Automated NLO calculations with **GoSam**

Gionata Luisoni

gionata.luisoni@durham.ac.uk

Institute for Particle Physics Phenomenology University of Durham Max-Planck-Institut für Physik München

In collaboration with:

G.Cullen, N. Greiner, G.Heinrich, P.Mastrolia, G.Ossola, T.Reiter, F. Tramontano

GoSam release: arXiv:1111.2034 [hep-ph] | http://gosam.hepforge.org/





HP2: High Precision for Hard Processes München, 04.09.2012

•• Motivation

- Progresses in NLO calculation:
 - pp $\longrightarrow W + 3$ jets
 - pp $\longrightarrow t\bar{t}b\bar{b}$
 - pp $\longrightarrow Z(\gamma) + 3$ jets
 - pp $\longrightarrow t\bar{t}jj$
 - pp $\longrightarrow W^+W^-b\bar{b}$
 - $e^+e^- \longrightarrow 5$ jets
 - pp $\longrightarrow W^+W^+jj$
 - pp $\longrightarrow Z(\gamma) / W + 4$ jets Blackhat (11)
 - pp $\longrightarrow b\bar{b}b\bar{b}$
 - pp $\longrightarrow W^+W^-jj$
 - $pp \longrightarrow 4$ jets







Blackhat (09) / Rocket (09)

Blackhat (10)

Rocket (10)

Rocket (10)

Blackhat (11)

HELAC-NLO (10)

Golem / Samurai (11)

Rocket (11) / GoSam (12)

Denner-Dittmaier (09) / HELAC-NLO (09)

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Denner-Dittmaier (09) / HELAC-NLO (09) Blackhat (10) HELAC-NLO (10) Denner-Dittmaier (10) / HELAC-NLO (10)

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Blackhat (09) / Rocket (09)

Blackhat (11)

Key Concept

• Automation in NLO calculations

• Different ingredients of a NLO calculation have also different levels of automation according to their complexity:



- Virtual corrections
- Automatized recently:
 - FEYNARTS/FORMCALC/LOOPTOOLS (public)

[Hahn et al.]

- HELAC-NLO (public) [Bevilacqua, Czakon, van Hameren, Papadopoulos, Pittau, Worek, 11]
- MadLoop [Hirschi,Fr
- [Hirschi,Frederix,Frixione,Garzelli, Maltoni,Pittau ,11]
 - OpenLoops [Cascioli, Maierhöfer, Pozzorini, 12]
- GoSam (public) [Cullen, Greiner, Heinrich, GL, Mastrolia, Ossola, Reiter, Tramontano, 11]

Dedicated programs also involve high level of automation: Denner-Dittmaier et al., VBFNLO (public), MCFM (public), NGLUON (public), BLACKHAT, ROCKET.

•• NLO evolution

- Evolution from collection of pre-coded processes...
 ... to generation of full NLO processes by the user "on the fly"!
- Possible thanks to pioneering works:
 - improvements on the computation of tensor integrals, [Binoth et al. GOLEM95; Denner, Dittmaier et al.]
 - application of unitarity to the computation of the one loop amplitudes, [Bern, Dixon, Kosower; Britto, Cachazo, Feng]
 - reduction at the integrand level.
 [Ossola, Papadopoulos, Pittau; Ellis, Giele, Kunszt, Melnikov]
- Automation allows
 - Self-organization / Process-independent framework / Avoid human mistakes / Focus on Pheno





•• The GoSam Project: phylosophy



Golem (General One Loop Evaluator of Matrix elements)

Samurai (Scattering Amplitudes from Unitarity based Reduction At Integrand level)

An automated amplitude generation based on Feynman diagrams

- Based upon:
 - Algebraic generation of D-dimensional integrands via Feynman diagrams
 - Reduction at the integrand level via D-dimensional extension of the OPP method
 - Generation on the fly of the full rational term





•• The GoSam Project: goals

- Main targets:
 - Provide an automated tool for stable evaluation of oneloop matrix elements
 - **Be general** and model independent (QCD, EW, MSSM, ...)
 - Interface with existing tools (MadEvent, Sherpa, POWHEG BOX, ...)
 - Build upon open source tools only (next slide)
 - Support open standards (for interfacing)





•• The GoSam Project: the codes

GoSam Project

GoSam: Python package to write code (fortran95)

Code generation

- Diagram generation:
 QGRAF [Nogueira 92]
- Algebra:

FORM [Vermaseren 91] SPINNEY [Cullen, Koch-Janusz, Reiter 10]

• Code generator:

HAGGIES [Reiter 09]

Yellow codes distributed separately







Generated code execution

• Loop integral reduction:

SAMURAI [Mastrolia, Ossola, Reiter, Tramontano 10]

GOLEM95 [Binoth, Cullen, Guillet, Heinrich, Pilon, Reiter 08]

PJFRY [Yundin]

• Scalar integral evaluation:

AVHOLO [van Hameren]

QCDLOOP [Ellis, Zanderighi]

GOLEM95C [Cullen, Guillet, Heinrich, Kleinschmidt, Pilon, Reiter, Rodgers 11]

All codes in gosam-contrib package

•• 3-Steps to the Loop Amplitude







•• Reduction methods

SAMURAI

[Mastrolia, Ossola, Reiter, Tramontano 10]

Reduction method can be choosen at runtime

Tensorial integrand-level reconstruction

[Heinrich, Ossola, Reiter, Tramontano 10]

with

- GOLEM95C [Binoth, Cullen, Guillet, Heinrich, Kleinschmidt, Pilon, Reiter, Rodgers 11]
- SAMURAI [Mastrolia, Ossola, Reiter, Tramontano 10]
- PJFry [Yundin]





•• Reduction: strategies



•• Reduction: strategies



•• A Walk through GoSam...

- GoSam as a standalone code
- Interfacing with an external Monte Carlo program:
 - The BLHA-interface [Comput.Phys.Commun. 181 (2010) 1612-1622, arXiv:1001.1307 [hep-ph]]
 Sherpa | Powheg Box | ...
 - An example with Sherpa/Powheg Box
 - The GoSam+Sherpa process packages





GoSam standalone: input card

Preparation of the input card "myprocess.rc":







GoSam standalone: input card

Preparation of the input card "myprocess.rc" (continued):

program options

extensions=samurai, golem95, dred

```
# abbrev.level=helicity # group , diagram
# abbrev.limit=0
```

form.bin=tform form.tempdir=/tmp fc.bin=gfortran -O2

```
golem95.fcflags=-I${HOME}/include/golem95
golem95.ldflags=-L${HOME}/lib/ -lgolem
```

Several other extension and options available. For further details check our user manual: http://www.hepforge.org/archive/gosam/gosam-1.0.pdf





•• GoSam standalone: generation/compilation

Generate code and compile

\$ gosam.py myprocess.rc python code generates fortran95 code

\$ make source

\$ make compile

Form & Haggies process diagrams to write code fortran95 code in compiled

ggttH : makevirt	_ O X
File Edit View Scrollback Bookmarks Settings Help	
luisonig@D22:ggttH\$./makevirt	<u>^</u>
> Creating code for virtual part	
> Generating code for amplitudes	
make -I Makelile.Source Source	
make[1]. Entering directory /Seratch/luisonig/GoSam_Processes/tth/ggtH/th/vitual/doc'	
make[2]: Nothing to be done for `source'.	
make[2]: Leaving directory `/scratch/luisonig/GoSam Processes/ttH/ggttH/ttH virtual/doc'	
make[2]: Entering directory '/scratch/luisonig/GoSam Processes/ttH/ggttH/ttH virtual/common'	
FORM is generating color.txt	
0.01 sec + 0.13 sec: 0.15 sec out of 0.10 sec	
haggies is generating color.f90	
haggies is generating model.f90	
FORM is generating version.out	
0.00 sec + 0.00 sec: 0.00 sec out of 0.00 sec	
haggies is generating version.f90	
make[2]: Leaving directory `/scratch/luisonig/GoSam_Processes/ttH/ggttH/ttH_virtual/common'	
make[2]: Entering directory '/scratch/luisonig/GoSam_Processes/ttH/ggttH/ttH_virtual/helicity0'	
Form is processing tree diagram 1 @ Helicity 0	
0.02 Sec + 0.11 Sec: 0.13 Sec out of 0.08 Sec	
Form is processing tree diagram 2 @ Helicity 0	
O DI Sec VOII Sec ULI Sec OLI DI UN SEC	
0.02 sec + 0.06 sec 0.09 sec out of 0.06 sec	
Form is processing tree diagram 4.0 Helicity 0	





•• GoSam standalone: documentation

- Check produced code with automatic generated documentation before the full generation/run
 - Documentation contains information about
 - the generated helicities
 - the colour basis
 - Loop diagrams are grouped into sets of diagrams which share loop propagators



luisonig

2012-02-19 (22:10:59)

Abstract

This process consists of 8 tree-level diagrams and 160 NLO diagrams. Golem has identified 15 groups of NLO diagrams by analyzing their oneloop integrals.

Index	1	2	3	4	5
0	_	_	0	_	_
1	_	_	0	—	+
2	_	_	0	+	_
3	_	_	0	+	+
4	_	+	0	_	_
5	_	+	0	_	+
6	_	+	0	+	_
7	_	+	0	+	+
$8 \rightarrow 4$	+	_	0	_	_
$9 \rightarrow 5$	+	_	0	_	+
$10 \rightarrow 6$	+	_	0	+	_
$11 \rightarrow 7$	+	_	0	+	+
12	+	+	0	_	_
13	+	+	0	_	+
14	+	+	0	+	_
15	+	+	0	+	+



•• GoSam standalone: documentation

J

5.4 Group 3 (5-Point)

General Information

The maximum effective rank in this group is 4.

$$r_{1} = -k_{2} + k_{5}, \quad m_{1} = m_{t} \\ r_{2} = -k_{2} \\ r_{3} = 0 \\ r_{4} = -k_{4}, \quad m_{4} = m_{t} \\ r_{5} = -k_{3} - k_{4}, \quad m_{5} = m_{t} \\ S = \begin{pmatrix} S_{1,1} & 0 & S_{1,3} & S_{1,4} & S_{1,5} \\ 0 & 0 & 0 & S_{2,4} & S_{2,5} \\ S_{3,1} & 0 & 0 & 0 & S_{3,5} \\ S_{4,1} & S_{4,2} & 0 & S_{4,4} & S_{4,5} \\ S_{5,1} & S_{5,2} & S_{5,3} & S_{5,4} & S_{5,5} \end{pmatrix} \\ S_{1,3} = -2m_{t}^{2} \\ S_{1,3} = -2m_{t}^{2} \\ S_{1,5} = -2m_{t}^{2} \\ S_{2,4} = s_{51} - s_{23} - s_{34} + m_{H}^{2} \\ S_{2,5} = s_{51} - m_{t}^{2} \\ S_{3,5} = -m_{t}^{2} + s_{44} \\ S_{4,4} = -2m_{t}^{2} \\ S_{4,5} = -2m_{t}^{2} \\ S_{4,5} = -2m_{t}^{2} \\ S_{5,5} = -2m_{t}^{2} \\ \end{pmatrix}$$

Sa≥ft

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G.Luisoni, 4th September 2012

Loop diagrams are grouped into sets of diagrams which share loop-propagators. A loop integral can be written as

$$\int \frac{\mathrm{d}^n k}{i\pi^{\frac{n}{2}}} \frac{\mathcal{N}(q)}{\prod_{j=1} N\left[(k+r_j)^2 - m_j^2 + im_j\Gamma_j + i\delta\right]} \tag{16}$$

For each group we list r_j , m_j and Γ_j . For m_j and Γ_j only non-vanishing symbols are listed. Furthermore, we give the matrix S which is defined as

$$S_{\alpha\beta} = (r_{\alpha} - r_{\beta})^2 - m_{\alpha}^2 + im_{\alpha}\Gamma_{\alpha} - m_{\beta}^2 + im_{\beta}\Gamma_{\beta}.$$
 (17)



GoSam standalone: code ready to use

Contributions divided in directories by helicity



• Example: $pp \rightarrow Ht\bar{t}$

Generation time: 1h 20min

Compilation time: 3h 6min

Time for 1 PS point: 280 ms

Machine: Intel Core Quad CPU Q6600 @ 2.4 GHz / 6 GB RAM

Process generated in DRED and converted to CDR at runtime

	E	p_x	p_{y}	p_z	
u/g	250.0	0.0	0.0	250.0	
\bar{u}/g	250.0	0.0	0.0	-250.0	
H	136.35582793693018	15.133871809486299	27.986733991031045	26.088703626953386	
t	181.47665951104506	20.889486679044587	-50.105625289561424	14.002628607367491	
\overline{t}	182.16751255202476	-36.023358488530903	22.118891298530357	-40.091332234320859	

parameters	result $gg \to t\bar{t}H$			
$\frac{1}{\sqrt{5}}$ 500 0 $\frac{1}{\sqrt{5}}$ 5	GoSam	Ref. [39]		
μ m_t N_{fh} 1	$a_0 \cdot 10^5$ 6.127399805961155	6.127400074872043		
$m_t = 172.6$ $\alpha_s = 0.1076395107858145$	c_0/a_0 9.006680638719660	9.006680836410272		
m_H 130 v 246.21835258713082	c_{-1}/a_0 2.986347664537282	2.9863477301662056 -6.000000131659877		

Comparison with MadLoop [Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau 11]



GoSam: further tested calculations

- $\bigcirc q\bar{q} \longrightarrow b\bar{b}b\bar{b}$
- $\bullet \ g\bar{g} \longrightarrow b\bar{b}b\bar{b}$
- $\ \, \circ \ \, q\bar{q} \longrightarrow t\bar{t}b\bar{b}$
- $\bullet \ g\bar{g} \longrightarrow t\bar{t}b\bar{b}$
- $\bullet \ u\bar{d} \longrightarrow W^+ ggg$
- $\bullet \ u\bar{u} \longrightarrow H t\bar{t}$
- $\bullet \ g\bar{g} \longrightarrow H t\bar{t}$
- $u\bar{d} \longrightarrow W + s\bar{s} \longrightarrow e^+\nu_e s\bar{s}$
- $u\bar{d} \longrightarrow W + gg \longrightarrow e^+\nu_e gg$
- $\bullet \ d\bar{d} \longrightarrow Z \, gg \longrightarrow e^+e^-gg$





• $u\bar{d} \longrightarrow W + b\bar{b} \longrightarrow e^+\nu_e b\bar{b}$ with massive b's • $ud \longrightarrow W + g \longrightarrow e^+ \nu_e g$ EW corrections $\bullet e^+e^- \longrightarrow Z \longrightarrow d\bar{d}q$ • $e^+e^- \longrightarrow Z \longrightarrow b\bar{b}q$ with massive b's • $u\bar{d} \longrightarrow W^+W^+ s\bar{c} \longrightarrow e^+\nu_e\mu^+\nu_\mu s\bar{c}$ $\bullet u\bar{u} \longrightarrow W^+W^+ c\bar{c} \longrightarrow e^+\nu_e\mu^+\nu_\mu c\bar{c}$ $\bullet u\bar{d} \longrightarrow W^+W^+ \bar{s}c \longrightarrow e^+ \overline{\nu_e \mu^+ \nu_\mu \bar{s}c}$ plus a large number of 2 to 2 processes

• GoSam: interface with MC

- GoSam supports the Binoth-Les-Houches-Accord (BLHA) standards to interface with Monte Carlo generators:
 - Monte Carlo program: Born / real corr. / sub. terms
 - One-loop Program (OLP): virtual corr.
 - Pre-runtime comunication via "order" and "contract" files
 - At runtime:
 - OLP_Start()
 - OLP_EvalSubProcess()

[arXiv:1001.1307 [hep-ph]]







•• BLHA-interface: order & contract



Dp. Dg > tt

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•• In practice: GoSam+ Sherpa

[In collaboration with M.Schonherr]

- Few steps needed to compute e.g. Z+1 jet @NLO:
 - <u>Prepare Sherpa card</u> according to your need and run it once
 - The "order" file and the necessary tree-level code is generated
 - <u>Run GoSam</u> feeding the "order" file and a configuration file with further needed inputs (paths / filtering options / ...)
 - After the virtual code is set up, <u>generate and compile</u> it with configure / make / make install
 - The produced library libgolem_olp.so must be added to the SHERPA_LDADD option in the Sherpa card
 High level of



High level of automation and optimization in the generated code

In practice: GoSam+ Sherpa



GoSam+Sherpa vs MCFM: W+1 jet



GoSam+Sherpa vs MCFM: W⁺ + W⁻



Qp. Qg≥±t

WAX-PLANCK-GESELLSCHAFT

GoSam+Sherpa vs MCFM: W⁻ + bb massive



GoSam+Sherpa vs Melia et al.: W⁺W⁺ + 2 jets



•• NLO analyses with Rivet

- Easy to perform phenomenological NLO analysis using e.g. GoSam+Sherpa in association with Rivet
 - LH-uncertainty study of W+1 jet [LH2011-proceedings]



GoSam+Sherpa Process Packages

http://gosam.hepforge.org/proc/

Process List:

- $p p / p \bar{p} \rightarrow W^{-}(\rightarrow e^{-} + \bar{\nu}_{e}) + jet$, wm1jet.tar.gz (437K)
- $p p / p \bar{p} \rightarrow W^+ (\rightarrow e^+ + \nu_e) + jet$, wp1jet.tar.gz (431K)
- $p p / p \bar{p} \rightarrow W^{-}(\rightarrow e^{-} + \bar{\nu}_{e}) + b \bar{b}$, wmbb.tar.gz (772K)
- $p p / p \bar{p} \to W^+(\to e^+ + \nu_e) + b \bar{b}$, wpbb.tar.gz (771K)
- $p p / p \bar{p} \rightarrow W^{-}(\rightarrow e^{-} + \bar{\nu}_{e}) + 2 jets$, wm2jets.tar.gz (3.49M)
- $p p / p \bar{p} \rightarrow W^+ (\rightarrow e^+ + \nu_e) + 2 j ets$, wp2jets.tar.gz (3.46M)
- $p p / p \bar{p} \to W^+(\to \mu^+ + \nu_\mu) + W^-(\to e^- + \bar{\nu}_e)$, wpwm.tar.gz (716K)
- $p p / p \bar{p} \to W^+(\to \mu^+ + \nu_\mu) + W^+(\to e^+ + \nu_e) + 2 jets_{, wpwp2jets.tar.gz} (3.76M)$

DEPENDENCIES

To run the process packages you need the following:

- Sherpa-1.4.0
- GoSam patch for Sherpa-1.4.0: linux, mac

• Interface with Sherpa 1.4.0 (March 2012) via BLHA-interface (--enable-lhole) with a little additional patch.

- Installation details on the webpage
- Only 3 steps for NLO:
 - download
 - un-tar package
 - run 'makecode' script
- Script for plots is also attached
- Example of interface with Rivet
- Soon possibility to shower
- gosam-contrib-1.0, we recommend to set the installation path using the option --prefix.





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wmbb : pas

File Edit View Scrollback Bookmarks Settings Help

luisonig@D22:wmbb\$ ls

gosam_process_wmbb-1.0.tar.gz makecode makeplots OLE_order.lh OLE_order.olc README Run_LO.dat Run_NLO.dat Sherpa_References.tex luisonig@D22:wmbb\$





GoSam+Sherpa Process Packages

http://gosam.hepforge.org/proc/



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osam process wmbb-1.0.tar.gz makecode makeplots OLE order.lh OLE order.olc README Run LO.dat Run NLO.dat Sherpa References.tex





•• GoSam+Powheg Box

- Powheg Box GoSam interface developed recently
 - [In collaboration with C.Oleari and P.Nason] Test examples against existing processes in the Powheg Box:



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•• BSM physics with GoSam

• New models can be added via FeynRules (UFO)[Christensen, Duhr]

LanHEP [Semenov]

Allows to compute one-loop corrections also for BSM phenomenology



Conclusions and Outlook

- **GoSam** is a code for the computation of one-loop multi-leg amplitudes
 - Based on Feynman diagrams
 - Uses D-dimensional reduction tecniques
 - Flexible and broadly applicable tool
 - Public
 - **Easy to interface** with MC event generator to perform full NLO calculations:
 - so far interfaced with:

SHERPA

POWHEG BOX

- Possibilities for precision studies using NLO parton-level matched with partonshower and with hadronization effects just around the corner
 - Possible to steer everything by just editing a single input card
- We look forward to interfacing with other tools and performing NLO analyses for the LHC





G.Luisoni, 4th September 2012

http://gosam.hepforge.org/







7 Ap. Dg > 1t

Reduction methods: Samurai [default]

[Mastrolia, Ossola, Reiter, Tramontano 10]

Integrals with μ^2 in the numerator

- OPP reduction algorithm [Ossola, Papadopoulos, Pittau 07]
- D-dimensional extension [Ellis, Giele, Kunszt, Melnikov 08]
- Coefficient of polynomials via DFT [Mastrolia et al. 08]
- Computation of the full rational term in one go [Internal GoSam algebraic handling]

For any one-loop amplitude:

$$\mathcal{A}_{n} = \int d^{d}\bar{q} \frac{\mathcal{N}(\bar{q},\epsilon)}{\bar{D}_{0}\bar{D}_{1}\cdots\bar{D}_{n-1}} \quad ; \quad \mathcal{N}(\bar{q},\epsilon) = N_{0}(\bar{q}) + \epsilon N_{1}(\bar{q}) + \epsilon^{2}N_{2}(\bar{q})$$
$$\bar{D}_{i} = (\bar{q} + p_{i})^{2} - m_{i}^{2} = (q + p_{i})^{2} - m_{i}^{2} - \mu^{2} \quad ; \quad \not{q} = \not{q} + \not{\mu} \quad ; \quad \bar{q}^{2} = q^{2} - \mu^{2}$$

Result of integration can be expressed as linear combination of scalar integrals: boxes, triangles, bubbles, tadpoles and rational terms

 $\mathcal{A}_{n} = \sum_{i_{0} < i_{1} < i_{2} < i_{3}}^{m-1} d(i_{0}i_{1}i_{2}i_{3})D_{0}(i_{0}i_{1}i_{2}i_{3}) + \sum_{i_{0} < i_{1} < i_{2}}^{m-1} c(i_{0}i_{1}i_{2})C_{0}(i_{0}i_{1}i_{2}) + \sum_{i_{0} < i_{1}}^{m-1} b(i_{0}i_{1})B_{0}(i_{0}i_{1}) + \sum_{i_{0}}^{m-1} a(i_{0})A_{0}(i_{0}) + \mathcal{R}$

Reduction methods: Tensorial Reconstr.

[Heinrich, Ossola, Reiter, Tramontano 10]

Tensorial reconstruction convoluted with tensor integrals:

Rewrite numerator function as linear combination of tensors

$$\mathcal{N}(q) = \sum_{r=0}^{R} C_{\mu_1 \dots \mu_r} q_{\mu_1} \dots q_{\mu_r}$$
$$C_{\mu_1 \dots \mu_r} q_{\mu_1} \dots q_{\mu_r} = \sum_{(i_1, i_2, i_3, i_4) \vdash r} \hat{C}_{i_1 \, i_2 \, i_3 \, i_4}^{(r)} \cdot (q_1)^{i_1} (q_2)^{i_2} (q_3)^{i_3} (q_4)^{i_4}$$

Determine the coefficients by sampling in q_{μ} in a bottom-up approach

if
$$q_{\mu} = (x, y, z, w)$$
 then $\mathcal{N}(q) = \mathcal{N}(x, y, z, w)$
d-O $q = (0, 0, 0, 0)$: $\mathcal{N}(0, 0, 0, 0) \equiv \mathcal{N}^{(0)} = C_0$

Level-1 4 systems, each sampling a monomial depending on one component of q_{μ} only $\mathcal{N}^{(1)}(q) \equiv \mathcal{N}(q) - \mathcal{N}^{(0)}$ Allows to avoid $q = (x, 0, 0, 0) \implies \mathcal{N}^{(1)}(x, 0, 0, 0) \equiv x C_1 + x^2 C_{11} + \ldots + x^R C_{11} \ldots 1$ $q = (0, y, 0, 0) \implies \mathcal{N}^{(1)}(0, y, 0, 0) \equiv y C_2 + y^2 C_{22} + \dots + y^R C_{22} \dots 2$

numerical instabilities due to vanishing Gram determinants



Leve



•• Derive & Numpolvec

- The latest version of GoSam also implements two new features to improve speed and precision:
 - **derive**: computes the numerator by expanding in a Taylor series

$$\mathcal{N}(\hat{q}) = \mathcal{N}(0) + \hat{q}^{\mu} \frac{\partial}{\partial \hat{q}_{\mu}} \mathcal{N}(\hat{q})|_{q=0} + \frac{1}{2!} \hat{q}^{\mu} \hat{q}^{\nu} \frac{\partial}{\partial \hat{q}_{\mu}} \frac{\partial}{\partial \hat{q}_{\nu}} \mathcal{N}(\hat{q})|_{q=0} + \dots$$

one-to-one correspondence between derivatives at $\hat{q} = 0$ and the coefficients of the tensor integrals

- numpolvec: uses numerical polarization vectors for external massless gauge bosons
 - This allows to reduce the code by generating only few helicities





[F.Tramontano 11]

 k_{γ}

k,

OPP integrand decomposition: 4-dim

□ At integrand level the structure is enriched by polynomial terms that integrate to zero (I multiplied with all the propagators)

$$\begin{split} N(q) &= \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} \left[d(i_0i_1i_2i_3) + \tilde{d}(q;i_0i_1i_2i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0 < i_1 < i_2}^{m-1} \left[c(i_0i_1i_2) + \tilde{c}(q;i_0i_1i_2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\ &+ \sum_{i_0 < i_1}^{m-1} \left[b(i_0i_1) + \tilde{b}(q;i_0i_1) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q;i_0) \right] \prod_{i \neq i_0}^{m-1} D_i \end{split}$$

□ A choice of q fulfilling 4-ple cut condition: $D_{i_0} = D_{i_1} = D_{i_2} = D_{i_3} = 0$ will single out just one polynomial

$$\Delta_{i_0 i_1 i_2 i_3} = \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right]$$

a can **only** be of the type
$$q.p$$

where $p = \varepsilon_{\alpha\beta\gamma} k_1^{\alpha} k_2^{\beta} k_3^{\gamma}$
[proof in OPP 2007]

- Once fitted such polynomial we can subtract it from both sides and repeat the game with another multiple cut condition -> recursive solution
- For each phase space point the only requirement for the reduction is the knowledge of the numerical value of the numerator function N for a small set of values of the loop momentum variable, solutions of the multiple cut conditions





•• OPP

[F.Tramontano 11]

Extension to D-dim

□ fix a parametric form for the loop momentum in terms of a linear combination of four known 4-vectors e_i suitably chosen

$$\vec{q} = \vec{q} + \mu$$
 $\vec{q}^2 = q^2 - \mu^2$ $q = x_1e_1 + x_2e_2 + x_3e_3 + x_4e_4$

the vanishing term (spurious term in the OPP terminology) are then polynomials of ${\bf x}_{\!_{i}}$ and μ^{2}

lacksquare The problem is to fit the coefficients in the polynomials Δ

$$\begin{split} N(\bar{q}) &= \sum_{i < < m}^{n-1} \Delta_{ijk\ell m}(\bar{q}) \prod_{h \neq i, j, k, \ell, m}^{n-1} \bar{D}_h + \sum_{i < < \ell}^{n-1} \Delta_{ijk\ell}(\bar{q}) \prod_{h \neq i, j, k, \ell}^{n-1} \bar{D}_h + \\ &+ \sum_{i < < k}^{n-1} \Delta_{ijk}(\bar{q}) \prod_{h \neq i, j, k}^{n-1} \bar{D}_h + \sum_{i < j}^{n-1} \Delta_{ij}(\bar{q}) \prod_{h \neq i, j}^{n-1} \bar{D}_h + \sum_{i}^{n-1} \Delta_{i}(\bar{q}) \prod_{h \neq i}^{n-1} \bar{D}_h \end{split}$$

✓ Example: 3-ple cut residue

$$\begin{split} \Delta_{ijk}(\bar{q}) &= c_{3,0}^{(ijk)} + c_{3,7}^{(ijk)} \mu^2 - \left((c_{3,1}^{(ijk)} + c_{3,8}^{(ijk)} \mu^2) x_4 + (c_{3,4}^{(ijk)} + c_{3,9}^{(ijk)} \mu^2) x_3 \right) (e_1 \cdot e_2) + \\ &+ \left(c_{3,2}^{(ijk)} x_4^2 + c_{3,5}^{(ijk)} x_3^2 \right) (e_1 \cdot e_2)^2 - \left(c_{3,3}^{(ijk)} x_4^3 + c_{3,6}^{(ijk)} x_3^3 \right) (e_1 \cdot e_2)^3 \; . \end{split}$$

with the 3 cut conditions: $D_i = D_i = D_k = 0$ one fixes x_1 , x_2 and the product x_3x_4





[F.Tramontano 11]

Amplitudes & Master Integrals

$$\begin{aligned} \mathcal{A}_{n} &= \sum_{i < j < k < \ell}^{n-1} \left\{ c_{4,0}^{(ijk\ell)} I_{ijk\ell}^{(d)} + \frac{(d-2)(d-4)}{4} c_{4,4}^{(ijk\ell)} I_{ijk\ell}^{(d+4)} \right\} & \int d^{d}\bar{q} \frac{\bar{q} \cdot e_{2}}{\bar{D}_{i}\bar{D}_{j}} = J_{ij}^{(d)} \\ &+ \sum_{i < j < k}^{n-1} \left\{ c_{3,0}^{(ijk)} I_{ijk}^{(d)} - \frac{(d-4)}{2} c_{3,7}^{(ijk)} I_{ijk}^{(d+2)} \right\} & \int d^{d}\bar{q} \frac{(\bar{q} \cdot e_{2})^{2}}{\bar{D}_{i}\bar{D}_{j}} = K_{ij}^{(d)} \\ &+ \sum_{i < j < k}^{n-1} \left\{ c_{2,0}^{(ij)} I_{ij}^{(d)} + c_{2,1}^{(ij)} J_{ij}^{(d)} + c_{2,2}^{(ij)} K_{ij}^{(d)} - \frac{(d-4)}{2} c_{2,9}^{(ij)} I_{ij}^{(d+2)} \right\} & d = 4 - 2\varepsilon \\ &+ \sum_{i < j}^{n-1} c_{1,0}^{(i)} I_{i}^{(d)} \end{aligned}$$

The sources of rational terms are the integrals with μ^2 powers in the numerator

They are generated by the reduction algorithm(R1), but could also be present ab initio in the numerator function as a consequence of the d-dimensional algebraic manipulations (R2)





 OPP



•• Rational term

[F.Tramontano 11]

More on the rational terms:

- Treatment strictly related the way the numerator function is furnished
 - > Diagramatic approach allows for the classification in two categories: R = R1 + R2
- R1 develops automatically performing the D-dimensional reduction of the tensors spanning the 4dimensional part of the loop momentum
- **R2** are present in the UV diagrams: bubbles, rank 2 and 3 triangles and rank4 boxes.
- At least two possibilities for R2 automatic computation:
 - for any fixed gauge theory calculate once and for all the contribution from all the diagrams that can generate R2 terms and define a set of tree level Feynman rules that give the R2 contribution for any process: MadLoop approach
 - Alternatively: construct the numerator function by implementing (few and universal) algebraic rules to get the R2 term on a diagram by diagram basis: GoSam approach





•• Rational term

GoSam offers different options for the computation of the R2 terms

Thanks to the fact that we generate analytic expressions for the *d*-dimensional numerator function $\bar{N}(\bar{q})$

- ▷ implicit: R₂ terms are kept in the numerator and reduced at runtime using the *d*-dimensional decomposition of the numerator
- explicit: R₂ terms are calculated analytically (without entering in the numerical decomposition)
- ▷ only: only the R₂ term is kept in the final result (this option does not require any additional libraries)
- \triangleright off: all R_2 terms are set to zero

R2 is a gauge dependent quantity





•• Precision tests

Use the decomposition of the numerator function $N(\bar{q})$ after determining all coefficients

$$\begin{split} \mathcal{N}(\bar{q}) &= \sum_{i < < m}^{n-1} \Delta_{ijk\ell m}(\bar{q}) \prod_{h \neq i, j, k, \ell, m}^{n-1} \bar{D}_h + \sum_{i < < \ell}^{n-1} \Delta_{ijk\ell}(\bar{q}) \prod_{h \neq i, j, k, \ell}^{n-1} \bar{D}_h + \\ &+ \sum_{i < < k}^{n-1} \Delta_{ijk}(\bar{q}) \prod_{h \neq i, j, k}^{n-1} \bar{D}_h + \sum_{i < j}^{n-1} \Delta_{ij}(\bar{q}) \prod_{h \neq i, j}^{n-1} \bar{D}_h + \sum_{i < j}^{n-1} \Delta_i(\bar{q}) \prod_{h \neq i}^{n-1} \bar{D}_h \end{split}$$

- **1** Global (N = N)-test
- **2** Local (N = N)-test
- 3 Power-test

Are those methods reliable in detecting unstable phase space points?





[G.Ossola EPS2001]

•• W⁺W⁻ + 2 jets @ NLO with GoSam

[Greiner, Heinrich, Mastrolia, Ossola, Reiter, Tramontano 12]

- Part A: no 3rd gen. quarks in fermion loops and VB attached to closed fermion loops,
- Part B: VB attached to closed fermion loops,
- Part C: 3rd gen. quarks in the loops.
 - previously unknown
- No b quarks in both initial and final state



[Melia, Melnikov, Rontsch, Zanderighi 11]



•• W+W-+2 jets @ NLO with GoSam





GoSam as standalone code

• When the full code is ready:

						ttH_	virtual : bash				-0	
File	Edit	View	Scrollback	Bookmarks	Settings	Help						
luiso	luisonig@D22:ttH virtual\$ ls							^				
codeg	en	diagr	ams-0.hh	diagrams	-1.log	helicity1	helicity14	helicity3	helicity6	Makefile.conf	model.hh	
conno	n	diagr	ams-0.log	doc		helicity12	helicity15	helicity4	helicity7	Makefile.source		
confi	g.sh	diagr	ams-1.hh	helicity	0	helicity13	helicity2	helicity5	Makefile	matrix		
luisonig@D22:ttH_virtual\$												

- Contributions divided in directories by helicity
- Many configuration switches (renorm/scalar loop/reduction strategy) in

common/config.f90

- QCD renormalization fully done
 - different parts can be steered from common/config.f90
 - different renormalization schemes implemented (DRED/tHV): can partially convert from one to another at runtime (DRED -> CDR) [DRED= dim. reduction, CDR= conv. Dim regulariz., tHV= tHooft-Veltman]
 - Yukawa coupling renormalization is missing!
- Model parameters in common/model.f90





•• Approching the Gram



Ap.Ag≥tt

JAX-PLANCK-GESELLSCHAFT

