Helicity amplitudes in high-energy factorization

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in collaboration with

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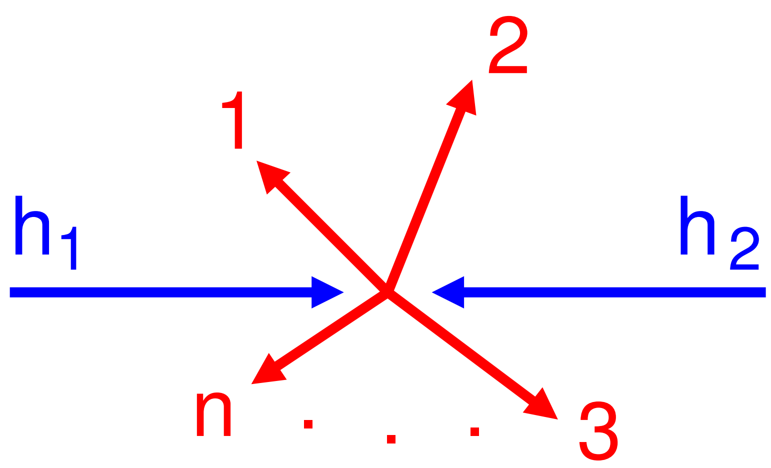
Outline

- high-energy factorization and off-shell scattering amplitudes
- helicity amplitudes for $g^* g \rightarrow n\, g$
- helicity amplitudes for $g^* g^* \rightarrow X$
- summary
- update ONELOOP

Hard scattering cross sections within collinear factorization



PDFs are related to the structure of the hadrons, universal to the scattering process

$$\sigma_{h_1,h_2\to n}(p_1,p_2) = \sum_{a,b}\int dx_1 dx_2\, f_a(x_1,\mu)\, f_b(x_2,\mu)\, \hat{\sigma}_{a,b\to n}(x_1p_1,x_2p_2;\mu)$$

$$\hat{\sigma}_{a,b\to n}(p_a,p_b;\mu) = \int d\Phi(p_a,p_b\to\{p\}_n)\, |\mathcal{M}_{a,b\to n}(p_a,p_b\to\{p\}_n;\mu)|^2\, \mathcal{O}(p_a,p_b,\{p\}_n)$$

Phase space (includes spin/color summation) governs the kinematics

Matrix element (squared) contains model parameters, governs the dynamics

Observable, imposes phase space cuts

High–energy, or kT, factorization

Gribov, Levin, Ryskin 1983
Catani, Ciafaloni, Hautmann 1991

$$\sigma_{h_1,h_2\to QQ} = \int d^2k_{1\perp}\,\frac{dx_1}{x_1}\,\mathcal{F}(x_1,k_{1\perp})\,d^2k_{2\perp}\,\frac{dx_2}{x_2}\,\mathcal{F}(x_2,k_{1\perp})\,\hat{\sigma}_{gg}\left(\frac{m^2}{x_1x_2s},\frac{k_{1\perp}}{m},\frac{k_{2\perp}}{m}\right)$$

- to be applied in the 3-scale regime $s \gg m^2 \gg \Lambda^2_{QCD}$
- reduces to collinear factorization for $s \gg m^2 \gg k_\perp^2$,
  but holds also for $s \gg m^2 \sim k_\perp^2$
- *unintegrated pdf* $\mathcal{F}$ may satisfy BFKL-eqn, CCFM-eqn, BK-eqn...
- typically associated with small-x physics
- relevant for forward physics, saturation physics, heavy-ion physics...
- $k_\perp$ gives a handle on the size of the proton
- it is known how to construct the necessary gauge invariant matrix elements with off-shell gluons Lipatov 1995, Antonov, Lipatov, Kuraev,Cherednikov 2005

Lipatov's effective action

Effective action in terms of quarks $\psi, \bar{\psi}$ gluons v_μ and reggeized gluons $A_\pm$.

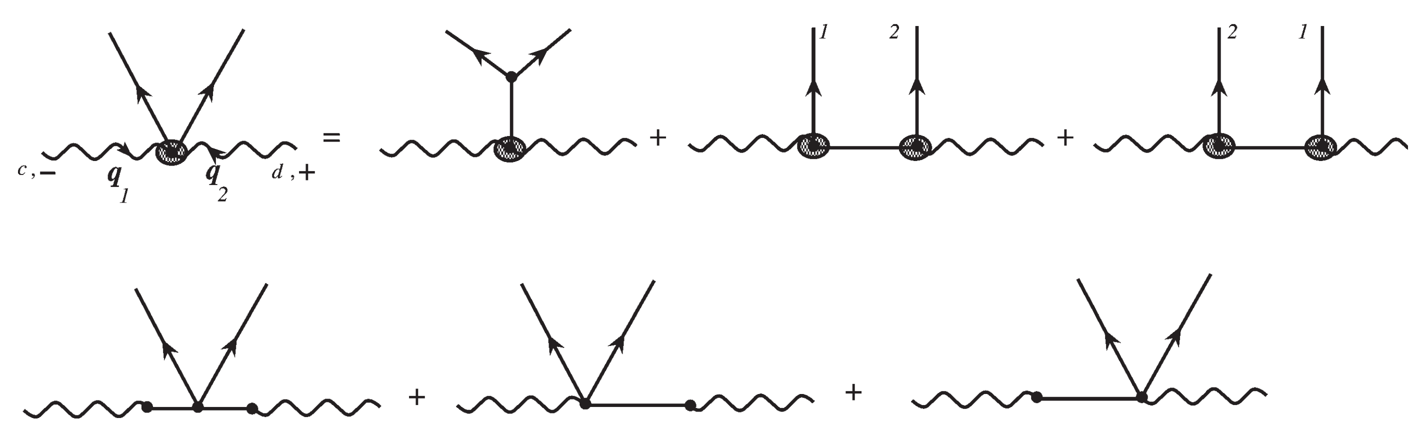
$$\mathcal{L} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{ind}}$$

$$\mathcal{L}_{\text{QCD}} = i\bar{\psi}\hat{D}\psi + \frac{1}{2}\text{Tr}\, G_{\mu\nu}^2 \qquad D_\mu = \partial_\mu + g v_\mu \qquad G_{\mu\nu} = \frac{1}{g}[D_\mu, D_\nu]$$

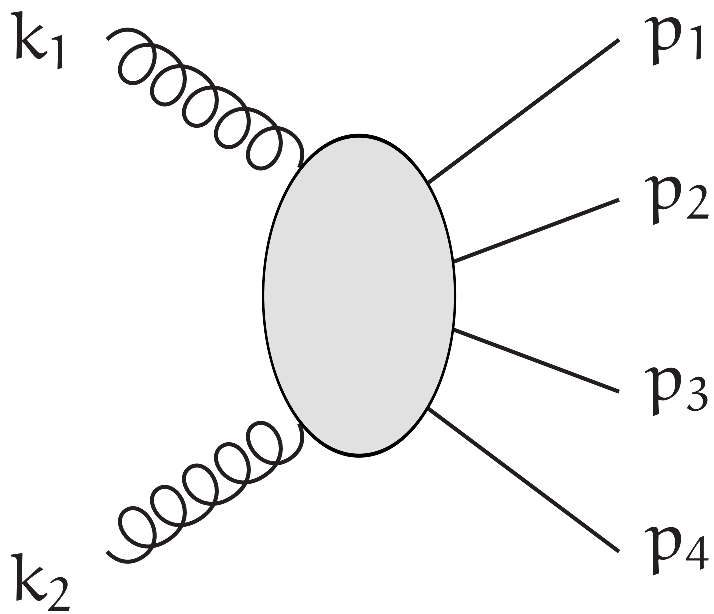
$$\mathcal{L}_{\text{ind}} = -\text{Tr}\left\{ \frac{1}{g}\partial_+ \left[\mathcal{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{x^+} v_+(y)\,dy^+ \right) \right] \cdot \partial_\sigma^2 A_-(x) + \frac{1}{g}\partial_- \left[\mathcal{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{x^-} v_-(y)\,dy^- \right) \right] \cdot \partial_\sigma^2 A_+(x) \right\}$$

$$k_\pm = (n_\mu^\pm)k^\mu \qquad (n^-)^2 = (n^+)^2 = 0 \qquad n^+ \cdot n^- = 2$$

Amplitudes are build up with the help of effective reggeon-gluon vertices.



Kinematical setup



$$k_1 + k_2 = p_1 + p_2 + p_3 + p_4$$

$$k_1 = x_1 P_A + k_{\perp 1} \qquad k_2 = x_2 P_B + k_{\perp 2}$$

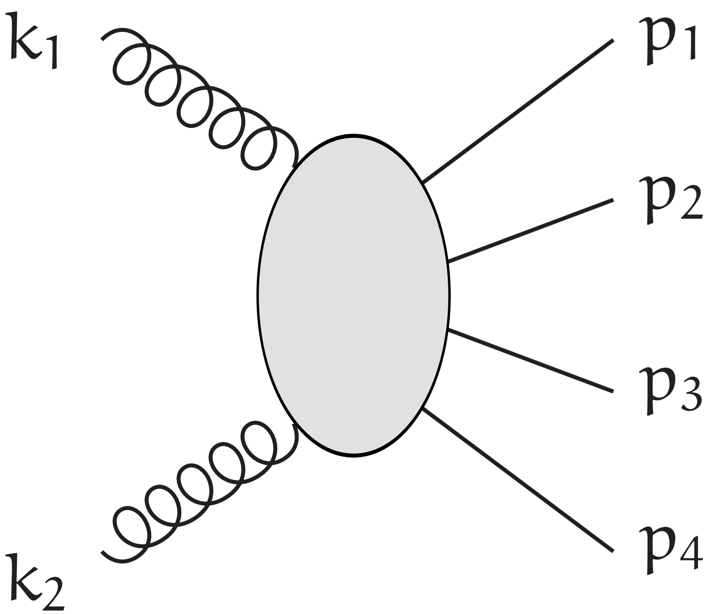
$$P_A \cdot k_{\perp 1} = P_A \cdot k_{\perp 2} = P_B \cdot k_{\perp 1} = P_B \cdot k_{\perp 2} = 0$$

$$P_A^2 = P_B^2 = 0$$

$$k_1^2 = k_{\perp 1}^2 \qquad k_2^2 = k_{\perp 2}^2$$

Off-shell initial-state gluons $\Longrightarrow$ what about gauge invariance?

Kinematical setup



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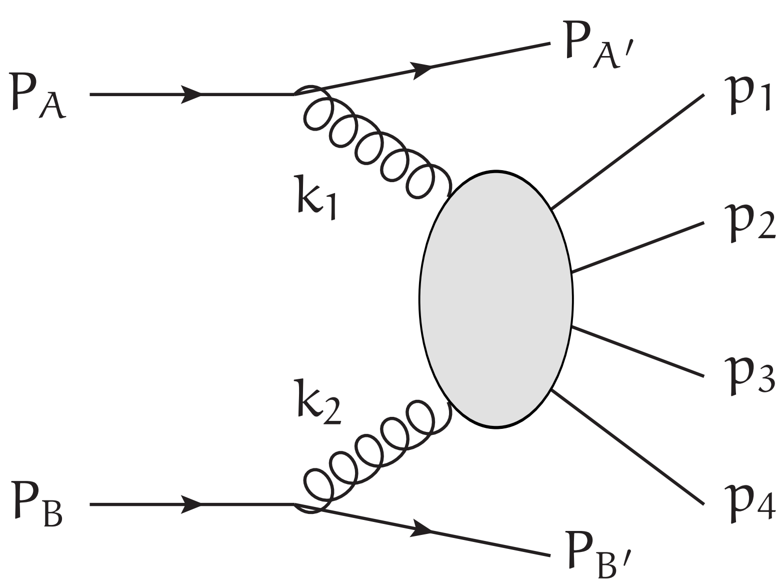
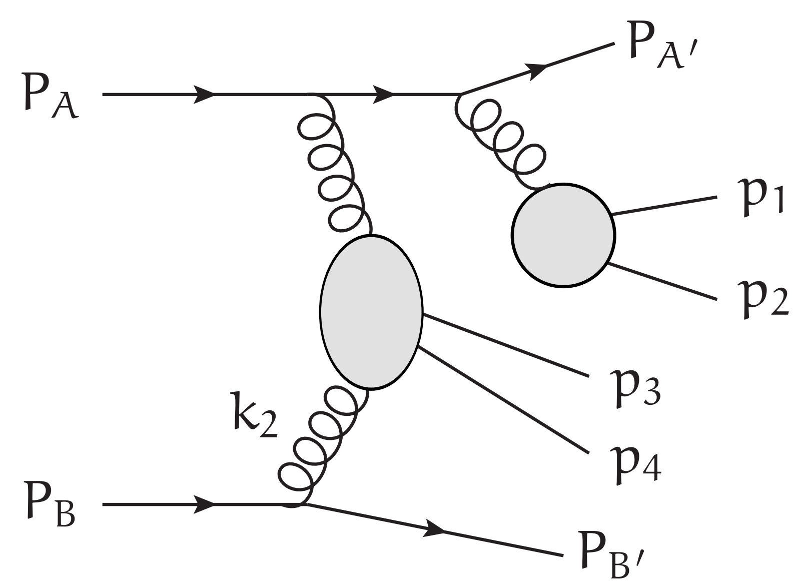
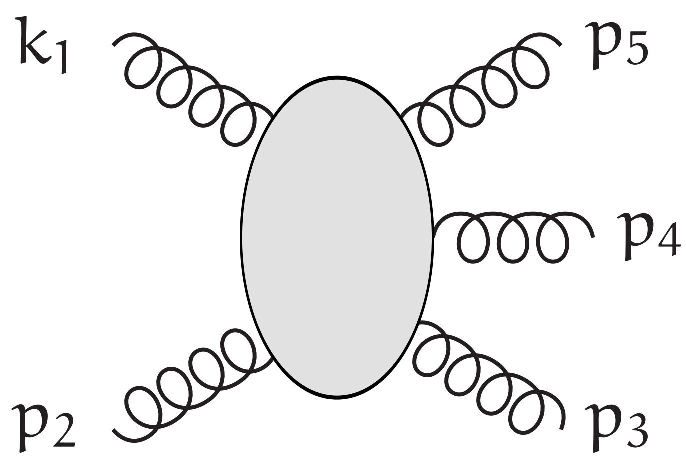




One off–shell gluon: $g^* g \rightarrow n\, g$

arXiv:1207.3332



Color-ordered (dual) amplitude

$$k_1 = p_1 + k_\perp$$
$$p_1 \cdot k_\perp = 0$$
$$p_i^2 = 0$$
$$k_1^2 = k_\perp^2$$

Use axial gauge with gauge vector p_1, and “polarization vector” for off-shell gluon

$$\text{gluon propagator} = \frac{-i}{p^2}\left[\eta^{\mu\nu} - \frac{p_1^\mu p^\nu + p^\mu p_1^\nu}{p_1 \cdot p}\right] \qquad \text{“}\varepsilon_1^\mu\text{”} = \frac{k_\perp^\mu}{|\vec{k}_\perp|}$$

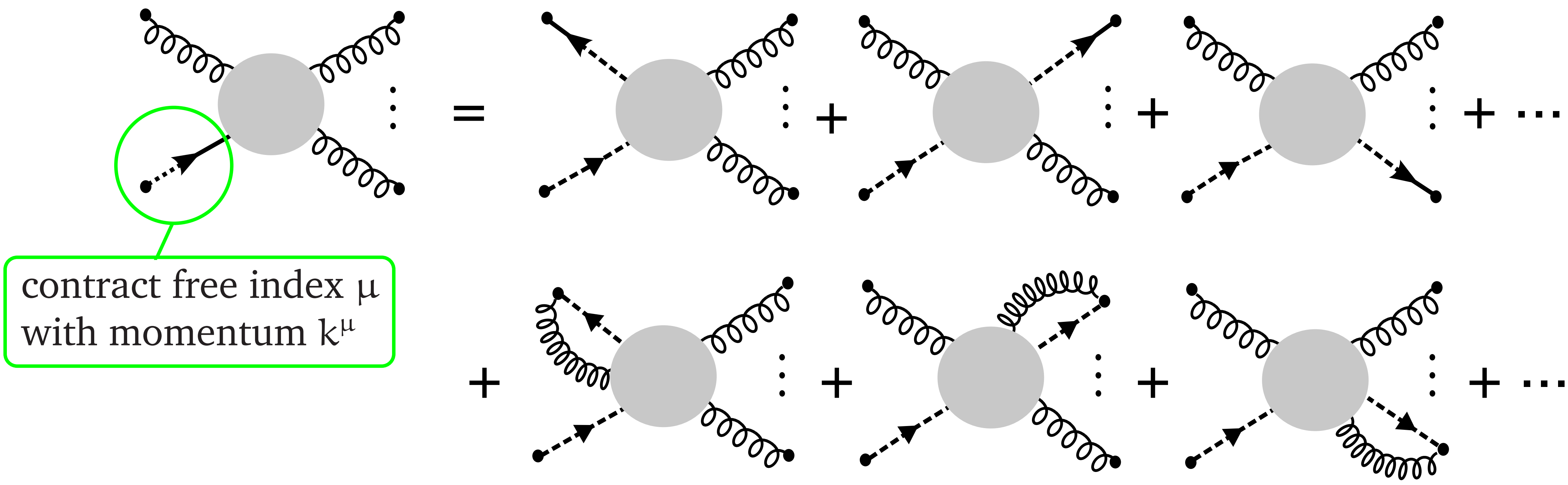
Then, the corrected n-gluon amplitude

$$\tilde{\mathcal{A}} = \mathcal{A} - \left(\frac{-g}{\sqrt{2}}\right)^{n-2} \frac{|\vec{k}_\perp|\; \varepsilon_2 \cdot p_1\; \varepsilon_3 \cdot p_1\; \cdots\; \varepsilon_n \cdot p_1}{p_2 \cdot p_1\; (p_2 + p_3) \cdot p_1\; \cdots\; (p_2 + p_3 + \cdots + p_{n-1}) \cdot p_1}$$

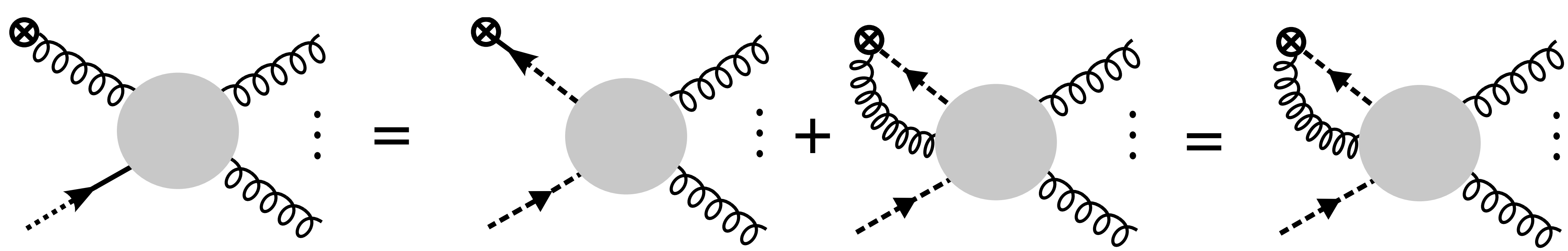
satisfies Ward identities $\tilde{\mathcal{A}}(\varepsilon_i \leftarrow p_i) = 0$.

Slavnov–Taylor identities

ST identities relate Green functions to each other, including external ghosts.

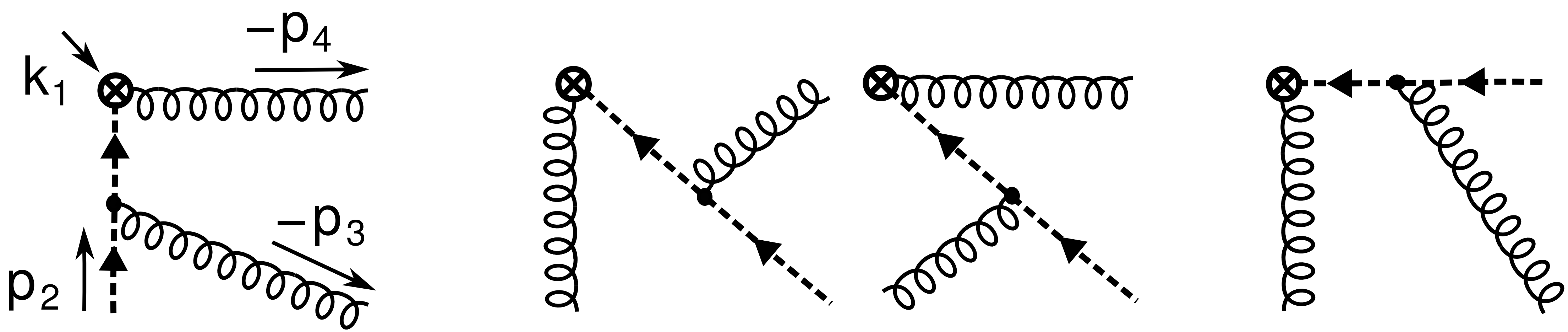


The relations involve amplitudes with external ghosts, which do not necessarily vanish in the axial gauge.



Construction of correction term for n=4

$$k_1 = p_1 + k_\perp \qquad k_1\cdot p_1 = 0 \qquad (p_2+p_3+p_4)\cdot p_1 = 0$$



$$\frac{\varepsilon_3\cdot p_1\ \varepsilon_4\cdot p_1}{(p_2+p_3)\cdot p_1} \qquad \frac{\varepsilon_2\cdot p_1\ \varepsilon_4\cdot p_1}{(k_1+p_2)\cdot p_1} \qquad \frac{\varepsilon_2\cdot p_1\ \varepsilon_4\cdot p_1}{(k_1+p_4)\cdot p_1} \qquad \frac{\varepsilon_2\cdot p_1\ \varepsilon_3\cdot p_1}{(k_1+p_2)\cdot p_1}$$

Mulitply with eikonal factors

$$\frac{\varepsilon_2\cdot p_1}{p_2\cdot p_1}\,\frac{\varepsilon_3\cdot p_1\ \varepsilon_4\cdot p_1}{(p_2+p_3)\cdot p_1} + \frac{\varepsilon_3\cdot p_1}{p_3\cdot p_1}\left[\frac{\varepsilon_2\cdot p_1\ \varepsilon_4\cdot p_1}{(k_1+p_2)\cdot p_1} + \frac{\varepsilon_2\cdot p_1\ \varepsilon_4\cdot p_1}{(k_1+p_4)\cdot p_1}\right] + \frac{\varepsilon_4\cdot p_1}{p_4\cdot p_1}\,\frac{\varepsilon_2\cdot p_1\ \varepsilon_3\cdot p_1}{(k_1+p_2)\cdot p_1}$$

$$= -\frac{\varepsilon_2\cdot p_1\ \varepsilon_3\cdot p_1\ \varepsilon_4\cdot p_1}{p_2\cdot p_1\ (p_2+p_3)\cdot p_1}$$

Helicity amplitudes

Now that the Ward identities hold, any reference vectors q_i can be chosen for the polarization vectors.

$$\varepsilon_-^\mu(p_i, q_i) = -\frac{[q_i|\gamma^\mu|p_i\rangle}{\sqrt{2}\,[q_i p_i]} \qquad \varepsilon_+^\mu(p_i, q_i) = \frac{\langle q_i|\gamma^\mu|p_i]}{\sqrt{2}\,\langle q_i p_i\rangle}$$

The correction term vanishes completely if we choose reference momentum $q_i = p_1$ for any polarization vector. Compact expressions for $n = 4$:

$$\tilde{\mathcal{A}}(2^-,3^-,4^-) = 0 \qquad \tilde{\mathcal{A}}(2^+,3^+,4^+) = 0$$

$$\tilde{\mathcal{A}}(2^-,3^-,4^+) = \frac{g^2}{\sqrt{2}}\,\frac{[3|\not{k}_T|1\rangle}{|\vec{k}_T|[31]}\,\frac{[41]^4}{[12][23][34][41]} \qquad \tilde{\mathcal{A}}(2^+,3^+,4^-) = \frac{g^2}{\sqrt{2}}\,\frac{\langle 1|\not{k}_T|3]}{|\vec{k}_T|\langle 13\rangle}\,\frac{\langle 41\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

$$\tilde{\mathcal{A}}(2^+,3^-,4^-) = \frac{g^2}{\sqrt{2}}\,\frac{[3|\not{k}_T|1\rangle}{|\vec{k}_T|[31]}\,\frac{[12]^4}{[12][23][34][41]} \qquad \tilde{\mathcal{A}}(2^-,3^+,4^+) = \frac{g^2}{\sqrt{2}}\,\frac{\langle 1|\not{k}_T|3]}{|\vec{k}_T|\langle 13\rangle}\,\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

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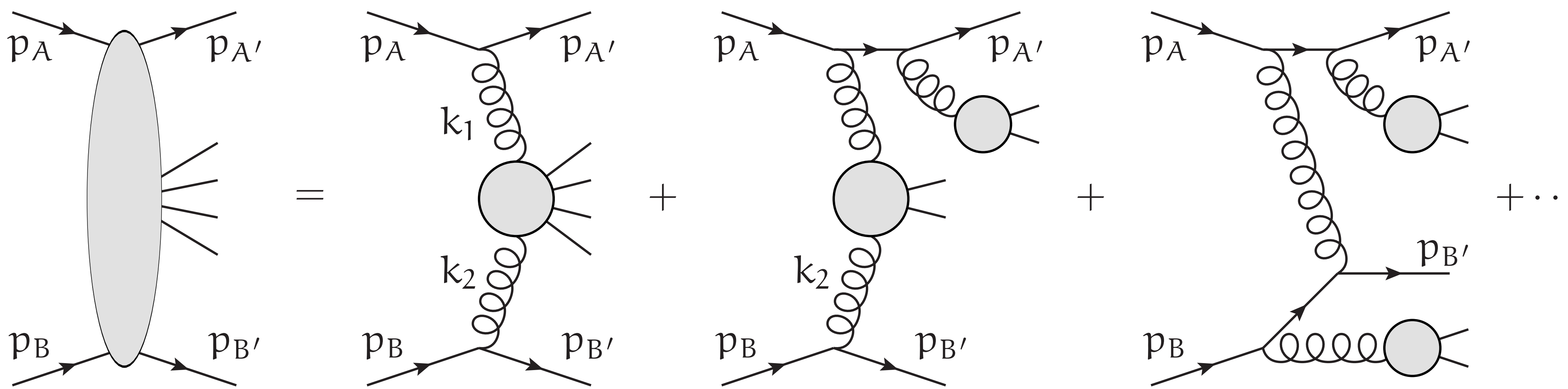
In agreement with Hautmann, Deak, Jung, Kutak 2009

Two off–shell initial–state gluons

Embed the process $g^*g^* \to X$ in the on-shell process $q_A q_B \to q_A q_B + X$.



$$\ell_1 = (E, 0, 0, E) \qquad \ell_2 = (E, 0, 0, -E)$$

$$p_A - p_{A'} = k_1 = x_1\ell_1 + k_{1\perp} + y_2\ell_2 \qquad p_B - p_{B'} = k_2 = x_2\ell_2 + k_{2\perp} + y_1\ell_1$$

The terms $y_2\ell_2$ and $y_1\ell_1$ are necessary to keep all quark momenta on-shell. Usually, one takes $\ell_1 = p_A$ and $\ell_2 = p_B$, and extracts the amplitude for $g^*g^* \to X$ by neglecting terms proportional to $y_{1,2}$. That is the *high-energy limit*.

Continuation to complex momenta

- we are just interested in a gauge invariant amplitude $\mathcal{A}(g^*g^* \to X)$
- the amplitude $\mathcal{A}(q_A q_B \to q_A q_B + X)$ must be gauge invariant, must be completely on-shell, but does not have to be physical
- introduce complex on-shell momenta $p_A, p_{A'}, p_B, p_{B'}$

$$\ell_3^\mu = \tfrac{1}{2}\langle \ell_2 - | \gamma^\mu | \ell_1 - \rangle \qquad \ell_4^\mu = \tfrac{1}{2}\langle \ell_1 - | \gamma^\mu | \ell_2 - \rangle$$

$$p_A^\mu = (\Lambda + x_1)\ell_1^\mu - \frac{\ell_4 \cdot k_{1\perp}}{\ell_1 \cdot \ell_2}\,\ell_3^\mu \qquad p_{A'}^\mu = \Lambda \ell_1^\mu + \frac{\ell_3 \cdot k_{1\perp}}{\ell_1 \cdot \ell_2}\,\ell_4^\mu$$

$$p_B^\mu = (\Lambda + x_2)\ell_2^\mu - \frac{\ell_3 \cdot k_{2\perp}}{\ell_1 \cdot \ell_2}\,\ell_4^\mu \qquad p_{B'}^\mu = \Lambda \ell_2^\mu + \frac{\ell_4 \cdot k_{2\perp}}{\ell_1 \cdot \ell_2}\,\ell_3^\mu$$

Now we have both the high-energy limit and on-shellness:

$$p_A^\mu - p_{A'}^\mu = x_1 \ell_1^\mu + k_{1\perp}^\mu \qquad p_B^\mu - p_{B'}^\mu = x_2 \ell_2^\mu + k_{2\perp}^\mu$$

$$p_A^2 = p_{A'}^2 = p_B^2 = p_{B'}^2 = 0$$

for any value of the dimensionless parameter Λ.

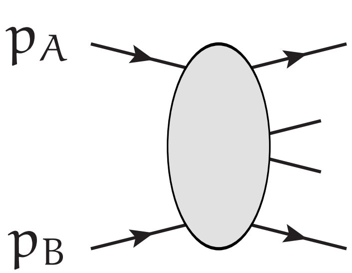
Extract physical amplitude

Assign spinors to quarks without breaking gauge invariance.

$$|\ell_3-\rangle = |\ell_1-\rangle \qquad \langle\ell_4-| = \langle\ell_1-| \qquad\Longrightarrow\qquad q_A(p_A) \to |\ell_1-\rangle \qquad q_A(p_{A'}) \to \langle\ell_1-|$$

$$|\ell_4-\rangle = |\ell_2-\rangle \qquad \langle\ell_3-| = \langle\ell_2-| \qquad\qquad q_B(p_B) \to |\ell_2-\rangle \qquad q_B(p_{B'}) \to \langle\ell_2-|$$

Take limit $\Lambda \to \infty$ to extract physical amplitude. This is not an approximation.



$$p_A = (\Lambda + x_1)\ell_1 + \kappa_{13}\ell_3$$
$$p_{A'} = \Lambda\ell_1 - \kappa_{14}\ell_4$$
$$p_A - p_{A'} = x_1\ell_1 + k_{1\perp}$$

$$p_B = (\Lambda + x_2)\ell_2 + \kappa_{24}\ell_4$$
$$p_{B'} = \Lambda\ell_2 - \kappa_{23}\ell_3$$
$$p_B - p_{B'} = x_2\ell_2 + k_{2\perp}$$

For for an A-quark line propagator, we get

$$\frac{\not{p}}{p^2} = \frac{(\Lambda + x)\not{\ell}_1 + y\not{\ell}_2 + \not{p}_\perp}{2(\Lambda + x)y\,\ell_1\cdot\ell_2 + p_\perp^2} \xrightarrow{\Lambda\to\infty} \frac{\not{\ell}_1}{2y\,\ell_1\cdot\ell_2} = \frac{\not{\ell}_1}{2\,\ell_1\cdot p}$$

Gluons will attach to A-quark line via eikonal couplings

$$\begin{aligned} &\langle\ell_1-|\,\gamma^{\mu_1}\not{\ell}_1\,\gamma^{\mu_2}\not{\ell}_1\,\cdots\,|\ell_1-\rangle \\ &= \langle\ell_1-|\,\gamma^{\mu_1}\,|\ell_1-\rangle\langle\ell_1-|\,\gamma^{\mu_2}\,|\ell_1-\rangle\langle\ell_1-|\,\cdots\,|\ell_1-\rangle \\ &= (2\ell_1^{\mu_1})(2\ell_1^{\mu_2})\cdots\;. \end{aligned}$$

Analogously for B-quark line.

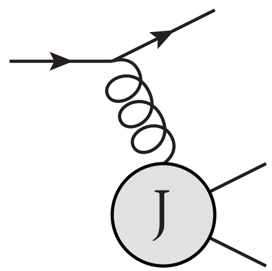
The prescription to get $\mathcal{A}(g^* g^* \to X)$

1. Consider the process $q_A\, q_B \to q_A\, q_B\, X$, where q_A, q_B are distinguishable massless quarks not occurring in X, and with momentum flow as if the momenta p_A, p_B of the initial-state quarks and $p_{A'}, p_{B'}$ of the final-state quarks are given by
$$p_A^\mu = k_1^\mu \quad , \quad p_B^\mu = k_2^\mu \quad , \quad p_{A'}^\mu = p_{B'}^\mu = 0 \,.$$
2. Interpret every vertex on the A-quark line as $g_s T^a_{ij}\, \ell_1^\mu$ instead of $-ig_s T^a_{ij}\, \gamma^\mu$.
3. Interpret every vertex on the B-quark line as $g_s T^a_{ij}\, \ell_2^\mu$ instead of $-ig_s T^a_{ij}\, \gamma^\mu$.
4. Interpret every propagator on the A-quark line as $\delta_{ij}/\ell_1 \cdot p$ instead of $i\delta_{ij}/\not{p}$.
5. Interpret every propagator on the B-quark line as $\delta_{ij}/\ell_2 \cdot p$ instead of $i\delta_{ij}/\not{p}$.
6. To get the correct collinear limit and power of coupling, multiply the amplitude with
$$\frac{i\, x_1 \sqrt{-k_{1\perp}^2}}{g_s} \times \frac{i\, x_2 \sqrt{-k_{2\perp}^2}}{g_s}$$

For the rest, normal Feynman rules apply.

In agreement with Lipatov's effective action.

One off–shell initial–state gluon



$$\ell_1^\mu \left(-\eta_\mu^\nu + \frac{k_\mu n^\nu + n_\mu k^\nu}{n\cdot k} - n^2 \frac{k_\mu k^\nu}{(n\cdot k)^2} \right) J_\nu = -\ell_1 \cdot J + \frac{\ell_1 \cdot k}{n\cdot k} n\cdot J$$

- the current $J = J_1$, with momentum k_1 and containing all particles except the q_A quarks, is attached to the A-quark line as $-\ell_1 \cdot J_1$, independently of the gauge.
- current conservation $k_1 \cdot J_1 = 0$ implies $-\ell_1 \cdot J_1 = \frac{1}{x_1} k_{1\perp} \cdot J_1$
- if we choose the gauge with $n^\mu = \ell_1^\mu$, then all contributions with currents attached to the A-quark line vanish, except
  - the one with J_1
  - and possibly a contribution for which all gluons attached to the A-quark line are on-shell

This is exactly in agreement with the found correction term for $g^* g \to n g$.

Summary

- high-energy factorization
- gauge invariant off-shell helicity amplitudes for $g^* g \to n\, g$
- gauge invariant off-shell helicity amplitudes for $g^* g^* \to X$
- in agreement with Lipatov's effective action
- implemented in a Monte Carlo program

OneLOop–3.3

```
use avh_olo
call olo( rslt ,m1 )                                            ! A0
call olo( rslt ,p1 ,m1,m2 )                                     ! B0
call olo( rslt ,p1,p2,p3 ,m1,m2,m3 )                            ! C0
call olo( rslt ,p1,p2,p3,p4,p12,p23 ,m1,m2,m3,m4 )              ! D0
call olo( rslt11,rslt00,rslt1,rslt0 ,p1 ,m1,m2 )                ! B11, B00, B1, B0
```

- single generic routine for
  - all scalar functions
  - all kinds of input (double precision, quadruple precision)
  - all types of input (real, complex, user-defined)
- can be used at multi-precision with `mpfun90`, `arprec`, `dd`, `qd`
- interface `cavh_olo.h` for `C++` with the `gcc` compilers
- all scripts to build compilable source file and library are in `python`