FLAVOUR IN WARPED EXTRA DIMENSIONS

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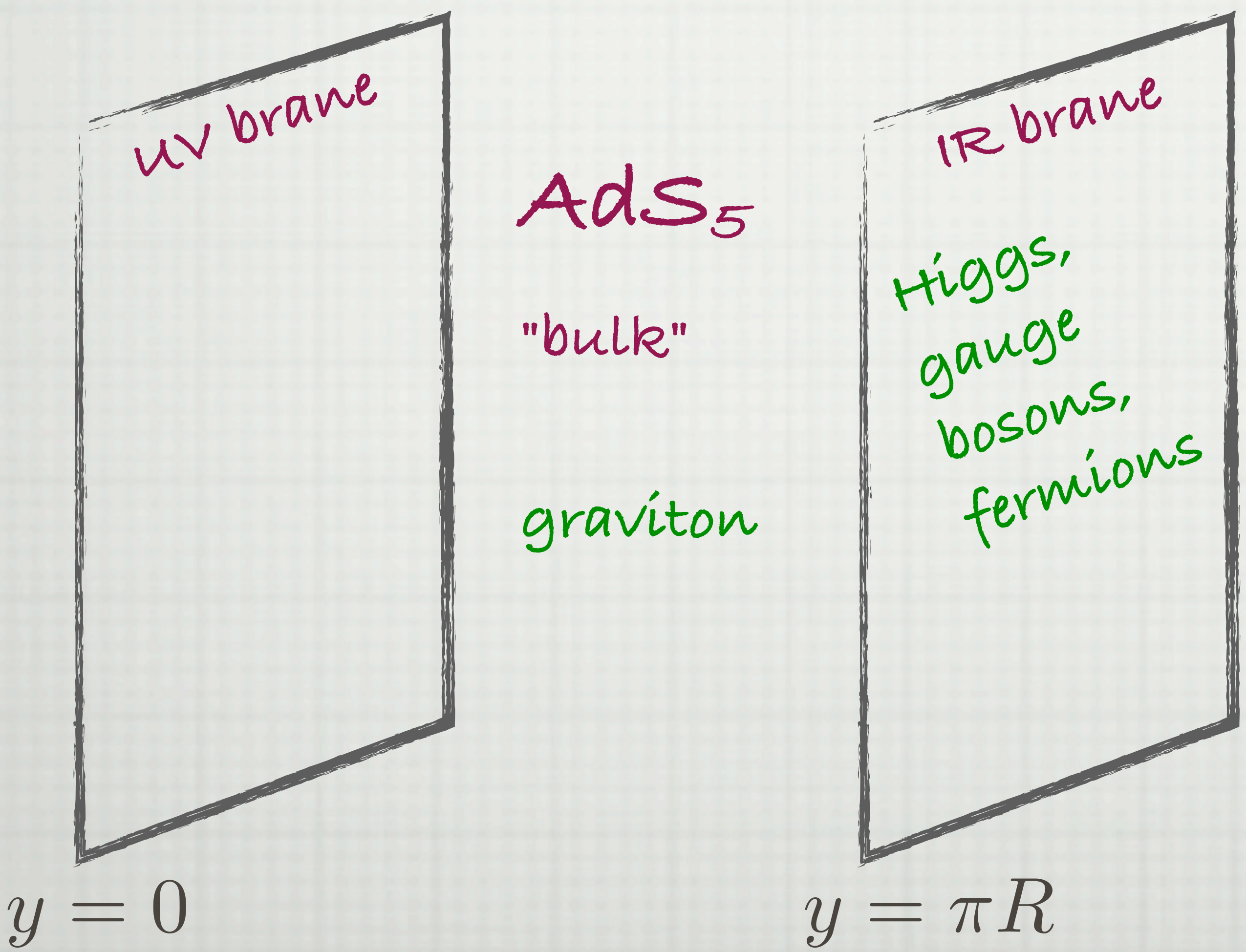
based on work done at the WEIZMANN INSTITUTE OF SCIENCE in collaboration with ORAM GEDALIA & GILAD PEREZ

WHY EXTRA DIMENSIONS?

Hierarchy Problem: Why is the scale of electroweak symmetry breaking so much smaller than the Planck scale?

Solution: **warped extra dimensions**

Randall & Sundrum '99



A slice of AdS_5 wedged between two flat 4D boundaries.

AdS_5 = 5D anti-de Sitter space:
maximally symmetric space of negative curvature.

WHY EXTRA DIMENSIONS?

space-time metric: $ds^2 = e^{-2ky}\eta_{\mu\nu}\mathrm{dx}^\mu \mathrm{dx}^\nu + \mathrm{dy}^2 \equiv \mathrm{g_{MN}dx^M dx^N}$

on the IR brane

Higgs action

$$g^{\mathrm{ind.}}_{\mu\nu} = e^{-2\pi Rk}\eta_{\nu\mu}$$

$$S_{\mathrm{Higgs}} = \int d^4x\sqrt{-g_{\mathrm{ind.}}}\left\{g^{\mu\nu}_{\mathrm{ind.}}\partial_\mu H^\dagger \partial_\nu H - \lambda\left(|H|^2 - v_0^2\right)^2\right\}$$

$$= \int d^4x\left\{e^{-2k\pi R}\eta^{\mu\nu}\partial_\mu H^\dagger \partial_\nu H - e^{-4k\pi R}\lambda\left(|H|^2 - v_0^2\right)\right\}$$

canonical normalization $e^{-k\pi R}H \to H$

$\Rightarrow$ effective Higgs vev v "warped down": $v = e^{-k\pi R}v_0$

$v_0 \approx M_{\mathrm{Planck}}$ $\qquad$ $kR \sim \mathcal{O}(10)$

PROBLEMS WITH EXTRA DIMENSIONS

[5D gauge couplings] = mass$^{-1/2}$

⇒ non-renormalisable

⇒ effective field theory (EFT)

⇒ other operators [coupling] = mass^{-a}

Mass scale that suppresses higher dimensional operators is also warped down!

$$\frac{1}{M^2}\bar{\Psi}_i\Psi_j\bar{\Psi}_k\Psi_l \to \frac{1}{(Me^{-\pi kR})^2}\bar{\Psi}_i\Psi_j\bar{\Psi}_k\Psi_l$$

$$\frac{1}{M}\nu\nu HH \to \frac{1}{Me^{-\pi kR}}\nu\nu HH$$

Proton decay!
FCNCs!
Neutrino masses!

SOLUTION: FERMIONS IN THE BULK

"SPLIT FERMION MODELS"

In 5D:

chiral mass terms (Yukawa couplings):

$$\mathcal{L} \supset \overline{Q_L}\, Y_u\, u_R\, H + \overline{Q_L}\, Y_d\, d_R\, H$$

vector-like mass terms

$$\mathcal{L} \supset C_Q \overline{Q_L} Q_L + C_u \overline{u_R} u_R + C_d \overline{d_R} d_R$$

Grossmann & Neubert '00
Arkani-Hamed & Schmaltz '00
Huber & Shafi '01
Gherghetta & Pomarol '00

FERMION PROFILES IN THE BULK

Ansatz for fermion wavefunction: separation of variables

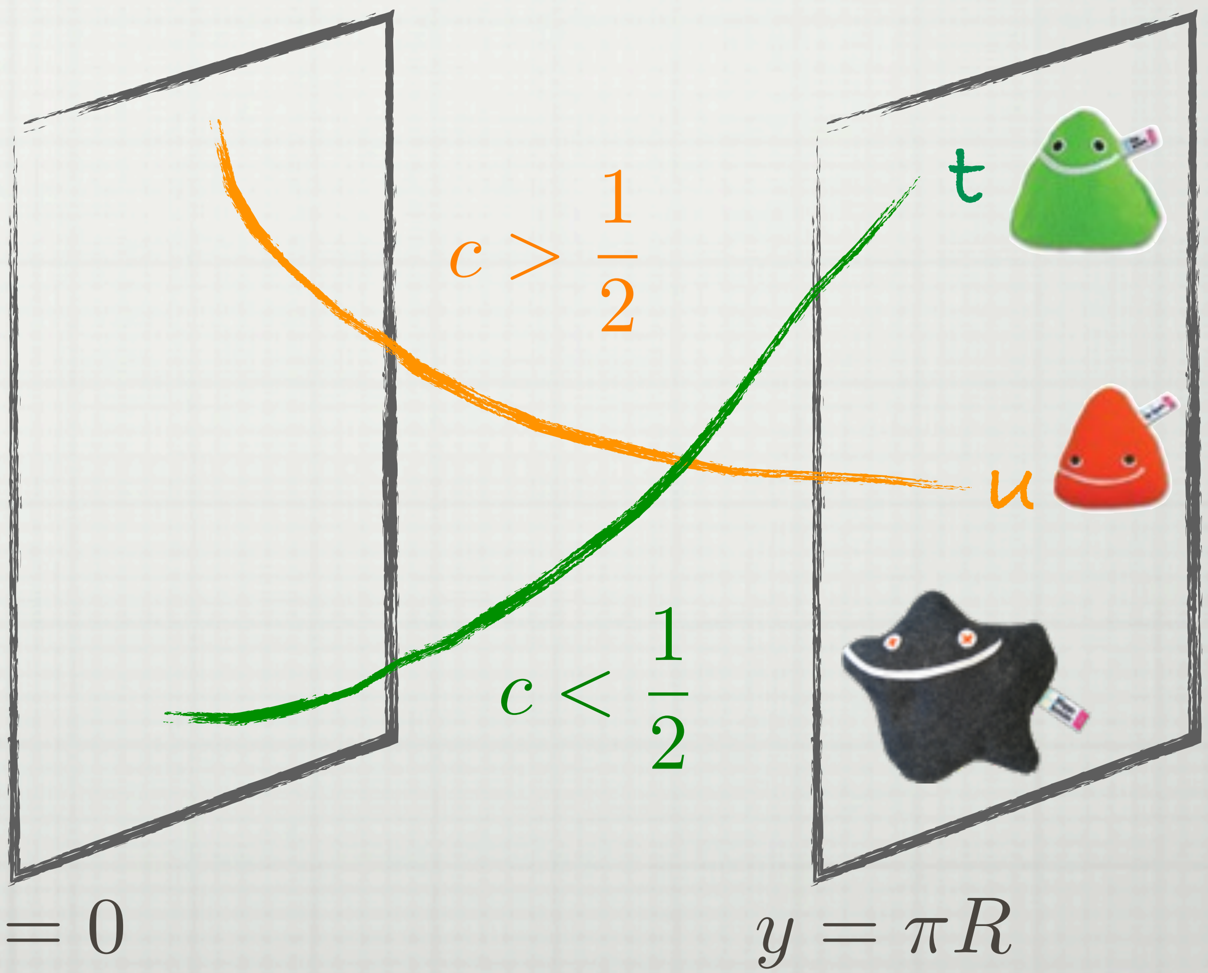
$$\Psi(x,y) = \frac{1}{\sqrt{\pi R}} \sum_n \Psi^{(n)}(x) f^{(n)}(y)$$

e.o.m. $\Rightarrow$ zero-mode

$$f^{(0)}(y) \propto e^{\left(\frac{1}{2}-c\right)ky}$$

flavour puzzle = Where does huge hierarchy in fermion masses come from?

solved

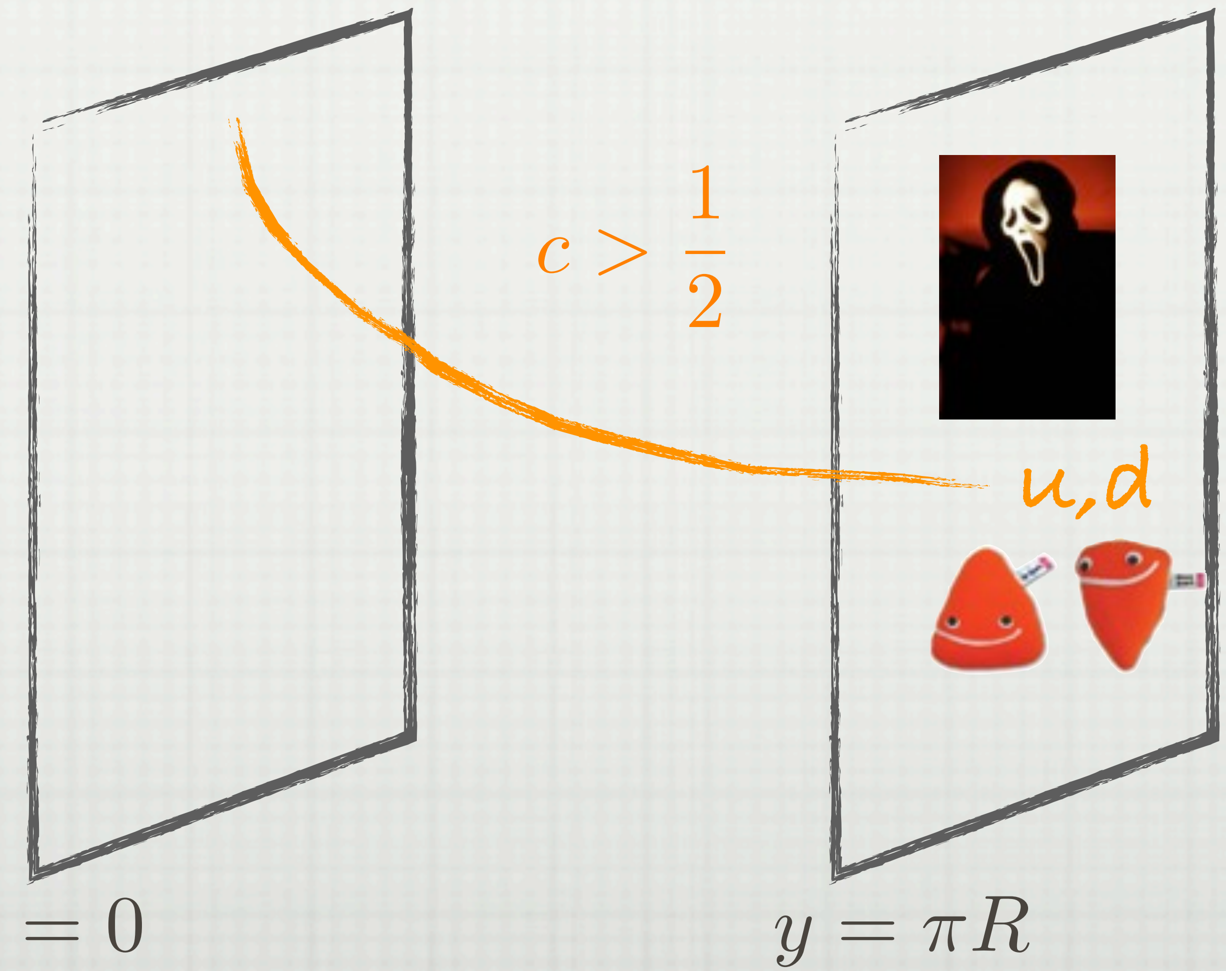


HIGHER DIMESIONAL OPERATORS IN SPLIT FERMION MODELS

Experimental bounds mainly from light fermions $\Rightarrow$ overlap with higher dimensional operators on IR brane suppressed

$$\frac{1}{M^2}\bar{\Psi}_i\Psi_j\bar{\Psi}_k\Psi_l$$

$$\frac{1}{(Me^{-k\pi R})^2}\bar{\Psi}_i\Psi_j\bar{\Psi}_k\Psi_l$$



COLLIDER SIGNATURE: KK EXCITATIONS

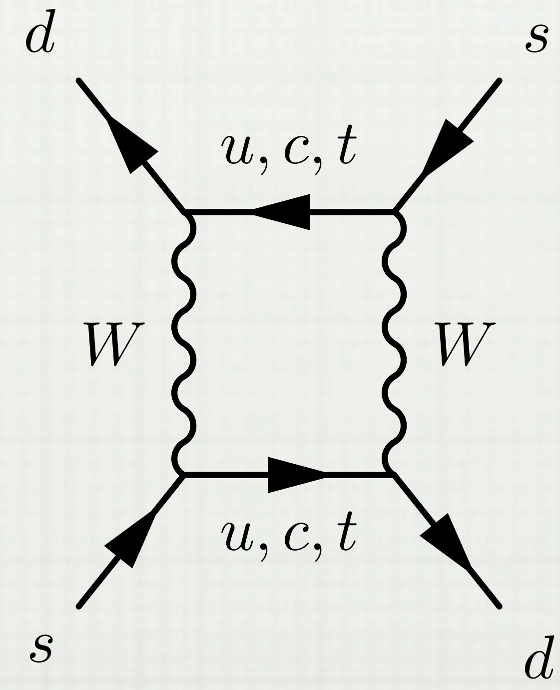
zero-modes = particles without momentum in x-dim

Kaluza-Klein (KK) states = particles with momentum in x-dim ⇒ appear as "KK tower" of heavy particles in 4D

REMINDER: FCNCs

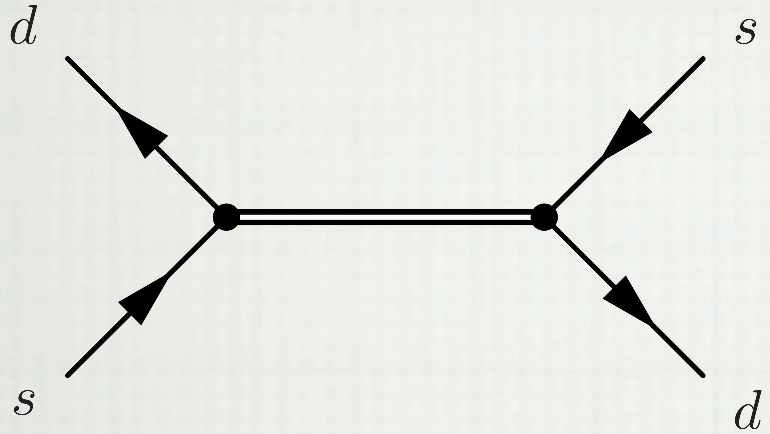
Flavour Changing Neutral Currents (FCNCs) = neutral processes that change the flavour of quarks, i.e. u↔c

SM: only at loop level



Experiments: meson-anti-meson mixing, rare decays ⇒ tight constraints

FCNCs VIA EXCHANGE OF KK GLUONS



Bulk masses $\mathcal{L} \supset C_Q \overline{Q_L} Q_L + C_u \overline{u_R} u_R + C_d \overline{d_R} d_R$

diagonal: $C_x = \mathrm{diag.}(c_{x_1}, c_{x_2}, c_{x_3})$

Zero mode wave function on IR brane:

$$f^{(0)}_{x_i}(\pi R) = e^{\left(\frac{1}{2} - c_{x_i}\right) k\pi R}$$

$$F_x = \mathrm{diag.}(f_{x_1}, f_{x_2}, f_{x_3})$$

Coupling of fermion zero modes to KK gauge bosons:

$$\mathcal{L} \supset G^{(n)} \left(\overline{Q_L} Q_L + \overline{U} U + \overline{D} D\right)$$

$$G^{(1)}_x \simeq g F^\dagger_x F_x$$

diagonal but not universal!

$$G^{(1)}_x = \mathrm{diag.}(...) \neq \mathrm{const.} \cdot \mathbb{1}$$

4D effective Yukawa couplings:

$$Y^{4D}_{u,d} \simeq F^\dagger_Q Y^{5D}_{u,d} F_{u,d}$$

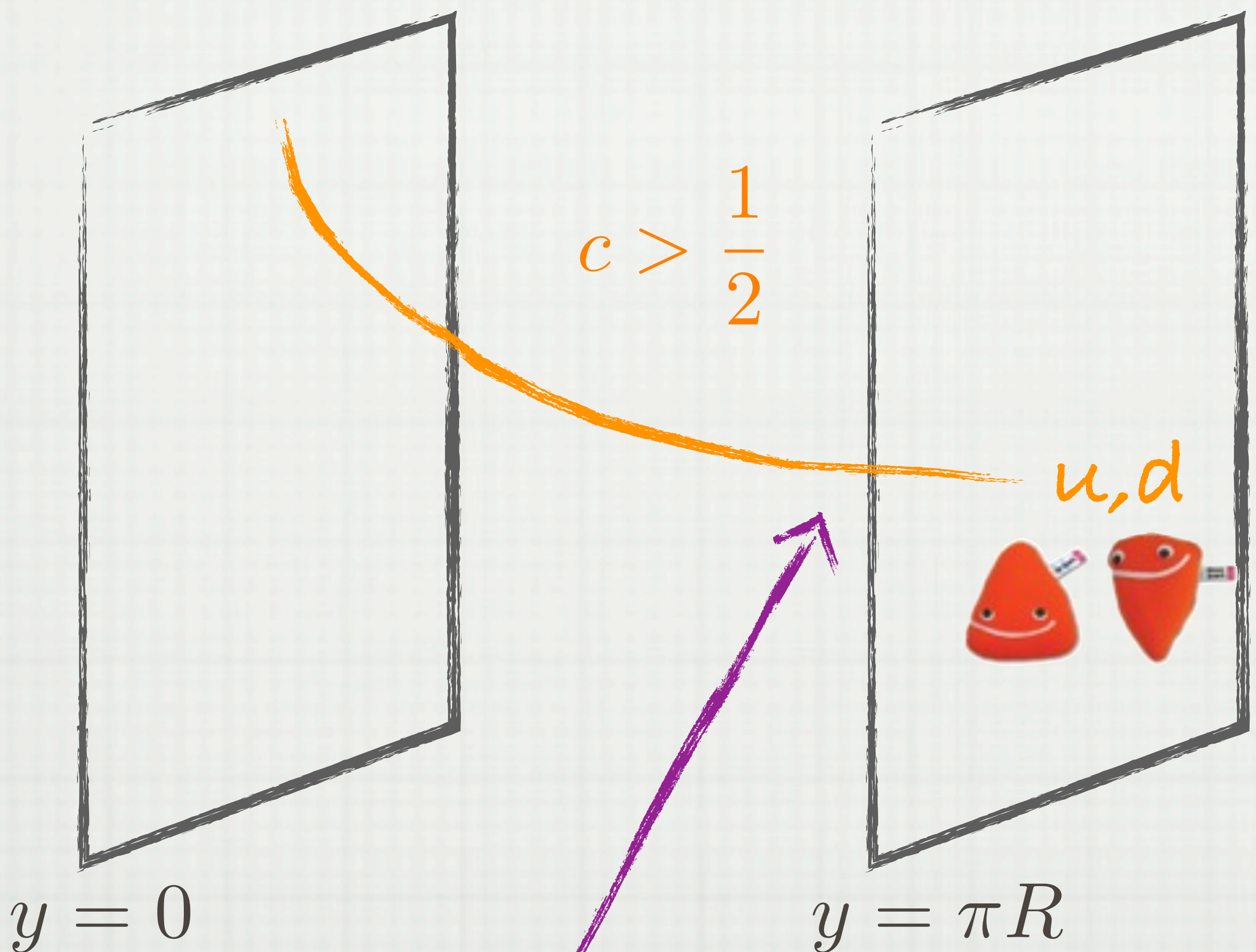
in mass basis:

$$(Y^{4\mathrm{D}}_{u,d})_{\mathrm{mass}} \simeq V_{(u,d)L} F^\dagger_Q Y^{5\mathrm{D}}_{u,d} F_{u,d} V^\dagger_{(u,d)R}$$

$$u_L \to V_{uL} u_L \qquad u_R \to V_{uR} u_R$$

$$G^{(1)}_{x(R,L)} \simeq g V_{x(R,L)} F^\dagger_x F_x V^\dagger_{x(R,L)} \neq \mathrm{diag.}(...)$$

RS GIM MECHANISM



$m_{KK} \gtrsim 10\text{TeV}$

KK gluons live near
IR brane $\Rightarrow$ overlap
with light quarks
suppressed

Gherghetta & Pomarol '00
Agashe, Perez & Soni '05

DEALING WITH FLAVOUR: SOME TECHNIQUES

SM without Yukawa couplings: symmetric under interchange of particle flavours

Idea: Yukawa couplings ≙ fields that transform under the flavour group ⇒ complete SM Lagrangian formally invariant under flavour symmetry

$SU(3)_Q \times SU(3)_u \times SU(3)_d$

Spurion analysis

$Q(3,1,1) \quad Y_u(3,\bar{3},1) \quad u(1,3,1)$

$\bar{Q} Y_u u H$

$$Y_u \to \Omega_{Q_u} Y_u \Omega_u^\dagger$$
$$Q \to \Omega_{Q_u} Q_u$$
$$u \to \Omega_u u$$

Flavour symmetry broken when Yukawas get vevs

Minimal Flavour Violation (MFV): only sources of flavour violation = Yukawa couplings

D'Ambrosio, Giudice, Isidori, Strumia '02

ALIGNMENT MODELS

Idea: Have bulk masses and Yukawas diagonal in the same basis (aligned).

Use: MFV + Flavour symmetry $SU(3)_Q \times SU(3)_d$

$$C_Q = \alpha_Q \mathbb{1} + \beta_Q Y_d Y_d^\dagger$$
$$C_d = \alpha_d \mathbb{1} + \beta_d Y_d^\dagger Y_d$$

Fitzpatrick, Perez, Randall '07
Csaki, Perez, Surujon, Weiler '09

Issues:

- no alignment in up sector
- radiative corrections: $\bar{Q} Y_u Y_u^\dagger Y_d d H$ spoils alignment
- Approx. alignment: FCNC suppression less than expected

OUR MODEL

Bulk gauge group: $\mathrm{SU(2)_R \times SU(2)_L \times U(1)_X}$

Idea: split Q_L to two doublets:

$$Q_u(2,2)_{\frac{2}{3}} \quad Q_d(2,2)_{-\frac{1}{3}} \quad u(1,1)_{\frac{2}{3}} \quad d(1,1)_{-\frac{1}{3}} \quad H(2,2)_0$$

flavour group: $SU(3)_{Q_u} \times SU(3)_{Q_d} \times SU(3)_u \times SU(3)_d$

$$\mathcal{L} \supset \overline{Q_u} Y_u^{5D} U H + \overline{Q_d} Y_d^{5D} D H$$

$$\mathcal{L} \supset \overline{Q_u} C_{Q_u} Q_u + \overline{Q_d} C_{Q_d} Q_d + \overline{U} C_u U + \overline{D} C_d D$$

~~MFV~~

Scalar potential $\Rightarrow$ alignment of Ys and Cs

- up and down sector aligned :)
- no CKM matrix :(
- two left-handed doublets :(

Braeuninger, Gedalia, Perez, to appear

OUR MODEL: MIXING ON THE UV BRANE

$$SU(2)_L \times U(1)_Y$$

Q_d, Q_u same quantum numbers $\Rightarrow$ mixing

$$\left(\overline{Q}_u, \overline{Q}_d\right)^i M_{ij} \widetilde{Q}^j = \left(\overline{Q}_u, \overline{Q}_d\right) \begin{pmatrix} M_u \\ M_d \end{pmatrix} \widetilde{Q}$$

3 massive (Planck-scale) + 3 massless quark doublets :)
alignment spoiled :(

To maintain sufficient alignment:
Assume small (O(0.1)) hierarchy between UV masses:

$$M_u = \begin{pmatrix} m_{u_1} & 0 & 0 \\ 0 & m_{u_2} & 0 \\ 0 & 0 & m_{u_3} \end{pmatrix} \qquad M_d = \epsilon \begin{pmatrix} m_{d_1} & 0 & 0 \\ 0 & m_{d_2} & 0 \\ 0 & 0 & m_{d_3} \end{pmatrix} \times V_X$$

FLAVOUR PHYSICS IN OUR MODEL

$$G_{dL}^{(1)} \sim \begin{pmatrix} f_{Q_1}^2 & f_{Q_1} f_{Q_2} \epsilon^2 & f_{Q_1} f_{Q_3} \epsilon^2 \\ f_{Q_1} f_{Q_2} \epsilon^2 & f_{Q_2}^2 & f_{Q_2} f_{Q_3} \epsilon^2 \\ f_{Q_1} f_{Q_3} \epsilon^2 & f_{Q_2} f_{Q_3} \epsilon^2 & f_{Q_3}^2 \end{pmatrix} \qquad G_{uL}^{(1)} \sim \begin{pmatrix} f_{Q_1}^2 & f_{Q_1} f_{Q_2} & f_{Q_1} f_{Q_3} \\ f_{Q_1} f_{Q_2} & f_{Q_2}^2 & f_{Q_2} f_{Q_3} \\ f_{Q_1} f_{Q_3} & f_{Q_2} f_{Q_3} & f_{Q_3}^2 \end{pmatrix}$$

$$G_{dR}^{(1)} \sim \begin{pmatrix} f_{d_1}^2 & f_{d_1} f_{d_2} \epsilon^2 & f_{d_1} f_{d_3} \epsilon^2 \\ f_{d_1} f_{d_2} \epsilon^2 & f_{d_2}^2 & f_{d_2} f_{d_3} \epsilon^2 \\ f_{d_1} f_{d_3} \epsilon^2 & f_{d_2} f_{d_3} \epsilon^2 & f_{d_3}^2 \end{pmatrix} \qquad G_{uR}^{(1)} \sim \begin{pmatrix} f_{u_1}^2 & f_{u_1} f_{u_2} & f_{u_1} f_{u_3} \\ f_{u_1} f_{u_2} & f_{u_2}^2 & f_{u_2} f_{u_3} \\ f_{u_1} f_{u_3} & f_{u_2} f_{u_3} & f_{u_3}^2 \end{pmatrix}$$

Additional suppression in down sector

RS GIM in up sector

$$V_{ij}^{\text{CKM}} \sim \frac{f_{Q_i}}{f_{Q_j}}$$ ✓

Stable under radiative corrections: Higher dim. operators cannot feed Y_u into Y_d due to flavour symmetry

CONCLUSIONS

- Warped extra dimensions: elegant way to solve hierarchy problem
- KK tower of heavy particles
- Fermions in the bulk:
  - no proton decay, fermion puzzle solved :)
  - FCNCs due to KK gluon exchange :(
    - alleviated by RS GIM $m_{KK} \sim 10$ TeV
    - alignment models $m_{KK} \sim 3$ TeV

BACK-UP SLIDES

OUR MODEL: MIXING ON THE UV BRANE

$$SU(2)_L \times U(1)_Y$$

Q_d, Q_u same quantum numbers $\Rightarrow$ mixing

$$\left(\overline{Q}_u, \overline{Q}_d\right)^i M_{ij} \widetilde{Q}^j = \left(\overline{Q}_u, \overline{Q}_d\right) \begin{pmatrix} M_u \\ M_d \end{pmatrix} \widetilde{Q}$$

$$V M V_R^\dagger = \text{diag.}(\ldots)$$

$$\begin{pmatrix} Q_L \\ Q_H \end{pmatrix} \equiv V \cdot \begin{pmatrix} Q_u \\ Q_d \end{pmatrix} \equiv \begin{pmatrix} A_u & A_d \\ B_u & B_d \end{pmatrix} \begin{pmatrix} Q_u \\ Q_d \end{pmatrix}$$

3 massive (Planck-scale) + 3 massless quark doublets :)

$$Q_{u,d} = A^\dagger_{u,d} Q_L + \cancel{B^\dagger_{u,d} Q_H}$$

$$\Rightarrow \overline{Q_L} \left(A_u C_{Q_u} A_u^\dagger + A_d C_{Q_d} A_d^\dagger \right) Q_L \equiv \overline{Q_L} C_Q Q_L$$

diagonalized by: $Q_L \to U_C Q_L$

OUR MODEL: MAINTAINING SUFFICIENT ALIGNMENT

To maintain sufficient alignment:

Assume small (O(0.1)) hierarchy between UV masses:

$$M_u = \begin{pmatrix} m_{u_1} & 0 & 0 \\ 0 & m_{u_2} & 0 \\ 0 & 0 & m_{u_3} \end{pmatrix} \qquad M_d = \epsilon \begin{pmatrix} m_{d_1} & 0 & 0 \\ 0 & m_{d_2} & 0 \\ 0 & 0 & m_{d_3} \end{pmatrix} \times V_X$$

$$A_d \sim \begin{pmatrix} 1 & \epsilon^2 & \epsilon^2 \\ 0 & 1 & \epsilon^2 \\ 0 & 0 & 1 \end{pmatrix} \qquad A_u \sim \begin{pmatrix} \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon \end{pmatrix}$$

$$C_Q = (\underbrace{A_u C_{Q_u} A_u^\dagger}_{\mathcal{O}(\epsilon^2)} + \underbrace{A_d C_{Q_d} A_d^\dagger}) \qquad \Rightarrow U_C \sim \begin{pmatrix} 1 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 1 & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}$$

diagonal: $\mathcal{O}(1) \times C_{Q_d}$

off-diagonal: $\mathcal{O}(\epsilon^2)$

OUR MODEL: YUKAWAS & CKM

quark wave-functions on IR brane

$$U_C \sim \begin{pmatrix} 1 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 1 & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix} \quad A_u \sim \begin{pmatrix} \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon \end{pmatrix}$$

quark wave-functions on IR brane

$$Y_u^{4D} = F_Q^\dagger U_C A_u Y_u^{5D} F_u$$

$$Y_d^{4D} = F_Q^\dagger U_C A_d Y_d^{5D} F_d$$

$$A_d \sim \begin{pmatrix} 1 & \epsilon^2 & \epsilon^2 \\ 0 & 1 & \epsilon^2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(Y_u^{4D})_{\text{mass}} = V_{uL} F_Q^\dagger U_C A_u Y_u^{5D} F_u V_{uR}^\dagger$$

all entries $\propto \epsilon$

$$(V_{uL})_{ij} \sim f_{Q^i}/f_{Q^j} \ (i<j)$$

$$(V_{uR})_{ij} \sim f_{u_i}/f_{u_j} \ (i<j)$$

$$(Y_d^{4D})_{\text{mass}} = V_{dL} F_Q^\dagger U_C A_d Y_d^{5D} F_d V_{dR}^\dagger$$

diagonal: unsuppressed,
off-diagonal: $\propto \epsilon^2$

$$(V_{dL})_{ij}^{\text{off-diag}} \sim \epsilon^2 f_{Q^i}/f_{Q^j}$$

$$(V_{dR})_{ij}^{\text{off-diag}} \sim \epsilon^2 f_{u_i}/f_{u_j}$$

$$V_{\text{CKM}} = V_{uL} V_{dL}^\dagger \quad V_{ij}^{\text{CKM}} \sim f_{Q_i}/f_{Q_j} \ (i<j)$$

OUR MODEL: KK GLUON COUPLINGS: LH QUARKS

$$G^{(1)}_{uL} = g_{s*} V_{uL} F^\dagger_Q U_C (A_d A^\dagger_d + A_u A^\dagger_u) U^\dagger_C F_Q V^\dagger_{uL}$$

$\sim$ unit matrix $+\,\epsilon^2$ corrections

$\sim f_{Q^i} f_{Q^j}$ with ϵ^2 suppression for off-diagonal elements

$(V_{uL})_{ij} \sim f_{Q^i}/f_{Q^j}$

$$G^{(1)}_{uL} \sim \begin{pmatrix} f^2_{Q_1} & f_{Q_1} f_{Q_2} & f_{Q_1} f_{Q_3} \\ f_{Q_1} f_{Q_2} & f^2_{Q_2} & f_{Q_2} f_{Q_3} \\ f_{Q_1} f_{Q_3} & f_{Q_2} f_{Q_3} & f^2_{Q_3} \end{pmatrix}$$

RS GIM in up sector

$$G^{(1)}_{dL} = g_{s*} V_{dL} F^\dagger_Q U_C (A_d A^\dagger_d + A_u A^\dagger_u) U^\dagger_C F_Q V^\dagger_{dL}$$

$\sim$ unit matrix $+\,\epsilon^2$ corrections

$\sim f_{Q^i} f_{Q^j}$ with ϵ^2 suppression for off-diagonal elements

$(V_{dL})_{ij} \sim \epsilon^2 f_{Q^i}/f_{Q^j}$

$$G^{(1)}_{dL} \sim \begin{pmatrix} f^2_{Q_1} & f_{Q_1} f_{Q_2} \epsilon^2 & f_{Q_1} f_{Q_3} \epsilon^2 \\ f_{Q_1} f_{Q_2} \epsilon^2 & f^2_{Q_2} & f_{Q_2} f_{Q_3} \epsilon^2 \\ f_{Q_1} f_{Q_3} \epsilon^2 & f_{Q_2} f_{Q_3} \epsilon^2 & f^2_{Q_3} \end{pmatrix}$$

Additional suppression in down sector

OUR MODEL: KK GLUON COUPLINGS: RH QUARKS

$$G^{(1)}_{uR} = g_{s*} V_{uR} F_u^\dagger F_u V_{uR}^\dagger$$

$$G^{(1)}_{uR} \sim \begin{pmatrix} f_{u_1}^2 & f_{u_1} f_{u_2} & f_{u_1} f_{u_3} \\ f_{u_1} f_{u_2} & f_{u_2}^2 & f_{u_2} f_{u_3} \\ f_{u_1} f_{u_3} & f_{u_2} f_{u_3} & f_{u_3}^2 \end{pmatrix}$$

RS GIM in up sector

$$G^{(1)}_{dR} = g_{s*} V_{dR} F_d^\dagger F_d V_{dR}^\dagger$$

$$G^{(1)}_{dR} \sim \begin{pmatrix} f_{d_1}^2 & f_{d_1} f_{d_2} \epsilon^2 & f_{d_1} f_{d_3} \epsilon^2 \\ f_{d_1} f_{d_2} \epsilon^2 & f_{d_2}^2 & f_{d_2} f_{d_3} \epsilon^2 \\ f_{d_1} f_{d_3} \epsilon^2 & f_{d_2} f_{d_3} \epsilon^2 & f_{d_3}^2 \end{pmatrix}$$

Additional suppression in down sector

Stable under radiative corrections:
Higher dim. operators cannot feed Y_u into Y_d due to flavour symmetry