Black Hole's Quantum N-Portrait

Gia Dvali

MPI Project Review

with: Cesar Gomez

Other directions with:

Pieter Lüst, Sarah Folkerts, Slava Mukhanov, Alex Kehagias, Andre Franca, Daniel Flassig, Konstantin Sluka, Nico Wintergerst

(Semi) Classically,
Black hole physics is
mysterious.

We show that
Quantum-mechanically it is
the simplest (in certain
sense):

$\frac{1}{N}$-coupled large-N
physics.

Black hole is a leaky Bose-condensate of soft gravitons:
(In Planck units)

Occupation number $= N$

Wave-length $= \sqrt{N}$

interaction strength $= 1/N$

Mass $= \sqrt{N}$

Temperature $= 1/\sqrt{N}$

Entropy $= N$

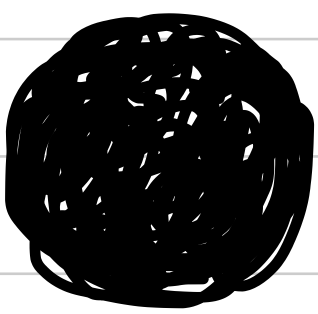
In $c=1$ units,

$$[G_N] = \ell/m$$

So we can define:

$$L_p^2 \equiv \hbar G_N \qquad M_p \equiv \frac{\hbar}{L_p}$$

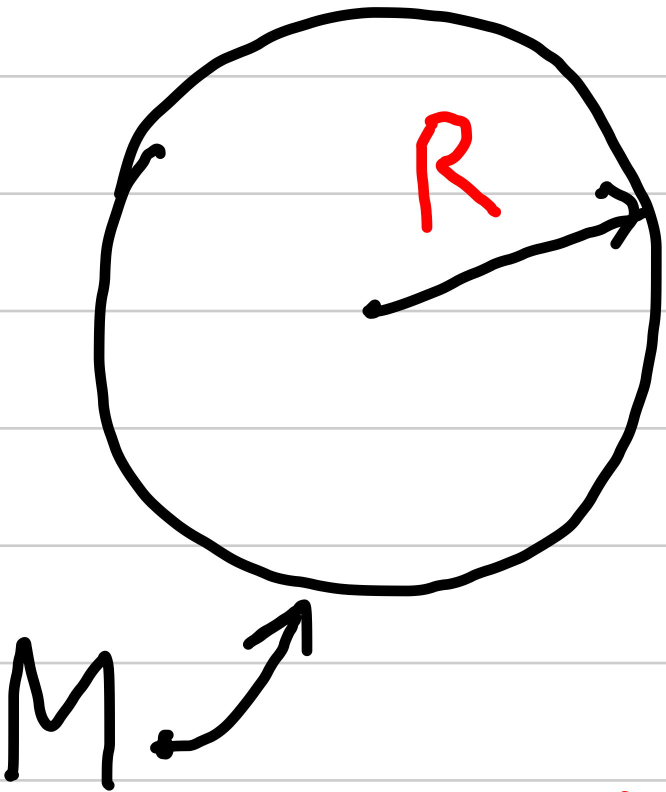
Both are quantum.



Source of mass M

Gravitational radius:

$$r_g \equiv M G_N = \frac{L_p^2}{\hbar M^{-1}}$$



Classical Newtonian Geometry

$$\phi(r) = -\frac{r_g}{r}$$

Gravitational energy

$$E_{grav} = M\frac{r_g}{R}$$

But, quantum-mechanically we should think about many gravitons with:

$\lambda = R$ and $E = \hbar/R$

Their occupation number is

$$N = \frac{E_{grav}}{E} = \frac{M r_g}{\hbar}$$

or

$$N = r_g^2 / L_p^2$$

$$= M^2 / M_p^2$$

$$= L_p^2 / \hbar^2 \bar{M}^2$$

For any source of size R occupation number of constituent weakly-coupled particles is maximum

$$N_{source-max} = \frac{MR}{\hbar}$$

and since $R \geq r_g$

for any fixed R,

N_{source} is maximazed by Black Holes

For $R = r_g$, graviton Bose-condensate becomes self-sustained.

$$\lambda = \sqrt{N} L_P$$

These gravitons are very weakly interacting:

$$\alpha_{gr} = \hbar G_N \lambda^{-2} = \frac{1}{N}$$

total mass

$$M = \sqrt{N} \frac{\hbar}{L_P}$$

Boundstate exist for any N, and it is leaky.

Hartree potential:

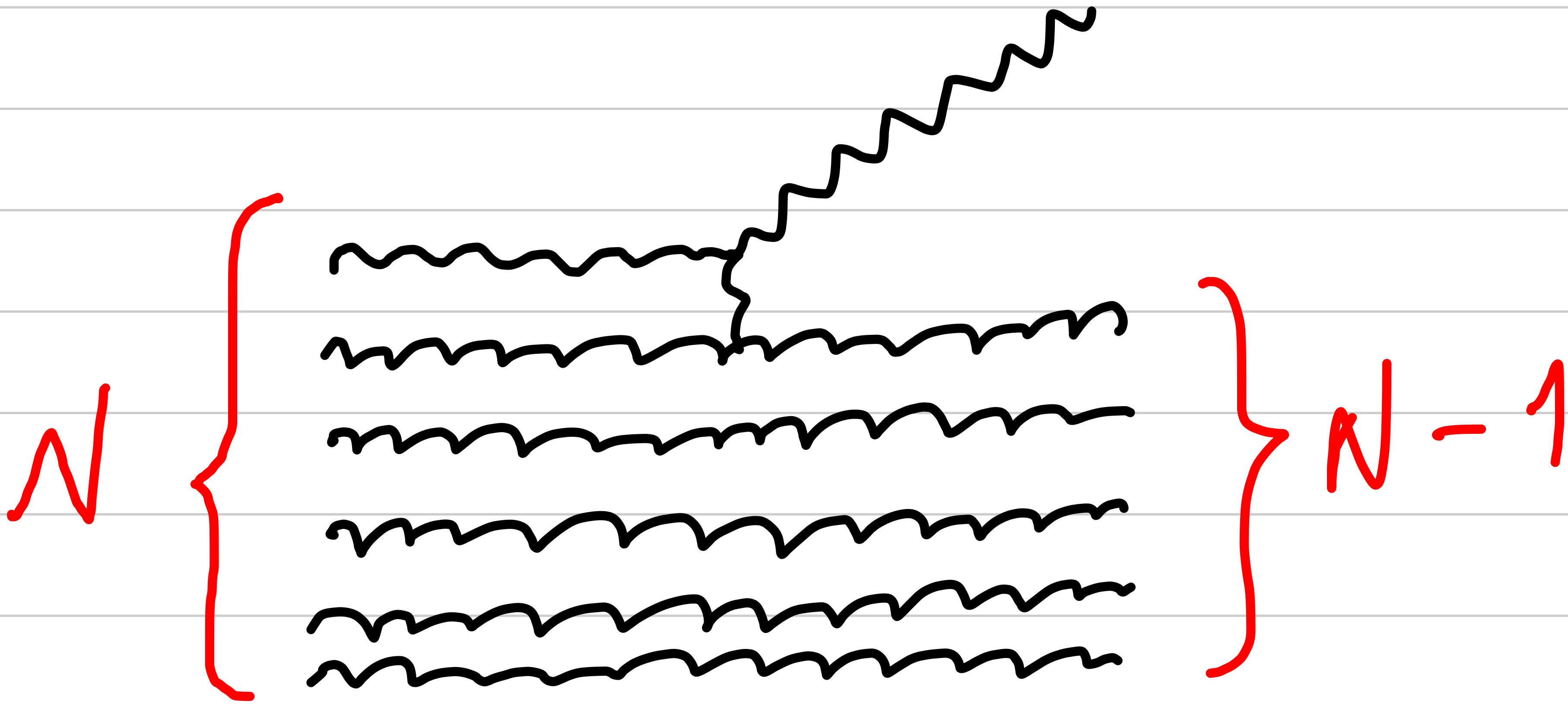
$$V = G_N N \frac{\hbar^2}{\lambda^2} \frac{1}{\lambda} = N \alpha_{gr} \frac{\hbar}{\lambda}$$

$$\alpha_{gr} = 1/N \qquad \lambda = \sqrt{N} L_p$$

$$V = \frac{\hbar}{\sqrt{N} L_p} = E_{escape}$$

Escape energy is just above!

Hawking radiation



$$\Gamma = \frac{\hbar}{\Delta t} = \frac{\hbar}{\sqrt{N} L_p}$$

$$\frac{dN}{dt} = -\frac{1}{\sqrt{N} L_p}$$

$$\tau = N^{3/2} L_p$$

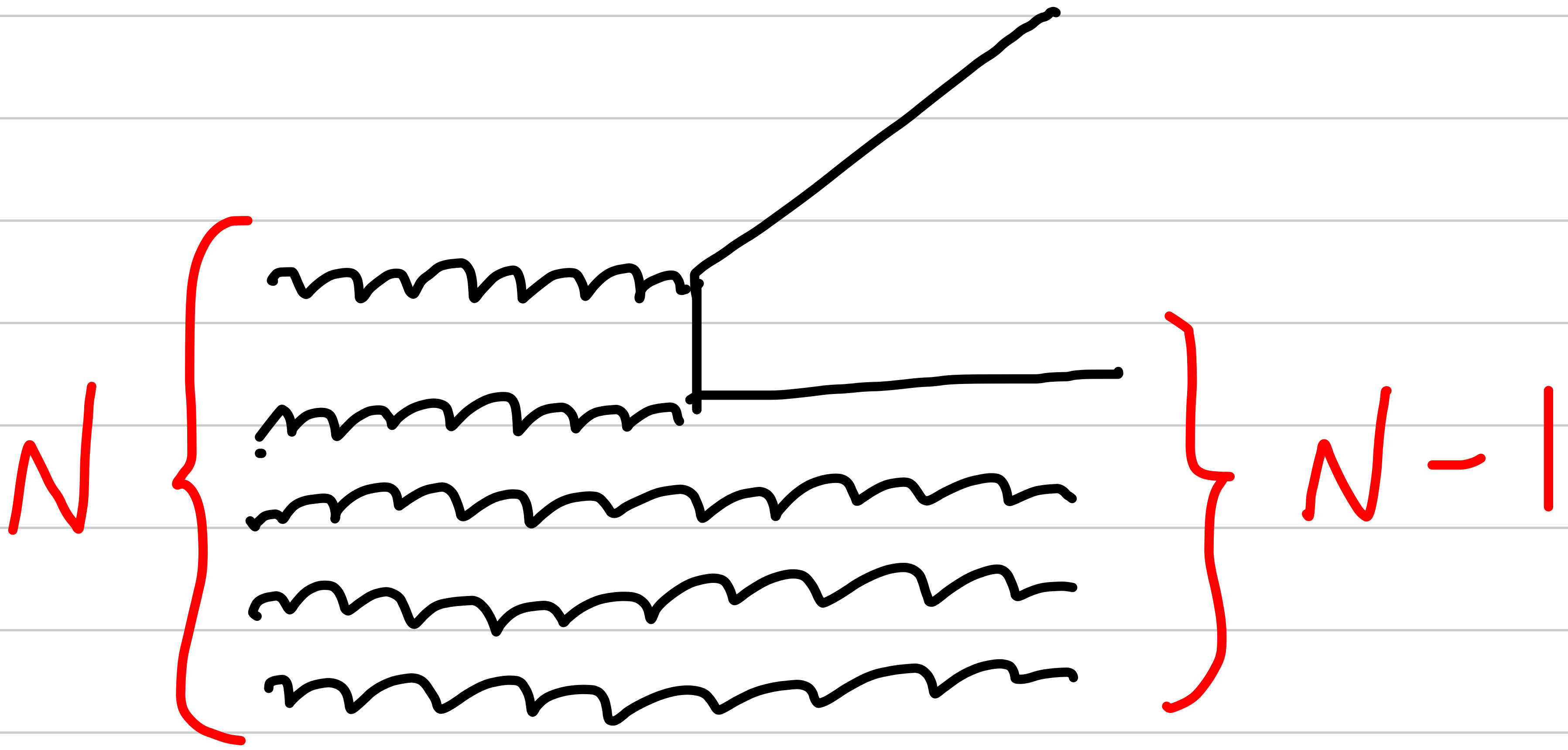
Bose-condensate is cold, but because of quantum depletion gravitons are radiated with a thermal spectrum of temperature

$$T = \frac{\hbar}{\sqrt{N}\, L_P}$$

$$\Downarrow$$

$$\frac{dM}{dt} = -\frac{T^2}{\hbar}$$

If there are extra species



$$\Gamma = \frac{\hbar}{\sqrt{N}\, L_p} N_{species}$$

Species bound:
Black holes cannot exist
for $N < N_{species}$!

Semi-classically this bound translates to the bound on r_g:

$$N > N_{species}$$

$$\Downarrow$$

$$r_g > L_N \equiv \sqrt{N_{species}}\, L_P$$

Solitons (e.g. Monopole):

Bose-condensate of W-bosons

$$\alpha \equiv \hbar g^2 \ll 1$$

$$M = \frac{M_W}{\alpha}$$

$$N = \frac{1}{\alpha}$$

$$\lambda = \frac{\hbar}{M_W}$$

Defining: $L_* \equiv \frac{\hbar}{v}$

Higgs VEV

Soliton N-picture:

Mass $\longrightarrow M = \sqrt{N_W}\,\frac{\hbar}{L_*}$

strength $\rightarrow \alpha = 1/N_W$

Number $= N_W$

wave-length $= \sqrt{N_W}\,L_*$

just like BH!

Now couple to gravity:

$$N = \frac{M^2}{M_p^2} \qquad N_W = \frac{1}{\hbar g}$$

$N_W > N \leftarrow$ Soliton

$N_W = N \leftarrow$ Extremal BH

$N_W < N \leftarrow$ Black hole

Explains non-singularity condition

$$Q_{Magnetic} \equiv \frac{1}{g} \leq \frac{M}{M_p}\sqrt{\hbar}$$

$$\Downarrow$$

$$N_W < N$$

For $N_W > N$, there are no BH!

Origin of Bekenstein entropy:

$N_f \sim N$ "flavors".

Because of order $\sim N$ subsets can form "unions".

As the number of non-interacting flavors is $\sim N$

BH wavefunction is a Hartree wave-function of N_f - flavors

$$\Psi_{BH} = \prod_j^{\sim N} \psi_j$$

$$n_{states} \sim \xi^N$$

Difference between BH-s and solitons:

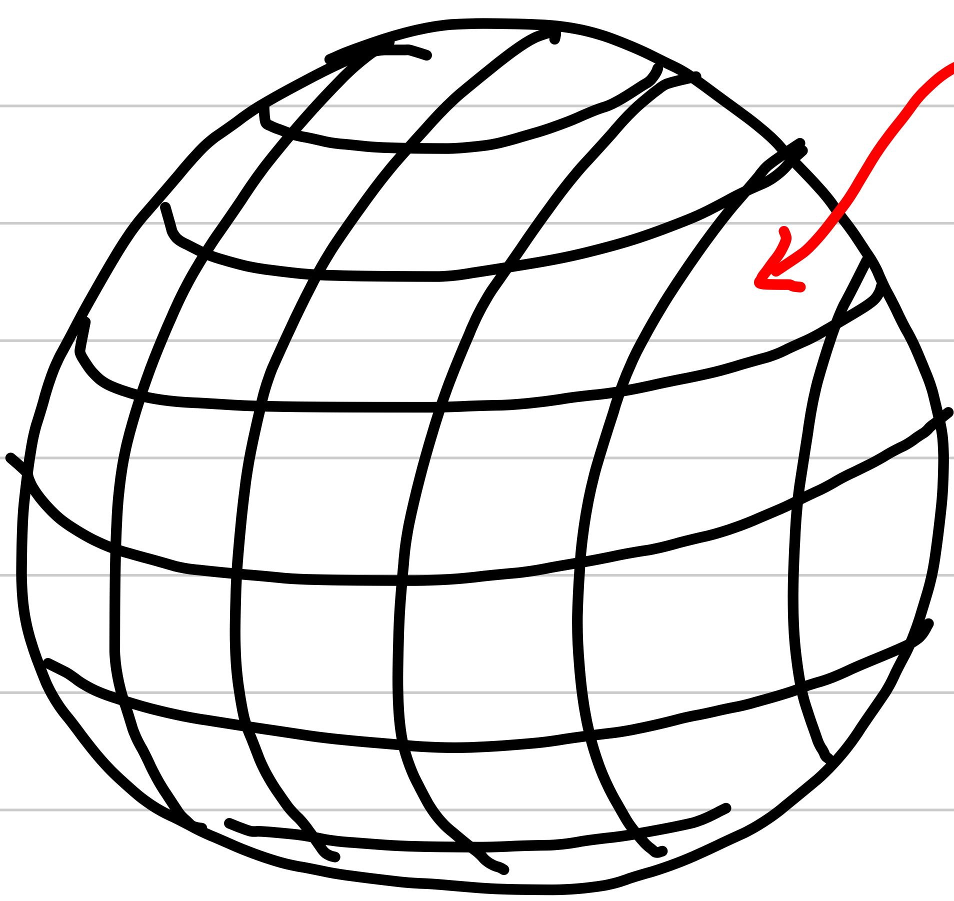
Solitons exist only for fixed N_W:

No energy self-sourcing!

Cannot be leaky!

We are learning that overpacked systems get oversimplified.

Origin of holography



L_p^2-pixel

$$N = \frac{r_g^2}{L_p^2}$$

It is interesting that generalizing our idea to AdS/dS-geometry, we get the same N-portrait:

In D-dimensions:

$$N = R^{D-2} / L_D^{D-2}$$

$$\lambda = N^{\frac{1}{D-2}} L_D$$

$$\alpha_D = \frac{1}{N}$$

Notice, that N coincides with the central charge of CFT

$$N_{CFT} = N = \left(\frac{R}{L_D}\right)^{D-2}$$