Towards NNLO corrections to top pair production at the LHC

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In collaboration with
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Introduction

Importance of top quark physics:

- Large Yukawa coupling. Sensitivity to electroweak symmetry breaking
- Large cross section for $t\bar{t}$ production at the LHC: $\sigma_{t\bar{t}}(14\,\mathrm{TeV}, p_T^{top} > 700\,\mathrm{GeV}) \approx 700\,\mathrm{fb}$
- Background to various new physics searches
- Preferred channel for the decay of potential new heavy resonances
- Forward-backward asymmetry at Tevatron

Need for a full NNLO calculation for $t\bar{t}$ production at the LHC:

- An experimental error of ~5% is expected for $\sigma_{t\bar{t}}$
- NLO[1] + N(N)LL[2] calculations give a theoretical uncertainty of ~10%
  - [1] Nason, Dawson, Ellis '88-'90; Kuijf, van Neerven, Smith '89-91
  - [2] Kidonakis, Sterman '97; Bonciani et al. '98; Cacciari et al. '08; Moch, Uwer '08; Kidonakis '08
  - Recently completed NNLL resummation: Ahrens et al. '11

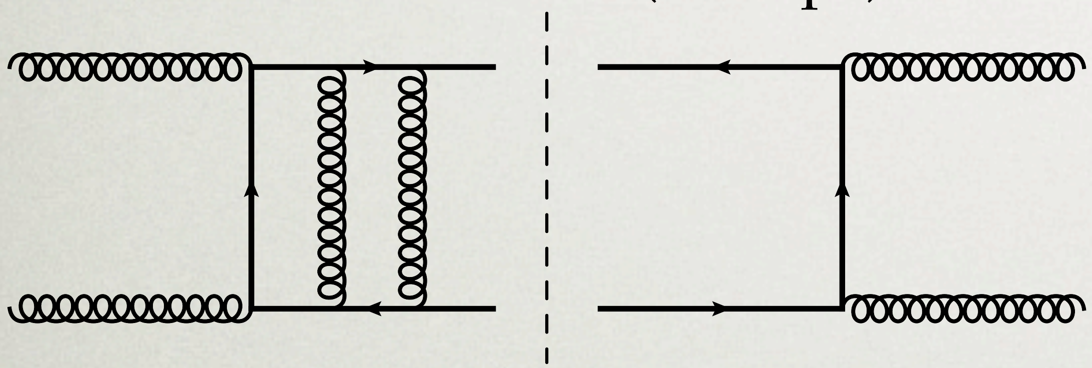
Introduction

Total cross section for $t\bar{t}$ production

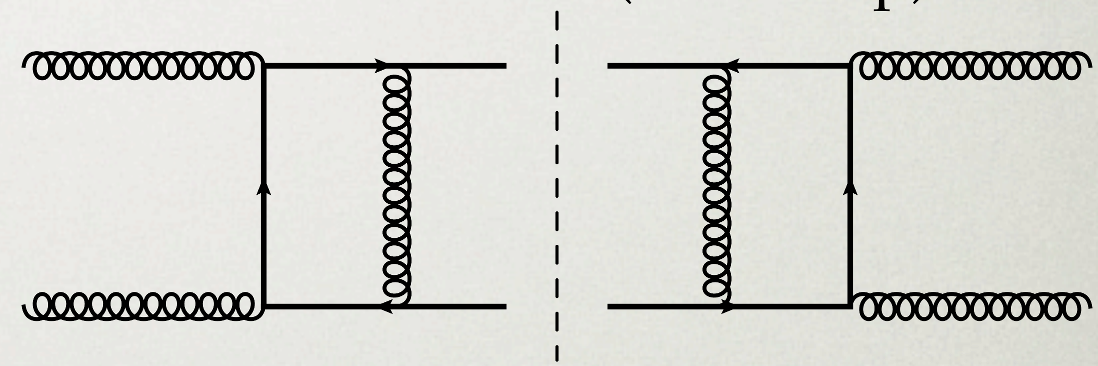
$$d\sigma_{pp\to t\bar{t}} = \sum_{a,b} \int_0^1 \frac{d\xi_1}{\xi_1} \frac{d\xi_2}{\xi_2}\, f_a(\xi_1,\mu_F)\, f_b(\xi_2,\mu_F)\, d\hat{\sigma}_{ab\to t\bar{t}}(\xi_1,\xi_2,\mu_F,\mu_R)$$

Four types of contributions to $d\hat{\sigma}_{ab\to t\bar{t}}$ at NNLO

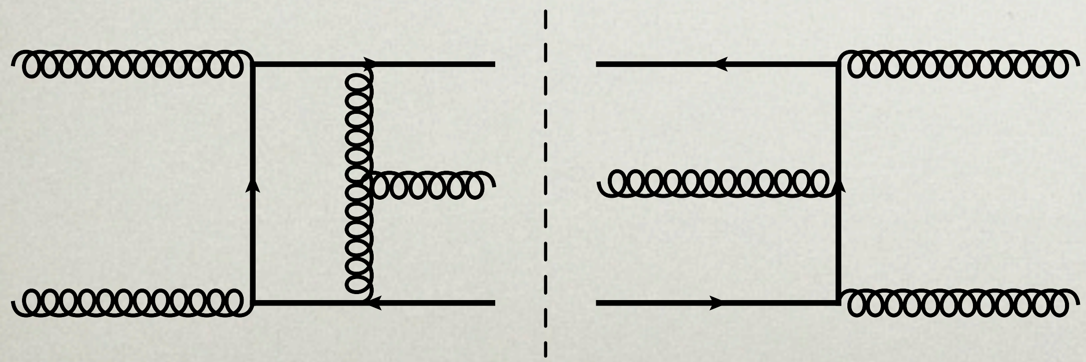
Double virtual (2-loops)



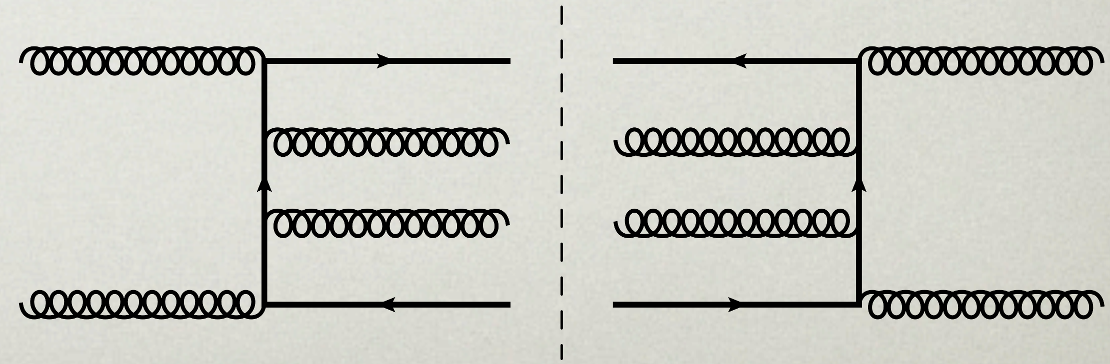
Double virtual (1 x 1-loop)



Real-virtual



Double real



Introduction

State of the art towards a full NNLO calculation

- 2-loop corrections
  - Matrix elements computed in the limit $s >> m_t^2$: Czakon, Mitov, Moch '07 -'08
  - Matrix elements for $q\bar{q}$ channel evaluated numerically: Czakon '08
  - All IR poles evaluated analytically: Ferroglia et al. '09
  - Fermionic and leading colour parts of the $q\bar{q}$ channnel, and leading colour part of the gg channel computed analytically: Bonciani et al. '08 -'09 -'11
- 1 x 1-loop corrections
  - Matrix element fully known: Korner et al. '06; Anastasiou, Aybat '08; Kniehl et al. '08
- Real-virtual corrections
  - IR structure of the massive one-loop amplitudes studied: Bierenbaum, Czakon, Mitov '11
- Double real corrections
  - Purely numerical implementation: sector decomposition + subtraction: Czakon '10 - '11
  - Antenna subtraction with massive fermions: Gehrmann-De Ridder, Ritzmann '09; GA, Gehrmann-De Ridder '11 -'12; Bernreuther, Bogner, Dekkers '11 → This talk
- Full NNLO corrections to the purely fermionic channels: Baernreuther, Czakon, Mitov '12

Introduction

In order to apply the antenna subtraction method to the double real corrections to $t\bar{t}$ production at hadron colliders we

- Fully extended the method at NLO (single unresolved NNLO) to treat massive quarks in hadronic collisions
  GA and Gehrmann-De Ridder, JHEP, 1104, 063 (2011)
  - Constructed NLO subtraction terms for $\sigma_{t\bar{t}}$ and $\sigma_{t\bar{t}+jet}$ (single unresolved part of $\sigma_{t\bar{t}}$ at NNLO)
  - Integrated the relevant massive 3-parton antenna functions

- Computed NNLO subtraction terms for the double real radiation processes
  GA and Gehrmann-De Ridder, JHEP, 1204, 076 (2012)
  GA and Gehrmann-De Ridder arXiv:1112.4736 [hep-ph]

$$q\bar{q} \to t\bar{t}q'\bar{q}' \quad q\bar{q} \to t\bar{t}q\bar{q} \quad qq' \to t\bar{t}qq' \quad qq \to t\bar{t}qq \quad gg \to t\bar{t}q\bar{q}$$

  - Integrated the relevant massive 4-parton antenna functions
    GA, Dekkers, Gehrmann-De Ridder (in preparation)

$\longrightarrow$ This talk

Antenna subtraction at NNLO

NNLO subtraction for m-jet production in hadronic collisions

$$
\begin{aligned}
\mathrm{d}\hat{\sigma}_{NNLO} = & \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\hat{\sigma}^{RR}_{NNLO} - \mathrm{d}\hat{\sigma}^{S}_{NNLO}\right) \\
& + \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\hat{\sigma}^{RV}_{NNLO} - \mathrm{d}\hat{\sigma}^{VS}_{NNLO}\right) + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{MF,1}_{NNLO} \\
& + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}^{VV}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}^{MF,2}_{NNLO} + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\hat{\sigma}^{S}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{VS}_{NNLO}
\end{aligned}
$$

- $\mathrm{d}\hat{\sigma}^{MF,1}_{NNLO}$, $\mathrm{d}\hat{\sigma}^{MF,2}_{NNLO}$: mass factorisation counterterms
- $\mathrm{d}\hat{\sigma}^{S}_{NNLO}$, $\mathrm{d}\hat{\sigma}^{VS}_{NNLO}$: subtraction terms
  - Approximate the double-real (real-virtual) radiation matrix element in all singular regions.
  - Can be integrated over a factorised form of the phase space making poles in ϵ explicit
- Each line is free of infrared poles and integration over the phase space can be carried out numerically in 4 dimensions

Antenna subtraction at NNLO

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&+ \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\hat{\sigma}^{RV}_{NNLO} - \mathrm{d}\hat{\sigma}^{VS}_{NNLO}\right) + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{MF,1}_{NNLO} \\
&+ \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}^{VV}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}^{MF,2}_{NNLO} + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\hat{\sigma}^{S}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{VS}_{NNLO}
\end{aligned}
$$

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Antenna subtraction at NNLO

Double real radiation corrections to $t\bar{t} + (m)jets$ production

$$\mathrm{d}\hat{\sigma}^{RR}_{NNLO}(p_1,p_2) = \mathcal{N}_{NNLO} \sum \mathrm{d}\Phi_{m+2}(p_Q,p_{\bar{Q}},p_5,\ldots,\, p_{m+4};p_1,p_2)$$
$$\times \frac{1}{S_{m+2}}\,|\mathcal{M}^0_{m+4}(p_Q,p_{\bar{Q}},p_5,\ldots,p_{m+4};p_1,p_2)|^2\; J_m^{(m+2)}(p_Q,p_{\bar{Q}},p_5,\ldots,p_{m+4})$$

Antenna subtraction terms

- Reproduce the behaviour of $|\mathcal{M}^0_{m+4}|^2$ in all their infrared limits
  - Based on the universal factorisation properties of $|\mathcal{M}^0_{m+4}|^2$ in these singular limits
    Campbell, Glover '98; Catani, Grazzini '99 -'00
- Constructed as products of antenna functions and reduced matrix elements with remapped momenta

Antenna subtraction at NNLO

Single unresolved limits of $|\mathcal{M}_{m+4}|^2$

$$|\mathcal{M}^0_{m+4}(\ldots,i,j,k,\ldots)|^2 \xrightarrow{p_j\to 0} \mathcal{S}(i,j,k)|\mathcal{M}^0_{m+3}(\ldots,i,k,\ldots)|^2$$
$$|\mathcal{M}^0_{m+4}(\ldots,i,j,k,\ldots)|^2 \xrightarrow{j||k} \frac{1}{s_{jk}}P_{jk\to l}(z)|\mathcal{M}^0_{m+3}(\ldots,i,l,\ldots)|^2$$

Subtracted with (in the final-final case)

$$X^0_{ijk}|\mathcal{M}^0_{m+3}(\ldots,I,K,\ldots)|^2$$

3-parton antenna functions X^0_{ijk}

- Two hard particles i,k (hard radiatiors) and an unresolved particle j
- Give the right unresolved factor (splitting function, soft eikonal factor) in each limit
- Derived from physical matrix elements (more on this later)

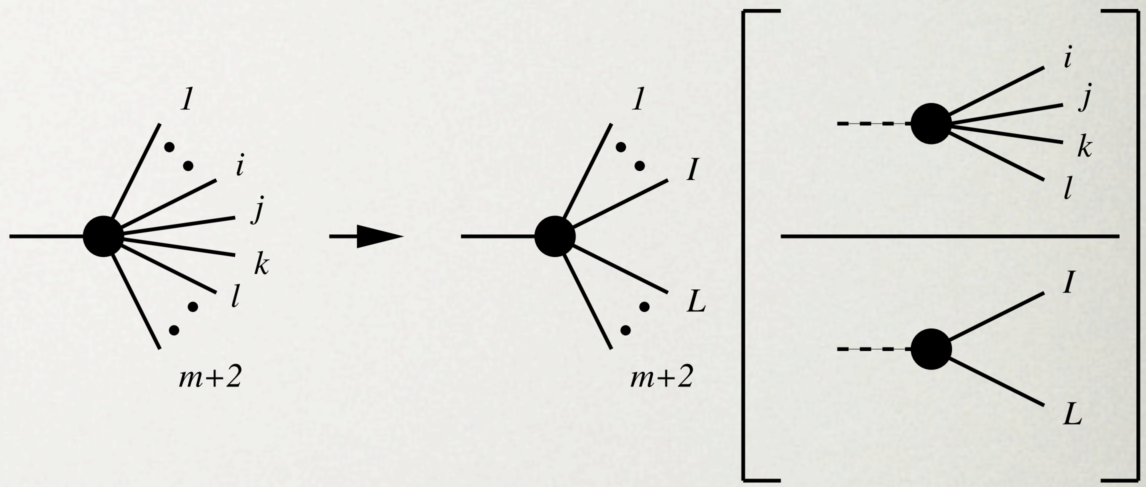
Phase space mapping for reduced matrix elements $\mathcal{M}^0_{m+3}(\ldots,I,K,\ldots)$

- (3 $\rightarrow$ 2) mapping required to define (p_I,p_K) from (p_i,p_j,p_k)
- Remapped kinematics reduces to Born kinematics in each limit

Antenna subtraction at NNLO

Double unresolved limits of $|\mathcal{M}_{m+4}|^2$

- Double soft limits
- Soft $q\bar{q}$ limits
- Triple collinear limits
- Soft and collinear limits
- Double collinear limits



Subtracted with (for final-final colour-connected configurations)

$$X^0_{ijkl}|\mathcal{M}^0_{m+2}(\ldots, I, L, \ldots)|^2$$

4-parton antenna functions X^0_{ijkl}

- Two hard particles i,l (hard radiatiors) and two unresolved particles j,k
- Give the right unresolved factor in each limit
- Derived from physical matrix elements (more on this later)

Phase space mapping for reduced matrix elements $\mathcal{M}^0_{m+2}(\ldots, I, L, \ldots)$

- (4 $\longrightarrow$ 2) mapping required to define (p_I, p_L) from (p_i, p_j, p_k, p_l)
- Remapped kinematics reduces to Born kinematics in each limit

Antenna functions

- Normalised physical colour-ordered matrix elements squared with two hard particles (radiators) and unresolved radiation emitted between them

- Divided into different types

  - According to the number of particles that they contain
    - Three-parton antennae $\longrightarrow$ One unresolved particle
    - Four-parton antennae $\longrightarrow$ Two unresolved particles

  - According to the type of hard radiators
    - Quark-antiquark antennae
    - Quark-gluon antennae
    - Gluon-gluon antennae

  - According to whether the hard radiators are in the initial or in the final state
    - Final-final antennae
    - Initial-final antennae
    - Initial-initial antennae

    All of them are needed in hadron collider observables

Subtraction terms for $t\bar{t}$ production

In the processes

$$q\bar{q} \to t\bar{t}q'\bar{q}' \quad q\bar{q} \to t\bar{t}q\bar{q} \quad qq' \to t\bar{t}qq' \quad qq \to t\bar{t}qq \quad gg \to t\bar{t}q\bar{q}$$

- Soft $q\bar{q}$ limits
- Initial-final triple collinear limits
- Double collinear initial-final limits
- Single final-final and initial-final collinear limits
- No collinear limits involving the heavy fermions. Mass acts as a regulator.

To subtract these limits we employ

- Four-parton antennae
  - Massive final-final: $B_4^0(Q, \bar{q}, q, \bar{Q})$
    Bernreuther, Bogner, Dekkers '11
  - Massive initial-final: $B_4^0(Q, \bar{q}, q, \hat{q}')$, $E_4^0(Q, q, \bar{q}, \hat{g})$, $\tilde{E}_4^0(Q, q, \bar{q}, \hat{g})$
    GA, Gehrmann-De Ridder '11-'12
  - Massless initial-initial: B-type, C-type, G-type $\longrightarrow$ See A. Gehrmann-De Ridder's talk
    Boughezal, Gehrmann-De Ridder, Ritzmann '10; Gehrmann, Gehrmann-De Ridder, Ritzmann '12
- Three-parton antennae
  - Massive and massless in all configurations (f-f, i-f, i-i)
    Gehrmann, Gehrmann-De Ridder, Glover '05; Daleo, Gehrmann, Maître '07;
    Gehrmann-De Ridder, Ritzmann '09; GA, Gehrmann-De Ridder '11

Subtraction terms for $t\bar{t}$ production

Example: $q\bar{q} \longrightarrow Q\bar{Q}q'\bar{q}'$

The colour decomposition reads

$$M_6^0(1_Q, 2_{\bar{Q}}, \hat{3}_{\bar{q}}, \hat{4}_q, 5_{q'}, 6_{\bar{q}'}) = g^4 \left(\delta_{i_1 i_6} \delta_{i_5 i_4} \delta_{i_3 i_2} \mathcal{M}_6^0(1_Q, 6_{\bar{q}'};; 5_{q'}, \hat{4}_q;; \hat{3}_{\bar{q}}, 2_{\bar{Q}}) + ... \right)$$

Squaring yields

$$|M_6^0(1_Q, 2_{\bar{Q}}, \hat{3}_{\bar{q}}, \hat{4}_q, 5_{q'}, 6_{\bar{q}'})|^2 = g^8 (N_c^2 - 1) \left(N_c |\mathcal{M}_6^0(1_Q, 6_{\bar{q}'};; 5_{q'}, \hat{4}_q;; \hat{3}_{\bar{q}}, 2_{\bar{Q}})|^2 + ... \right)$$

Unresolved limits of $|\mathcal{M}_6^0(1_Q, 6_{\bar{q}'};; 5_{q'}, \hat{4}_q;; \hat{3}_{\bar{q}}, 2_{\bar{Q}})|^2$

- Soft $q\bar{q}$ limit $p_5, p_6 \to 0$ (hard radiators: $1_Q, \hat{4}_q$)
- Triple collinear limit $\hat{4}||5||6$
- Single collinear limit $5||6$

Subtraction terms for $t\bar{t}$ production

<u>Example</u>: $q\bar{q} \longrightarrow Q\bar{Q}q'\bar{q}'$

The subtraction terms for this amplitude are

$$\begin{aligned} \mathrm{d}\hat{\sigma}^{S}_{NNLO} \quad \propto \quad & E_3^0(1_Q, 5_{q'}, 6_{\bar{q}'})|\mathcal{M}_5^0((\widetilde{15})_Q, (\widetilde{56})_g, \hat{4}_q; ; \hat{3}_{\bar{q}}, 2_{\bar{Q}})|^2 J_2^{(3)}(p_1, p_2, \widetilde{p_{56}}) \\ & + \Big(B_4^0(1_Q, 6_{\bar{q}'}, 5_{q'}, \hat{4}_q) - E_3^0(1_Q, 5_{q'}, 6_{\bar{q}'}) A_3^0((\widetilde{15})_Q, (\widetilde{56})_g, \hat{4}_q) \Big) \\ & \quad \times |\mathcal{M}_4^0((\widetilde{156})_Q, 2_{\bar{Q}}, \hat{3}_{\bar{q}}, \hat{\hat{4}}_q)|^2 J_2^{(2)}(\widetilde{p_{156}}, p_2) \end{aligned}$$

- $E_3^0(1_Q, 5_{q'}, 6_{\bar{q}'})|\mathcal{M}_5^0(\ldots)|^2 J_2^{(3)}(\ldots)$
  - Subtracts the single collinear limit $5||6$
  - Introduces a spurious collinear singularity $\hat{4}||(\widetilde{56})$

- $B_4^0(1_Q, 6_{\bar{q}'}, 5_{q'}, \hat{4}_q)|\mathcal{M}_4^0(\ldots)|^2 J_2^{(2)}(\ldots)$
  - Subtracts the soft $q\bar{q}$ limit and the triple collinear limit
  - Introduces a spurious collinear singularity $5||6$

- $E_3^0(1_Q, 5_{q'}, 6_{\bar{q}'}) A_3^0((\widetilde{15})_Q, (\widetilde{56})_g, \hat{4}_q)|\mathcal{M}_4^0(\ldots)|^2 J_2^{(2)}(\ldots)$
  - Subtracts both spurious collinear limits

Subtraction of soft $q\bar{q}$ limits

In leading colour pieces, the subtraction of $q\bar{q}$ limits follows the general antenna subtraction scheme

- Sub-amplitudes squared factorise as

$$|\mathcal{M}_n^0(..., a, c_{\bar{q}};; d_q, b, ...)|^2 \stackrel{p_c, p_d \to 0}{\longrightarrow} \mathcal{S}_{acdb}(m_a, m_b)|\mathcal{M}_{n-2}^0(..., a, b, ...)|^2$$

- Soft factor Bernreuther, Bogner, Dekkers '11

$$\begin{aligned}\mathcal{S}_{acdb}(m_a, m_b) \quad = \quad & \frac{2(s_{ab}s_{cd} - s_{ac}s_{bd} - s_{bc}s_{ad})}{s_{cd}^2(s_{ac}+s_{ad})(s_{bc}+s_{bd})} + \frac{2}{s_{cd}^2}\left[\frac{s_{ac}s_{ad}}{(s_{ac}+s_{ad})^2} + \frac{s_{bc}s_{bd}}{(s_{bc}+s_{bd})^2}\right] \\ & - \frac{2m_a^2}{s_{cd}(s_{ac}+s_{ad})^2} - \frac{2m_b^2}{s_{cd}(s_{bc}+s_{bd})^2}\end{aligned}$$

- Hard radiators (a,b) can be immediately identified from the colour connection

- This singularity is subtracted with one four-parton antenna

$$X_4^0(a, c_{\bar{q}}, d_q, b)|\mathcal{M}_{n-2}^0(..., A, B, ...)|^2$$

since

$$X_4^0(a, c_{\bar{q}}, d_q, b) \stackrel{p_c, p_d \to 0}{\longrightarrow} \mathcal{S}_{acdb}(m_a, m_b)$$

Subtraction of soft $q\bar{q}$ limits

In certain subleading colour pieces there are sub-amplitudes like

$$|\mathcal{M}_n^0(..., a; ; d_q, c_{\bar{q}})|^2$$

- Colour factor at the amplitude level contains δ_{i_c, i_d}
- Soft $q\bar{q}$ pair splits from a photon-like propagator
- Hard radiators cannot be identified from the colour connection
- Soft $q\bar{q}$ limit cannot be subtracted with a single antenna function

To subtract the soft $q\bar{q}$ limits in these types of subleading colour contributions we examine the universal factorisation properties of colour ordered matrix elements at the amplitude level

Subtraction of soft $q\bar{q}$ limits

In the leading colour case

$$\mathcal{M}_n^0(\ldots,a,c_{\bar{q}};;d_q,b,\ldots) \xrightarrow{p_c,p_d\to 0} [\bar{u}_d\gamma_\mu v_c]\left(J_a^\mu(p_c,p_d)-J_b^\mu(p_c,p_d)\right)\mathcal{M}_{n-2}^0(\ldots,a,b,\ldots)$$

- Soft currents $J_i^\mu(p_j,p_k)=\frac{p_i^\mu}{s_{jk}(s_{ij}+s_{ik})}$
- Squaring $[\bar{u}_d\gamma_\mu v_c]\left(J_a^\mu(p_c,p_d)-J_b^\mu(p_c,p_d)\right)$ we obtain $\mathcal{S}_{acdb}(m_a,m_b)$

In the subleading colour case, all hard fermions can radiate the soft $q\bar{q}$ pair

$$\mathcal{M}_n^0(\ldots,a;;d_q,c_{\bar{q}}) \xrightarrow{p_c,p_d\to 0} [\bar{u}_d\gamma_\mu v_c]\Bigg(\sum_{i\in\{q\}} J_i^\mu(p_c,p_d)-\sum_{j\in\{\bar{q}\}} J_j^\mu(p_c,p_d)\Bigg)\mathcal{M}_{n-2}^0(\ldots,a)$$

Squaring this gives

$$|\mathcal{M}_n^0(\ldots,a;;d_q,c_{\bar{q}})|^2 \xrightarrow{p_c,p_d\to 0} \Bigg(\sum_{\substack{i\in\{q\}\\ j\in\{\bar{q}\}}}\mathcal{S}_{icdj}(m_i,m_j)-\frac{1}{2}\sum_{\substack{(i,j)\in\{q\}\\ i\neq j}}\mathcal{S}_{icdj}(m_i,m_j) -\frac{1}{2}\sum_{\substack{(i,j)\in\{\bar{q}\}\\ i\neq j}}\mathcal{S}_{icdj}(m_i,m_j)\Bigg)|\mathcal{M}_{n-2}^0(\ldots,a)|^2$$

Subtraction of soft $q\bar{q}$ limits

Therefore, to subtract soft $q\bar{q}$ limits in these subleading colour pieces we need a combination of several antennae

$$
\begin{aligned}
\mathrm{d}\hat{\sigma}^{S}_{NNLO} \quad \sim \quad & \sum_{\substack{i\in\{q\}\\ j\in\{\bar{q}\}}} X_4^0(i,c,d,j)|\mathcal{M}^0_{n-2}(...,A)_{(i,j)}|^2 \\
& -\frac{1}{2}\sum_{\substack{(i,j)\in\{q\}\\ i\neq j}} X_4^0(i,c,d,j)|\mathcal{M}^0_{n-2}(...,A)_{(i,j)}|^2 \\
& -\frac{1}{2}\sum_{\substack{(i,j)\in\{\bar{q}\}\\ i\neq j}} X_4^0(i,c,d,j)|\mathcal{M}^0_{n-2}(...,A)_{(i,j)}|^2
\end{aligned}
$$

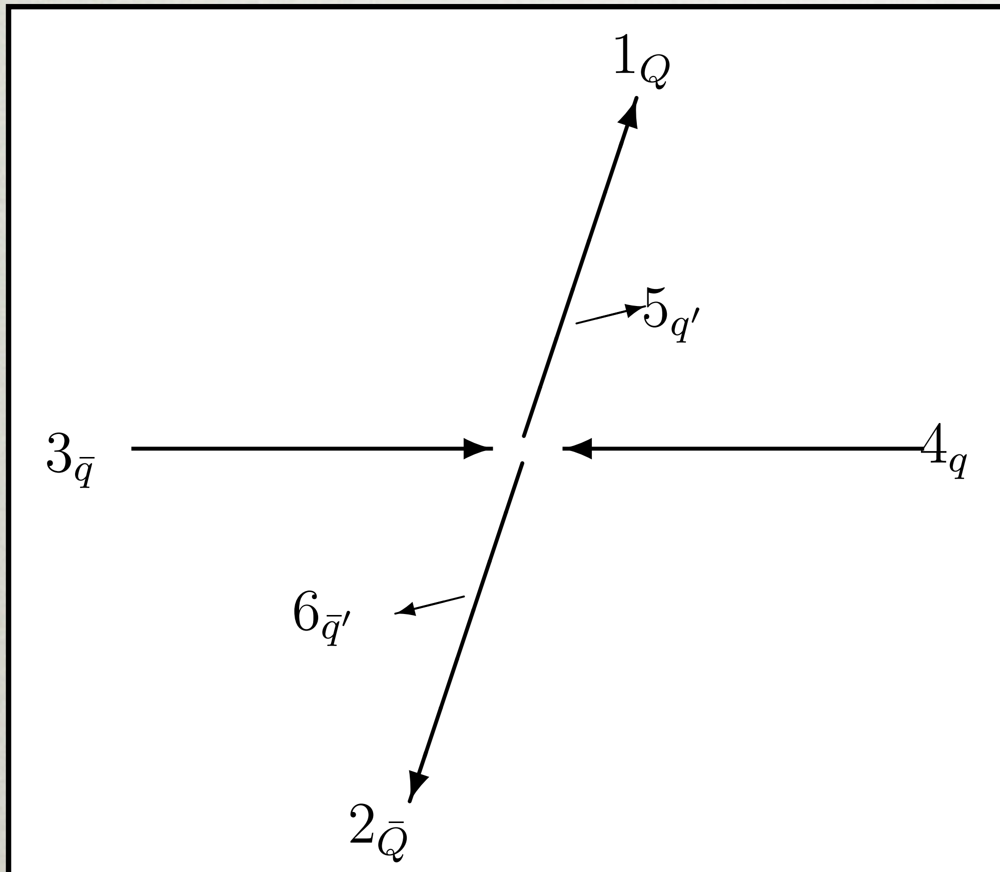
Crucial: this subtraction term also reproduces the triple collinear limits involving the soft $q\bar{q}$ pair without introducing any spurious singularity

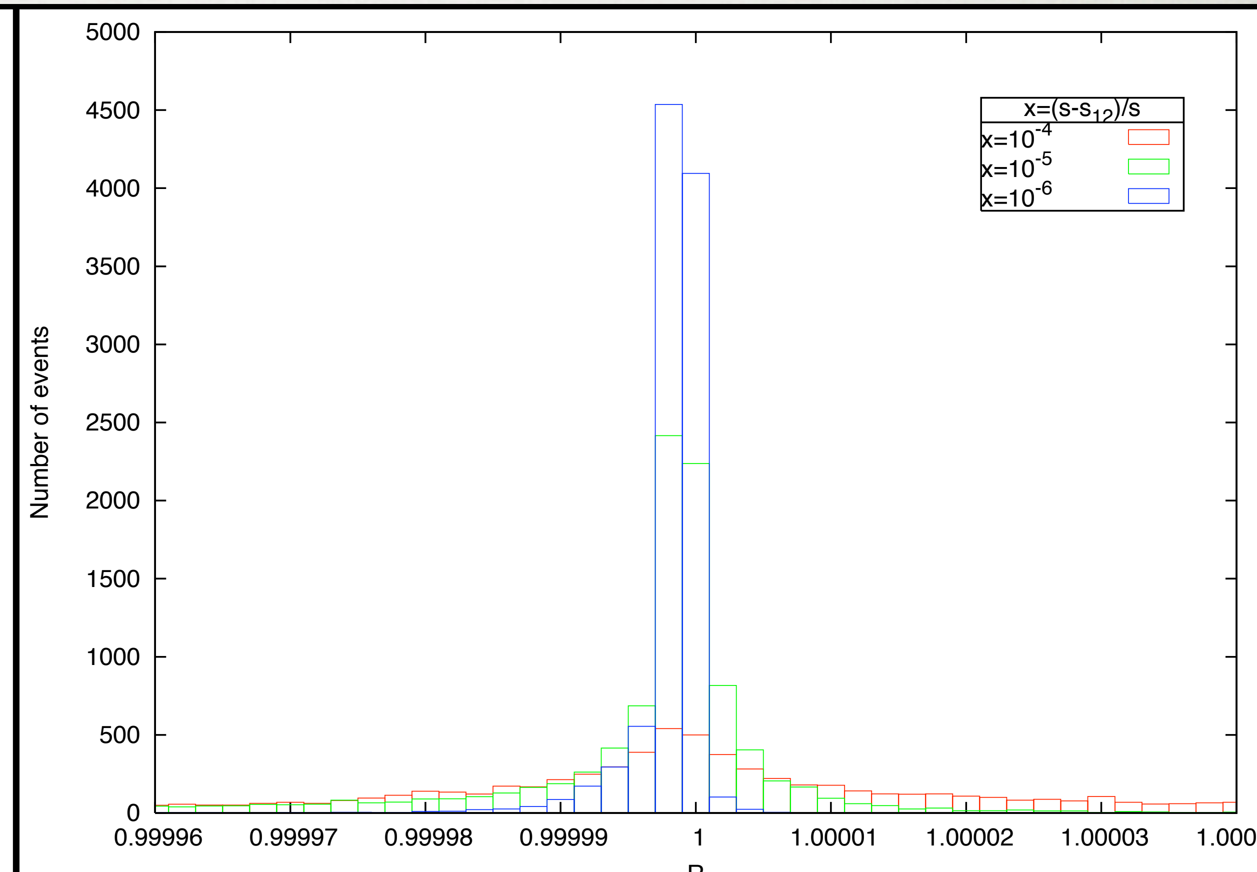
Numerical checks

To test our subtraction terms

- Generate phase space points in the vicinity of each limit
- Define a variable x for each limit that controls the proximity of the phase space points to the singularity
- Compute the ratio $R = \mathrm{d}\hat{\sigma}^{RR}_{NNLO}/\mathrm{d}\hat{\sigma}^{S}_{NNLO}$ for several values of x

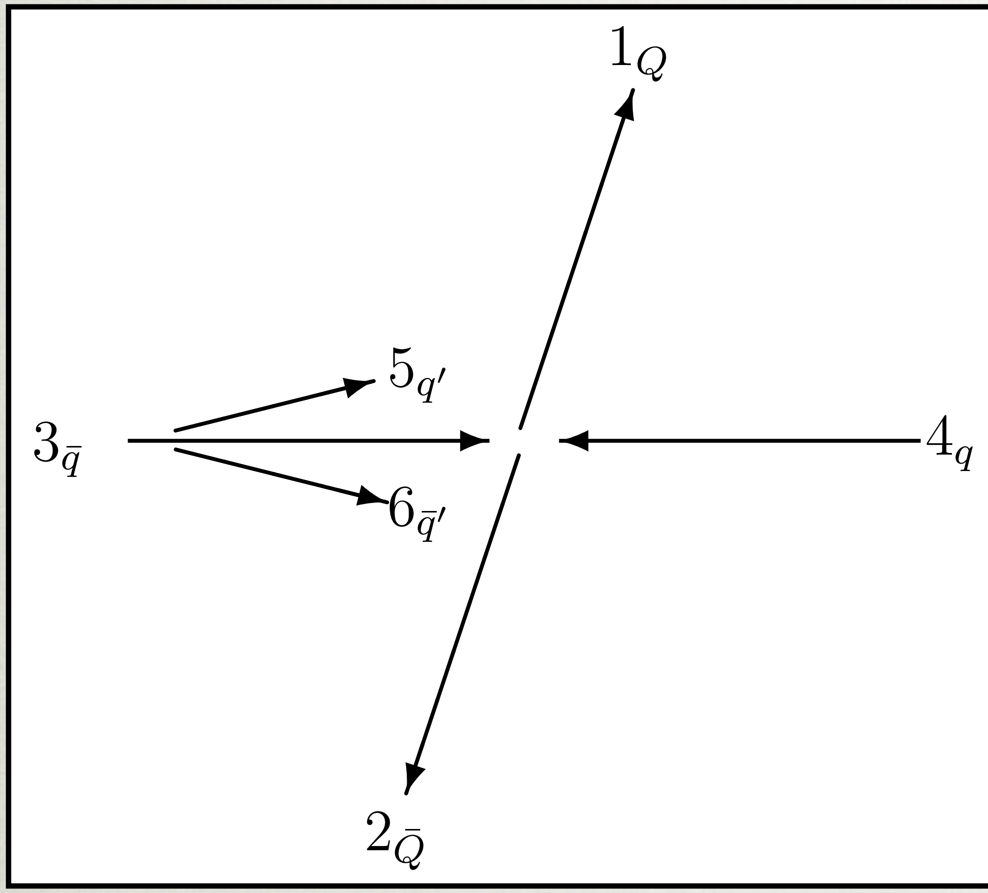
Soft $q\bar{q}$ limit in $q\bar{q} \longrightarrow Q\bar{Q}q'\bar{q}'$ $\qquad x = (\hat{s} - s_{12})/\hat{s}$

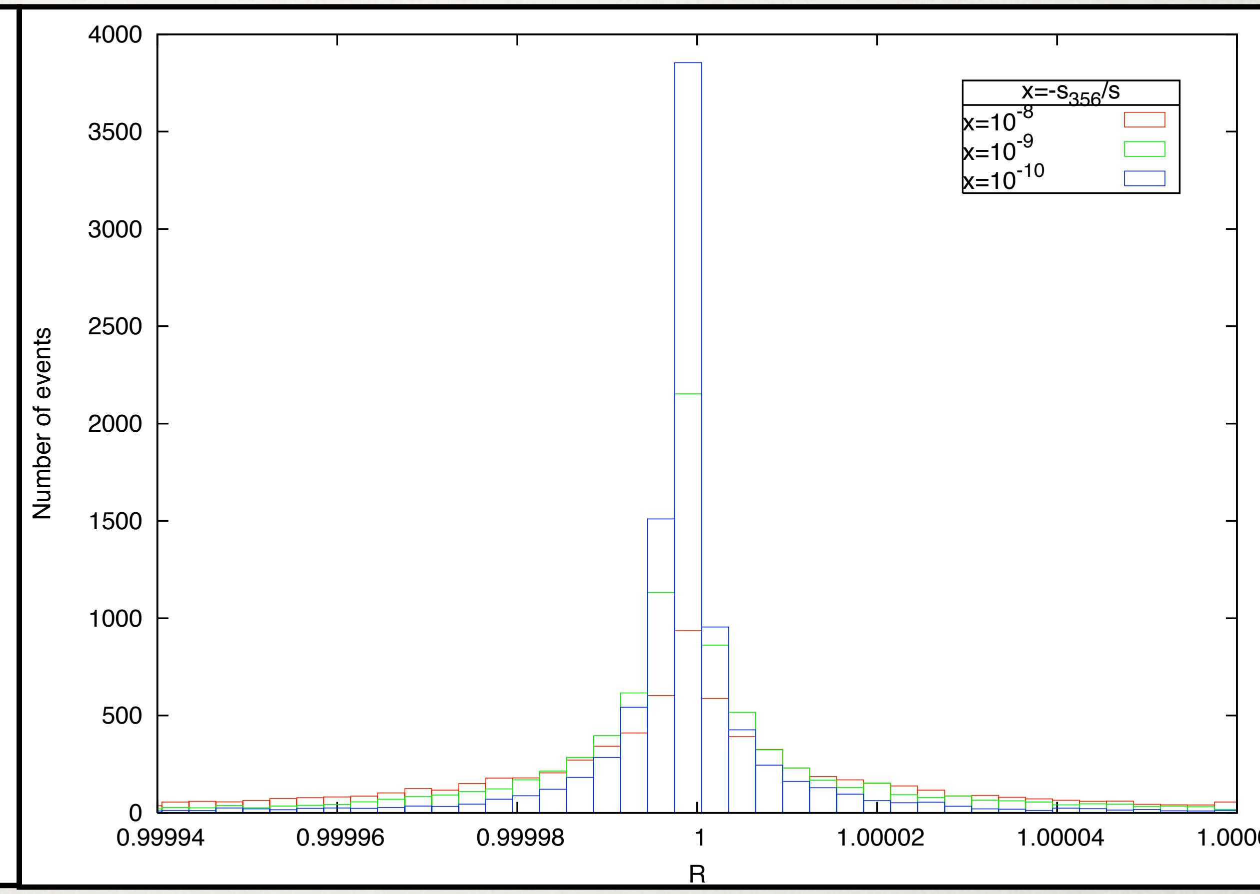




Numerical checks

Triple collinear limit $\hat{3}||5||6$ in $q\bar{q} \longrightarrow Q\bar{Q}q'\bar{q}'$ $\qquad x = -s_{356}/\hat{s}$





Histrograms become more sharply peaked around R=1 as we get closer to the singularities by making x smaller

Similar results are obtained in all limits for all the other partonic processes

$\Rightarrow$ $\mathrm{d}\hat{\sigma}^{S}_{NNLO}$ is a correct approximation of $\mathrm{d}\hat{\sigma}^{RR}_{NNLO}$ in all singular regions!

Integrated antennae

Subtraction terms need to be integrated

- Factorise the phase space into a product of an antenna phase space and a reduced phase space (different factorisation for f-f, i-f, i-i configurations)
- Integrate antenna functions over the corresponding antenna phase space analytically
- Leave the reduced matrix elements unintegrated

Schematically, we do the following (for subtraction terms with 4-parton antennae)

Antenna phase space. Different for (f-f,i-f,i-i)

$$\mathrm{d}\Phi_{m+2} = \mathrm{d}\Phi_m \cdot \mathrm{d}\Phi_{X_4}$$

$$\Rightarrow \int \mathrm{d}\Phi_{m+2}\, X_4^0\, |\mathcal{M}_{m+2}|^2\, J_m^{(m)} = \int \mathrm{d}\Phi_{X_4}\, X_4^0 \cdot \int \mathrm{d}\Phi_m\, |\mathcal{M}_{m+2}|^2\, J_m^{(m)}$$

$$= \underbrace{\mathcal{X}_4^0 \cdot \int \mathrm{d}\Phi_m\, |\mathcal{M}_{m+2}|^2\, J_m^{(m)}}_{\text{Integrated subtraction term}}$$

Integrated antenna
(explicit poles in ϵ)

The integrated subtraction term can be then combined with the double virtual and mass factorisation terms cancelling IR poles. Integration over $\mathrm{d}\Phi_m$ can be safely done in d=4.

Integrated massive final-final antennae

- Phase space factorises as

$$\mathrm{d}\Phi_{m+2}(p_3,\ldots,p_i,p_j,p_k,p_l,\ldots,p_{m+4};p_1,p_2) \;=\; \mathrm{d}\Phi_m(p_3,\ldots,p_I,p_L,\ldots,p_{m+2};p_1,p_2)\,\mathrm{d}\Phi_{X_{ijkl}}(p_i,p_j,p_k,p_l;p_I+p_L)$$

- Massive antenna phase space related to the massive 1→4 phase space

$$\mathrm{d}\Phi_4(p_i,p_j,p_k,p_l;p_I,p_L) = P_2(q^2,m_Q)\times \mathrm{d}\Phi_{X_{ijkl}}(p_i,p_j,p_k,p_l;p_I,p_L)$$

Inclusive 2-particle phase space with $q = p_I + p_L$

- Integrated antennae

$$\mathcal{X}^0_{ijkl} = \frac{1}{[C(\epsilon)]^2}\int \mathrm{d}\Phi_{X_{ijkl}} X^0_{ijkl} \qquad\qquad C(\epsilon) = (4\pi)^\epsilon \frac{e^{-\epsilon\gamma}}{8\pi^2}$$

  - Depend on two variabes $\mathcal{X}^0_{ijkl} = \mathcal{X}^0_{ijkl}(q^2,m_Q^2)$

- Used to evaluate the integrated antennae $\mathcal{B}^0_{Q\bar{q}q\bar{Q}}$

Bernreuther, Bogner, Dekkers ’11

Integrated massive initial-final antennae

- Phase space factorises as

$$\mathrm{d}\Phi_{m+2}(p_3,\ldots,p_j,p_k,p_l,\ldots,p_{m+4};p_1,p_2) = \int_0^1 \frac{\mathrm{d}x}{x}\mathrm{d}\Phi_m(p_3,\ldots,\widetilde{p_{jkl}},\ldots,p_{m+2};xp_1,p_2) \times \mathrm{d}\Phi_{X_{1,jkl}}(p_j,p_k,p_l;p_1,q)$$

$$x = \frac{Q^2+m_j^2+m_k^2+m_l^2}{2p_1\cdot q} \qquad \widetilde{p_{jkl}} = p_j+p_k+p_l-(1-x)p_1$$

- Massive antenna phase space related to the massive 2→3 phase space

$$\mathrm{d}\Phi_{X_{1,jkl}}(p_j,p_k,p_l;p_1,q) = \frac{Q^2+m_j^2+m_k^2+m_l^2}{2\pi}\mathrm{d}\Phi_3(p_j,p_k,p_l;p_1,q)$$

- Integrated antennae

$$\mathcal{X}^0_{1,jkl} = \frac{1}{[C(\epsilon)]^2}\int \mathrm{d}\Phi_{X_{1,jkl}} X^0_{1,jkl}$$

- Used to evaluate the integrated antennae $\mathcal{B}^0_{q',Q\bar{q}q}$, $\mathcal{E}^0_{g,Q\bar{q}q}$, $\tilde{\mathcal{E}}^0_{g,Q\bar{q}q}$

  GA, Dekkers, Gehrmann-De Ridder ’12 (in preparation)

Integrated massive initial-final antennae

Integration of initial-final 4-parton massive antennae $\mathcal{B}^0_{q',Q\bar{q}q}$, $\mathcal{E}^0_{g,Q\bar{q}q}$, $\tilde{\mathcal{E}}^0_{g,Q\bar{q}q}$

- DIS-like 2→3 kinematics with a massive particle in the final state

$$q + p_2 \to p_1 + p_3 + p_4 \qquad p_1^2 = m_Q^2 \qquad p_2^2 = p_3^2 = p_4^2 = 0 \qquad q^2 < 0$$

- We use the following variables to parametrise the kinematics

$$Q^2 = -q^2 \qquad y = 1 - \frac{Q^2 + m_Q^2}{2p_2 \cdot q} \qquad z = \frac{m_Q^2}{(q+p_2)^2} \qquad \Rightarrow \quad \mathcal{X}^0_{i,jkl} = \mathcal{X}^0_{i,jkl}(Q^2, y, z)$$

- Express the phase space integrals as cuts of two-loop four-point functions with two off-shell legs in forward scattering kinematics
- Reduce to master integrals

$$\left.\begin{aligned} I_{[0]} &= \int \mathrm{d}\Phi_3(p_1,p_2,p_3;p_2,q) \\ I_{[-8]} &= \int \mathrm{d}\Phi_3(p_1,p_2,p_3;p_2,q)((p_1+p_3)^2 - m_Q^2) \end{aligned}\right\} \text{Effectively } 1\to 3 \text{ integrals. Known from Gehrmann-De Ridder, Ritzmann '10}$$

$$\left.\begin{aligned} I_{[4]} &= \int \mathrm{d}\Phi_3(p_1,p_2,p_3;p_2,q)\frac{1}{(q-p_1)^2} \\ I_{[4,9]} &= \int \mathrm{d}\Phi_3(p_1,p_2,p_3;p_2,q)\frac{1}{(q-p_1)^2((p_1-p_2)^2 - m_Q^2)} \end{aligned}\right\} \text{Computed using differential equations (NEW)}$$

Summary and conclusions

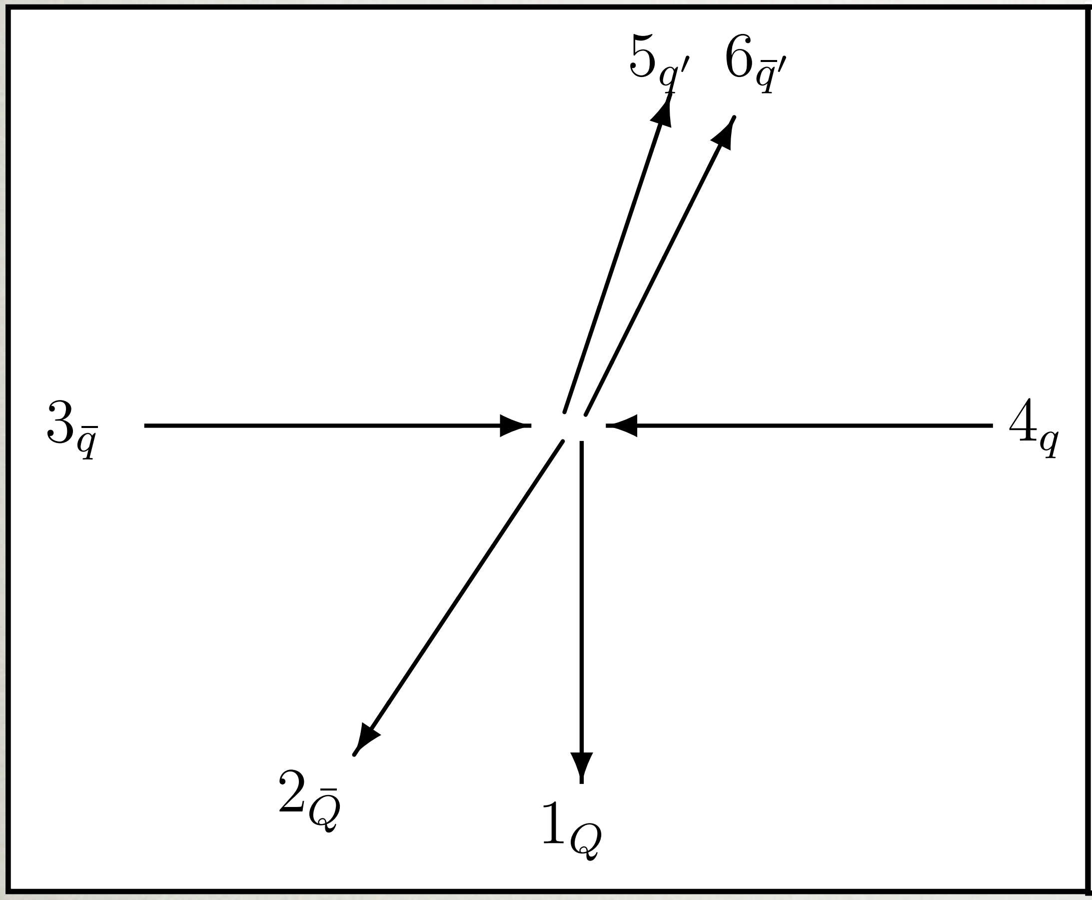
- We applied the antenna subtraction method at NNLO to evaluate the double real corrections to heavy quark pair production in hadronic collisions
- We built subtraction terms for the processes

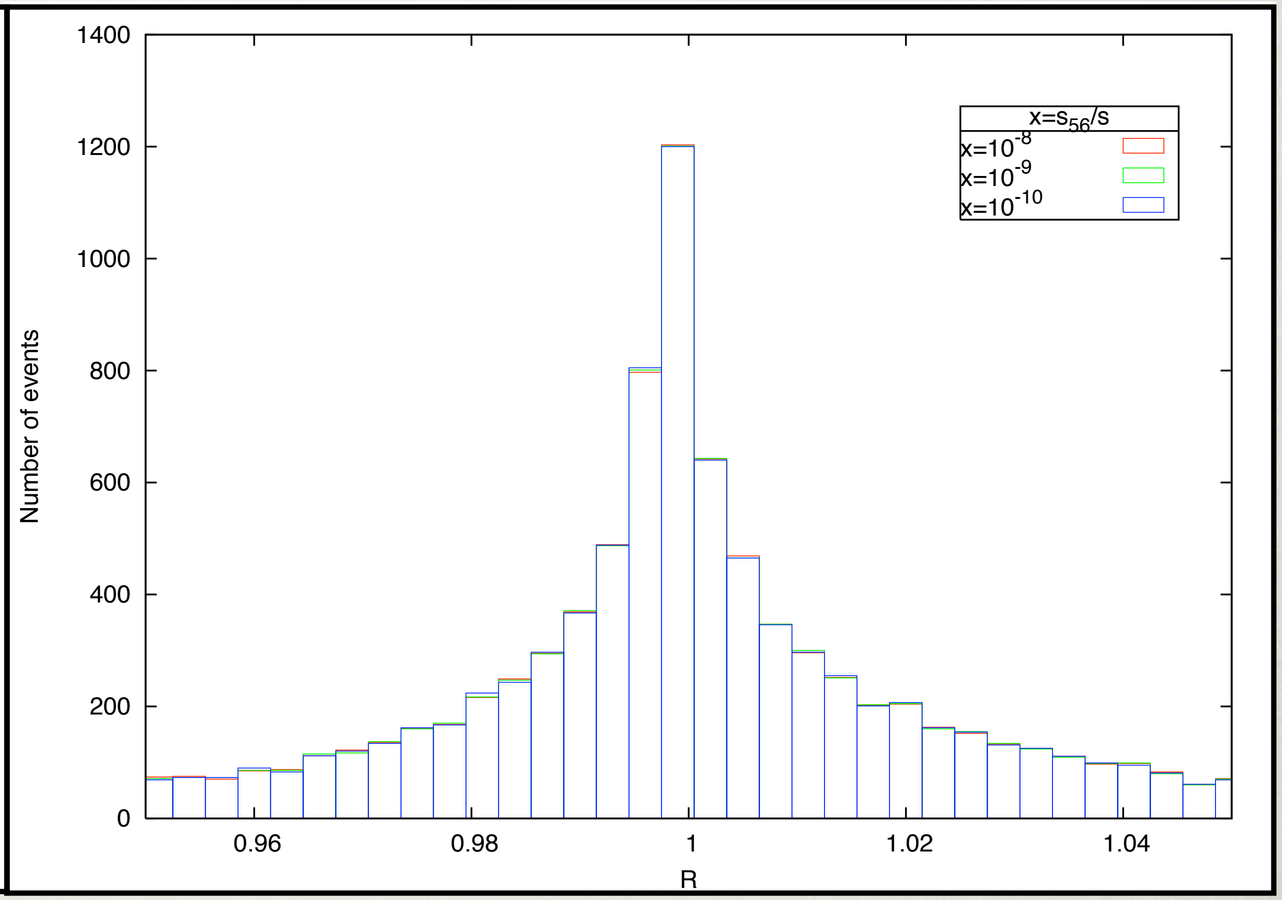
$$q\bar{q} \to t\bar{t}q'\bar{q}' \quad q\bar{q} \to t\bar{t}q\bar{q} \quad qq' \to t\bar{t}qq' \quad qq \to t\bar{t}qq \quad gg \to t\bar{t}q\bar{q}$$

- We tested numerically that our subtraction terms correctly approximate the double real radiation matrix elements in all their unresolved limits: non trivial check on our extension of the antenna subtraction method to treat massive final states
- We reduced the corresponding initial-final massive four-parton antenna to master integrals and evaluated these integrals using differential equations. Integrated antennae to come out soon
- Next tasks
  - Construct subtraction terms for the remaining partonic processes
  - Tackle the real-virtual contributions

Spin correlations

Single collinear limit $5||6$ in $q\bar{q} \longrightarrow Q\bar{Q}q'\bar{q}'$ $\qquad x = s_{56}/\hat{s}$





- The peaks around R=1 are not very sharp
- The peaks do not become sharper as we make x smaller

Spin correlations

In building our subtraction terms we assumed

$$|\mathcal{M}_m(\ldots,\bar{q},q,\ldots)|^2 \xrightarrow{q||\bar{q}} \frac{1}{s_{q\bar{q}}} P_{q\bar{q}\to G}(z)|\mathcal{M}_{m-1}(\ldots,G,\ldots)|^2$$

We used this factorisation

- To subtract single collinear limits from the real radiation matrix elements
- To remove spurious single collinear limits from 4-parton antennae

However, in limits arising from a gluon splitting angular correlations are also present

In these types of limits amplitudes actually factorise as

$$\begin{aligned} |\mathcal{M}_n^0(\ldots,\bar{q},q,\ldots)|^2 \quad &\xrightarrow{q||\bar{q}} \quad \frac{1}{s_{q\bar{q}}} P^{\mu\nu}_{q\bar{q}\to G}(z,k_\perp)|\mathcal{M}^0_{n-1}(...,G,...)|^2_{\mu\nu} \\ &= \quad \frac{1}{s_{q\bar{q}}} P_{q\bar{q}\to G}(z)|\mathcal{M}_{m-1}(\ldots,G,\ldots)|^2 + ang \end{aligned}$$

- $P^{\mu\nu}_{q\bar{q}\to G}(z,k_\perp) = -g^{\mu\nu} + 4z(1-z)\frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2}$: tensorial splitting functiong
- ang : angular correlation terms

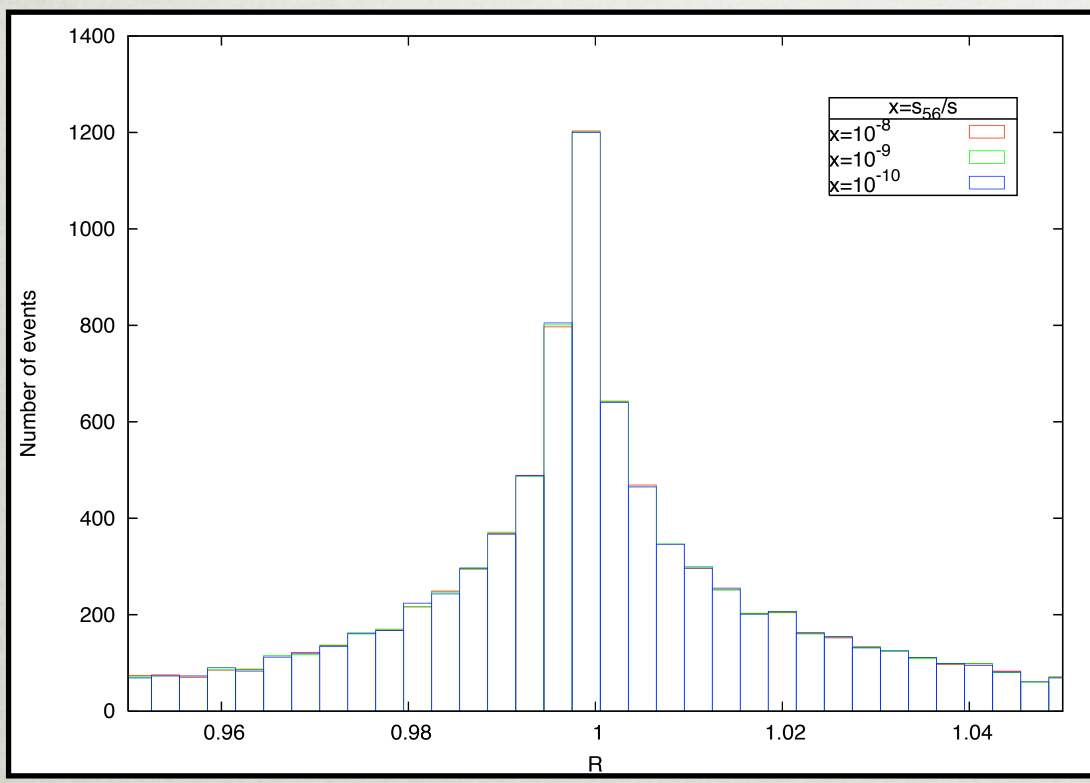
Spin correlations

With a suitable parametrisation of the kinematics in the limit it can be shown that

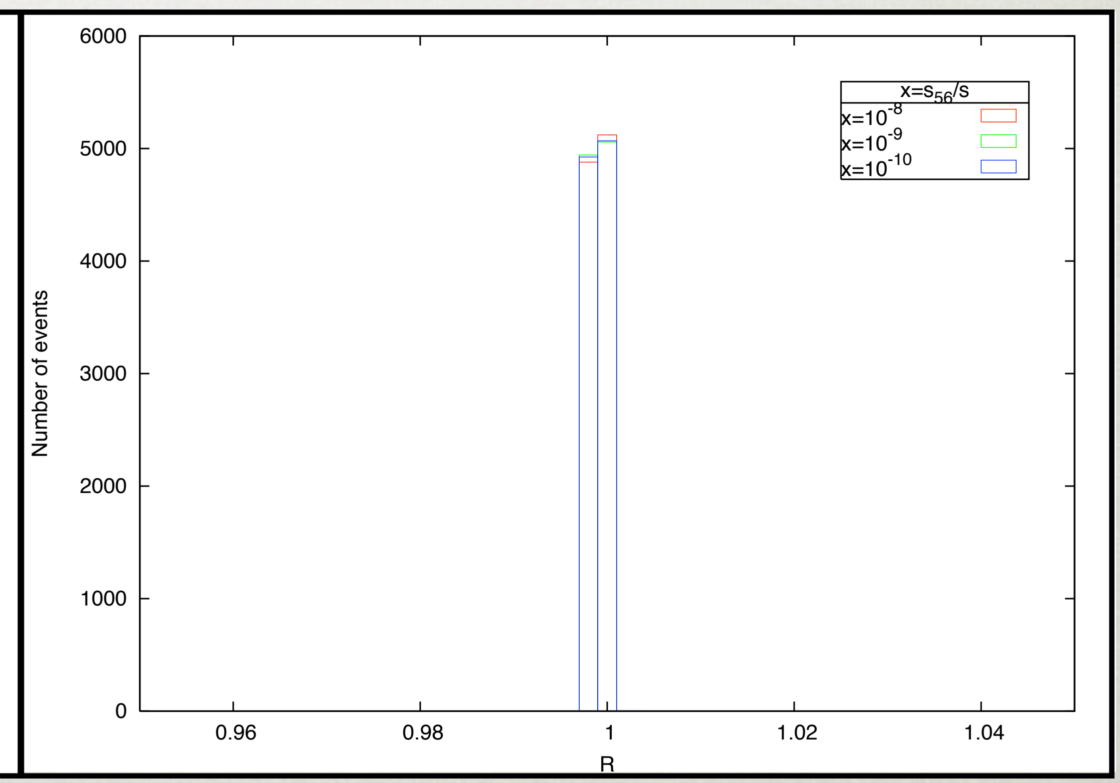
$$ang \sim \cos(2\phi + \alpha)$$

Azimuthal angle of the collinear pair
about the collinear axis

$\Longrightarrow$ The angular correlation terms corresponding to a given phase space point cancel against those corresponding to the same point with the collinear pair rotated about the collinear axis by $\pi/2$



Without azimuthal averaging



With azimuthal averaging

Antenna functions

- Normalised physical matrix elements with two hard particles (radiators) and unresolved radiation emitted between them
- Divided into different types
  - According to the type of hard radiators
  - According to the number of particles that they contain
  - According to whether the hard radiators are in the initial or in the final state

Antenna functions

- Normalised physical matrix elements with two hard particles (radiators) and unresolved radiation emitted between them
- Divided into different types
  - According to the type of hard radiators
    - Quark-antiquark antennae: calculated from $|\mathcal{M}^0(\gamma^* \to q\bar{q} + X)|^2 / |\mathcal{M}^0(\gamma^* \to q\bar{q})|^2$

      E.g. $$A_3^0(q,g,\bar{q}) = \frac{|\mathcal{M}^0(\gamma^* \to qg\bar{q})|^2}{|\mathcal{M}^0(\gamma^* \to q\bar{q})|^2}$$

    - Quark-gluon antennae: calculated from $|\mathcal{M}^0(\tilde{\chi} \to \tilde{g} + X)|^2 / |\mathcal{M}^0(\tilde{\chi} \to \tilde{g}g)|^2$

      E.g. $$D_3^0(q,g,g) = \frac{|\mathcal{M}^0(\tilde{\chi} \to \tilde{g}gg)|^2}{|\mathcal{M}^0(\tilde{\chi} \to \tilde{g}g)|^2}$$

    - Gluon-gluon antennae: calculated from $|\mathcal{M}^0(H \to X)|^2 / |\mathcal{M}^0(H \to gg)|^2$

      E.g. $$F_3^0(g,g,g) = \frac{|\mathcal{M}^0(H \to ggg)|^2}{|\mathcal{M}^0(H \to gg)|^2}$$

  - According to the number of particles that they contain
  - According to whether the hard radiators are in the initial or in the final state

Antenna functions

- Normalised physical matrix elements with two hard particles (radiators) and unresolved radiation emitted between them
- Divided into different types
  - According to the type of hard radiators
  - <u>According to the number of particles that they contain</u>

    -Three-parton antennae X^0_{ijk}
    ✓ One unresolved particle j
    ✓ Subtract single unresolved limits

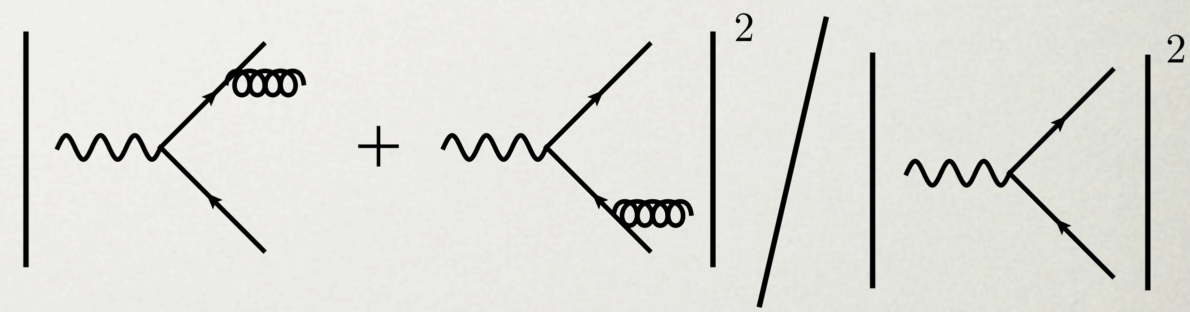
    - Four-parton antennae X^0_{ijkl}
    ✓ Two unresolved particles j,k
    ✓ Subtract (colour-connected) double unresolved limits

  - According to whether the hard radiators are in the initial or in the final state

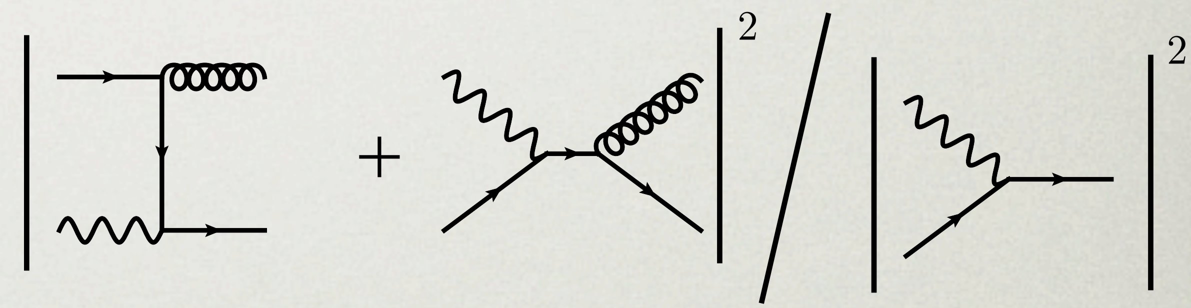
Antenna functions

- Normalised physical matrix elements with two hard particles (radiators) and unresolved radiation emitted between them
- Divided into different types
  - According to the type of hard radiators
  - According to the number of particles that they contain
  - According to whether the hard radiators are in the initial or in the final state

- Final-final antennae $A_3^0(q, g, \bar{q}) =$



-Initial-final antennae $A_3^0(q, g, \hat{q}) =$



-Initial-initial antennae $A_3^0(\hat{\bar{q}}, g, \hat{q}) =$