Top-pair production in hadron collisions at NNLL

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in collaboration with
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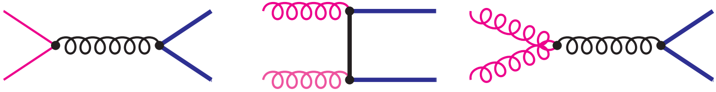
Cross Section



$$\sigma_{t\bar{t}}(s) = \sum_{i,j} \int_{4m_t^2}^{s} \mathrm{d}\hat{s}\; \mathcal{L}_{ij}(s,\hat{s},\mu_f)\; \hat{\sigma}_{ij}(\hat{s},\mu_f,\mu_r)$$

two ways to compute cross section:

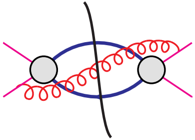
- fixed order
- sum dominant contributions to all orders

Dominant Terms

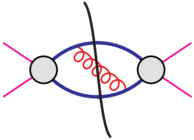
consider threshold limit: $\beta = \sqrt{1 - 4m_t^2/\hat{s}} \to 0$

Laenen et al. 1991; Catani et al.; Berger, Contopanagos; Kidonakis et al. 1996; Bonciani et al. 1998

Soft corrections:

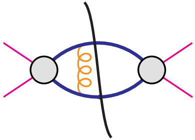


$\sim \alpha_s \ln^2 \beta$



$\sim \alpha_s \ln \beta$

Coulomb corrections:

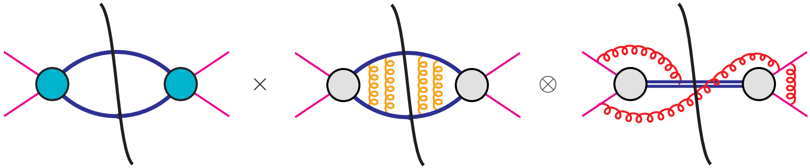


$\sim \alpha_s / \beta$

Power counting: $\alpha_s/\beta \sim \alpha_s \ln \beta \sim 1$

Soft and Coulomb resummation: Beneke, Falgari, Schwinn 2009,2010

$$\hat{\sigma}_{ij} = \sum_R H^R_{ij} \int \mathrm{d}\omega\, J^R(E - \omega/2)\, W^R(\omega)$$



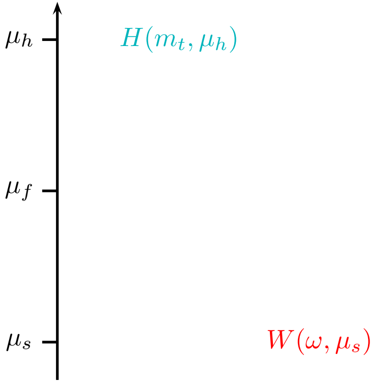
$$\hat{\sigma} \quad \propto \quad \hat{\sigma}^{(0)} \sum_k \left(\frac{\alpha_s}{\beta}\right)^k \exp[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{LL}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{NLL}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{NNLL}} + \ldots]$$

$$\times \{1\,(\text{LL, NLL})\,;\, \alpha_s, \beta\,(\text{NNLL})\,;\ldots\}$$

Resummation of Soft Logarithms

Becher, Neubert 2006; Becher, Neubert, Xu 2007; Ferroglia Neubert, Pecjak, Yang 2009;
Beneke, Falgari, Schwinn 2009; Czakon, Mitov, Sterman 2009

- hard and soft function obey RGEs



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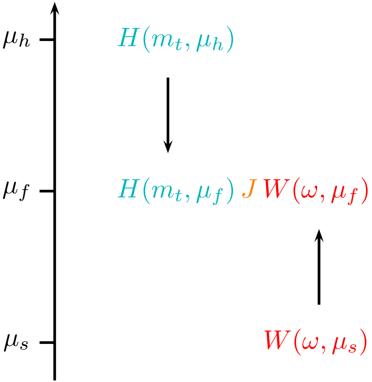
- hard and soft function obey RGEs
- solve RGEs in momentum space
- evolve H from $\mu_h = 2m_t$ to μ_f
- evolve W from $\mu_s = 2m_t\beta^2$ to μ_f



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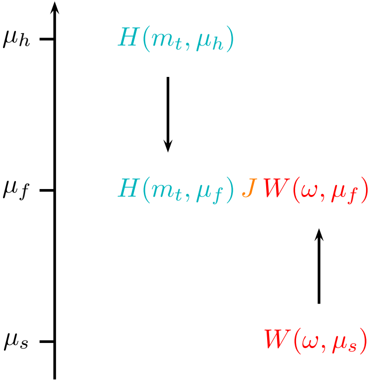
- evolve H from $\mu_h = 2m_t$ to μ_f
- evolve W from $\mu_s = 2m_t\beta^2$ to μ_f
- resummation of $\ln\beta$



NB: Resummation can also be done in Mellin space: $\ln\beta \to \ln N$

Sterman 1987; Catani, Trentadue 1989

Fixed vs. Running Soft Scale

fixed soft scale: Becher, Neubert, Xu 2007

- minimises relative fixed-order 1-loop soft correction to $\sigma_{t\bar{t}}$
- resums logarithms in hadronic cross section
- does not predict partonic cross section

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running soft scale: Beneke, Falgari, Klein, Schwinn 2011

- divide β integration into two regions
- $\beta < \beta_{\text{cut}}$: small ambiguities, $\mu_s = 2m_t\beta_{\text{cut}}^2$
- $\beta > \beta_{\text{cut}}$: no large logarithms, $\mu_s = 2m_t\beta^2$
- Tevatron: $\beta_{\text{cut}} = 0.35$; LHC: $\beta_{\text{cut}} = 0.54$

Resummation of Coulomb Corrections

Resummation of Coulomb corrections from non-relativistic Green's function

Fadin, Khoze 1987; Peskin, Strassler 1990; . . .

$$\left[-\frac{\vec{\nabla}^2}{m_t} - C_F \frac{\alpha_s}{r}\right] G_C^{(0)}(\vec{r}, \vec{r}\,', E) = \delta(\vec{r} - \vec{r}\,')$$

$$J(E) = 2\,\mathrm{Im}\left[G_C^{(0)}(0,0;E) + \ldots\right]$$

- includes bound states below threshold ($E < 0$)
- depends on Coulomb scale: $\mu_C \sim m_t \beta$
- NNLL: need higher order Coulomb and non-Coulomb corrections

Matching to Fixed Order Cross Section

- $t\bar{t}$ production at Tevatron and LHC is not close to threshold: $\beta \approx 0.4$
- NLO: threshold expansion gives reasonable estimate of integral over β
- match to fixed order result to improve behaviour at large β

$$\sigma^{\text{NLO+NNLL}} = \sigma^{\text{NNLL}} - \sigma^{\text{NNLL}}\big|_{\text{NLO}} + \sigma^{\text{NLO}}$$

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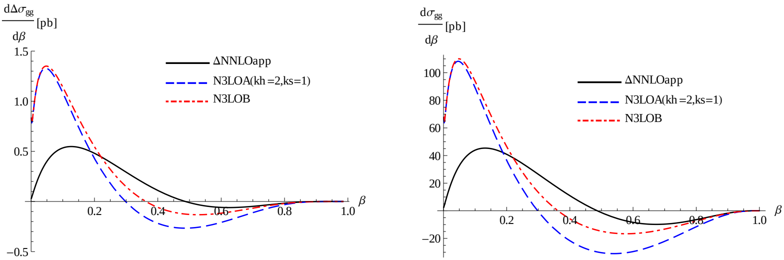
$$
\begin{aligned}
\sigma^{\text{NLO+NNLL}} &= \sigma^{\text{NNLL}} - \sigma^{\text{NNLL}}\big|_{\text{NLO}} + \sigma^{\text{NLO}} \\
\sigma^{\text{NNLO}_{\text{app}}+\text{NNLL}} &= \sigma^{\text{NNLL}} - \sigma^{\text{NNLL}}\big|_{\text{NNLO}} + \sigma^{\text{NLO}} + \sigma^{\text{NNLO}_{\text{app}}}
\end{aligned}
$$

$\sigma^{\text{NNLO}_{\text{app}}}$ contains all singular terms in β at NNLO

Necessity of Resummation

$$\frac{d\Delta\sigma_{gg}}{d\beta} = \frac{8\beta m_t^2}{s(1-\beta^2)^2}\,\mathcal{L}_{gg}(\beta)\,\Delta\hat{\sigma}_{gg}(\beta)$$



Beneke, Falgari, Klein, Schwinn 2011

- potentially large corrections at N^3LO
- $1/\beta^4$ term at N^4LO

$\rightsquigarrow$ resummation is necessary

Results at NNLL

total cross section: $\sigma_{t\bar{t}}$

distributions: $\frac{\mathrm{d}\sigma_{t\bar{t}}}{\mathrm{d}M_{t\bar{t}}}, \frac{\mathrm{d}\sigma_{t+X}}{\mathrm{d}p_T}, \ldots$

$\mathrm{NNLO_{app}}$

Moch, Uwer 2008; Langenfeld, Moch, Uwer 2009;

Beneke, Czakon, Falgari, Mitov, Schwinn 2009

Ahrens, Ferroglia, Neubert, Pecjak, Yang 2010, 2011;

Kidonakis 2010

NNLL soft gluon resummation

Cacciari, Czakon, Mangano, Mitov, Nason 2011

Ahrens, Ferroglia, Neubert, Pecjak, Yang 2010, 2011

NNLL soft + Coulomb resummation

Beneke, Falgari, Klein, Schwinn 2011

TOPIXS

TOP-pair Inclusive X Section

http://users.ph.tum.de/t31software/topixs/

- user-friendly program for combined soft and gluon resummation
- very flexible choice of input parameters ($\sqrt{s}$, m_t, ...)
- fully automatic evaluation of PDF+α_s error for predefined PDF sets
- current version: `TOPIXS` 1.1

other programs are available:

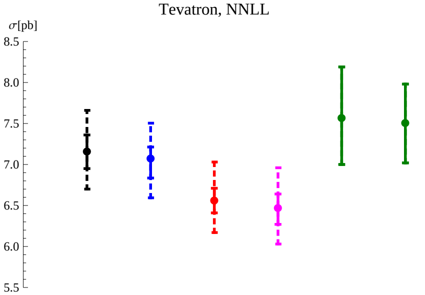
`HATHOR` [Aliev et al. 2010], `TopNNLO` [Ahrens et al. 2011], `Top++` [Czakon, Mitov 2011]

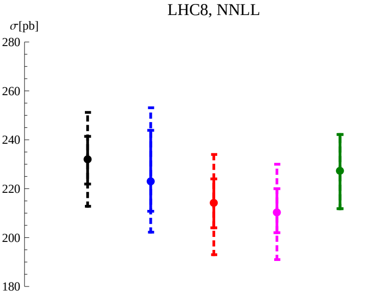
$\sigma_{t\bar{t}}$ [pb]	Tevatron	LHC @ 7 TeV	LHC @ 8 TeV
NLO	$6.68\,^{+0.36}_{-0.75}\,^{+0.23}_{-0.22}$	$158.1\,^{+19.5}_{-21.2}\,^{+6.8}_{-6.2}$	$226.2\,^{+27.8}_{-29.7}\,^{+9.2}_{-8.3}$
NNLO$_{\text{app}}$	$7.00\,^{+0.21}_{-0.31}\,^{+0.29}_{-0.25}$	$160.9\,^{+11.1}_{-11.5}\,^{+7.2}_{-6.7}$	$229.8\,^{+16.5}_{-16.7}\,^{+9.7}_{-9.0}$
NNLL	$7.15\,^{+0.21}_{-0.20}\,^{+0.30}_{-0.25}$	$162.4\,^{+6.7}_{-6.9}\,^{+7.3}_{-6.8}$	$231.8\,^{+9.6}_{-9.9}\,^{+9.8}_{-9.1}$

m_t=173.3 GeV, MSTW 2008 NLO/NNLO [Beneke, Falgari, Klein, JP, Schwinn, Ubiali, Yan 2012]

- correction beyond NLO: $\sim 7\%$ at Tevatron, $\sim 3\%$ at LHC
- exact NNLO $q\bar{q}$ result reduces theory uncertainty at Tevatron to $\sim \pm 3\%$ [Bärnreuther, Czakon, Mitov 2012]
- theory uncertainty at LHC $\sim \pm 5\%$
- PDF+α_s uncertainty $\sim \pm 5\%$

Comparison of NNLL Results



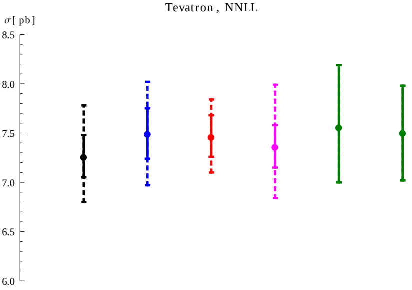


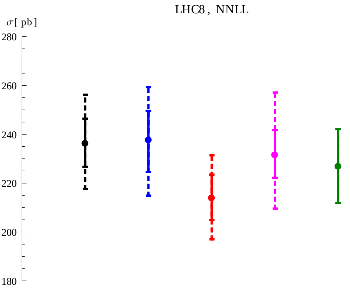
Beneke, Falgari, Klein, JP, Schwinn, Ubiali, Yan 2012 ($m_t = 173.3$ GeV)
`Top++ 1.3` ($m_t = 173.3$ GeV)
Ahrens, Ferroglia, Neubert, Pecjak, Yang 2011 (1PI, $m_t = 173.1$ GeV)
Ahrens, Ferroglia, Neubert, Pecjak, Yang 2011 (PIM, $m_t = 173.1$ GeV)
D0 2011; CDF 2009; CMS result from ICHEP 2012

error bars: solid – theory, dashed – PDF($+\alpha_s$)

PDF set: MSTW 2008 NNLO

Comparison of PDF Sets





MSTW 2008
NNPDF 2.1
ABM 2011
CT 2010
D0 2011; CDF 2009; CMS result from ICHEP 2012

error bars: solid – theory, dashed – PDF+α_s

$\alpha_s(M_Z) = 0.118$, $m_t = 173.3$ GeV, NNLL$_2$ computed with TOPIXS 1.1 using NNLO PDFs

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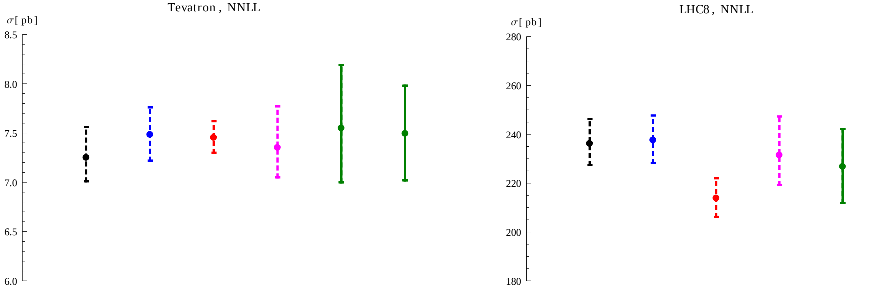

MSTW 2008
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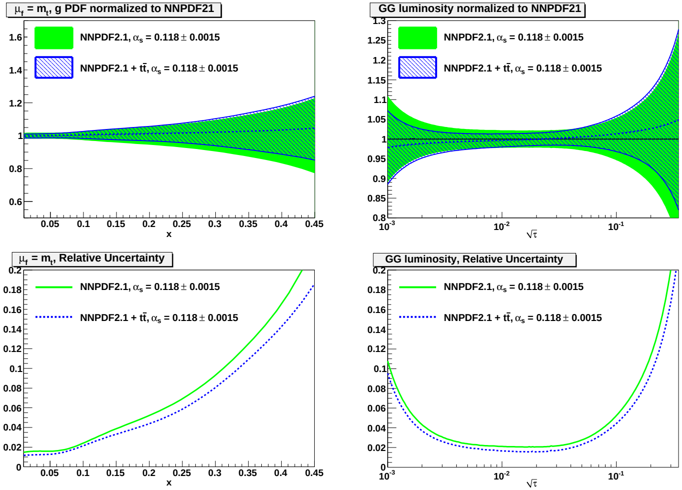
Gluon Distribution



Reweighting of NNPDF 2.1 gluon distribution using 350 replicas and $\sigma_{t\bar{t}}^{\mathrm{exp}} = 173.23 \pm 9.59$ pb, $m_t = 173.3$ GeV, $\alpha_s = 0.118 \pm 0.0015$

Summary

- total cross section is available at NNLL
- public programs are available for analyses

- theory uncertainty $\sim \pm 5\%$ ($\pm 3\%$ at Tevatron)
- PDF+α_s uncertainty $\sim \pm 5\%$
- good agreement between PDF sets, except ABM 2011 at LHC
- LHC data should already have impact on PDF fits

Kinematic Ambiguity in Fixed Soft Scale

cross section depends on energy $E = \sqrt{\hat{s}} - 2m_t$

threshold limit: $E \approx m_t\beta^2 + \ldots$

$\rightsquigarrow$ soft corrections can be expressed in terms of $\ln(E/\mu_s)$ or $\ln(m_t\beta^2/\mu_s)$

expressions agree in threshold limit, but lead to large differences at Tevatron and LHC energies:

	$\ln(E/\mu_s)$	$\ln(m_t\beta^2/\mu_s)$
Tevatron	$\mu_s = 52$ GeV	$\mu_s = 35$ GeV
LHC (8 TeV)	$\mu_s = 103$ GeV	$\mu_s = 60$ GeV

	Tevatron	LHC8
Beneke et al. 2012	$7.15\,^{+0.21}_{-0.20}\,^{+0.30}_{-0.25}$	$231.8\,^{+9.6}_{-9.9}\,^{+9.8}_{-9.1}$
top++ 1.3	$7.06\,^{+0.15}_{-0.23}\,^{+0.29}_{-0.24}$	$222.7\,^{+21.2}_{-11.9}\,^{+9.2}_{-8.6}$
Ahrens et al. 2011, 1PI	$6.55\,^{+0.16}_{-0.14}\,^{+0.32}_{-0.24}$	$214\,^{+10}_{-10}\,^{+10}_{-11}$
Ahrens et al. 2011, PIM	$6.46\,^{+0.18}_{-0.19}\,^{+0.32}_{-0.24}$	$210\,^{+10}_{-8}\,^{+10}_{-11}$
D0 2011	$7.56\,^{+0.63}_{-0.56}$	
CDF 2009	$7.50\,^{+0.48}_{-0.48}$	
CMS 2012		227^{+15}_{-15}

Comparison of PDF Sets — Numbers

	Tevatron	LHC8
MSTW 2008	$7.26\,^{+0.22}_{-0.21}\,^{+0.30}_{-0.25}$	$236.5\,^{+9.9}_{-9.8}\,^{+9.8}_{-9.1}$
NNPDF 2.1	$7.49\,^{+0.26}_{-0.25}\,^{+0.27}_{-0.27}$	$238.0\,^{+11.6}_{-13.4}\,^{+9.7}_{-9.7}$
ABM 2011	$7.46\,^{+0.22}_{-0.20}\,^{+0.16}_{-0.16}$	$214.1\,^{+9.4}_{-9.2}\,^{+7.9}_{-7.9}$
CT 2010	$7.36\,^{+0.22}_{-0.21}\,^{+0.41}_{-0.31}$	$231.9\,^{+9.8}_{-9.7}\,^{+15.4}_{-12.6}$
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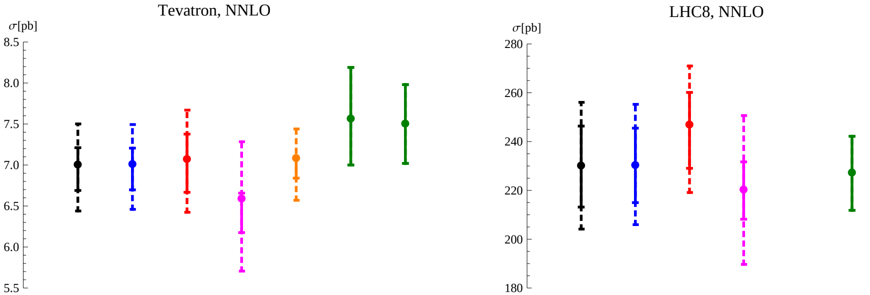
Comparison of NNLO$_{(app)}$ Results



TOPIXS 1.0 (m_t = 173.3 GeV)
Top++ 1.3 (m_t = 173.3 GeV)
HATHOR 1.3 (m_t = 173.3 GeV)
TopNNLO (m_t = 173.3 GeV)
Kidonakis 2010 (m_t = 173 GeV)
D0 2011; CDF 2009; CMS result from ICHEP 2012

error bars: solid – theory, dashed – PDF($+\alpha_s$)

PDF set: MSTW 2008 NNLO

Comparison of NNLO$_{(app)}$ Results — Numbers

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top++ 1.3	$7.00\,^{+0.20}_{-0.31}\,^{+0.29}_{-0.24}$	$230.2\,^{+15.3}_{-15.2}\,^{+9.8}_{-9.0}$
HATHOR 1.3	$7.07\,^{+0.31}_{-0.40}\,^{+0.29}_{-0.24}$	$246.8\,^{+13.4}_{-17.7}\,^{+10.8}_{-9.9}$
TopNNLO	$6.59\,^{+0.07}_{-0.41}\,^{+0.63}_{-0.47}$	$220.0\,^{+11.7}_{-11.8}\,^{+19.0}_{-18.5}$
Kidonakis 2010	$7.08\,^{+0.00}_{-0.24}\,^{+0.36}_{-0.27}$	
D0 2011	$7.56\,^{+0.63}_{-0.56}$	
CDF 2009	$7.50\,^{+0.48}_{-0.48}$	
CMS 2012		227^{+15}_{-15}