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Towards Jet Cross Sections at NNLO

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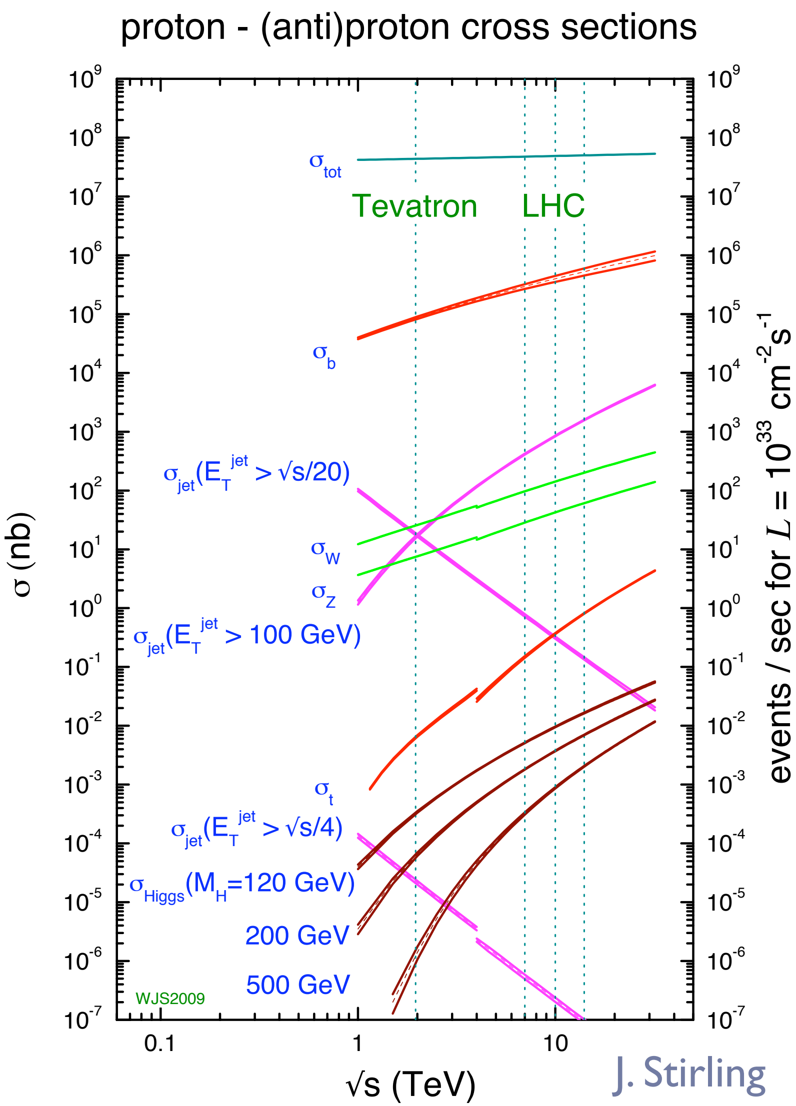
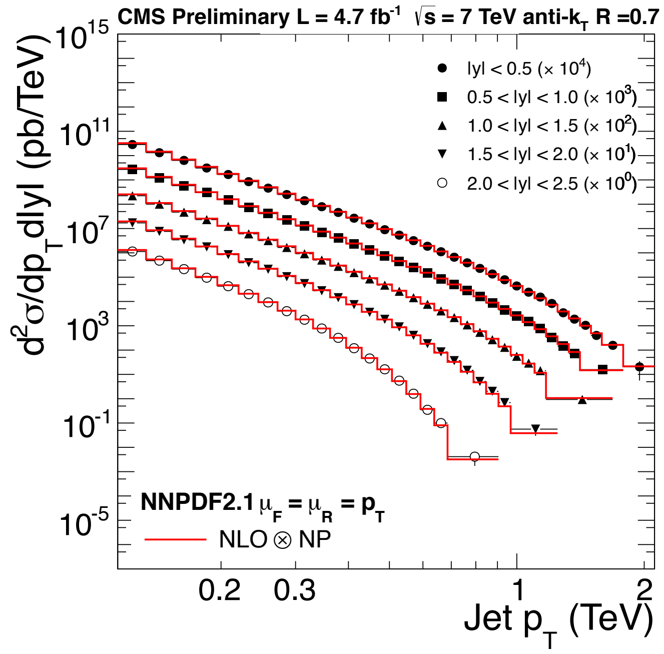
Expectations at LHC

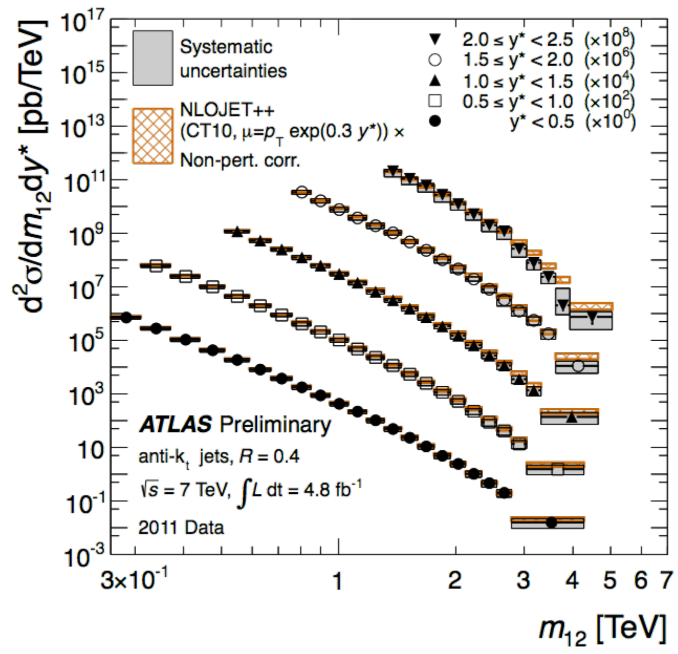
- Large production rates for Standard Model processes
  - single jet inclusive and differential di-jet cross section will be measured to per cent accuracy
- Allow precision determinations
  - strong coupling constant
  - parton distributions
- Provided theory description is known to the same precision: NNLO



J. Stirling

Inclusive jet and dijet cross sections

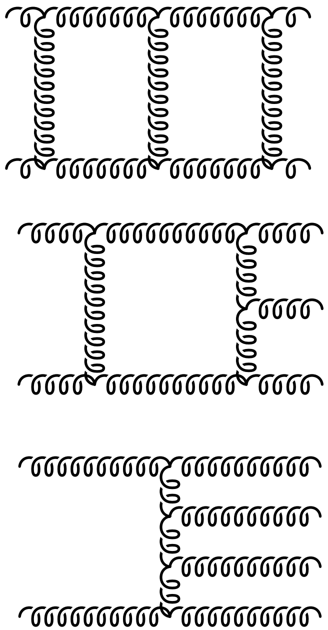




- Data can be used to constrain parton distributions
  - NNLO parton distribution fits currently include DIS structure functions and inclusive Drell-Yan cross sections
  - Inclusion of jet data in NNLO parton distribution fits requires NNLO corrections to jet cross sections

NNLO calculations

- Require three principal ingredients (here: pp → 2j)
  - two-loop matrix elements
    - explicit infrared poles from loop integral
      - known for all massless 2 → 2 processes
  - one-loop matrix elements
    - explicit infrared poles from loop integral
    - and implicit poles from single real emission
      - usually known from NLO calculations
  - tree-level matrix elements
    - implicit poles from double real emission
      - known from LO calculations



- Infrared poles cancel in the sum
- Challenge: combine contributions into parton-level generator
- need method to extract implicit infrared poles

NNLO calculations

- Solutions
  - sector decomposition: expansion in distributions, numerical integration (T. Binoth, G. Heinrich; C. Anastasiou, K. Melnikov, F. Petriello; M. Czakon)
    - applied to NNLO corrections to Higgs and vector boson production (C. Anastasiou, K. Melnikov, F. Petriello)
  - subtraction: add and subtract counter-terms: process-independent approximations in all unresolved limits, analytical integration
    - several well-established methods at NLO
    - q_T subtraction at NNLO applied to Higgs and vector boson production, associated H+W production, diphoton production (S. Catani, M. Grazzini; with L. Cieri, G. Ferrera, D. de Florian, F. Tramontano)
    - antenna subtraction at NNLO for jet observables in e^+e^- collisions (T. Gehrmann, N. Glover, AG)

NNLO methods: new developments

- sector decomposition combined with subtraction
  - use sector decomposition to compute integrated subtraction terms numerically (M. Czakon; R. Boughezal, K. Melnikov, F. Petriello)
    - applied to top quark pair production (P. Bärnreuther, M. Czakon, A. Mitov)
- non-linear mappings (C. Anastasiou, F. Herzog, A. Lazopoulos)
    - applied to Higgs decay into bottom quarks (C. Anastasiou, F. Herzog, A. Lazopoulos)
    - applied to Higgs production in botton fusion (S. Bühler, F. Herzog, A. Lazopoulos, R. Müller)

- This talk:
  NNLO antenna subtraction for jet observables in hadronic collisions

NNLO Subtraction

- Structure of NNLO m-jet cross section at hadron colliders

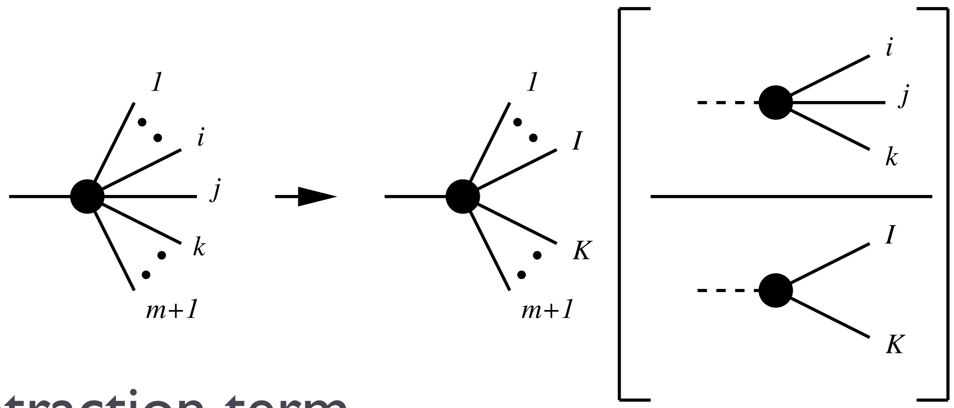
$$\begin{aligned}
\mathrm{d}\hat\sigma_{NNLO} = &\int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\hat\sigma^{RR}_{NNLO} - \mathrm{d}\hat\sigma^{S}_{NNLO}\right) \\
&+ \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\hat\sigma^{RV}_{NNLO} - \mathrm{d}\hat\sigma^{VS}_{NNLO} + \mathrm{d}\hat\sigma^{MF,1}_{NNLO}\right) \\
&+ \int_{\mathrm{d}\Phi_{m}} \left(\mathrm{d}\hat\sigma^{VV}_{NNLO} + \mathrm{d}\hat\sigma^{MF,2}_{NNLO}\right) + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\hat\sigma^{S}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat\sigma^{VS}_{NNLO}
\end{aligned}$$

- with:
  - Partonic contributions: $\mathrm{d}\hat\sigma^{RR}_{NNLO}$ $\mathrm{d}\hat\sigma^{RV}_{NNLO}$ $\mathrm{d}\hat\sigma^{VV}_{NNLO}$
  - Subtraction terms for double real radiation: $\mathrm{d}\hat\sigma^{S}_{NNLO}$
  - Subtraction terms for one-loop real radiation: $\mathrm{d}\hat\sigma^{VS}_{NNLO}$
  - Mass factorization terms: $\mathrm{d}\hat\sigma^{MF,1}_{NNLO}$ $\mathrm{d}\hat\sigma^{MF,2}_{NNLO}$

- Challenge: construction and integration of subtraction terms

Antenna subtraction

- **Subtraction terms constructed from antenna functions**
  - Antenna function contains all emission between two partons



  - NLO subtraction term

$$d\hat{\sigma}^S_{NLO} = \int d\Phi_{m+1}(p_1,\ldots,p_{m+1};q) \sum_j X^0_{ijk} |\mathcal{M}_m|^2 J_m^{(m)}(p_1,\ldots,p_I,p_K,\ldots,p_{m+1})$$

  - Phase space factorization

$$d\Phi_{m+1}(p_1,\ldots,p_{m+1};q) = d\Phi_m(p_1,\ldots,\tilde{p}_I,\tilde{p}_K,\ldots,p_{m+1};q) \cdot d\Phi_{X_{ijk}}(p_i,p_j,p_k;\tilde{p}_I+\tilde{p}_K)$$

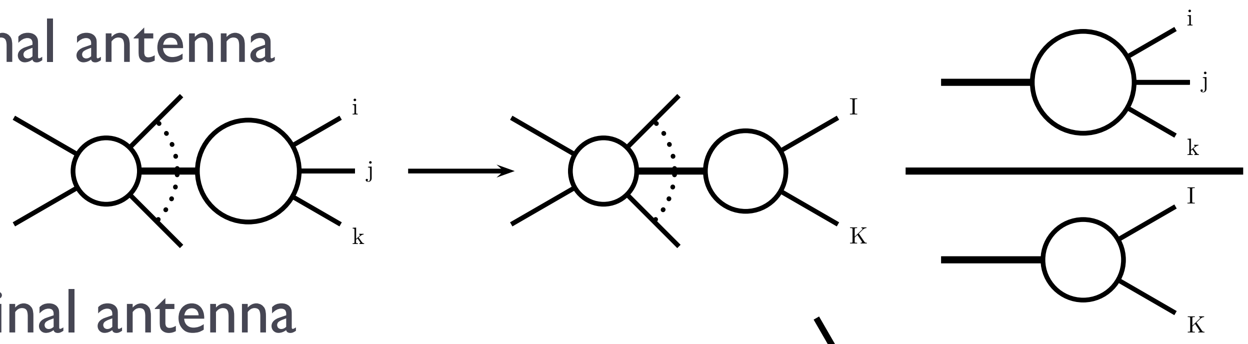
  - Integrated subtraction term $\mathcal{X}_{ijk} = \int d\Phi_{X_{ijk}} X_{ijk}$

Antenna functions

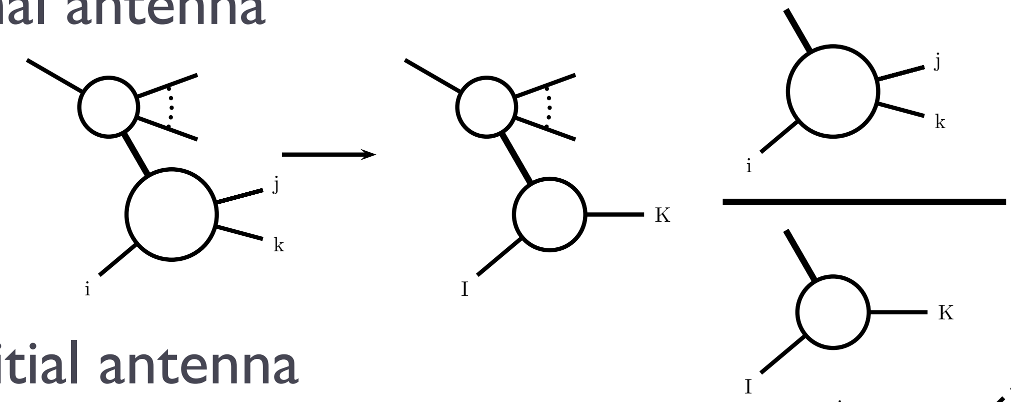
- Colour-ordered pair of hard partons (radiators)
  - quark-antiquark pair
  - quark-gluon pair
  - gluon-gluon pair
- NLO (D. Kosower; J. Campbell, M. Cullen, N. Glover)
  - Three-parton antenna: one unresolved parton X_3^0
- NNLO (T. Gehrmann, N. Glover, AG)
  - Four-parton antenna: two unresolved partons X_4^0
  - Three-parton antenna at one loop: X_3^1
  - Products of NLO antenna functions: $X_3^0 \otimes X_3^0$
  - Soft antenna function S

Antenna subtraction: incoming hadrons

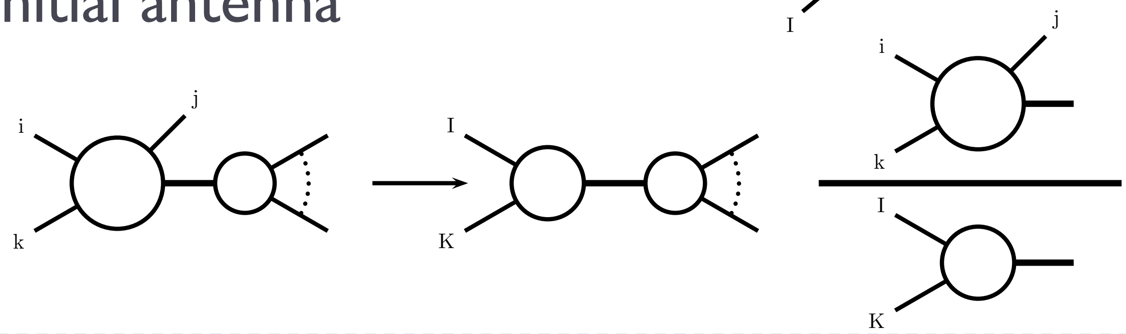
- **Three antenna types** (NLO: A. Daleo, T. Gehrmann, D. Maitre)
  - Final-final antenna



  - Initial-final antenna



  - Initial-initial antenna



Initial-initial antenna functions

- are crossings of final-final antennae: four-parton case

quark-antiquark antennae	
A_4^0	$A_4^0(\widehat{q},\widehat{g},g,\overline{q})$, $A_4^0(\widehat{q},g,\widehat{g},\overline{q})$, $A_4^0(\widehat{q},g,g,\widehat{\overline{q}})$, $A_4^0(q,\widehat{g},\widehat{g},\overline{q})$
$\widetilde{A}_4^0$	$\widetilde{A}_4^0(\widehat{q},\widehat{g},g,\overline{q})$, $\widetilde{A}_4^0(\widehat{q},g,g,\widehat{\overline{q}})$, $\widetilde{A}_4^0(q,\widehat{g},\widehat{g},\overline{q})$
B_4^0	$B_4^0(\widehat{q},\widehat{q}',\overline{q}',\overline{q})$, $B_4^0(\widehat{q},q',\overline{q}',\widehat{\overline{q}})$, $B_4^0(q,\widehat{q}',\widehat{\overline{q}}',\overline{q})^*$
C_4^0	$C_4^0(\widehat{q},\widehat{\overline{q}},q,\overline{q})$, $C_4^0(\widehat{q},\overline{q},\widehat{q},\overline{q})$, $C_4^0(q,\widehat{\overline{q}},\widehat{q},\overline{q})^*$, $C_4^0(q,\overline{q},\widehat{q},\widehat{\overline{q}})^*$

quark-gluon antennae	
D_4^0	$D_4^0(\widehat{q},\widehat{g},g,g)$, $D_4^0(\widehat{q},g,\widehat{g},g)$, $D_4^0(q,\widehat{g},\widehat{g},g)$, $D_4^0(q,\widehat{g},g,\widehat{g})$
E_4^0	$E_4^0(\widehat{q},\widehat{q}',\overline{q}',g)$, $E_4^0(\widehat{q},q',\overline{q}',\widehat{g})$, $E_4^0(q,\widehat{q}',\widehat{\overline{q}}',g)$, $E_4^0(q,\widehat{q}',\overline{q}',\widehat{g})$,
$\widetilde{E}_4^0$	$\widetilde{E}_4^0(\widehat{q},\widehat{q}',\overline{q}',g)$, $\widetilde{E}_4^0(\widehat{q},q',\overline{q}',\widehat{g})$, $\widetilde{E}_4^0(q,\widehat{q}',\widehat{\overline{q}}',g)$, $\widetilde{E}_4^0(q,\widehat{q}',\overline{q}',\widehat{g})$

gluon-gluon antennae	
F_4^0	$F_4^0(\widehat{g},\widehat{g},g,g)$, $F_4^0(\widehat{g},g,\widehat{g},g)$
G_4^0	$G_4^0(\widehat{g},\widehat{q},\overline{q},g)$, $G_4^0(\widehat{g},q,\widehat{\overline{q}},g)$, $G_4^0(\widehat{g},q,\overline{q},\widehat{g})$, $G_4^0(g,\widehat{q},\widehat{\overline{q}},g)$
$\widetilde{G}_4^0$	$\widetilde{G}_4^0(\widehat{g},\widehat{q},\overline{q},g)$, $\widetilde{G}_4^0(\widehat{g},q,\overline{q},\widehat{g})$, $\widetilde{G}_4^0(g,\widehat{q},\widehat{\overline{q}},g)$
H_4^0	$H_4^0(\widehat{q},\widehat{\overline{q}},q',\overline{q}')$, $H_4^0(\widehat{q},\overline{q},\widehat{q}',\overline{q}')$

Integrated NNLO antenna functions

- Analytical integration over unresolved part of phase space only
  - phase space integrals reduced to masters (C. Anastasiou, K. Melnikov)
  - Final-final: $q \to k_1 + k_2 + k_3(+k_4)$, one scale: q^2
    - $1 \to 4$ tree level (4 master integrals)
    - $1 \to 3$ one loop (3 master integrals)
  - Initial-final: $q + p_1 \to k_1 + k_2(+k_3)$, two scales: q^2, x

    (A. Daleo, T. Gehrmann, G. Luisoni, AG)
    - $2 \to 3$ tree level (9 master integrals)
    - $2 \to 2$ one loop (6 master integrals)
  - Initial-initial: $p_1 + p_2 \to q + k_1(+k_2)$, three scales: q^2, x_1, x_2
    - $2 \to 3$ tree level (20 master integrals)

      (R. Boughezal, M. Ritzmann, AG; T. Gehrmann, M. Ritzmann, AG)
    - $2 \to 2$ one loop (5 master integrals) (T. Gehrmann, P.F. Monni)

Integrated initial-initial antennae

- Tree-level antenna functions X_4^0
  - Kinematics: $p_1 + p_2 \to q + k_j + k_k$
  - Phase space factorization (A. Daleo, T. Gehrmann, D. Maitre)

$$d\Phi_{m+2}(k_1,\ldots,k_{m+2};p_1,p_2) = d\Phi_m(\tilde{k}_1,\ldots,\tilde{k}_i,\tilde{k}_l,\ldots,\tilde{k}_{m+1};x_1p_1,x_2p_2)$$
$$\delta(x_1-\hat{x}_1)\,\delta(x_2-\hat{x}_2)\,[dk_j]\,[dk_k]\,dx_1\,dx_2$$

$$\hat{x}_1 = \left(\frac{s_{12}-s_{j2}-s_{k2}}{s_{12}}\;\frac{s_{12}-s_{1j}-s_{1k}-s_{j2}-s_{k2}+s_{jk}}{s_{12}-s_{1j}-s_{1k}}\right)^{\frac{1}{2}}$$
$$\hat{x}_2 = \left(\frac{s_{12}-s_{1j}-s_{1k}}{s_{12}}\;\frac{s_{12}-s_{1j}-s_{1k}-s_{j2}-s_{k2}+s_{jk}}{s_{12}-s_{j2}-s_{k2}}\right)^{\frac{1}{2}}$$

  - Fix x_1,x_2 by imposing collinear limits; Lorentz boost to frame with

$$x_1p_1 + x_2p_2 \to \tilde{q}\;; \qquad \tilde{q}^2 = q^2$$

  - Integration: $2\to3$ particle phase space with x_1,x_2 fixed

Integrated initial-initial antennae

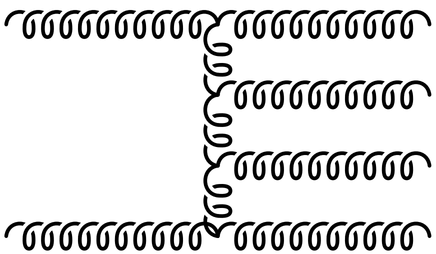
- Integration of tree-level antenna functions X_4^0
  - Express phase space integrals as masters with x_1,x_2 fixed
  - Distinguish
    - Hard region $x_1,x_2 \neq 1$: transcendentality 2
    - Collinear regions $x_1=1, x_2\neq 1$ or $x_1\neq 1,x_2=1$: transcendentality 3
    - Soft region $x_1=x_2=1$: transcendentality 4
  - Determine master integrals from differential equations in x_1,x_2
    - Antenna functions with secondary fermion pair: 10 masters
      (R. Boughezal, M. Ritzmann, AG)
    - Full set of antennae now completed: contains 20 masters
      (T. Gehrmann, M. Ritzmann, AG)
- Integrated initial-initial antennae all known: X_3^0,X_3^1,X_4^0

Jet production at NNLO

Double real radiation at NNLO for $pp \to 2j$

- Contributions from all tree-level $2 \to 4$ processes
- Test case: $gg \to gggg$ (N. Glover, J. Pires)

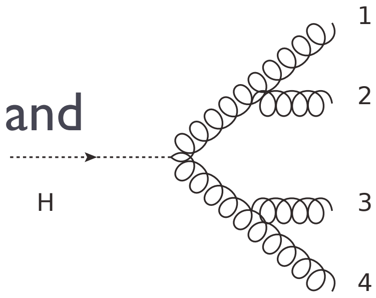
$$
\begin{aligned}
\mathrm{d}\sigma^{R}_{NNLO} \;=\;& N^2\, N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 \mathrm{d}\Phi_4(p_3,\ldots,p_6;p_1,p_2)\Bigg(\\
& \frac{2}{4!} \sum_{P(i,j,k,l)\in(3,4,5,6)} A_6^0(\hat{1}_g,\hat{2}_g,i_g,j_g,k_g,l_g) J_2^{(4)}(p_i,\ldots,p_l) \\
& + \frac{2}{4!} \sum_{P(i,j,k,l)\in(3,4,5,6)} A_6^0(\hat{1}_g,i_g,\hat{2}_g,j_g,k_g,l_g) J_2^{(4)}(p_i,\ldots,p_l) \\
& + \frac{2}{4!} \sum_{P_C(i,j,k,l)\in(3,4,5,6)} A_6^0(\hat{1}_g,i_g,j_g,\hat{2}_g,k_g,l_g) J_2^{(4)}(p_i,\ldots,p_l)\Bigg)
\end{aligned}
$$



- three topologies according to initial state gluon positions

Antenna subtraction for gg → gg at NNLO

- Double real radiation: gg→gggg (N. Glover, J.Pires)
  - Subtraction terms involve only gluon-gluon antennae in all three configurations (initial-initial, initial-final, final-final)
    - F_4^0 for colour-connected double unresolved limits
    - $F_3^0 \cdot F_3^0$ for oversubtracted single unresolved limits and colour unconnected double unresolved limits
    - $F_3^0 \cdot S$ for large-angle soft gluon radiation
    - F_3^0 for single unresolved limits



- antenna subtraction terms constructed, implemented and tested in all unresolved limits

Integrated double real subtraction terms

- Integrated antennae combine with either real-virtual (m+1 partons) or double virtual (m partons) channel

$$\int \mathrm{d}\Phi_{m+2}\,\mathrm{d}\sigma^{S}_{NNLO} = \int \mathrm{d}\Phi_{m+1} \int_1 \mathrm{d}\sigma^{S,1}_{NNLO} + \int \mathrm{d}\Phi_{m} \int_2 \mathrm{d}\sigma^{S,2}_{NNLO}$$

  - one-particle integrals

$\int_1 \mathrm{d}\sigma^{S,1}_{NNLO}$ contains $\mathcal{F}_3^0\,|M_{m+1}|^2$ $\quad \mathcal{F}_3^0\,F_3^0\,|M_m|^2$ $\quad \mathcal{S}\,F_3^0\,|M_m|^2$

  - two-particle integrals

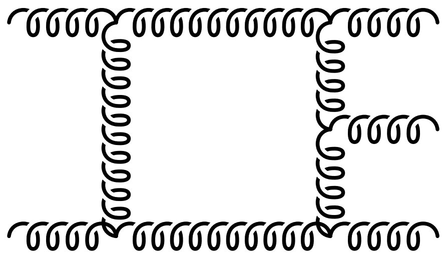
$\int_2 \mathrm{d}\sigma^{S,2}_{NNLO}$ contains $\mathcal{F}_4^0\,|M_m|^2$ $\quad \mathcal{F}_3^0 \otimes \mathcal{F}_3^0\,|M_m|^2$

- Integrated antennae depend on momentum fractions x_1, x_2 of initial state partons

Jet production at NNLO

Real-virtual radiation at NNLO for $pp \to 2j$

- Contributions from all one-loop $2 \to 3$ processes
- Test case: $gg \to ggg$ (N. Glover, J. Pires, AG)



$$
\begin{aligned}
\mathrm{d}\hat{\sigma}^{RV}_{NNLO} =& N^2\, N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 \mathrm{d}\Phi_3(p_3,\ldots,p_5;p_1,p_2)\Bigg(\\
& \frac{2}{3!} \sum_{P(i,j,k)\in(3,4,5)} A_5^1(\hat{1}_g,\hat{2}_g,i_g,j_g,k_g) J_2^{(3)}(p_i,p_j,p_k) \\
& + \frac{2}{3!} \sum_{P(i,j,k)\in(3,4,5)} A_5^1(\hat{1}_g,i_g,\hat{2}_g,j_g,k_g) J_2^{(3)}(p_i,p_j,p_k)\Bigg)
\end{aligned}
$$

- two topologies according to initial state gluon positions
- one-loop matrix elements contain explicit infrared poles

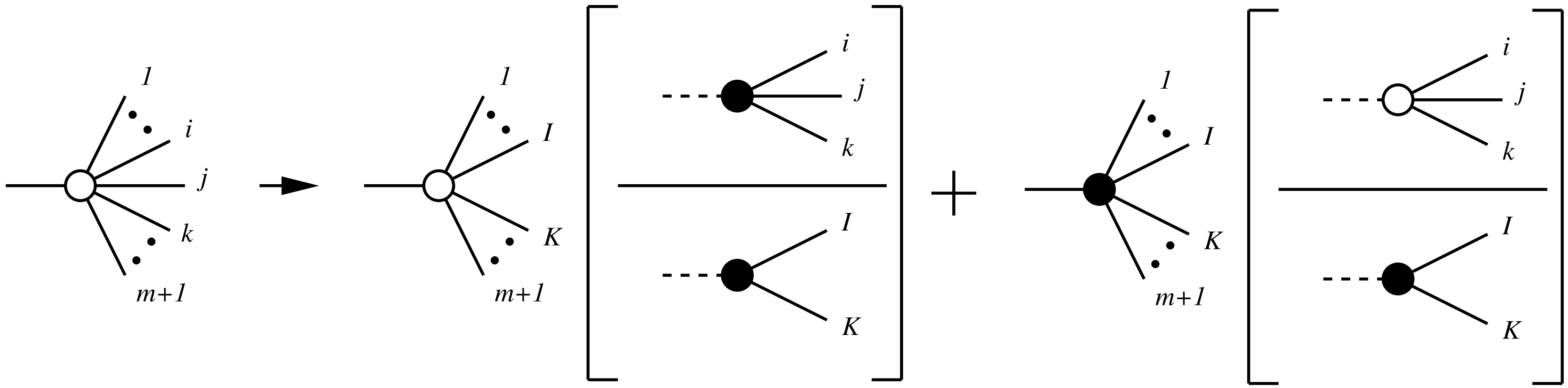
Real-virtual subtraction for gg→gg

- Single unresolved limit of one-loop amplitudes

(Z. Bern, L.D. Dixon, D. Dunbar, D. Kosower)

$$Loop_{m+1} \xrightarrow{j\ unresolved} Split_{tree} \times Loop_m + Split_{loop} \times Tree_m$$

- Accordingly: $Split_{tree} \rightarrow X^0_{ijk} \qquad Split_{loop} \rightarrow X^1_{ijk}$



$$d\sigma^{VS,a}_{NNLO} = \mathcal{N}\, d\Phi_{m+1}(p_3,\ldots,p_{m+3};p_1,p_2) \sum_j X^0_3 |\mathcal{M}^{(1)}_{m+2}|^2\, J^{(m)}_m(p_3,\ldots,p_{m+2})$$

$$+\mathcal{N}\, d\Phi_{m+1}(p_3,\ldots,p_{m+3};p_1,p_2) \sum_j X^1_3 |\mathcal{M}^{(0)}_{m+2}|^2\, J^{(m)}_m(p_3,\ldots,p_{m+2})$$

Real-virtual subtraction for gg→gg

- Structure of subtraction term $\mathrm{d}\sigma_{NNLO}^{VS} = \mathrm{d}\sigma_{NNLO}^{VS,a} + \mathrm{d}\sigma_{NNLO}^{VS,b}$
  - $\mathrm{d}\sigma_{NNLO}^{VS,a}$ approaches $\mathrm{d}\sigma_{NNLO}^{RV}$ in all single unresolved limits
  - $\mathrm{d}\sigma_{NNLO}^{VS,b}$ removes oversubtraction of explicit and implicit poles

$$\mathrm{d}\sigma_{NNLO}^{VS,b} = \mathcal{N}\ \mathrm{d}\Phi_{m+1}(p_3,\ldots,p_{m+3};p_1,p_2) \sum_{ik} \mathcal{X}_3^0(s_{ik}) \sum X_3^0 |\mathcal{M}_{m+2}|^2\ J_m^{(m)}$$

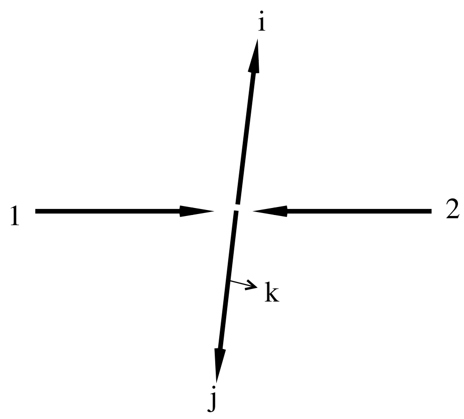
  - such that:

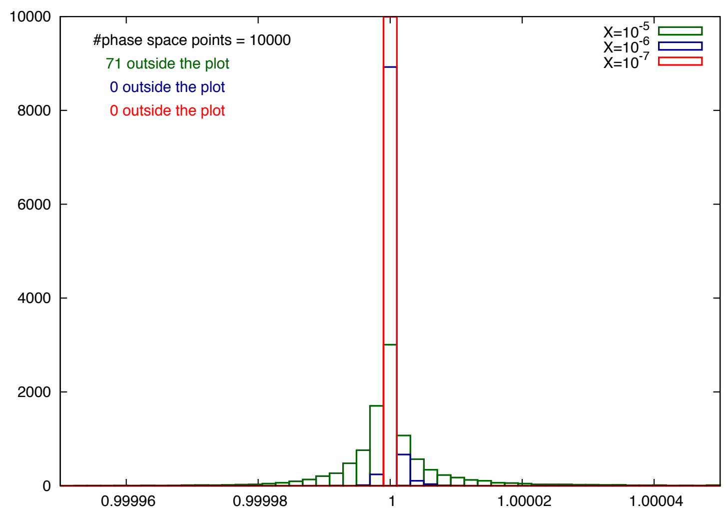
$$\boxed{\mathcal{P}oles\left(\mathrm{d}\hat{\sigma}_{NNLO}^{RV} - \mathrm{d}\hat{\sigma}_{NNLO}^{VS} - \int_1 \mathrm{d}\hat{\sigma}_{NNLO}^{S,1} - \mathrm{d}\hat{\sigma}_{NNLO}^{MF,1}\right) = 0}$$

  - strong check on explicit pole cancellation in real-virtual channel

Real-virtual subtraction for $gg \to gg$

- Check of the subtraction terms
  - choose scaling parameter x for each limit
  - generate phase space trajectories into each limit
  - require reconstruction of two hard jets
  - compute ratio (matrix element)/(subtraction term): $|M_{RV}|^2/S_{term}$
  - Example: soft limit : $s_{ij} \simeq s$





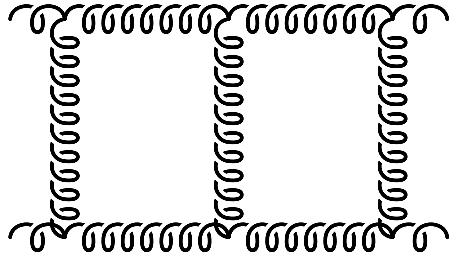
- Ratio approaches unity in all unresolved limits
- Strong check on implementation of subtraction terms

Jet production at NNLO

Double-virtual radiation at NNLO for $pp \to 2j$

- Contributions from all one-loop 2 → 2 processes
- Test case: $gg \to gg$ (N. Glover, J. Pires, T.Gehrmann, AG)

$$\mathrm{d}\hat{\sigma}^{VV}_{NNLO} = N^2\, N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 \mathrm{d}\Phi_3(p_3, p_4; p_1, p_2) \Bigg(\frac{2}{2!} \sum_{P(i,j)\in(3,4)} A_4^2(\hat{1}_g, \hat{2}_g, i_g, j_g) J_2^{(2)}(p_i, p_j) + \frac{1}{2!} \sum_{P(i,j)\in(3,4)} A_4^2(\hat{1}_g, i_g, \hat{2}_g, j_g) J_2^{(2)}(p_i, p_j) \Bigg)$$



- two topologies according to initial state gluon positions
- contains (two-loop*tree) and (one-loop)^2
- explicit infrared poles up to $1/\epsilon^4$ from loop integrals

Double virtual channel

- Explicit poles of double virtual contribution cancel with
  - integrated double real subtraction terms

$$\int_2 \mathrm{d}\sigma^{S,2}_{NNLO} \quad \text{of the form} \quad \mathcal{F}^0_4 \, |M_m|^2 \qquad \mathcal{F}^0_3 \otimes \mathcal{F}^0_3 \, |M_m|^2$$

  - integrated one-loop subtraction terms

$$\int_1 \mathrm{d}\sigma^{VS}_{NNLO} \quad \text{of the form} \quad \mathcal{F}^1_3 \, |M_m|^2 \qquad \mathcal{F}^0_3 \, |M_m|^2_{1l}$$

  - mass factorization counter terms $\mathrm{d}\sigma^{MF,2}_{NNLO}$
  - In all three configurations (final-final, initial-final, initial-initial)

Double virtual channel

- For purely gluonic contributions to $pp \to 2j$, we obtain

$$\boxed{\mathcal{P}oles\left(\mathrm{d}\hat{\sigma}^{VV}_{NNLO} + \int_2 \mathrm{d}\hat{\sigma}^{S,2}_{NNLO} + \int_1 \mathrm{d}\hat{\sigma}^{VS}_{NNLO} + \mathrm{d}\hat{\sigma}^{MF,2}_{NNLO}\right) = 0}$$

- Highly non-trivial check of analytic cancellation of infrared singularities between double-real, real-virtual and double-virtual corrections
- Proof of principle for NNLO antenna subtraction method applied to hadronic collisions

Conclusions

- NNLO antenna subtraction method generalized to hadronic collisions
  - completed analytic integration of all antenna functions for one or two partons in the initial state: full set of integrated antennae now available in all configurations
- Proof-of-principle implementation for $gg \to gg$ contribution to $pp \to 2j$
  - subtraction terms in double real and real virtual channel
    - constructed and implemented
    - observe point-wise convergence for matrix element/subtraction term
  - Double virtual channel
    - observe analytical cancellation of all infrared poles

- Parton–level generator : In progress