A gauge/gravity approach to the FQHE

Constantin Greubel

Max-Planck-Institut für Physik (Werner Heisenberg Institut)

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Outline

- Motivation
- Introduction
  - What is the FQHE?
  - How to model the FQHE?
- Model and Calculations
  - Brane embedding
  - Chemical potential
  - Excitations
- Results
  - Spectral function
  - QNMs
- Conclusion

Can string theory describe condensed matter?

FQHE

The Fractional Quantum Hall Effect was a revolutionary discovery
many explanations brought forward to describe it (Laughlin, Halperin, Wilczek, Haldane)
but still not fully understood for now 30 years
effective theories in field theoretic language (Chern-Simons)

AdS/CFT correspondence

(super-) gravity description $\leftrightarrow$ quantum field theory description
relates regimes with weak/strong coupling

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relates regimes with weak/strong coupling

$\Rightarrow$ gauge/gravity useful here!

The Fractional Quantum Hall Effect

- discovered in 1982 by Tsui, Störmer by experiment
- 2-dim. probe in strong magnetic field at very low temperature
- electrons in collective excitation, strongly correlated
- (quasi-)particles with fractional charge and statistics
- first discovery of a series of new quantum systems where no Landau-Ginzburg approach was viable
- topological phase transition
- still not fully explained

Main observable

filling fraction ν is the number of electronic filled states per Landau level

Laughlin: $\nu = 1/m$ for m odd, hierarchical: $\nu = p/q$

How to model the FQHE?

- use ABJM model (Aharony, Bergman, Jafferis and Maldacena) [0806.1218]
- gauge/gravity duality (holography)
- relates (UV completion of) string theory on AdS_4 to CFT_3 with Chern-Simons theory ($\mathcal{N} = 6$)
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Ingredients for the model

Objects in (type IIA) string theory, called *branes*
Multidim. objects (Dp-branes) which source fields and hook strings

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For our gauge/gravity duality: N_c D2-branes
For our condensed matter model: one D8-brane

Branes and quarks

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- $\mathbb{CP}^3$ is the compact space
- $AdS_4 - BH$ is noncompact: gives 1+2 at bdy + radial coord.

Why $AdS - BlackHole$? Finite temperature!

	0	1	2	3	4	5	6	7	8	9
D2	$\times$	$\times$	$\times$							
D8	$\times$	$\times$		$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$

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strings and quarks

symmetry reasons: flat embedding
consistent with derivation by the action (see next slide)
quarks in the FT correspond to open strings in ST
flat embedding give us massless quarks

Action and the gauge field

Introduce $U(1)$ gauge field A_μ living on D8-brane gives rise to field strength $F_{\mu\nu}$

DBI-action

$$S_{\mathrm{D8}} \propto \int \mathrm{d}\sigma^9 \sqrt{-\det(\mathcal{P}[g] + 2\pi\alpha' F)} + WZ - \mathrm{terms}$$

where:

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Rough direction:

- 0. ~~Solve for embedding function~~
- 1. Solve for gauge field A_μ
- 2. consider fluctuations for D8-brane and its gauge field
- 3. Solve for embedding and gauge field fluctuations

Chemical potential

AdS/CFT dictionary

acquired by matching symmetries, calculating observables on both sides
tells us how to relate quantities through the correspondence:

The $U(1)$ gauge field introduces chemical potential μ, density d

(One D8-brane, therefore interpretation as baryon density
isospin density possible, by extending gauge group to $U(N_f)$)

Asymptotics of gauge field

Compute the EOM for the gauge field (only A_t component of interest)
At the holographic boundary of AdS $\quad A_t = \mu_B + \ldots$
Invert this relation to express action in terms of density d_B

Scalar and vector excitations

Scalar fluctuations

Fluctuations of the D8-brane
remember: D8 has only one direction transverse: y
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Equations of motion for fluctuations

Linearized EOMs $\Rightarrow$ expand the DBI-action to 2nd order in fluctuations
rewrite EOMs by using Fouriers trick in the form of ODEs
$\partial_r^2(\delta y) + \mathfrak{C}_1 \partial_r(\delta y) + \mathfrak{C}_2(\delta y) = 0$
solve them by numerical integration

The spectral function

Definition

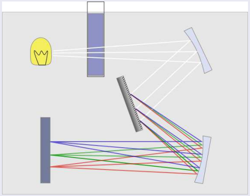
The spectral function is related to the retarded correlator G^R by

$$\mathfrak{R} = -2\mathrm{Im}\, G^R \quad \text{where}$$

$$G^R(k) = -\mathrm{i} \int \mathrm{d}x^3\, \mathrm{e}^{ikx}\, \theta(x^0) \langle [J(x), J(0)] \rangle$$

with $$G^R = \frac{\delta^2 S_{\mathrm{SUGRA}}}{\delta \tilde{A}^2}\Big|_{\mathrm{boundary}}$$

[Son, Starinets, '02]



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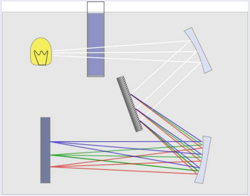

In our coordinates

$$\mathfrak{R}(\omega) \propto \mathrm{Im}\, \frac{\partial_r y}{r^3 y}\Big|_{\mathrm{boundary}}$$

QNM vs. density

Quasinormal Modes:
like the normal frequencies for oscillator, but with damping

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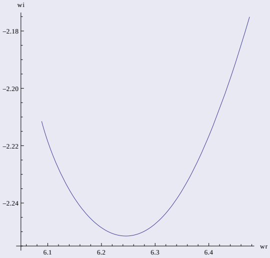
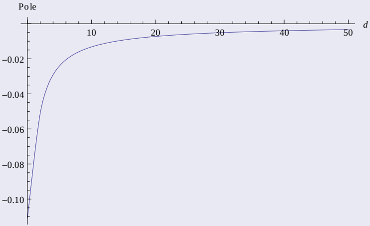
The scalar and vector QNM





QNM vs. density

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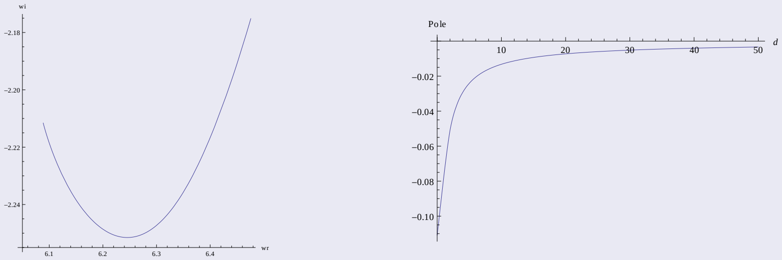

- imaginary part of the modes in all regimes below 0
- important check for stability

Conclusion

Summary

- AdS/CFT and the FQHE
- Chemical potential and baryon density
- Scalar and vector fluctuations on the brane
- Spectral function and QNMs

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Outlook

- compute further observables: conductivity, ...
- extend action by a self-dual field strength
- make process of edge current dynamical

Conclusion

Thank you for your attention!