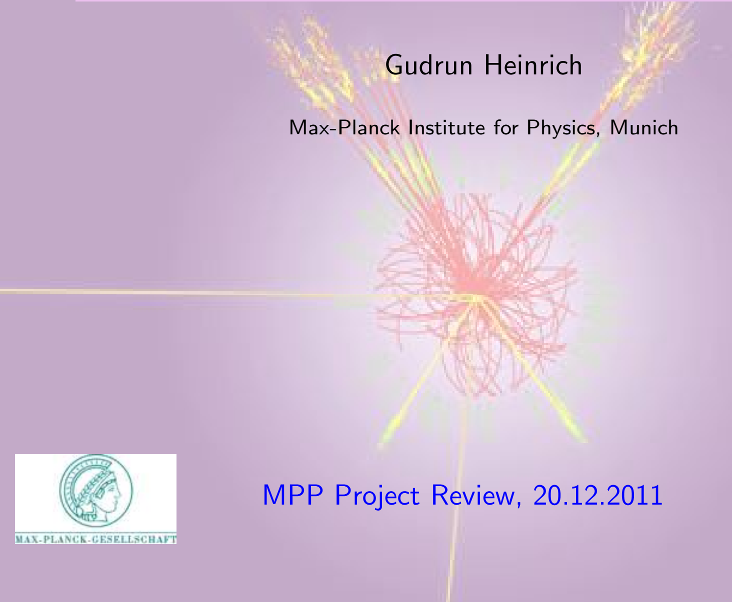
Collider Physics and Higher Order Corrections

Gudrun Heinrich

Max-Planck Institute for Physics, Munich



MPP Project Review, 20.12.2011

Outline

Outline

1. Motivation
2. Automation of one-loop calculations
    - FeynArts/FormCalc/LoopTools
      T. Hahn, E. Mirabella
    - GoSam
      N. Greiner, GH, P. Mastrolia, T. Reiter et al.,
3. Beyond one loop
    - calculation of multi-scale two-loop integrals
      S. Borowka, GH, J. Carter (IPPP Durham)
    - FeynHiggs T. Hahn, S. Heinemeyer, W. Hollik et al.
4. Summary

not covered in this talk:

one- and two-loop corrections in electroweak sector and MSSM
Hahn, Hollik, Landwehr, Lindert, Mirabella, Pagani, Passehr, Sturm, Weber

cut-based reduction of two-loop scattering amplitudes
Mastrolia, Ossola, Peraro, van Deurzen

Automation of one-loop multi-leg calculations

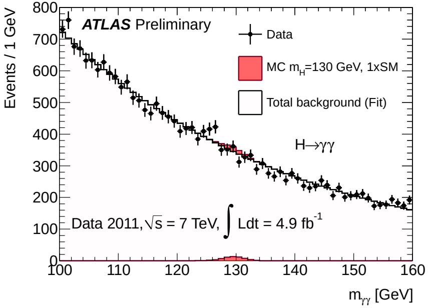
members based at MPP:

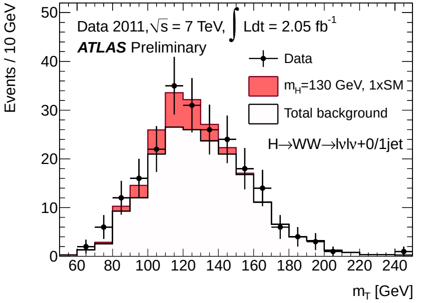
Nicolas Greiner
Pierpaolo Mastrolia (+ Tiziano Peraro, Hans van Deurzen)
Thomas Reiter
Johann Felix von Soden-Fraunhofen
GH

plus

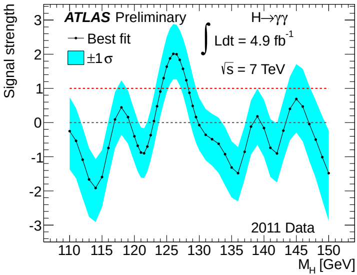
Gavin Cullen (Edinburgh → Desy Zeuthen)
Gionata Luisoni (IPPP Durham)
Giovanni Ossola (City University New York)
Mark Rodgers (IPPP Durham)
Francesco Tramontano (CERN)

Motivation: Higgs or Hype ?



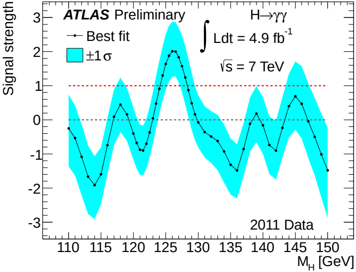


Higgs $\to \gamma\gamma$



background: from data and ...

Higgs → $\gamma\gamma$



background: from data and . . .

. . . theory predictions (NLO), e.g. diphox

The PHOX Family

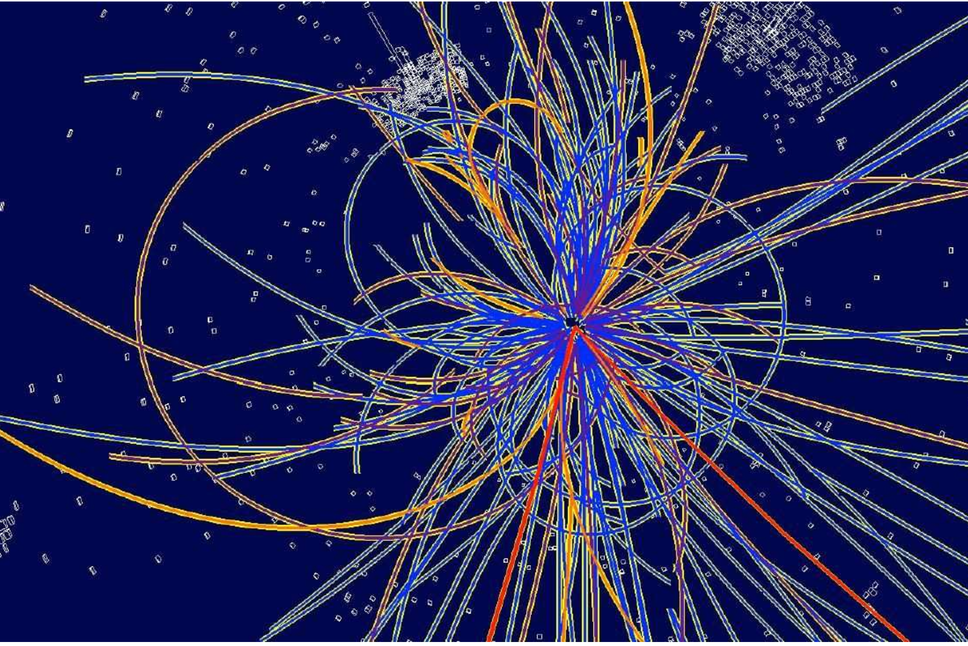
NLO Monte Carlo programs (partonic event generators) to calculate cross sections for the production of large-p_T photons

http://lappth.in2p3.fr/PHOX_FAMILY/main.html

F. Arleo, P. Aurenche, T. Binoth, M. Fontannaz, J.Ph. Guillet, GH, E. Pilon, M. Werlen

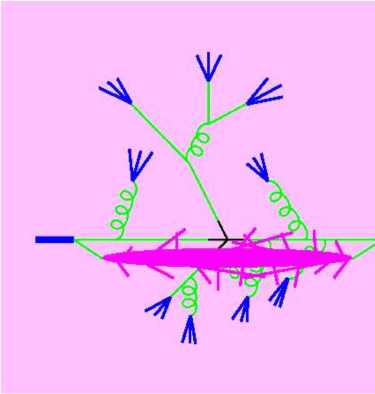
- DIPHOX
  $h_1\, h_2 \to \gamma\ \gamma\ + X$, $h_1\, h_2 \to \gamma\ h_3\ + X$, $h_1\, h_2 \to h_3\ h_4\ + X$
- JETPHOX
  $h_1\, h_2 \to \gamma\ \text{jet}\ + X$, $h_1\, h_2 \to \gamma\ + X$
  $h_1\, h_2 \to h_3\ \text{jet}\ + X$, $h_1\, h_2 \to h_3 + X$
- EPHOX
  $\gamma\, p \to \gamma\ \text{jet}\ + X$, $\gamma\, p \to \gamma\ + X$
  $\gamma\, p \to h\ \text{jet}\ + X$, $\gamma\, p \to h + X$
- TWINPHOX
  $\gamma\,\gamma \to \gamma\ \text{jet}\ + X$, $\gamma\,\gamma \to \gamma\ + X$

Hadron collider events



Generic event

1. hard interaction
   $\hat{\sigma} = \alpha_s^k \, \hat{\sigma}^{\mathrm{LO}} + \alpha_s^{k+1} \, \hat{\sigma}^{\mathrm{NLO}} + \ldots$
   calculable order by order
   in perturbation theory
2. parton shower
   soft and collinear branching,
   treatment within perturbative
   QCD framework
3. hadronization
   non-perturbative models,
   fits to data
4. (underlying event)



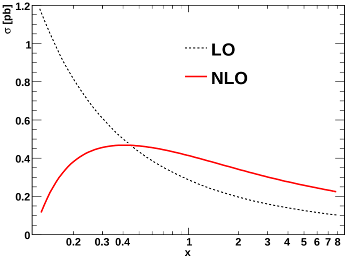
Perturbative expansion

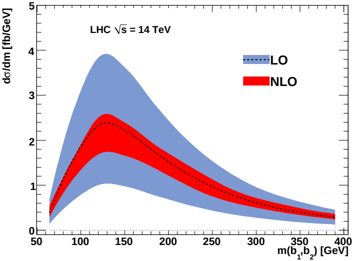
$$\hat{\sigma} = \alpha_s^k(\mu)\,\left[\,\hat{\sigma}^{\rm LO} + \alpha_s(\mu)\,\hat{\sigma}^{\rm NLO}(\mu) + \alpha_s^2(\mu)\,\hat{\sigma}^{\rm NNLO}(\mu) + \ldots\,\right]$$

μ-dependence comes from truncation of perturbative series

truncation at LO:

$\Rightarrow$ large renormalisation/factorisation scale dependence





example $pp \to b\bar{b}b\bar{b}$ to NLO

Binoth, Greiner, Guffanti, Guillet, Reiter, Reuter '09, '11 (PRL)

multi-particle final states

- to establish signals of New Physics
- to measure model parameters

  Leading Order is not sufficient !

- at LHC: typically multi-particle final states

  $\Rightarrow$ calculations of higher orders increasingly difficult

multi-particle final states

- to establish signals of New Physics
- to measure model parameters

  Leading Order is not sufficient !

- at LHC: typically multi-particle final states

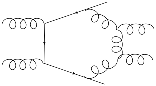
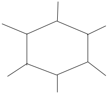
  $\Rightarrow$ calculations of higher orders increasingly difficult

- example for time scale to add one parton:
  $pp \to 2$ jets at NLO (4-point process): Ellis/Sexton 1986
  $pp \to 3$ jets at NLO (5-point): Bern et al, Kunszt et al '93-95
  $pp \to 4$ jets at NLO (6-point): 19.12.2011
  Bern, Diana, Dixon, Febres Cordero, Höche, Kosower, Ita, Maitre, Ozeren

One-loop methods

basically two categories:

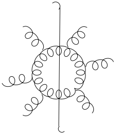
- methods based on Feynman diagrams: tensor reduction

 →  $+$ integrals with less legs

non-trivial tensor structure

scalar 6-point function

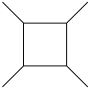
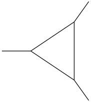
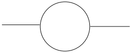
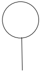
- "unitarity cuts"

$\mathcal{A} = \sum_{\text{cuts}} \int d\,PS$  $+ \ \mathcal{R}$

common to both:

reduction to set of basis integrals (4-,. . ., 1-point functions)

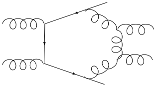
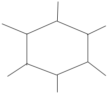
(known analytically)

$\mathcal{A} = C_4$  $+ C_3$  $+ C_2$  $+ C_1$  $+ \mathcal{R}$

One-loop methods

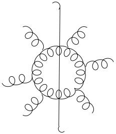
basically two categories:

- methods based on Feynman diagrams: tensor reduction

 $\longrightarrow$  $+$ integrals with less legs

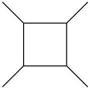
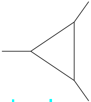
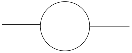
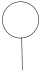
non-trivial tensor structure — scalar 6-point function

- "unitarity cuts"

$$\mathcal{A} = \sum_{\mathrm{cuts}} \int d\, PS$$  $+ \quad \mathcal{R}$

common to both:

reduction to set of basis integrals (4-,. . ., 1-point functions)
(known analytically)

$\mathcal{A} = \ C_4$  $+\ C_3$  $+\ C_2$  $+\ C_1$  $+\ \mathcal{R}$

enormous progress in the last couple of years

Automated NLO Tools

One-loop automation

- FeynArts/FormCalc/LoopTools (public) Thomas Hahn et al
- GRACE Fujimoto et al.
- MadLoop/ aMC@NLO Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau
  uses CutTools (public) Ossola, Papadopoulos, Pittau and MadFKS
- Helac-NLO (public) Bevilacqua, Czakon, van Hameren, Papadopoulos, Pittau, Worek
- GoSam (public) Cullen, Greiner, GH, Luisoni, Mastrolia, Ossola, Reiter, Tramontano
  uses Samurai (public) Mastrolia, Ossola, Reiter, Tramontano and
  golem95 (public) Binoth, Cullen, Guillet, GH, Kleinschmidt, Pilon, Reiter, Rodgers

Automated NLO Tools

One-loop automation

- FeynArts/FormCalc/LoopTools (public) Thomas Hahn et al
- GRACE Fujimoto et al.
- MadLoop/ aMC@NLO Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau
  uses CutTools (public) Ossola, Papadopoulos, Pittau and MadFKS
- Helac-NLO (public) Bevilacqua, Czakon, van Hameren, Papadopoulos, Pittau, Worek
- GoSam (public) Cullen, Greiner, GH, Luisoni, Mastrolia, Ossola, Reiter, Tramontano
  uses Samurai (public) Mastrolia, Ossola, Reiter, Tramontano and
  golem95 (public) Binoth, Cullen, Guillet, GH, Kleinschmidt, Pilon, Reiter, Rodgers

automation of subtraction for IR divergent real radiation

- MadDipole Frederix, Greiner, Gehrmann 08
- TevJet Seymour Tevlin 08
- AutoDipole Hasegawa, Moch, Uwer 08,09
- Helac-Phegas Czakon, Papadopoulos, Worek 09; polarized
- MadFKS Frederix, Frixione, Maltoni, Stelzer 09

Golem-Samurai (GoSam)

General One-Loop Evaluator of Matrix elements &

Scattering Amplitudes from Unitarity based Reduction At Integrand level

[Cullen, Greiner, GH, Luisoni, Mastrolia, Ossola, Reiter, Tramontano]

http://projects.hepforge.org/gosam/

arXiv: 1111.6534 [hep-ph]

Golem-Samurai (GoSam)

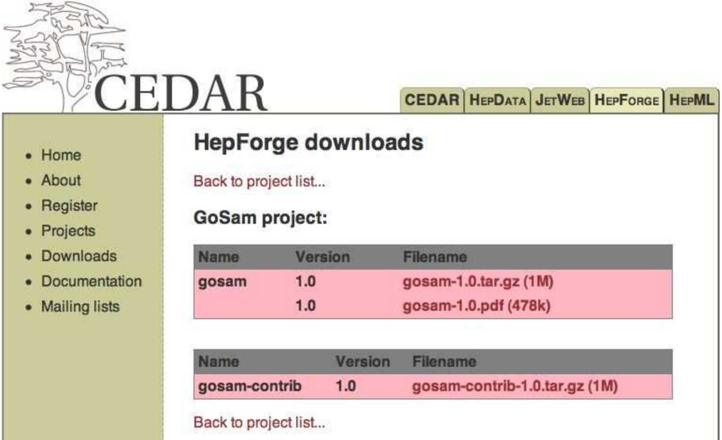
General One-Loop Evaluator of Matrix elements &

Scattering Amplitudes from Unitarity based Reduction At Integrand level

[Cullen, Greiner, GH, Luisoni, Mastrolia, Ossola, Reiter, Tramontano]

http://projects.hepforge.org/gosam/

arXiv: 1111.6534 [hep-ph]



Golem-Samurai (GoSam)

General One-Loop Evaluator of Matrix elements &

Scattering Amplitudes from Unitarity based Reduction At Integrand level

[Cullen, Greiner, GH, Luisoni, Mastrolia, Ossola, Reiter, Tramontano]

http://projects.hepforge.org/gosam/

arXiv: 1111.6534 [hep-ph]

- algebraic generation of D-dimensional integrands based on Feynman diagrams
  - QCD, EW, BSM

Golem-Samurai (GoSam)

General One-Loop Evaluator of Matrix elements &

Scattering Amplitudes from Unitarity based Reduction At Integrand level

[Cullen, Greiner, GH, Luisoni, Mastrolia, Ossola, Reiter, Tramontano]

http://projects.hepforge.org/gosam/

arXiv: 1111.6534 [hep-ph]

- algebraic generation of D-dimensional integrands based on Feynman diagrams
  - QCD, EW, BSM
- reduction by D-dimensional extension of cut-based method options:
  - Samurai for reduction Mastrolia, Ossola, Reiter, Tramontano '10
  - traditional tensor reduction (using golem95 library)
  - tensorial reduction at integrand level GH, Ossola, Reiter, Tramontano '10

Golem-Samurai (GoSam)

General One-Loop Evaluator of Matrix elements &

Scattering Amplitudes from Unitarity based Reduction At Integrand level

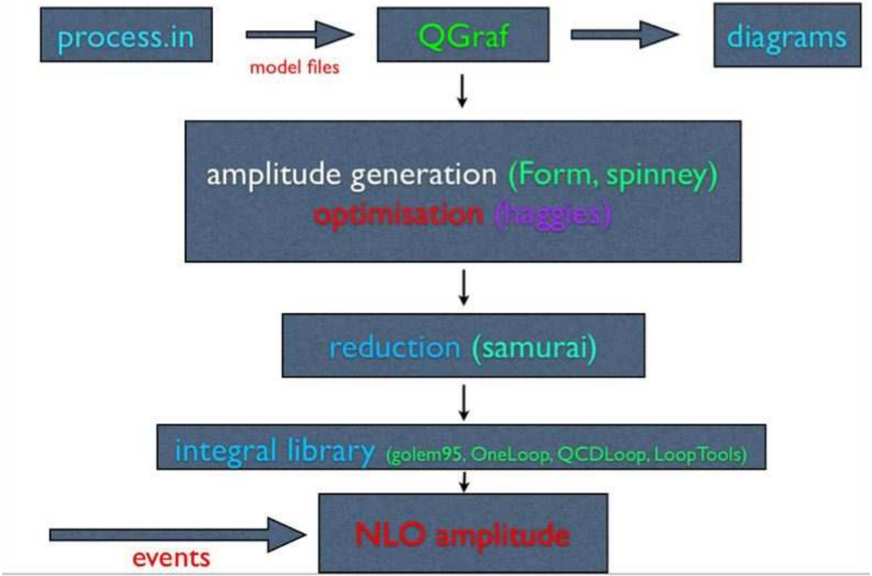
[Cullen, Greiner, GH, Luisoni, Mastrolia, Ossola, Reiter, Tramontano]

http://projects.hepforge.org/gosam/

arXiv: 1111.6534 [hep-ph]

- algebraic generation of D-dimensional integrands based on Feynman diagrams
  - QCD, EW, BSM
- reduction by D-dimensional extension of cut-based method options:
  - Samurai for reduction Mastrolia, Ossola, Reiter, Tramontano '10
  - traditional tensor reduction (using golem95 library)
  - tensorial reduction at integrand level GH, Ossola, Reiter, Tramontano '10
- interface with existing tools for real radiation (MadGraph/MadEvent, Sherpa, Powheg, . . .)

GoSam structure



GoSam

usage:

- edit "input card"

```
in= u,d~
out= nmu, mu+, e-, ne~, s~, c
model=smdiag
```

models can be added via **FeynRules** (Duhr) or **LanHEP** (Semenov)

```
order=gw,4,4; order=gs,2,4
zero=mB,mC,mS,mU,mD,me,mmu
one=gs,e
helicities=-+-+-++-
extensions=samurai, dred
```

- gosam.py process.in

usage:

- edit "input card"

```
in= u,d~
out= nmu, mu+, e-, ne~, s~, c
model=smdiag
    models can be added via FeynRules (Duhr) or LanHEP (Semenov)
order=gw,4,4; order=gs,2,4
zero=mB,mC,mS,mU,mD,me,mmu
one=gs,e
helicities=-+-+-++-
extensions=samurai, dred
```

- gosam.py process.in
- make doc ⇒ documentation and diagram pictures

GoSam

usage:

- edit "input card"

```
in= u,d~
out= nmu, mu+, e-, ne~, s~, c
model=smdiag
    models can be added via FeynRules (Duhr) or LanHEP (Semenov)
order=gw,4,4; order=gs,2,4
zero=mB,mC,mS,mU,mD,me,mmu
one=gs,e
helicities=-+-+-++-
extensions=samurai, dred
```

- gosam.py process.in
- make doc ⇒ documentation and diagram pictures
- make source ⇒ source files

GoSam

usage:

- edit "input card"

```
in= u,d~
out= nmu, mu+, e-, ne~, s~, c
model=smdiag
    models can be added via FeynRules (Duhr) or LanHEP (Semenov)
order=gw,4,4; order=gs,2,4
zero=mB,mC,mS,mU,mD,me,mmu
one=gs,e
helicities=-+-+-++-
extensions=samurai, dred
```

- gosam.py process.in
- make doc ⇒ documentation and diagram pictures
- make source ⇒ source files
- make compile ⇒ fully compiled code

Example $u\,\bar{d} \to W^- W^+ \,\bar{s}c \to e^-\bar{\nu}_e \mu^+ \nu_\mu \,\bar{s}c$

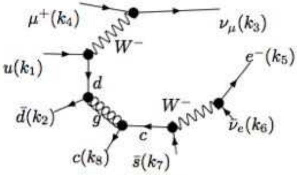


Diagram 1

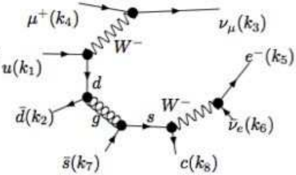


Diagram 2

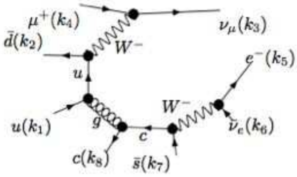


Diagram 3

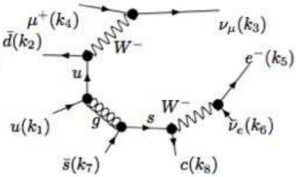


Diagram 4

5 One-Loop Diagrams

General Information

QGraf Setup

Example $u\,\bar{d} \to W^- W^+ \bar{s} c \to e^- \bar{\nu}_e \mu^+ \nu_\mu \bar{s} c$

NLO sample diagrams

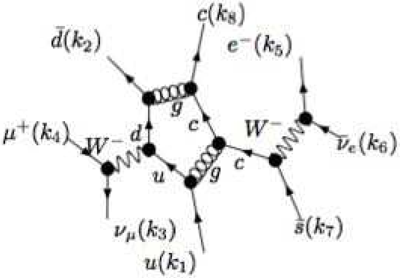


Diagram 8

$S' = S^{\{4\}}_{Q\to -q-(k3-k2+k4)}$, rk = 3

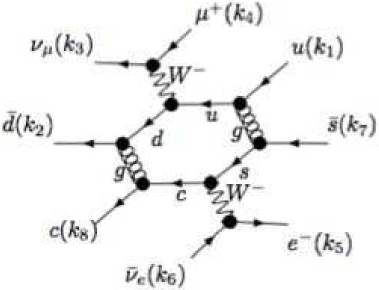
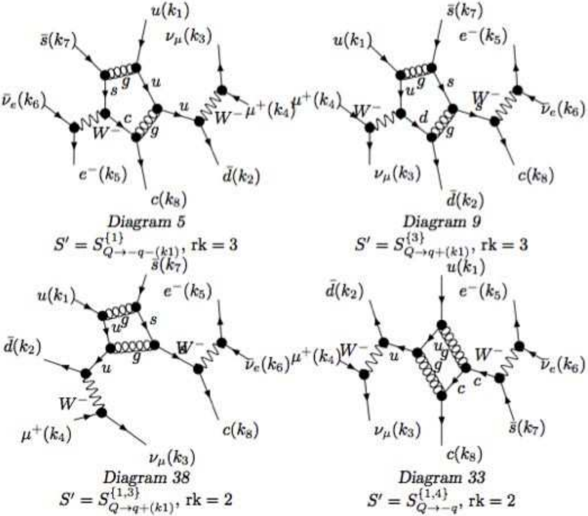


Diagram 1

$S' = S_{Q\to q+(k1)}$, rk = 4

Example $u\,\bar{d} \to W^- W^+ \,\bar{s}c \to e^- \bar{\nu}_e \mu^+ \nu_\mu \,\bar{s}c$



Example $u\,\bar{d} \to W^- W^+ \,\bar{s} c \to e^- \bar{\nu}_e \mu^+ \nu_\mu \,\bar{s} c$

code generation:

```
Form is processing loop diagram 80 @ Helicity 0
  2.72 sec out of 2.74 sec
Haggies is processing abbreviations for loop diagram 80 @ Helicity 0
Form is processing loop diagram 81 @ Helicity 0
  0.71 sec out of 0.73 sec
Haggies is processing abbreviations for loop diagram 81 @ Helicity 0
Form is processing loop diagram 82 @ Helicity 0
  0.73 sec out of 0.75 sec
Haggies is processing abbreviations for loop diagram 82 @ Helicity 0
Form is processing loop diagram 83 @ Helicity 0
  0.70 sec out of 0.71 sec
Haggies is processing abbreviations for loop diagram 83 @ Helicity 0
Form is processing loop diagram 84 @ Helicity 0
  0.73 sec out of 0.73 sec
Haggies is processing abbreviations for loop diagram 84 @ Helicity 0
Form is processing loop diagram 85 @ Helicity 0
```

Example $u\,\bar{d} \to W^- W^+\,\bar{s}c \to e^-\bar{\nu}_e\mu^+\nu_\mu\,\bar{s}c$

```
=====================================================
                 GoSam-1.0
=====================================================

#    NLO/LO, finite part:  -15.9157511871461 2
#    NLO/LO, single pole:    7.587050495888512
#    NLO/LO, double pole:  -5.333333333333234

 CPU time (secs):    1.2997999999999991E-002
```

result compared with
Melia, Melnikov, Rontsch, Zanderighi (MMRZ) 1104.2327 [hep-ph]

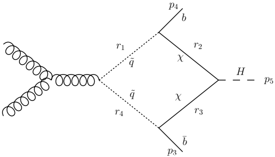
NLO/LO	GoSam	MMRZ
$1/\epsilon^2$	-5.333333333	-5.333333
$1/\epsilon$	7.5870504959	7.587051
finite	-15.915751119	-15.91575

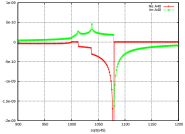
Tested 5- or 6-point processes

- $pp \to t\bar{t}\,H$
- $pp \to b\bar{b}b\bar{b}$
- $pp \to W^+W^-\,b\bar{b}$
- $gg, u\bar{u} \to t\bar{t}b\bar{b}$
- $pp \to W^+W^+\,jj \to e^+\nu_e\mu^+\nu_\mu\,jj$
- $pp \to W^+W^-\,jj \to e^-\bar{\nu}_e\mu^+\nu_\mu\,jj$
- $u\,\bar{d} \to W^+\,s\bar{s} \to e^+\nu_e\,s\bar{s}$
- $u\,\bar{d} \to W^+\,gg \to e^+\nu_e gg$
- $d\,\bar{d} \to Z\,gg \to e^+e^-\,gg$
- $u\,\bar{d} \to W^+\,b\bar{b} \to e^+\nu_e\,b\bar{b}$ also with massive b's
- $u\,\bar{d} \to W^+\,g\,g\,g$
- $e^+e^- \to e^+e^-\,\gamma$
- $\gamma\gamma \to \gamma\gamma\gamma\gamma$
- $u\,\bar{d} \to W^+\,g \to e^+\nu_e\,g$ EW corrections
- plus a large number of $2 \to 2$ processes

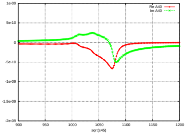
golem95 integral library

Example: production of a heavy neutral MSSM Higgs and a $b\bar{b}$ pair with unstable particles (squarks, neutralinos) in the loop





real masses



complex masses

contained in golem95C library: 1101.5595 [hep-ph]

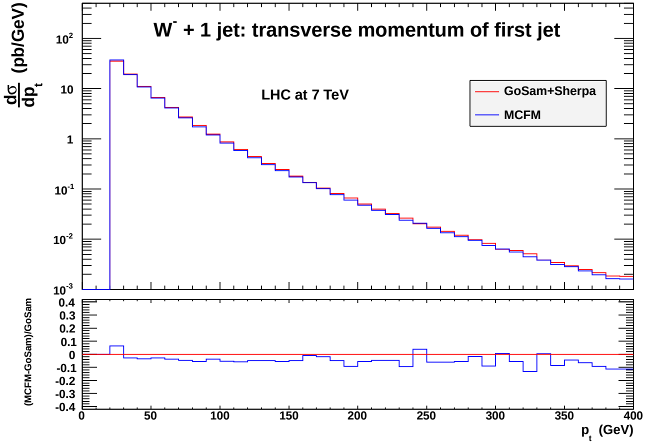
Binoth, Cullen, Guillet, GH, Kleinschmidt, Pilon, Reiter, Rodgers

http://projects.hepforge.org/~golem/95/

currently being extended to integrals with rank $>$ number of propagators (needed e.g. in models involving gravitons) by Johann Felix von Soden-Fraunhofen

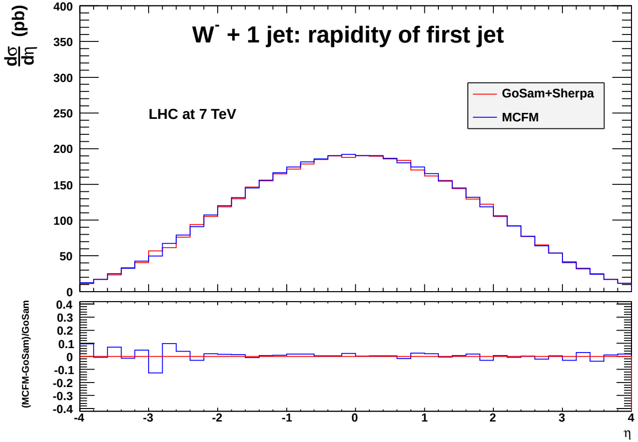
Interface

- standard interface to real radiation programs (Binoth Les Houches Accord) implemented
- tested with Sherpa and Powheg
- example $pp \to W$+jet [figures by G. Luisoni]



Interface

- standard interface to real radiation programs (Binoth Les Houches Accord) implemented
- tested with Sherpa and Powheg
- example $pp \to W$+jet [figures by G. Luisoni]



Example MSSM: $pp \to \chi_1^0 \chi_1^0$

NLO SUSY-QCD corrections

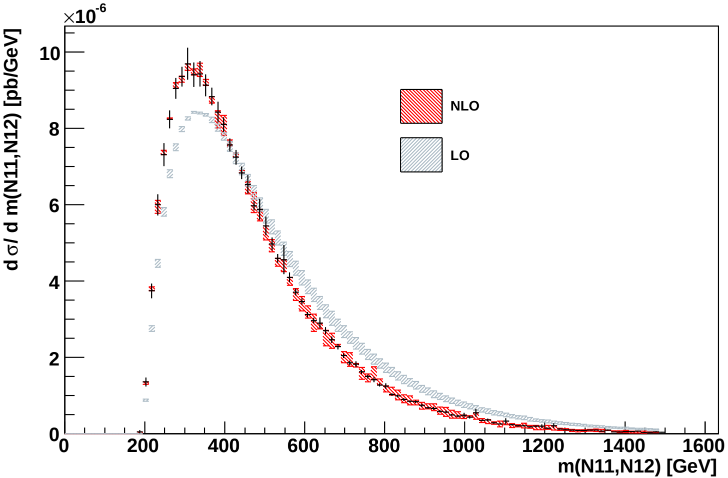
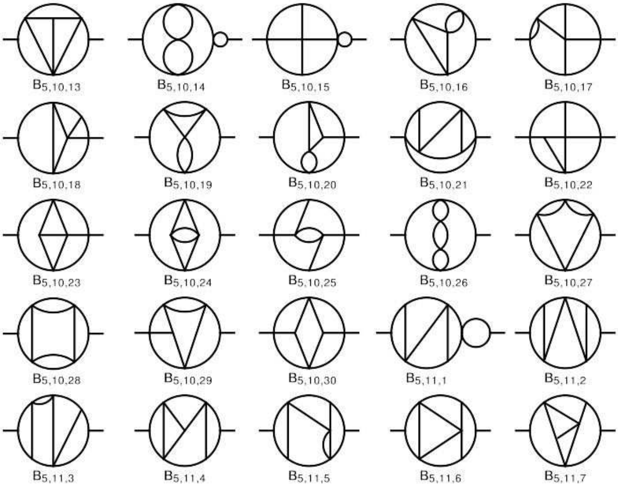


figure by G. Cullen, N. Greiner

Beyond One Loop

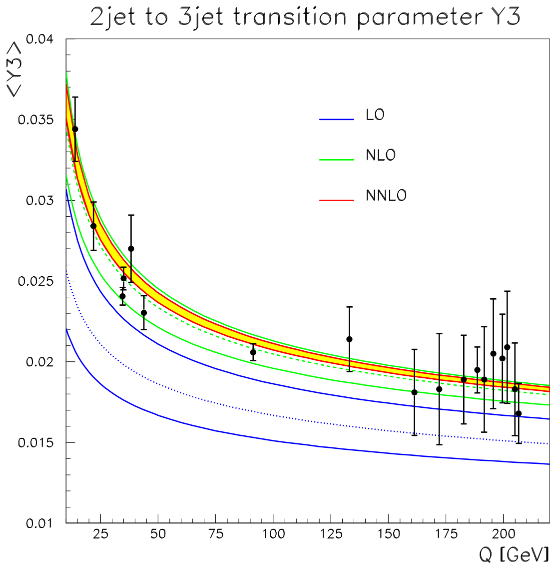


NNLO

- full NNLO cross sections:
  - e^+e^-: partonic event generator program EERAD3 for 3-jet observables in e^+e^- annihilation

    [A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, GH '07]

    [S. Weinzierl '08/'09]



example:

3-jet observable

in e^+e^- annihilation

[A. Gehrmann-De Ridder,
T. Gehrmann, N. Glover, GH '09]

uncertainty bands:

$M_Z/2 < \mu < 2\,M_Z$

NNLO

- full NNLO cross sections:
  - e^+e^-: partonic event generator program EERAD3 for 3-jet observables in e^+e^- annihilation

    [A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, GH '07]

    [S. Weinzierl '08/'09]
  - hadronic collisions:
    - one colour-neutral final state particle (W/Z, Higgs)

      Anastasiou, Dixon, Melnikov, Petriello; Grazzini, Catani, DeFlorian, Cieri, Ferrera
    - $t\bar{t}$, W^+W^-, $\gamma\gamma$, V+jet, dijet under construction

NNLO

- full NNLO cross sections:
  - e^+e^-: partonic event generator program EERAD3 for 3-jet observables in e^+e^- annihilation

    [A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, GH '07]

    [S. Weinzierl '08/'09]
  - hadronic collisions:
    - one colour-neutral final state particle (W/Z, Higgs)

      Anastasiou, Dixon, Melnikov, Petriello; Grazzini, Catani, DeFlorian, Cieri, Ferrera
    - $t\bar{t}$, W^+W^-, $\gamma\gamma$, V+jet, dijet under construction
- different methods, a promising one is based on

  sector decomposition

  [Binoth, GH '00-'11, Anastasiou et al '03-'11, Czakon '10/'11]

Sector Decomposition

- allows to extract UV and IR singularities from (dimensionally regulated) parameter integrals in an automated way
- produces a Laurent series in ϵ
- coefficients are finite parameter integrals
  $\Rightarrow$ integrate numerically
- can be applied to multi-loop integrals and phase space integrals

Sector Decomposition

public programs:

- sector_decomposition (uses `Ginac`) Bogner, Weinzierl '07
- FIESTA (uses `Mathematica`) A. Smirnov, V.Smirnov,M. Tentyukov '08
- SecDec (uses `Mathematica` and Fortran/C) Jon Carter, GH '10

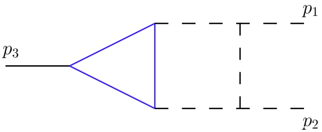
http://projects.hepforge.org/secdec

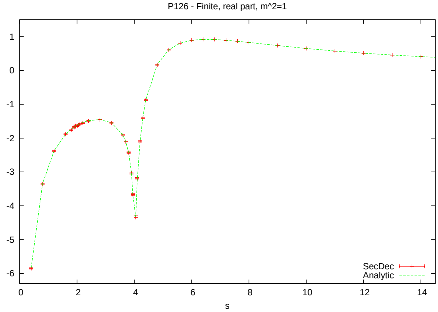
limitation until recently:
multi-scale integrals limited to Euclidean region
(e.g. no thresholds)

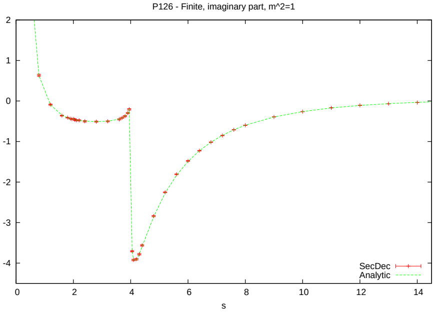
extension of SecDec to general kinematics under construction

method: integration in complex plane, automated contour deformation in multi-dimensional integration parameter space
S. Borowka, J. Carter, GH

2-loop example with threshold







S. Borowka, J. Carter, GH

Summary

- automation is necessary to have NLO predictions as a standard at LHC

Summary

- automation is necessary to have NLO predictions as a standard at LHC
- at MPI powerful and flexible automated tools for one-loop amplitudes have been developed:
  FeynArts/FormCalc/LoopTools and GoSam

Summary

- automation is necessary to have NLO predictions as a standard at LHC
- at MPI powerful and flexible automated tools for one-loop amplitudes have been developed:
  FeynArts/FormCalc/LoopTools and GoSam
- GoSam: very powerful for multi-leg amplitudes

Summary

- automation is necessary to have NLO predictions as a standard at LHC
- at MPI powerful and flexible automated tools for one-loop amplitudes have been developed: FeynArts/FormCalc/LoopTools and GoSam
- GoSam: very powerful for multi-leg amplitudes
- standard interface to real radiation programs (BLHA)

Summary

- automation is necessary to have NLO predictions as a standard at LHC
- at MPI powerful and flexible automated tools for one-loop amplitudes have been developed: FeynArts/FormCalc/LoopTools and GoSam
- GoSam: very powerful for multi-leg amplitudes
- standard interface to real radiation programs (BLHA)
- not limited to Standard Model

Summary

- automation is necessary to have NLO predictions as a standard at LHC
- at MPI powerful and flexible automated tools for one-loop amplitudes have been developed: FeynArts/FormCalc/LoopTools and GoSam
- GoSam: very powerful for multi-leg amplitudes
- standard interface to real radiation programs (BLHA)
- not limited to Standard Model
- GoSam is publicly available at

http://projects.hepforge.org/gosam/

Summary

- automation is necessary to have NLO predictions as a standard at LHC
- at MPI powerful and flexible automated tools for one-loop amplitudes have been developed: FeynArts/FormCalc/LoopTools and GoSam
- GoSam: very powerful for multi-leg amplitudes
- standard interface to real radiation programs (BLHA)
- not limited to Standard Model
- GoSam is publicly available at

http://projects.hepforge.org/gosam/

- useful beyond one loop: program SecDec available at

http://projects.hepforge.org/secdec/

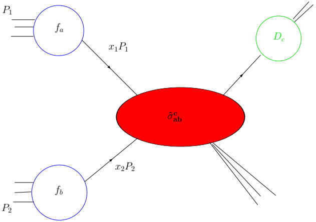
Phenomenology group members

Wolfgang Hollik

Thomas Hahn
GH
Pierpaolo Mastrolia (Humboldt Fellow)
Nicolas Greiner
Edoardo Mirabella
Thomas Reiter
Christian Sturm
Marcus Weber
Sophia Borowka
Hans van Deurzen
Ananda Landwehr
Davide Pagani
Sebastian Passehr
Tiziano Peraro
Johann Felix von Soden-Fraunhofen

backup slides

factorisation



$$\sigma_{pp\to X} = \sum_{a,b,c} f_a(x_1,\mu_f^2) f_b(x_2,\mu_f^2) \otimes \hat{\sigma}_{ab}\left(p_1,p_2,\frac{Q^2}{\mu_f^2},\frac{Q^2}{\mu_r^2},\alpha_s(\mu_r^2)\right)$$
$$\otimes D_{c\to X}(z,\mu_f^2) + \mathcal{O}(\Lambda/Q)$$

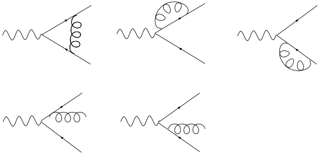
f_a, f_b: parton distribution functions (from fits to data)

$\hat{\sigma}_{ab}$: partonic hard scattering cross section

calculable order by order in perturbation theory

$D_{c\to X}(z,\mu_f^2)$: describing the final state e.g. fragmentation function, jet observable, etc.

ingredients for m-particle observable at NLO



virtual part (one-loop integrals):

$$\mathcal{A}^V_{NLO} = A_2/\epsilon^2 + A_1/\epsilon + A_0^{(v)}$$

$$d\sigma^V \sim Re\left(\mathcal{A}^\dagger_{LO}\,\mathcal{A}^V_{NLO}\right)$$

real radiation part: soft/collinear emission of massless particles

$\Rightarrow$ need subtraction terms

$$\Rightarrow \int_{\text{sing}} d\sigma^S = -A_2/\epsilon^2 - A_1/\epsilon + A_0^{(r)}$$

$$\sigma^{NLO} = \underbrace{\int_{m+1}\left[d\sigma^R - d\sigma^S\right]_{\epsilon=0}}_{\text{numerically}} + \underbrace{\int_m \left[\underbrace{d\sigma^V}_{\text{cancel poles}} + \underbrace{\int_s d\sigma^S}_{\text{analytically}}\right]_{\epsilon=0}}_{\text{numerically}}$$

NLO calculations

exploit modular structure

Tree Modules		One-Loop Module		IR Modules
$\|\mathcal{A}^{LO}\|^2$	$\oplus$	$2\,Re(\mathcal{A}^{LO\,\dagger}\mathcal{A}^{NLO,virt})$	$\oplus$	integrated IR subtraction terms
$\|\mathcal{A}^{NLO,real}\|^2$			$\ominus$	soft/collinear subtraction terms

NLO calculations

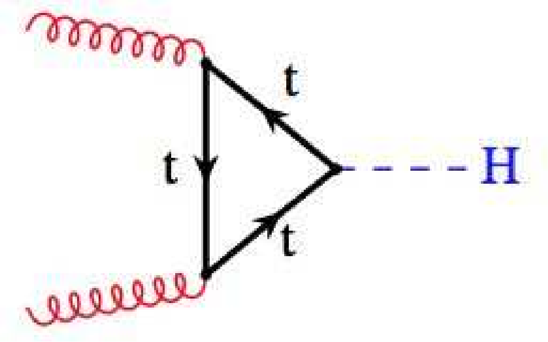
exploit modular structure

Tree Modules		One-Loop Module		IR Modules
$\lvert\mathcal{A}^{LO}\rvert^2$	$\oplus$	$2\,Re(\mathcal{A}^{LO\,\dagger}\mathcal{A}^{NLO,virt})$	$\oplus$	integrated IR subtraction terms
$\lvert\mathcal{A}^{NLO,real}\rvert^2$			$\ominus$	soft/collinear subtraction terms

has been bottleneck so far

$gg \to H$, $H \to \gamma\gamma$

"leading order" already means one loop
$\Rightarrow$ two-loop corrections are important



two-loop electroweak corrections:
S. Actis, G. Passarino, Christian Sturm, S. Uccirati '08
with 4th generation:
Denner, Dittmaier, Mück, Passarino, Spira, Sturm, Uccirati, Weber '11