} d\sigma_{LO}^{V} \otimes \Sigma(q_T/M)dq_T^2
\]
The final result valid also for $q_T = 0$ is:

$$d\sigma^V_{(N)NLO} = \mathcal{H}^V_{(N)NLO} \otimes d\sigma^V_{LO} + \left[ d\sigma^{V+\text{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right],$$

where

$$\mathcal{H}^V_{NNLO} = \left[ 1 + \frac{\alpha_S}{\pi} \mathcal{H}^V(1) + \left( \frac{\alpha_S}{\pi} \right)^2 \mathcal{H}^V(2) \right].$$

- The choice of the counter-term has some arbitrariness but it must behave $d\sigma^{CT}_{q_T \rightarrow 0} \rightarrow d\sigma^V_{LO} \otimes \Sigma(q_T/M)dq_T^2$ where $\Sigma(q_T/M)$ is universal.

- $d\sigma^{CT}$ regularizes the $q_T = 0$ singularity of $d\sigma^{V+\text{jets}}$: double real and real-virtual NNLO contributions, while (the finite part of) two-loops virtual corrections are contained in $\mathcal{H}^V_{NNLO}$.

- Final state partons only appear in $d\sigma^{V+\text{jets}}$ so that NNLO IR-safe cuts are included in the NLO computation: observable-independent NNLO extension of the subtraction formalism.
The final result valid also for $q_T = 0$ is:

$$d\sigma_{(N)NLO}^V = \mathcal{H}_{(N)NLO}^V \otimes d\sigma_{LO}^V + \left[ d\sigma_{(N)LO}^{V+\text{jets}} - d\sigma_{(N)LO}^{CT} \right],$$

where $\mathcal{H}_{NNLO}^V = \left[ 1 + \frac{\alpha_S}{\pi} \mathcal{H}^{V(1)} + \left( \frac{\alpha_S}{\pi} \right)^2 \mathcal{H}^{V(2)} \right]$

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- Final state partons only appear in $d\sigma_{V+\text{jets}}$ so that NNLO IR-safe cuts are included in the NLO computation: observable-independent NNLO extension of the subtraction formalism.
The final result valid also for $q_T = 0$ is:

\[
d\sigma^{V,(N)NLO}_{(N)NLO} = \mathcal{H}^{V,(N)NLO} \otimes d\sigma^{V,LO} + \left[ d\sigma^{V+jets,(N)LO}_{(N)LO} - d\sigma^{CT,(N)LO}_{(N)LO} \right] ,
\]

where \( \mathcal{H}^{V,NNLO} = \left[ 1 + \frac{\alpha_S}{\pi} \mathcal{H}^{V(1)} + \left( \frac{\alpha_S}{\pi} \right)^2 \mathcal{H}^{V(2)} \right] \).

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  \( d\sigma^{CT, q_T \to 0} \to d\sigma^{V,LO} \otimes \Sigma(q_T/M)dq_T^2 \) where \( \Sigma(q_T/M) \) is universal.

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where $\mathcal{H}^{V}_{NNLO} = \left[ 1 + \frac{\alpha_S}{\pi} \mathcal{H}^{V(1)} + \left( \frac{\alpha_S}{\pi} \right)^2 \mathcal{H}^{V(2)} \right]$.

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- Final state partons only appear in $d\sigma^{V+\text{jets}}$ so that NNLO IR-safe cuts are included in the NLO computation: observable-independent NNLO extension of the subtraction formalism.
Associated WH production in NNLO QCD

G.F., Grazzini, Tramontano arXiv:1107.1164

A NLO calculation for \( h_1 h_2 \rightarrow V + X \) requires:

- \( d\sigma_{LO}^{V+\text{jets}} \) (and \( d\sigma_{LO}^{V} \)).
- \( \mathcal{H}_{V}^{(1)} \) [de Florian, Grazzini (’01)]: contains the finite-part of the one-loop amplitude \( c\bar{c} \rightarrow V \).
- \( d\sigma_{LO}^{CT} \): depends by the (universal) \( q_T \)-resummation coeff. \( A_1 \) and \( B_1 \).

A NNLO calculation for \( h_1 h_2 \rightarrow V + X \) requires also:

- \( d\sigma_{NLO}^{V+\text{jets}} \).
- \( \mathcal{H}_{V}^{(2)} \): contains the finite-part of the two-loops amplitude \( c\bar{c} \rightarrow V \).
- \( d\sigma_{NLO}^{CT} \): depends by the (universal) \( q_T \)-resummation coeff. \( A_2 \) and \( B_2 \).

WH production at NNLO within \( q_T \)-subtraction:

- \( d\sigma_{NLO}^{WH+\text{jets}} \).
- \( \mathcal{H}_{DY}^{(2)} \) [Catani, Cieri, de Florian, G.F., Grazzini (’12)].

Fully-exclusive NNLO calculation, implemented in the parton-level Monte Carlo code: [G.F., Grazzini, Tramontano (’11)].
Associated WH production in NNLO QCD

G.F., Grazzini, Tramontano arXiv:1107.1164

- A NLO calculation for $h_1 h_2 \to V + X$ requires:
  - $d\sigma^{V+\text{jets}}_{\text{LO}}$ (and $d\sigma^V_{\text{LO}}$).
  - $\mathcal{H}^V(1)$ [de Florian, Grazzini ('01)]: contains the finite-part of the one-loop amplitude $c\bar{c} \to V$.
  - $d\sigma^{CT}_{\text{LO}}$: depends by the (universal) $q_T$-resummation coeff. $A_1$ and $B_1$.

- A NNLO calculation for $h_1 h_2 \to V + X$ requires also:
  - $d\sigma^{V+\text{jets}}_{\text{NLO}}$.
  - $\mathcal{H}^V(2)$: contains the finite-part of the two-loops amplitude $c\bar{c} \to V$.
  - $d\sigma^{CT}_{\text{NLO}}$: depends by the (universal) $q_T$-resummation coeff. $A_2$ and $B_2$.

- WH production at NNLO within $q_T$-subtraction:
  - $d\sigma^{WH+\text{jets}}_{\text{NLO}}$.
  - $\mathcal{H}^{DY(2)}$ [Catani, Cieri, de Florian, G.F., Grazzini ('12)].

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**Fully-exclusive NNLO calculation**, implemented in the parton-level Monte Carlo code: [G.F., Grazzini, Tramontano(’11)]
Numerical results at the LHC and the Tevatron
Selection strategy of [Butterworth et al. (’08)]: search a large-$p_T$ Higgs boson through a collimated $b\bar{b}$ pair decay.

Cuts:
Leptons: $p_T^{l} > 30\text{GeV}$, $|\eta^{l}| < 2.5$,
$\not{p}_T > 30\text{GeV}$, $p_T^{W} > 200\text{GeV}$.
Jets: Cambridge/Aachen algorithm with $R=1.2$.
Fat jet (contain the $b\bar{b}$) $p_T^{J} > 200\text{GeV}$, $|\eta^{J}| < 2.5$
Jet veto: No other jets with $p_T > 20\text{GeV}$ and $|\eta| < 5$.

Large negative higher-order corrections: NLO (NNLO) effects -52%/-36% (-6%/-19%), depending on the scale choice (factor two around $\mu_F = \mu_R = m_W + m_H$).

Jet veto strongly affect the higher order corrections ⇒ stability of fixed order calculation challenged.

$pp \rightarrow WH(\rightarrow l\nu b\bar{b})$

$p_T$ spectra of the fat jet at the LHC@14TeV for $m_H = 120\text{GeV}$ at LO (dots), NLO (dashes) and NNLO (solid).
Outline  |  Associated VH production  |  $q_T$-subtraction  |  VH production at NNLO  |  Conclusions

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Cuts:
Leptons: $p_T^l > 20 \text{GeV}, |\eta^l| < 2, p_T^{\text{miss}} > 20 \text{GeV}$. 
Jets: $k_T$ algorithm with $R=0.4$.
Exactly two jets (with $p_T > 20 \text{GeV}$ and $|\eta| < 2$) at least one of them has to be a $b$ jet (with $|\eta| < 1$).

Higher-order corrections: NLO (NNLO) effects from $+13\%$ to $+30\%$ (from $-1\%$ to $+4\%$) depending on the scale choice (factor two around $\mu_F = \mu_R = m_W + m_H$). The scale dependence is at the level of about $\pm 1\%$ both at NLO and NNLO.

The shape of the distribution is stable against perturbative corrections. Perturbative expansion under good control.

$p\bar{p} \rightarrow WH(\rightarrow l\nu b\bar{b})$
$p_T$ spectra of the dijet system at the Tevatron for $m_H = 120 \text{GeV}$ at LO (dots), NLO (dashes) and NNLO (solid).
Cuts:
Leptons: $p_T^l > 20 \text{GeV}, |\eta^l| < 2$, $p_T^{\text{miss}} > 20 \text{GeV}$.  
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NEW: associated $ZH$ production at NNLO:

G.F., Grazzini, Tramontano (in preparation)

$gg \rightarrow HZ$ top-loop $\sim g^2 \lambda_t^2 \alpha_S^2$ (non DY-like) corrections included.

- Cuts (we follow CMS analysis):
  - Leptons: $p_T^l > 20 \text{GeV}, |\eta^l| < 2.5$,
  - $75 < m_{ll} < 105 \text{GeV}, p_T^{ll} > 100 \text{GeV}$.
  - Jets: anti-$k_T$ algorithm with $R=0.5$.
  - Two $b$-jets (with $p_T > 20 \text{GeV}, |\eta| < 2.5$ and $p_T^{bb} > 100 \text{GeV}$).
  - Jet veto: extra jet radiation is vetoed if $p_T > 20 \text{GeV}$ and $|\eta| < 2.5$.

- Higher-order corrections: NLO (NNLO) effects:
  - without jet-veto $+20\% (+14\%)$; with jet-veto $-20\% (+14\%)$.
  - Effect of the jet-veto: $-33\%$ both at NLO and NNLO.

$pp \rightarrow ZH(\rightarrow 2l b\bar{b})$

$p_T$ spectra of the $b\bar{b}$ system at the LHC for $m_H = 125 \text{GeV}$ at NLO and NNLO with and without the jet veto.
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- Cuts (we follow CMS analysis):
  - Leptons: $p_T^l > 20$ GeV, $|\eta^l| < 2.5$, $75 < m_{ll} < 105$ GeV, $p_T^{ll} > 100$ GeV.
  - Jets: anti-$k_T$ algorithm with $R=0.5$.
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Distributions in $p_{T,H}$ for $pp \rightarrow WH \rightarrow l\nu H$ (NNLO QCD + NLO EW) and for $pp \rightarrow ZH \rightarrow ll/\nu\nu H$ (NLO QCD + NLO EW) at $\sqrt{s} = 7$ TeV.

Boosted setup: $|\eta_l| < 2.5$, $p_{T,l} > 20$ GeV, $p_{T,\nu} > 25$ GeV, $p_{T,H} > 200$ GeV, $p_{T,W/Z} > 190$ GeV.

We produced similar results at $\sqrt{s} = 8$ TeV.
Conclusions

- Calculation of **NNLO QCD corrections to VH production** in hadron collision using the $q_T$-subtraction formalism, included in a **fully-exclusive** parton-level Monte Carlo code.

- NNLO corrections can be important:
  - *large and negative:* $\sim -20\%$ for the $WH$ fat-jet analysis at the LHC@14 TeV when a jet veto is applied;
  - *sizeable* for the CMS analysis at the LHC;
  - *moderate* for the $WH$ Tevatron analysis.

- **Outlook/Work in progress:**
  - Public release of the parton-level numerical code.
  - Inclusion of the higher-order QCD correction to $H \rightarrow b\bar{b}$ decay.
  - Extension to the $ZH$ production.
  - Inclusion of $H \rightarrow WW/ZZ \rightarrow 2l2\nu/4l$ decay.
  - Comparison with parton-shower Monte Carlo predictions.
  - Study of the NNLO uncertainty band: **first reliable estimate** of perturbative uncertainty.