Universal Scaling of Holographic Superconductors

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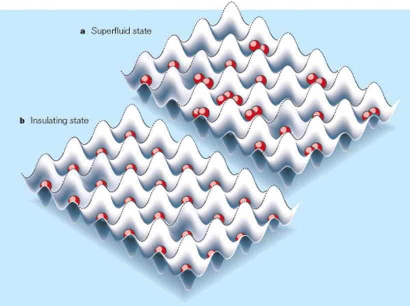
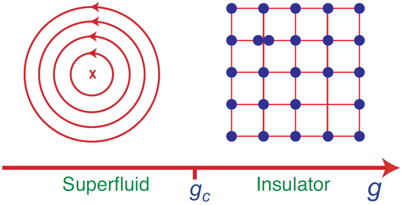
Condensed Matter/Quantum Matter Systems

Superfluid-insulator quantum phase transitions
e.g. with ultracold ^{87}Rb atoms in an optical lattice



[Henk T. C. Stoof, Nature 415, 25-26]

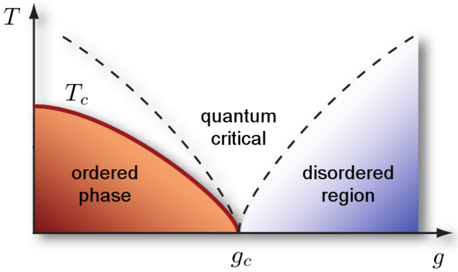


[Subir Sachdev, 1108.1197]

Outline

Outline

Continuous Quantum Phase Transitions



- Driven by quantum fluctuations
- Non-analyticity in ground state energy $E(g)$ at $T = 0$
- Near critical coupling g_c

$$\Delta \sim J\,|g - g_c|^{z\nu}$$
$$\xi^{-1} \sim \Lambda\,|g - g_c|^{\nu}$$
$$\Delta \sim \xi^{-z}$$

- Quantum critical point $\longrightarrow$ quantum critical region for $T > 0$ with two different regimes:

$$\Delta > k_\mathrm{B} T \qquad \tau \gg \frac{\hbar}{k_\mathrm{B} T} \qquad \text{(effectively classical)}$$

$$\Delta < k_\mathrm{B} T \qquad \tau \sim \frac{\hbar}{k_\mathrm{B} T} \qquad \text{(quantum critical)}$$

Universal Scaling & Quantum Criticality

Quantum Critical Region (QCR)

- τ independent of microscopic energy scale J
  $\longrightarrow$ Quantum & thermal fluctuations determine the dynamics
  $\Longrightarrow$ Universal lower timescale set by $\tau_\hbar = \hbar/k_B T$

- QCR effectively describe by a universal QFT depending on:
  - Symmetry of the order parameter (order $\leftrightarrow$ disorder)
  - Dimensionality of space(time)/lattice
  - Conserved currents

Mapping of Quantum Models to Classical Models

d dimensional quantum mechanics evolving in imaginary time $\Leftrightarrow$ $d+1$ dimensional classical statistical mechanics with geometry $\mathbb{R}^d \times S^1$ and 'temperature' g

S^1 is defined by the temperature of the quantum system via $\tau = \hbar/k_B T$

Universal Scaling & Quantum Criticality

- "Proof":

$$Z \propto \mathrm{tr}\, \mathrm{e}^{-\beta H} = \sum_i \mathrm{e}^{-\beta H(\{s_i\})} = \sum_i \left\langle \phi_i(\tau) \right| \mathrm{e}^{-\frac{\tau \mathcal{H}}{\hbar}} \left| \phi_i(0) \right\rangle$$
$$= \sum_i \left\langle \phi_i(t) \right| \mathrm{e}^{-\mathrm{i}t\frac{\mathcal{H}}{\hbar}} \left| \phi_i(0) \right\rangle$$

with $\phi(\tau + \hbar\beta) = \phi(\tau)$ and $\tau = \hbar/k_B T = \hbar\beta = \mathrm{i}t \longrightarrow \tau \in [0, \hbar\beta]$.

$O(N)$ Vector Model/Classical Statistical Model/Quantum Model

$N = 1$	Ising model	Quantum Ising model
$N = 2$	XY model	$O(2)$ quantum rotor model boson Hubbard model
$N = 3$	Heisenberg model	$O(3)$ quantum rotor model
$N = 4$	Higgs sector of SM	???

Universal Scaling & Quantum Criticality

- Mapping between QFT/Classical Field Theories

$$
\begin{array}{ccc}
d \text{ dimensional QM} & \xrightarrow[\text{evolution}]{\text{imaginary time}} & (d+1) \text{ statistical model} \\
\Lambda\xi \to \infty \Big\downarrow J/\Delta \to \infty & & \text{lattice spacing} \Big\downarrow a \to 0 \\
\text{QFT near critical point} & \xleftarrow[\text{formalism}]{\text{path integral}} & D \text{ dimensional FT}
\end{array}
$$

Field Theory of the $O(N)$ Vector Model

$$Z = \sum_i \mathrm{e}^{-\beta H(\{s_{i\alpha}\})} \approx \int \mathcal{D}\varphi_\alpha(x)\, \mathrm{e}^{-S[\varphi_\alpha(x)]}$$

$$S[\varphi_\alpha(x)] = \int \mathrm{d}^{d+1}x \left[\frac{1}{2}\left(\nabla\varphi_\alpha(x)\right)^2 + \frac{r}{2}\varphi_\alpha(x)^2 + \frac{u}{4!}\varphi_\alpha(x)^4 \right]$$

- Potential arise from averaging over microscopic degrees of freedom
- Quantum Ising/rotor models have $z = 1 \longrightarrow$ relativistic field theories

Solutions of Critical QFT for $D < 4$ and Renormalization

- Scaling dimension:

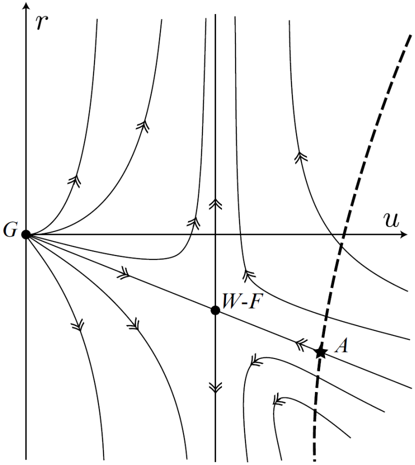
$$\dim \varphi_\alpha(x) = \frac{D-2}{2}$$

- RG flow equations for couplings

$$\frac{\mathrm{d}r}{\mathrm{d}\lambda} = 2r \qquad \frac{\mathrm{d}u}{\mathrm{d}\lambda} = (4-D)u$$

- For $D > 4$: Trivial fixed point $r_* = 0 = u_* \longrightarrow$ free theory

- For $D \leq 4$ corrections to RG flow: $r \sim |g - g_c| \neq 0 \longrightarrow$ vicinity of QCP is strongly coupled



So far:

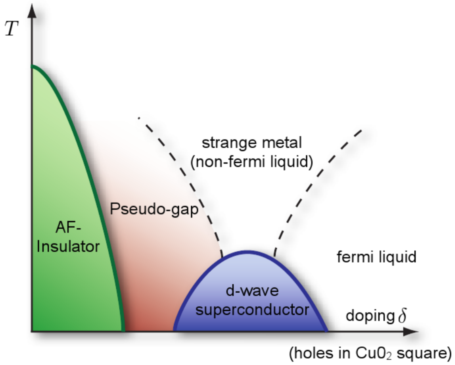
- Universal QFT is invariant under conformal transformations $\longrightarrow$ CFT
- For $D > 4$ we have a free CFT but for $D < 4$ it is strongly coupled
- For $T = 0$ analytical continuation to real time controllable

Problems for $D < 4$ and $T > 0$:

- Quantum-Classical mapping gives rise to imaginary time correlation functions
  - Analytical continuation is ill-defined especially for $t \gg \hbar/k_{\mathrm{B}}T$
  - No classical analog of quantum interference effects $\longrightarrow$ Quantum phase coherence time τ_φ (in QCR $\tau_\varphi \sim \hbar/k_{\mathrm{B}}T$)
- Interactions destroy quasi-particles pictures:
  - How to define properties/dynamics/transport in QCR?
  - Description for strongly interacting regimes without reference to quasi-particles $\longrightarrow$ Gauge/Gravity Duality

Outline

Properties of Conventional/High T_c Superconductors



Conventional Superconductors

- Infinite DC conductivity
- Ideal diamagnet
- Superconducting/normal phase transition
- Cooper pairing via phonon exchange

Unresolved Problems in High T_c Superconductors

- Unknown pairing mechanism between electrons
- Pseudo-gap between insulator/superfluid phase
- QCP below superconducting dome $\longrightarrow$ Is strange metal phase a QCR?

Homes' Law

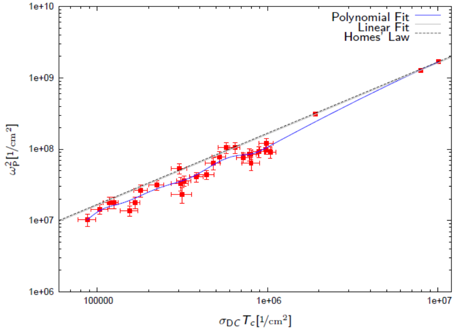
- All superconductors seem to show an universal scaling law

$$\rho_s \propto \sigma_{\mathrm{DC}}(T_c)\,T(_c)$$

[Homes et al., cond-mat/0410719]

- Applying $\rho_s \equiv \omega_{\mathrm{Ps}}^2$ and $\sigma_{\mathrm{DC}} = \omega_{\mathrm{P}}^2 \tau / 4\pi$:

$$\omega_{\mathrm{Ps}}^2 \propto \frac{\omega_{\mathrm{P}}^2 \tau_c}{4\pi} T_c$$



- Assuming the sum-rules are valid (no missing spectral weight) $\omega_{\mathrm{Ps}} = \omega_{\mathrm{P}} \longrightarrow \tau_c T_c = \text{const.}$
- Possibility to check timescale in strange metal phase of order $\tau \sim \hbar / k_{\mathrm{B}} T \longrightarrow$ "nearly" perfect fluid (universal momentum transport)

Outline

Gauge/Gravity Duality

Conjectures of Gauge/Gravity Duality

QFT in d dimensional spacetime	$\Leftrightarrow$	Gravity in $d+1$ dimensional spacetime
Conformal Field Theory in d spacetime dimensions	$\Leftrightarrow$	Gravity in $d+1$ dimensional Anti-de-Sitter spacetime
$N \to \infty$ gauge theories in d spacetime dimensions	$\Leftrightarrow$	Classical gravity in $d+1$ spacetime dimensions

Renormalization Flow and AdS-Space

Conjecture 2 of Gauge/Gravity Duality

Conformal Field Theory in d spacetime dimensions	$\Leftrightarrow$	Gravity in $d+1$ dimensional Anti-de-Sitter spacetime

- Coupling constants g are locally dependent on energy/length scale u

$$\frac{\mathrm{d}g(u)}{\mathrm{d}u} = R\big(g(u)\big)$$

- Put length scale u on the same footing as spacetime coordinates:
  - Scale invariance at RG fixed points $R(g) \equiv 0$:

$$x \to \lambda x, \qquad u \to \lambda u \qquad \longrightarrow \qquad \mathrm{d}s^2 = \frac{L^2}{u^2}\left(\mathrm{d}t^2 + \mathrm{d}\mathbf{x}^2 + \mathrm{d}u^2\right)$$

  - The RG fixed point AdS_{d+1} space isometries $\equiv$ symmetry of CFT_d:

Poincaré symmetry $\oplus$ scale invariance	$\longrightarrow$	$SO(d,2)$ conformal group in d dimensions

Holographic Principle

Conjecture 3 of Gauge/Gravity Duality

$N \to \infty$ gauge theories in d spacetime dimensions $\Leftrightarrow$ Classical gravity in $d+1$ spacetime dimensions

Holographic Principle

#(degrees of freedom) in $(d+1)$ dimensional gravity $=$ #(degrees of freedom) in d dimensional gauge theory

- (Maximal) entropy in gravity

$$S_{\text{gravity}} = \frac{2\pi A_{\text{boundary}}}{\kappa^2} \quad \longrightarrow \quad \#(\text{gravity}_{d+1}) = \infty = \#(\text{QFT}_d)$$

- Regulation $\longrightarrow$ putting field theory on a lattice:

$$S_{\text{QFT}} \sim (R\Lambda)^{d-1} N^2 \sim \frac{(RL\Lambda)^{d-1}}{\kappa^2} \qquad \longrightarrow \qquad \frac{L^{d-1}}{\kappa^2} \sim N^2$$

Gauge/Gravity Dictionary

Basic Dictionary

Operator $\mathcal{O}$ in CFT $\Leftrightarrow$ Dynamical field φ in AdS-spacetime

Sourcing $\mathcal{O}$ is dual to boundary condition $\varphi_{(0)} = J$ at infinity

$$Z_{d+1}\left[\varphi(x,u) \to \varphi_{(0)}(x)\right] = \left\langle \mathrm{e}^{\mathrm{i}\int \mathrm{d}^d x \varphi_{(0)}(x)\mathcal{O}(x)} \right\rangle_{\mathrm{QFT}}$$

Advanced Dictionary

energy momentum tensor $T_{\mu\nu}$	$\Leftrightarrow$	metric g_{ab}
global symmetry current J_μ		local gauge symmetry field A_a
scalar operator $\mathcal{O}_\mathrm{B}$		scalar field φ
fermionic operator $\mathcal{O}_\mathrm{F}$		fermionic field ψ

Gauge/Gravity Dictionary

Vacuum Expectation Values & Correlators...

...are calculated using functional derivatives with respect to $\varphi_{(0)}$

$$\langle \mathcal{O} \rangle = \frac{\delta}{\delta \varphi_{(0)}} \left\langle \mathrm{e}^{\mathrm{i} \int \mathrm{d}^d x \varphi_{(0)}(x) \mathcal{O}(x)} \right\rangle \Big|_{\varphi_{(0)}=0} = \frac{\delta Z_{d+1}[\varphi \to \varphi_{(0)}]}{\delta \varphi_{(0)}}$$

$$\langle \mathcal{O}(x) \mathcal{O}(y) \rangle = \frac{\delta^2}{\delta \varphi_{(0)}^2} Z_{d+1}[\varphi \to \varphi_{(0)}] \approx \frac{\delta^2}{\delta \varphi_{(0)}^2} \mathrm{e}^{-S_{d+1}[\varphi \to \varphi_{(0)}]}$$

- Using Hamilton-Jacobi theory the conjugated momentum is

$$\Pi_{(0)} = \frac{\delta S_{d+1}[\varphi \to \varphi_{(0)}]}{\delta \varphi_{(0)}} \quad \longrightarrow \quad \frac{\delta^2 S_{d+1}[\varphi \to \varphi_{(0)}]}{\delta \varphi_{(0)}^2} = \frac{\delta \Pi_{(0)}}{\delta \varphi_{(0)}}$$

- Solution with general boundary conditions ($\Delta(\Delta - d) = (mL)^2$)

$$\varphi(u \to 0) = \varphi_{(0)}^{\Delta_-} + \varphi_{(1)} u^{\Delta_+} + \dots \quad \longrightarrow \quad \langle \mathcal{O}(x) \mathcal{O}(y) \rangle = \frac{\Delta_-}{L} \frac{\varphi_{(1)}}{\varphi_{(0)}}$$

Outline

Holographic S-Wave Superconductor

Einstein-Maxwell Theory

Einstein Maxwell action with charged scalar field Ψ and $\alpha = {}^{\kappa^2}/_{e^2 L^2}$

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^{d+1}x \sqrt{-g} \left[R - 2\lambda + \frac{2\kappa^2}{e^2} \left(F^2 - |\nabla\Psi - \mathrm{i}A\psi|^2 - V(|\psi|) \right) \right]$$

Equations of Motion

$$R_{ab} - \frac{1}{2} R g_{ab} + \frac{d(d-1)}{2L^2} g_{ab} = \alpha^2 L^2 T_{ab}[A, \Psi], \qquad \nabla_a F^{ab} = j_b[A, \Psi]$$

Solutions to Equations of Motion

	$\alpha = 0$ (probe limit)	$\alpha \neq 0$ (backreaction)
$\Psi = 0$	AdS Schwarzschild BH	AdS Reissner Nordström BH
$\Psi \neq 0$	AdS Schwarzschild BH	Scalar Hair AdS RN BH

Linear Response Theory

- Weak and short perturbations:

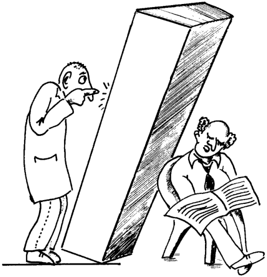
$$\delta\mathcal{H} = \int \mathrm{d}^d x\, \varphi_i(t,x)\mathcal{O}^i(t,x)$$

- The response of the system is

$$\delta\left\langle \mathcal{O}^i(x)\right\rangle = \int \mathrm{d}^{d+1}y\, G_{\mathrm{R}}^{ij}(x,y)\varphi_j(y)$$

- Causal structure:

$$G_{\mathrm{R}}^{ij}(x,y) = \mathrm{i}\theta(t_x - t_y)\left\langle\left[\mathcal{O}^i(x), \mathcal{O}^j(y)\right]\right\rangle$$



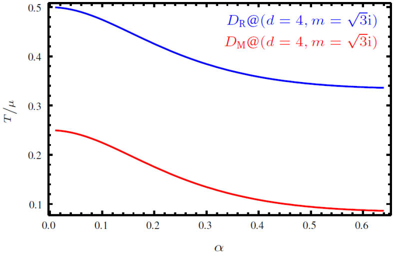
[Alexei Tsvelik - QFT in CMP]

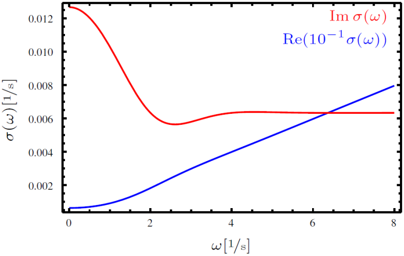
- Fluctuation-Dissipation theorem:
  - Calculate correlators of fluctuations about background solution
  - Charged black hole with finite temperature provides damping

    $\longrightarrow$ determines dissipative transport coefficients

Transport Coefficients

- Interesting transport coefficient for different time scales "explaining" Homes' law $\tau_c T_c = const$.
  - Momentum diffusion $D_{\mathrm{M}} \longrightarrow$ Shear mode pole in $\langle T_{xy}, T_{xy} \rangle$
  - Charge diffusion $D_{\mathrm{R}} \longrightarrow$ Relation between $\chi(\omega, \mathbf{k}) = -\frac{i\mathbf{k}^2}{\omega}\sigma(\omega, \mathbf{k})$
  - Plasma frequency $\omega_{\mathrm{P}} \longrightarrow$ Dielectric function $\epsilon(\omega, \mathbf{k}) = 1 - \frac{\omega_{\mathrm{P}}^2}{\omega^2}$





$$D_{\mathrm{R}} = \frac{1}{4\pi T}\frac{(2-\bar{Q}^2)(2+\bar{Q}^2)}{2(1+\bar{Q}^2)}, \qquad D_{\mathrm{M}} = \frac{1}{4\pi T}\left(1 + \frac{\mu}{T}(\bar{Q})\frac{n}{s}(\bar{Q})\right)^{-1}$$

Thank You
for
Listening!