An Introduction to SUSY breaking
-
From Theory to the LHC

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Overview

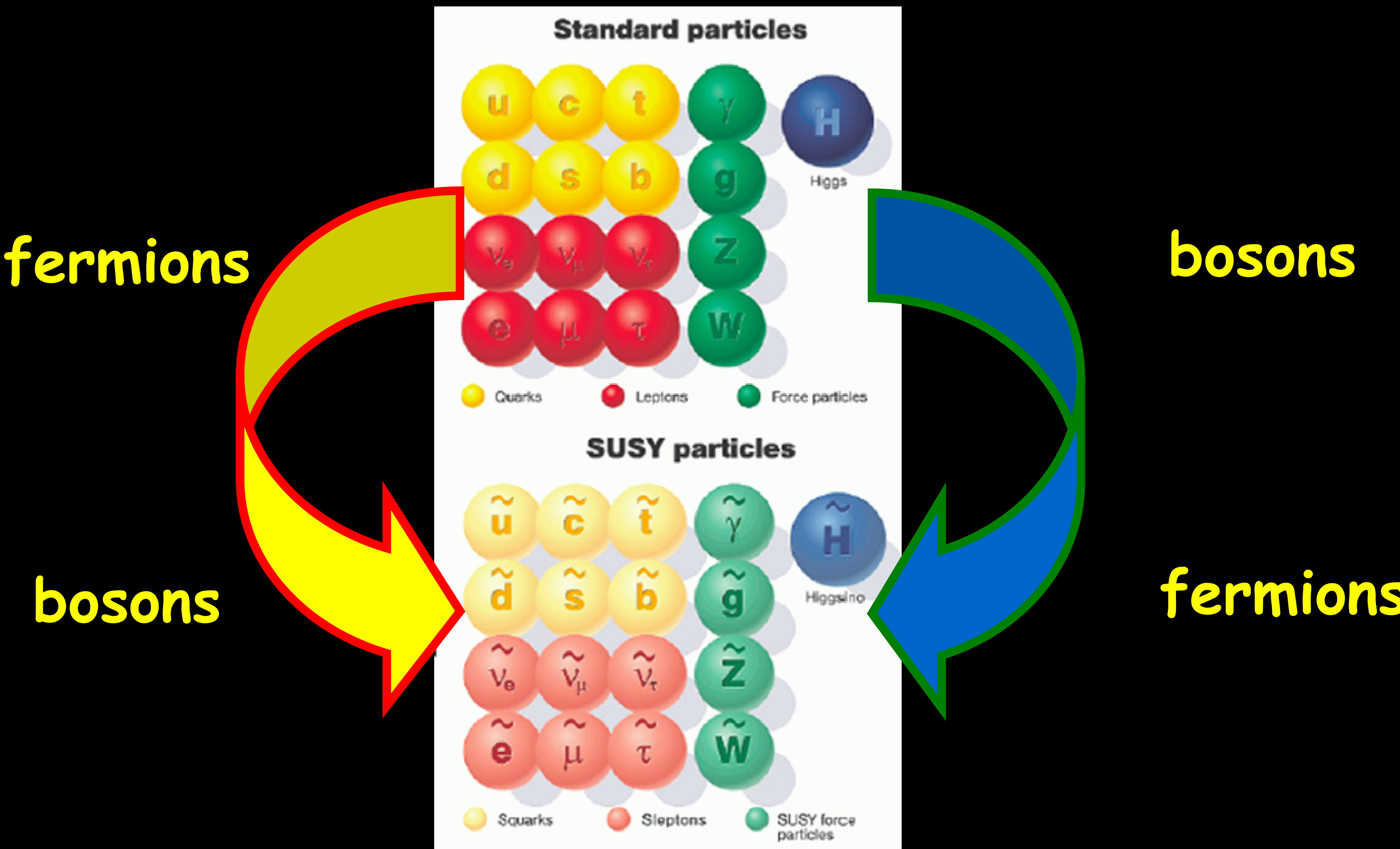
- 1. SUSY (breaking) basics
    - Reminder: SUSY
    - ~~SUSY~~
    - Simple model
- 2. SUSY breaking for real
    - O'Raifeartaigh Model
    - Theory and pheno constraints and features
    - R-axion, Meta stability I
- 3. SUSY breaking and the SM
    - Hidden Sector ~~SUSY~~
    - Gauge + Gravity mediation
    - Meta stability II
- 4. SUSY @ the LHC
    - RG Evolution
    - Direct searches
    - ~~SUSY~~ and the Higgs

SUSY (breaking) basics

Teaser: Why SUSY?

SUSY

- Symmetry between bosons and fermions



Why SUSY I

1. SUSY (even when softly broken) removes quadratic divergencies in the scalar sector.

   → *improves consistency of the theory, helps with the hierarchy problem*

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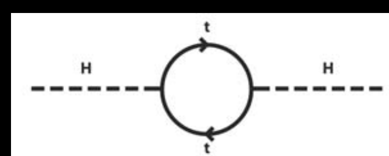
Without SUSY:

$v_{EW}^2 \sim m_{H,tree}^2 +$ 

$\sim \Lambda_{UV}^2 \sim M_P^2$

$\gg v_{EW}^2 \sim (100\,\mathrm{GeV})^2$

➔ We need $O(10^{32})$ finetuning!!

Why SUSY I

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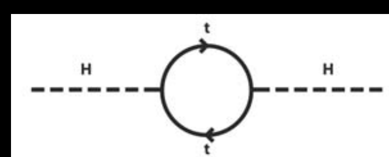
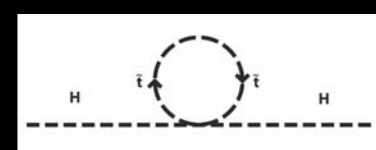
SUSY:

$v^2_{EW} \sim m^2_{H,tree} +$



+



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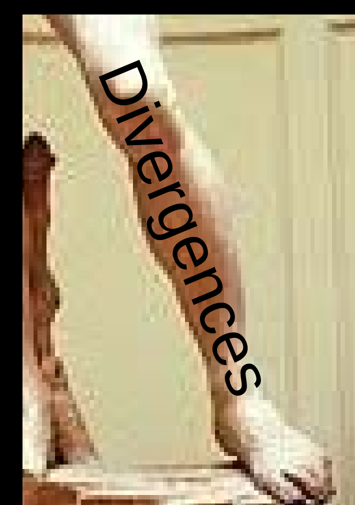
➔ No Finetuning



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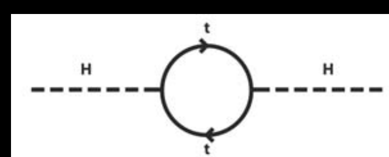
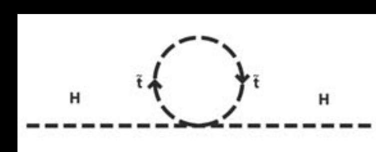
~~SUSY~~:

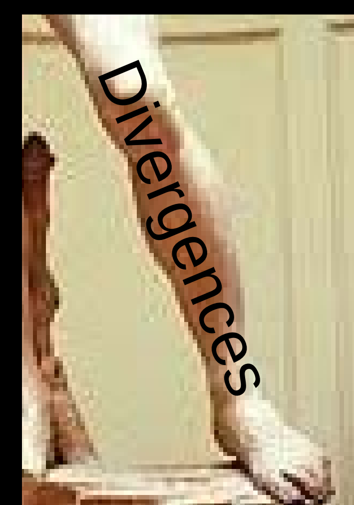
$v_{EW}^2 \sim m_{H,tree}^2 +$ 

 + 

$\sim m_{stop}^2 \sim m_{squark}^2 \sim \mathrm{TeV}^2$

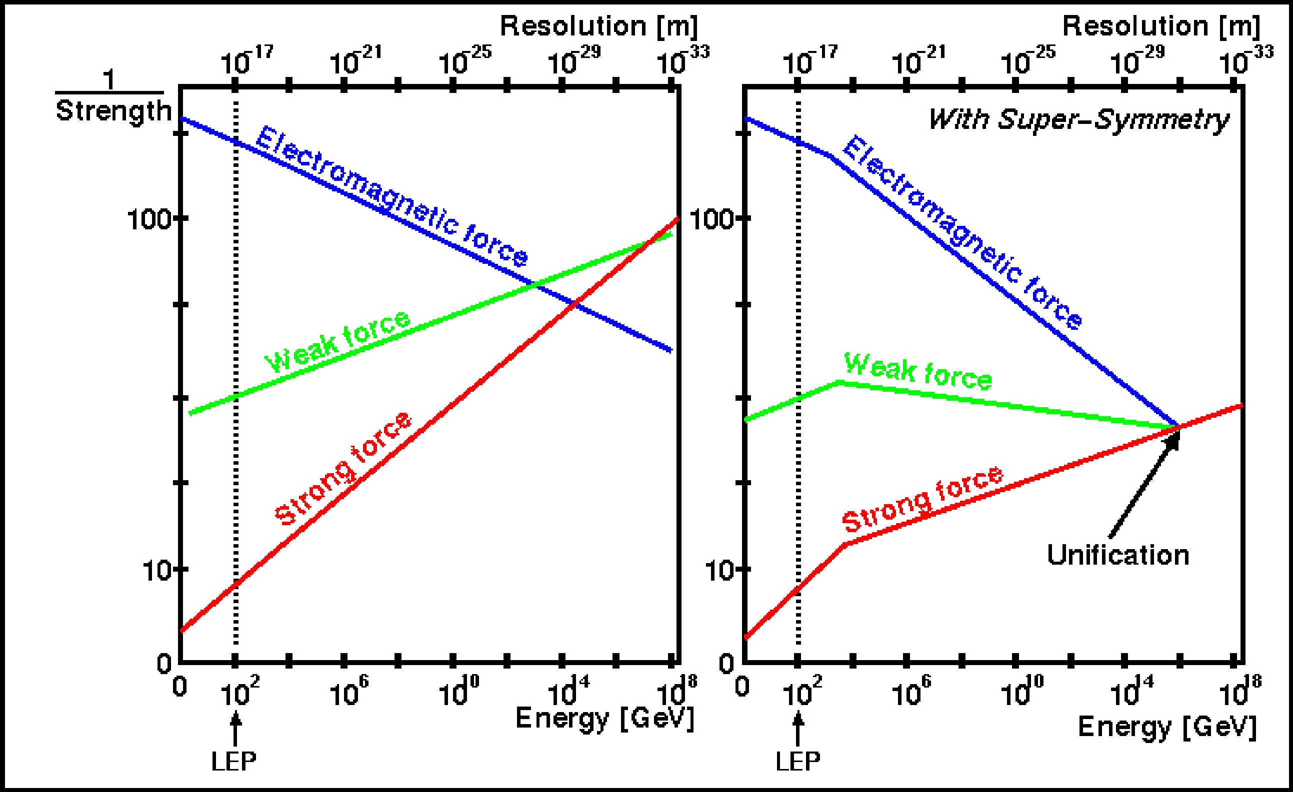
$\sim v_{EW}^2 \sim (100\,\mathrm{GeV})^2$

➔ We need O(100) finetuning!!



Why SUSY II

2. SUSY improves the
   unification of the
   SM gauge couplings
   → *Grand Unification*

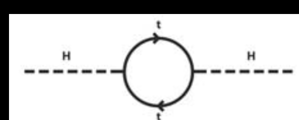


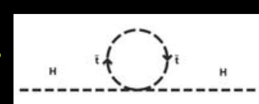
Why SUSY III

3. Supersymmetry breaking triggers electroweak symmetry breaking in the Standard Model

   → *explain electroweak symmetry breaking*

~~SUSY~~:

$v_{EW}^2 \sim m_{H,tree}^2 +$ 

+ 

$\sim -m_{\mathrm{stop}}^2$

Why SUSY IV

4. Supersymmetry
   can explain Dark Matter
   Neutralino LSPs in gravity mediation;
   gravitino LSPs in gauge mediation...
   $\rightarrow$ *Dark Matter and*
   Cosmology applications

~~SUSY~~ @ (0.1-1)TeV scale
➔ Many new particles
with such masses

Lightest can be stable
and can be produced in
the early universe
➔ DM candidate

Why SUSY V

5. Supersymmetry is required in string theory
   (a UV-complete underlying description unifying with quantum gravity)

   → *SUSY appears as natural ingredient.*

Why SUSY V

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Why SUSY VI

6. Its pretty!

Unique extension
of Poincare algebra

Why NOT SUSY

Because it must be broken!

Because…

…we haven't seen the **selectron**

Spin 0, charge=-1

$$m_{\tilde{e}} = 511\,\mathrm{keV}$$

Why NOT SUSY

Because it must be broken!

Because...

...we haven't seen the **selectron**

Spin 0, charge=-1

$$\cancel{m_{\tilde{e}} = 511\,\mathrm{keV}}$$

Disclaimer: Not complete!!!!!!!

1) Mainly SUSY field theory
2) Only passing comments on (super-)gravity

Why? I don't understand gravity ;-).

SUSY basics

Poincare group

- **Translations + LT**

$$\begin{aligned} \text{translations} \quad &: \quad \Phi(x) \longrightarrow \Phi'(x) = e^{i\,a^\rho P_\rho}\Phi(x) \\ \text{LT} \quad &: \quad \Phi(x) \longrightarrow \Phi'(x) = e^{\frac{i}{2}\,\omega^{\rho\sigma} M_{\rho\sigma}}\Phi(x) \end{aligned}$$

- **P, M fulfill Poicare algebra**

$$\begin{aligned} [P^\rho, P^\sigma] &= 0 \\ [P^\rho, M^{\nu\sigma}] &= i(g^{\rho\nu} P^\sigma - g^{\rho\sigma} P^\nu) \\ [M^{\mu\nu}, M^{\rho\sigma}] &= -i(g^{\mu\rho} M^{\nu\sigma} + g^{\nu\sigma} M^{\mu\rho} - g^{\mu\sigma} M^{\nu\rho} - g^{\nu\rho} M^{\mu\sigma}) \end{aligned}$$

Poincare group

- E.g. for a spin ½ field

$$P^\rho = i\,\partial^\rho\,; \qquad M^{\rho\sigma} = i(x^\rho\partial^\sigma - x^\sigma\partial^\rho) + \frac{i}{4}[\gamma^\rho,\gamma^\sigma]$$

Can we make it bigger?

- Nature seems to respect Poincare group

- Natural to ask for bigger symmetry
- Example gauge symmetry

$$\begin{aligned}[T^a, T^b] &= i f^{abc} T^c \\ [T^a, P^\rho] &= 0 \\ [T^a, M^{\rho\sigma}] &= 0\end{aligned}$$

Fairly trivial extension

Can we make it bigger?

- Nature seems to respect Poincare group

- Natural to ask for bigger symmetry
- Example gauge symmetry

$$\begin{aligned}
[T^a, T^b] &= i\, f^{abc} T^c \\
[T^a, P^\rho] &= 0 \\
[T^a, M^{\rho\sigma}] &= 0
\end{aligned}$$

- Can we have non-trivial one?

SUSY is non-trivial ext. of Poincare

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- Not with commutators/bosonic generators (Coleman Mandula)

➔ Need anticommutators/fermionic generators

$$Q_\alpha |\mathrm{bos}\rangle = |\mathrm{ferm}\rangle_\alpha\,; \qquad Q_\alpha |\mathrm{ferm}\rangle^\alpha = |\mathrm{bos}\rangle\,;$$

Action of symmetry transforms bosons ←→ fermions

Supersymmetry!

SUSY is non-trivial ext. of Poincare

- Commutators and anticommutators

$$
\begin{aligned}
[Q_\alpha, P^\rho] &= 0 \\
\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} &= 2(\sigma^\rho)_{\alpha\dot{\beta}} P_\rho \\
[M^{\rho\sigma}, Q_\alpha] &= -i(\sigma^{\rho\sigma})_\alpha{}^\beta Q_\beta \\
\{Q_\alpha, Q_\beta\} &= \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0
\end{aligned}
$$

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\{Q_\alpha, Q_\beta\} &= \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0
\end{aligned}
$$

Restrict ourselves to so-called N=1 SUSY

→ 2Q and 2$\bar{Q}$

Superspace

- We have added 4 additional fermionic "one-index" generators
  ➔ indexwise similar to translations P^μ

- It makes sense to add additional coordinated to our space-time
  to allow "supertranslations"

$$x^\mu \longrightarrow X = (x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$$

$$[x^\mu] = -1$$

$$[\theta^\alpha] = [\bar{\theta}^{\dot{\alpha}}] = -\frac{1}{2}$$

Super-field

- Can consider fields(=functions) on superspace

$$\Omega(X) = \Omega(x, \theta, \bar{\theta})$$

- The most general superfield

$$\begin{aligned}\Omega(x,\theta,\bar{\theta}) &= c(x) + \theta\psi(x) + \bar{\theta}\bar{\psi}'(x) + (\theta\theta)\, F(x) + (\bar{\theta}\bar{\theta})\, F'(x) + \theta\sigma^{\mu}\bar{\theta}\, v_{\mu}(x) \\ &+ (\theta\theta)\, \bar{\theta}\bar{X}'(x) + (\bar{\theta}\bar{\theta})\, \theta\lambda(x) + (\theta\theta)\, (\bar{\theta}\bar{\theta})\, D(x)\end{aligned}$$

"Supertranslations"

- SUSY transformation + translation

$$S(a,\zeta,\bar{\zeta}) \equiv e^{i\left(\zeta^\alpha Q_\alpha + \bar{\zeta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}} + a^\mu P_\mu\right)}$$

- Two times... (check as homework ← SUSY algebra)

$$S(a,\zeta,\bar{\zeta})S(x,\theta,\bar{\theta}) = S(x^\mu + a^\mu + i\,\zeta\sigma^\mu\bar{\theta} - i\,\theta\sigma^\mu\bar{\zeta},\, \theta+\zeta,\, \bar{\theta}+\bar{\zeta})$$

" $S(A)S(X) = S(A+X)$ "

Super-field transformations

- Pure translation

$$\phi(x) \to S(a,0,0)\phi(x)S^{-1}(a,0,0) = e^{ia^\mu P_\mu}\phi(x)e^{-ia^\mu P_\mu} = \phi(x+a)$$

- SUSY + translation

$A \cdot P$

$$\begin{aligned}\Omega(x,\theta,\bar\theta) \quad &\to \quad e^{i\left(\zeta^\alpha Q_\alpha + \bar\zeta_{\dot\alpha}\bar Q^{\dot\alpha} + a^\mu P_\mu\right)}\,\Omega(x,\theta,\bar\theta)\,e^{-i\left(\zeta^\alpha Q_\alpha + \bar\zeta_{\dot\alpha}\bar Q^{\dot\alpha} + a^\mu P_\mu\right)} \\ &= \quad \Omega(x^\mu + a^\mu + i\,\zeta\sigma^\mu\bar\theta - i\,\theta\sigma^\mu\bar\zeta,\, \theta+\zeta,\, \bar\theta+\bar\zeta)\end{aligned}$$

X

$A + X$

Super-field transformations II

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- SUSY + translation

$A \cdot P$

$$\Omega(x,\theta,\bar\theta) \;\to\; e^{i\left(\zeta^\alpha Q_\alpha + \bar\zeta_{\dot\alpha}\bar Q^{\dot\alpha} + a^\mu P_\mu\right)}\,\Omega(x,\theta,\bar\theta)\,e^{-i\left(\zeta^\alpha Q_\alpha + \bar\zeta_{\dot\alpha}\bar Q^{\dot\alpha} + a^\mu P_\mu\right)}$$
$$= \Omega(x^\mu + a^\mu + i\,\zeta\sigma^\mu\bar\theta - i\,\theta\sigma^\mu\bar\zeta,\, \theta+\zeta,\, \bar\theta+\bar\zeta)$$

X $\qquad$ $A + X$

- Implement P, Q as differential operators

$$\Omega(x^\mu + a^\mu + i\,\zeta\sigma^\mu\bar\theta - i\,\theta\sigma^\mu\bar\zeta,\, \theta+\zeta,\, \bar\theta+\bar\zeta) = e^{-i\left(\zeta^\alpha Q_\alpha + \bar\zeta_{\dot\alpha}\bar Q^{\dot\alpha} + a^\mu P_\mu\right)}\Omega(x,\theta,\bar\theta)$$

Super-field transformations III

- Expanding both sides for infinitesimal A

$$\Omega(x^\mu + a^\mu + i\,\zeta\sigma^\mu\bar\theta - i\,\theta\sigma^\mu\bar\zeta,\, \theta+\zeta,\, \bar\theta+\bar\zeta)$$

$$= \Omega + \left(a^\mu + i\,\zeta\sigma^\mu\bar\theta - i\,\theta\sigma^\mu\bar\zeta\right)\partial_\mu\Omega + \zeta^\alpha\partial_\alpha\Omega - \bar\zeta_{\dot\alpha}\bar\partial^{\dot\alpha}\Omega$$

$$= \Omega - i\left(\zeta^\alpha Q_\alpha + \bar\zeta_{\dot\alpha}\bar Q^{\dot\alpha} + a^\mu P_\mu\right)\Omega$$

➔

$$\begin{aligned} P_\mu &= i\partial_\mu \\ Q_\alpha &= i\,\partial_\alpha - \sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\,\partial_\mu = iD_\alpha \\ \bar Q_{\dot\alpha} &= -i\,\bar\partial_{\dot\alpha} + \theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\,\partial_\mu = i\bar D_{\dot\alpha} \end{aligned}$$

Representation of SUSY algebra

Chiral superfield

- Take most general superfield

$$\begin{aligned}\Omega(x,\theta,\bar\theta) &= c(x)+\theta\psi(x)+\bar\theta\bar\psi'(x)+(\theta\theta)\,F(x)+(\bar\theta\bar\theta)\,F'(x)+\theta\sigma^\mu\bar\theta\,v_\mu(x)\\ &+ (\theta\theta)\,\bar\theta\bar\lambda'(x)+(\bar\theta\bar\theta)\,\theta\lambda(x)+(\theta\theta)\,(\bar\theta\bar\theta)\,D(x)\end{aligned}$$

- Apply covariant constraint

$$\bar D_{\dot\alpha}\,\phi(x,\theta,\bar\theta)=0$$

- ➔ Left handed chiral superfield

$$\begin{aligned}\phi(x,\theta,\bar\theta) &= \varphi(x)+\sqrt2\theta\psi(x)-i\theta\sigma^\mu\bar\theta\,\partial_\mu\varphi(x)+\frac{i}{\sqrt2}(\theta\theta)(\partial_\mu\psi(x)\sigma^\mu\bar\theta)\\ &- \frac14(\theta\theta)(\bar\theta\bar\theta)\partial^\mu\partial_\mu\varphi(x)-(\theta\theta)F(x)\end{aligned}$$

Chiral superfield

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- **Left handed superfield**

$$\begin{aligned}\phi(x,\theta,\bar\theta) &= \varphi(x)+\sqrt{2}\theta\psi(x)-i\theta\sigma^\mu\bar\theta\,\partial_\mu\varphi(x)+\frac{i}{\sqrt{2}}(\theta\theta)(\partial_\mu\psi(x)\sigma^\mu\bar\theta)\\ &- \frac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\partial^\mu\partial_\mu\varphi(x)-(\theta\theta)F(x)\end{aligned}$$

- **Transformations**

$$\begin{aligned}\delta\varphi &= \sqrt{2}\,\zeta\psi\\ \delta\psi_\alpha &= -\sqrt{2}\,F\,\zeta_\alpha - i\sqrt{2}\,\sigma^\mu_{\alpha\dot\alpha}\bar\zeta^{\dot\alpha}\,\partial_\mu\varphi\\ \delta F &= -i\sqrt{2}\,\partial_\mu\psi\sigma^\mu\bar\zeta = \partial_\mu\left(-i\sqrt{2}\,\psi\sigma^\mu\bar\zeta\right)\end{aligned}$$

Total derivative!!!

SUSY breaking… finally made it there!

$$\begin{aligned} \delta\varphi &= \sqrt{2}\zeta\psi \\ \delta\psi_\alpha &= -\sqrt{2}F\zeta_\alpha - i\sqrt{2}\sigma^\mu_{\alpha\dot{\alpha}}\bar{\zeta}^{\dot{\alpha}}\partial_\mu\varphi \\ \delta F &= -i\sqrt{2}\partial_\mu\psi\sigma^\mu\bar{\zeta} = \partial_\mu\left(-i\sqrt{2}\psi\sigma^\mu\bar{\zeta}\right) \end{aligned}$$

- Which vev's break SUSY?
  - A φ vev?

$$\varphi = const, \quad \psi = 0, \quad F = 0$$

$$\rightarrow \quad \delta\varphi = 0, \quad \delta\psi = 0, \quad \delta F = 0$$

SUSY preserved!

SUSY breaking

$$
\begin{aligned}
\delta\varphi &= \sqrt{2}\,\zeta\psi \\
\delta\psi_\alpha &= -\sqrt{2}\,F\,\zeta_\alpha - i\sqrt{2}\,\sigma^\mu_{\alpha\dot\alpha}\bar\zeta^{\dot\alpha}\,\partial_\mu\varphi \\
\delta F &= -i\sqrt{2}\,\partial_\mu\psi\sigma^\mu\bar\zeta = \partial_\mu\left(-i\sqrt{2}\,\psi\sigma^\mu\bar\zeta\right)
\end{aligned}
$$

- Which vev's break SUSY?
  - An F vev?

$$\varphi = 0, \quad \psi = 0, \quad F \neq 0$$

$\rightarrow$
$$\delta\varphi = 0, \quad \delta\psi = -\sqrt{2}F\zeta_\alpha \neq 0, \quad \delta F = 0$$
$\rightarrow$ ~~SUSY~~

Vector superfield

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- Same story different constraint

$$V = V^\dagger$$

Vector!

$$\begin{aligned}
V(x,\theta,\bar\theta) &= c(x) + i\,\theta\chi(x) - i\,\bar\theta\bar\chi(x) + \theta\sigma^\mu\bar\theta\, v_\mu(x) + i\,(\theta\theta)N(x) - i\,(\bar\theta\bar\theta)N^\dagger(x) \\
&+ i\,(\theta\theta)\bar\theta\left(\bar\lambda(x) + \frac{i}{2}\partial_\mu\chi(x)\sigma^\mu\right) - i\,(\bar\theta\bar\theta)\theta\left(\lambda(x) - \frac{i}{2}\sigma^\mu\partial_\mu\bar\chi(x)\right) \\
&+ \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)\left(D(x) - \frac{1}{2}\partial^\mu\partial_\mu c(x)\right) \qquad (4
\end{aligned}$$

- D transforms like total derivative!
- Vev for D breaks SUSY!

SUSY breaking

~~SUSY~~

1) Non-vanishing F-terms of χSF

2) Non-vanishing D-terms of vector SF

SUSY Lagrangians

SUSY Lagrangians

- F terms of χSF and
  D term of VSF
  transform as total derivatives.

➔ $\int d^4x F$ and $\int d^4x D$

Are invariant under SUSY transformations

➔ Can be used to build SUSY Lagrangians!

How to extract F and D?

- Use Grassmann integration.

$$F = \phi\Big|_{\theta\theta} = \int d^2\theta\,\phi$$

$$D = V\Big|_{\theta\theta\bar{\theta}\bar{\theta}} = \int d^2\theta\, d^2\bar{\theta}\, V$$

Combinations of χ superfields.

- ϕ_i χSF

➔ $W(\phi_i)$, holomorphic is a χSF

➔ $K(\phi^\dagger, \phi)$, real is a VSF

Combinations of χ superfields.

- ϕ_i χSF

➔ $W(\phi_i)$, holomorphic is a χSF

Superpotential

➔ $K(\phi^\dagger,\phi)$, real is a VSF

Kähler potential

Construct Lagrangian

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- We can combine

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_D$$

- with

$$\mathcal{L}_F = \int \mathrm{d}^2\theta\, W(\phi_i) + \int \mathrm{d}^2\bar{\theta}\, W^\dagger(\phi_i^\dagger)$$

$$\mathcal{L}_D = \int d^2\theta d^2\bar{\theta}\, K(\phi_i^\dagger, \phi_i)$$

Example Wess-Zumino model

- Just one field

$$W(\phi) = a\phi + \frac{m}{2}\phi^2 + \frac{y}{3!}\phi^3$$

$$K(\phi^\dagger, \phi) = \phi^\dagger \phi$$

➔

$$\begin{aligned}
\mathcal{L}_{D,\mathrm{WZ}} &= F^\dagger F + (\partial_\mu \varphi)(\partial^\mu \varphi)^\dagger + \frac{i}{2}\psi\sigma^\mu(\partial_\mu \bar{\psi}) - \frac{i}{2}(\partial_\mu \psi)\sigma^\mu \bar{\psi} \\
\mathcal{L}_{F,\mathrm{WZ}} &= -a\,F - m\,\varphi F - \frac{m}{2}(\psi\psi) - \frac{y}{2}\varphi\varphi F - \frac{y}{2}\varphi(\psi\psi) + \mathrm{h.c.}
\end{aligned}$$

Example Wess-Zumino model

- Just one field

$$W(\phi) = a\phi + m\phi^2 + y\phi^3$$

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→ Kinetic terms for bosons and fermions

$$\mathcal{L}_{D,\mathrm{WZ}} = F^\dagger F + (\partial_\mu \varphi)(\partial^\mu \varphi)^\dagger + \frac{i}{2}\psi\sigma^\mu(\partial_\mu \bar{\psi}) - \frac{i}{2}(\partial_\mu \psi)\sigma^\mu \bar{\psi}$$

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Example Wess-Zumino model

- Just one field

$$W(\phi) = a\phi + \frac{m}{2}\phi^2 + \frac{y}{3!}\phi^3$$

$$K(\phi^\dagger, \phi) = \phi^\dagger \phi$$

➔ Kinetic terms for bosons and fermions

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\mathcal{L}_{F,\mathrm{WZ}} &= -a\,F - m\,\varphi F - \frac{m}{2}(\psi\psi) - \frac{y}{2}\varphi\varphi F - \frac{y}{2}\varphi(\psi\psi) + \mathrm{h.c.}
\end{aligned}$$

No kinetic term for F! ➔ integrate out

Integrate out F

- Quadratic in F → enough to solve EOM

$$0 = \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu F)} - \frac{\partial \mathcal{L}}{\partial F} = -\frac{\partial \mathcal{L}}{\partial F} = -F^\dagger + a + m\,\varphi + \frac{y}{2}\varphi\varphi$$

→ $$F^\dagger F - \left(a\,F + m\,\varphi\,F + \frac{y}{2}\varphi\varphi\,F + \text{h.c.}\right) = -\left|a + m\,\varphi + \frac{y}{2}\varphi\varphi\right|^2 = -\left|\frac{\partial W(\varphi)}{\partial\varphi}\right|^2$$

→ All the terms with F together give potential for the scalar field

$$V(\varphi) = \left|\frac{\partial W(\varphi)}{\partial\varphi}\right|^2 = |F|^2$$

Note: SUSY breaking

- $F \neq 0$ ➔ ~~SUSY~~

- $V(\phi) > 0$ ➔ ~~SUSY~~

- If SUSY broken: positive vacuum energy!!!

Combined Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\mathrm{WZ}} \;=\;& (\partial_\mu \varphi)\,(\partial^\mu \varphi)^\dagger + \frac{i}{2}\,\psi\sigma^\mu(\partial_\mu\bar{\psi}) - \frac{i}{2}\,(\partial_\mu\psi)\sigma^\mu\bar{\psi} \\
-\;& |M|^2\varphi\varphi^\dagger - \frac{|y|^2}{4}\varphi\varphi\varphi^\dagger\varphi^\dagger - \left(\frac{M}{2}\psi\psi + \frac{M^* y}{2}\varphi\varphi\varphi^\dagger + \frac{y}{2}\varphi\,\psi\psi + \mathrm{h.c.}\right)
\end{aligned}
$$

More χSF

$$
\begin{aligned}
\mathcal{L} \quad &= \quad (\partial_\mu \varphi_i)(\partial^\mu \varphi_i)^\dagger + \frac{i}{2}\psi_i \sigma^\mu (\partial_\mu \bar{\psi}_i) - \frac{i}{2}(\partial_\mu \psi_i)\sigma^\mu \bar{\psi}_i \\
&- \quad \sum_i \left| \frac{\partial W(\varphi_i)}{\partial \varphi_i} \right|^2 - \frac{1}{2}\left(\frac{\partial^2 W(\varphi_i)}{\partial \varphi_i \, \partial \varphi_j} \right) \psi_i \psi_j - \frac{1}{2}\left(\frac{\partial^2 W^\dagger(\varphi_i)}{\partial \varphi_i^\dagger \, \partial \varphi_j^\dagger} \right) \bar{\psi}_i \bar{\psi}_j
\end{aligned}
$$

- **Bosonic masses**

$$V = \left(\phi^{*j} \quad \phi_j \right) \mathrm{m}_{\mathrm{S}}^2 \begin{pmatrix} \phi_i \\ \phi^{*i} \end{pmatrix}$$

$$m_S^2 = \begin{pmatrix} W^*_{jk} W^{ik} & W^*_{ijk} W^k \\ W^{ijk} W^*_k & W^*_{ik} W^{jk} \end{pmatrix}$$

- **Fermionic masses**

$$m_F = W^{ji}$$

In the one field model

- **Bosonic masses**

$$m_S^2 = \begin{pmatrix} |m|^2 & -(yF)^\dagger \\ -yF & |m|^2 \end{pmatrix}$$

- **Fermionic masses**

$$m_F = m$$

In the one field model: ~~SUSY~~

- Bosonic masses

$$m_S^2 = \begin{pmatrix} |m|^2 & -(yF)^\dagger \\ -yF & |m|^2 \end{pmatrix}$$

- Fermionic masses

$$m_F = m$$

- F=0 → SUSY → $m_{fermion} = m_{scalar}$
- F≠0 → ~~SUSY~~ → $m_{fermion} \neq m_{scalar}$

SUSY breaking

first attempts

F-term breaking

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- We have seen $F \neq 0$ ➔ SUSY breaking
- F and $V(\phi)$ are connected

$$V(\varphi) = \left| \frac{\partial W(\varphi)}{\partial \varphi} \right|^2 = |F|^2$$

- Vacuum needs

$$\frac{\partial V(\varphi)}{\partial \varphi_i^*} = 0 = W_{ij}^* W^i = -W_{ij}^* F_i^\dagger$$

If we can find solution for $F^\dagger_i = W^i = 0$ ➔ SUSY vacuum!

F-term breaking

- If F_i=0 possible ➔ No ~~SUSY~~
- Example: one field model

$$W' = \frac{\partial W}{\partial \varphi} = a + m\varphi + \frac{1}{2} y \varphi^2 = 0$$

Always solution as long as m or y $\neq$ 0
➔ Has SUSY vacuum

F-term breaking

- Example II: one field model m=y=0

$$W' = \frac{\partial W}{\partial \varphi} = a \neq 0$$

➔ breaks SUSY ☺.

But: $\mathcal{L} = a^2 = \text{field independent}$

➔ No masses, no interactions, only vacuum energy ☹

Summary I

Summary

- SUSY is nice but must be broken

- $F \neq 0$ or $D \neq 0$ do the trick
  → Boson and fermion masses can(!) differ

- $F=0$ possible → no SUSY breaking
  → SUSY breaking is difficult to arrange!

SUSY breaking

first attempts

F-term breaking

- We have seen $F \neq 0$ ➔ SUSY breaking

- F and $V(\phi)$ are connected

$$V(\varphi) = \left| \frac{\partial W(\varphi)}{\partial \varphi} \right|^2 = |F|^2$$

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$$\frac{\partial V(\varphi)}{\partial \varphi_i^*} = 0 = W_{ij}^* W^i = -W_{ij}^* F_i^\dagger$$

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- Example: one field model

$$W' = \frac{\partial W}{\partial \varphi} = a + m\varphi + \frac{1}{2} y \varphi^2 = 0$$

Always solution as long as m or y ≠ 0
→ Has SUSY vacuum

F-term breaking

- Example II: one field model m=y=0

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➔ breaks SUSY ☺.

But: $\mathcal{L} = a^2 = \text{field independent}$

➔ No masses, no interactions, only vacuum energy ☹

On the way to better

SUSY breaking

Sucessful SUSY breaking

- O'Raifeartaigh model (OR model)

$$W_{\mathrm{OR}}(\phi_i) = -a\,\phi_1 + m\,\phi_2\phi_3 + \frac{y}{2}\phi_1\phi_3^2$$

$$\Rightarrow \quad \begin{aligned} W_1 &= -a + \tfrac{y}{2}\phi_3^2 \\ W_2 &= m\phi_3 \\ W_3 &= m\phi_2 + y\phi_3 \end{aligned}$$

Successful SUSY breaking

- O'Raifeartaigh model

$$W_{\rm OR}(\phi_i) = -a\,\phi_1 + m\,\phi_2\phi_3 + \frac{y}{2}\phi_1\phi_3^2$$

Try to find SUSY vac:
Solve $W_i=0$

$$\Rightarrow\quad \begin{aligned} W_1 &= -a + \tfrac{y}{2}\phi_3^2 \\ W_2 &= m\phi_3 \\ W_3 &= m\phi_2 + y\phi_3 \end{aligned}$$

$$W_2 = m\phi_3 \longrightarrow \phi_3 = 0$$

$$W_1 = -a + \tfrac{y}{2}\phi_3^2 \longrightarrow W_1 = -a \neq 0$$

$\Downarrow$

~~SUSY~~ ☺

Properties of OR model

- Potential

$$V_{\rm OR} = \sum_i \left|\frac{\partial W_{\rm OR}(\varphi_i)}{\partial \varphi_i}\right|^2 = \left|a - \frac{y}{2}\varphi_3^2\right|^2 + |m\,\varphi_3|^2 + |m\,\varphi_2 + y\,\varphi_1\varphi_3|^2$$

- For $a < \frac{m^2}{y}$ (check homework)

  Absolute minimum at

$$\phi_2 = \phi_3 = 0, \qquad \phi_1 = \text{arbitrary}$$

Properties of OR model

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- For $a < \dfrac{m^2}{y}$ (check homework)

  Absolute minimum at

$$\phi_2 = \phi_3 = 0, \qquad \phi_1 = \text{arbitrary}$$

(pseudo-)modulus

(Nearly) massless, A plague of many a SUSY model!

Fermion mass matrix

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$$m_F = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & m \\ 0 & m & 0 \end{pmatrix}$$

$$\phi_1 = 0$$

- One massless fermion, ψ_1, and two with mass $m_\pm = \frac{1}{2} y\phi_1 \pm \sqrt{m^2 + \frac{y^2\phi_1^2}{4}}$ (comb. of ψ_2, ψ_3).

Fermion mass matrix

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- One massless fermion, ψ_1, and two with mass (comb. of ψ_2, ψ_3). $m_\pm = \frac{1}{2} y\phi_1 \pm \sqrt{m^2 + \frac{y^2\phi_1^2}{4}}$

For later: $\phi_1 = 0$ → $|m_\pm| = m$

The Goldstino

The Goldstino

- We broke a global fermionic symmetry
  ➔ Expect massess fermion: goldstino!

- ψ_1 is the goldstino!

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- ψ_1 is the goldstino!

- Goldstino direction

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \sim \begin{pmatrix} -W_1^\star \\ -W_2^\star \\ -W_3^\star \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \begin{pmatrix} F_1 \neq 0 \\ 0 \\ 0 \end{pmatrix}$$

The Goldstino: general case (F-terms only)

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- SUSY breaking → Goldstino

- Goldstino direction

$$\tilde{G} = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \dots \end{pmatrix} = \begin{pmatrix} -W_1^\star \\ -W_2^\star \\ \dots \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \\ \dots \end{pmatrix}$$

The Goldstino: general case (F-terms only)

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- SUSY breaking ➔ Goldstino

- Goldstino direction

$$\tilde{G} = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \cdots \end{pmatrix} = \begin{pmatrix} -W_1^\star \\ -W_2^\star \\ \cdots \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \\ \cdots \end{pmatrix}$$

- Proof:

$$m_F^{ij} \tilde{G}_j = W^{ij} W_j^* = \frac{\partial}{\partial \phi_i} W^j W_j^* = \frac{\partial}{\partial \phi_i} V(\phi) = 0$$

Minimum of the potential

SUSY breaking
is difficult

Back to OR model

The bosonic mass matrix

$$m_B^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & |m|^2 + my^*\phi_1^* & m^* y\phi_1 & 0 & 00 & \\ 0 & 0 & |m|^2|y|^2 + |\phi_1|^2 & 0 & 0 & -ay \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & |m|^2 + my^*\phi_1^* & 0 \\ 0 & 0 & -a^* y & 0 & \phi_1 m^* y & |m|^2 + |y|^2|\phi_1|^2 \end{pmatrix}$$

- **At ϕ_1=0**

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & |m|^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & |m|^2|y|^2 & 0 & 0 & -ay \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & |m|^2 & 0 \\ 0 & 0 & -a^* y & 0 & 0 & |m|^2 \end{pmatrix}$$

The bosonic mass matrix

- At $\phi_1=0$

$$
= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & |m|^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & |m|^2|y|^2 & 0 & 0 & -ay \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & |m|^2 & 0 \\ 0 & 0 & -a^*y & 0 & 0 & |m|^2 \end{pmatrix}
$$

- Eigenvalues $2*0, 2*|m|^2$
- $|m|^2+|y||a|$, $|m|^2-|y||a|$

The bosonic mass matrix

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- At ϕ_1=0

$$
= \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & |m|^2 & 0 & 0 & 00 & \\
0 & 0 & |m|^2|y|^2 & 0 & 0 & -ay \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & |m|^2 & 0 \\
0 & 0 & -a^*y & 0 & 0 & |m|^2
\end{pmatrix}
$$

- Eigenvalues $2*0, 2*|m|^2$ $\longrightarrow$ match fermions
- $|m|^2+|y||a|$, $|m|^2-|y||a|$

~~SUSY~~ $\sim F\neq 0$

No fermionic counterparts with these masses

The Supertrace

a challenge for (tree level) ~~SUSY~~

The Supertrace

- Sum over bosonic and fermionic degrees of freedom,
- + for bosons, - for fermions

$$\mathrm{STr}(M^2) = \sum (-1)^{2s}(2s+1)m_s^2 = 0$$

Counts d.o.f.

The Supertrace in OR model

- Fermions: 2*(0, 2*|m|^2)
- Bosons: $2*0, 2*|m|^2,$

  $|m|^2+|y|^2|a|^2,\ |m|^2-|y|^2|a|^2$

➔ $$\mathrm{STr} M^2 = 0$$

($= 2 \times 0 + 2 \times |m|^2 + |m|^2 + |y||a| + |m|^2 - |y||a| - 2 \times 0 - 2 \times 2 \times |m|^2$)

Supertrace: In general

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$$m_F = W^{ij}$$

$$m_B^2 = \begin{pmatrix} W^{jk} W^*_{ik} & W^*_{ijk} W^k \\ W^{ijk} W^*_k & W^*_{jk} W^{ik} \end{pmatrix}$$

$$= \begin{pmatrix} m_F^\dagger m_F & W^*_{ijk} W^k \\ W^{ijk} W^*_k & m_F m_F^\dagger \end{pmatrix}$$

Supertrace: In general

$$m_F = W^{ij}$$

$$m_B^2 = \begin{pmatrix} W^{jk}W^*_{ik} & W^*_{ijk}W^k \\ W^{ijk}W^*_k & W^*_{jk}W^{ik} \end{pmatrix}$$

$$= \begin{pmatrix} m_F^\dagger m_F & W^*_{ijk}W^k \\ W^{ijk}W^*_k & m_F m_F^\dagger \end{pmatrix}$$

Only bits relevant for trace
Independent of ~~SUSY~~

~~SUSY~~

Supertrace: In general

$$m_F = W^{ij}$$

$$m_B^2 = \begin{pmatrix} W^{jk}W^*_{ik} & W^*_{ijk}W^k \\ W^{ijk}W^*_k & W^*_{jk}W^{ik} \end{pmatrix}$$

$$= \begin{pmatrix} m_F^\dagger m_F & W^*_{ijk}W^k \\ W^{ijk}W^*_k & m_F m_F^\dagger \end{pmatrix}$$

$$\rightarrow \mathrm{STr}M^2 = \mathrm{Tr}(m_F^\dagger m_F) + \mathrm{Tr}(m_F m_F^\dagger) - 2\mathrm{Tr}(m_F^\dagger m_F)$$

$$= 0 \;!!$$

The evil Supertrace!

- For conserved quantities
  particles can only mix if same "charges"

  ➔ Mass matrices block diagonal
  (with blocks of same "charge")

  ➔ $\mathrm{STr} M^2 = 0$ for each block!!!

The evil Supertrace II

- $\mathrm{STr}\, M^2 = 0$ for each "charge" multiplet

- Example:
  - electric charge =-1/3
  - Triplett under $SU(3)_c$

$$m^2_{\tilde{d}} + m^2_{\tilde{s}} + m^2_{\tilde{b}} = 2(m^2_d + m^2_s + m^2_b)$$

$$= 2(5\mathrm{GeV})^2$$

This is RULED OUT!

How to avoid the evil supertrace?

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- 4^{th} (or more) generation

$$m^2_{b_4} \gg (100\,\mathrm{GeV})^2$$

$$m^2_{\tilde{d}} + m^2_{\tilde{s}} + m^2_{\tilde{b}} + m^2_{\tilde{b}_4} = 2(m^2_d + m^2_s + m^2_b + m^2_{b_4})$$

➔

$$\gg (100\,\mathrm{GeV})^2$$

- Unnatural
- In trouble with LHC Higgs searches (see later)

How to avoid the evil supertrace?

- 4^{th} (or more) generation

- Loop level
  - Hidden Sector ~~SUSY~~
  - Strongly coupled

How to avoid the evil supertrace?

- 4^{th} (or more) generation

- Loop level
  - Hidden Sector ~~SU~~SY → Tomorrow
  - Strongly coupled
    (ugly, difficult)

R-symmetry

What is R-symmetry?

- Naively:
  - preserve SUSY
  - fermions and bosons of the same mutiplet transform in the same way under symmetry

What is R-symmetry?

- Naively:
  - preserve SUSY
  - fermions and bosons of the same mutiplet transform in the same way under symmetry

Not always True!!!!

➔ R-Symmetry

R-symmetry and the superpotential

- The Lagrangian needs to be invariant

$$\mathcal{L}_F = \int \mathrm{d}^2\theta\, W(\phi_i) + \int \mathrm{d}^2\bar{\theta}\, W^\dagger(\phi_i^\dagger)$$

- Transformation

$$W(\phi_i) \longrightarrow \exp(2i\alpha), \qquad \theta \longrightarrow \exp(i\alpha)$$

is a symmetry, R-symmetry $U(1)_R$

R-symmetry and the superpotential

- The Lagrangian needs to be invariant

$$\mathcal{L}_F = \int \mathrm{d}^2\theta\, W(\phi_i) + \int \mathrm{d}^2\bar{\theta}\, W^\dagger(\phi_i^\dagger)$$

- Transformation

$$W(\phi_i) \longrightarrow \exp(2i\alpha) W(\phi_i), \qquad d\theta \longrightarrow \exp(-i\alpha) d\theta$$

$$\theta \longrightarrow \exp(i\alpha)\theta$$

is a symmetry, R-symmetry $U(1)_R$

➔ Charge of W=2 charge of θ=1

R-symmetry, fermions and bosons

- Superfield of charge X

$$
\begin{aligned}
\phi(x,\theta,\bar\theta) &= \varphi(x) + \sqrt{2}\theta\psi(x) - i\theta\sigma^\mu\bar\theta\,\partial_\mu\varphi(x) + \frac{i}{\sqrt{2}}(\theta\theta)(\partial_\mu\psi(x)\sigma^\mu\bar\theta) \\
&- \frac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\partial^\mu\partial_\mu\varphi(x) - (\theta\theta)F(x)
\end{aligned}
$$

➔ Charge of boson =X

Charge of fermion =X-1

Back to OR model

OR model has an R-symmetry

- The superpotential needs R-charge =2

$$W_{\rm OR}(\phi_i) = -a\,\phi_1 + m\,\phi_2\phi_3 + \frac{y}{2}\phi_1\phi_3^2$$

➔ $[\phi_1]=2$, $[\phi_2]=2$, $[\phi_3]=0$ does the trick

➔ OR model has R-symmetry

OR model has an R-symmetry

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- The superpotential needs R-charge =2

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➔ $[\phi_1]=2$, $[\phi_2]=2$, $[\phi_3]=0$ does the trick

➔ OR model has R-symmetry

Note: If we impose additional Z_2 symmetry S
with $S\phi_1=\phi_1$, $S\phi_2=-\phi_2$, $S\phi_3=-\phi_3$, $SW=W$
➔ No other terms allowed in W

➔ "Generic" superpotential

The
Nelson-Seiberg Theorem

Nelson-Seiberg Theorem

- Assume generic superpotential

→ If we want ~~SUSY~~ (via F-terms)

  - R-symmetry is necessary
  - spontaneously broken R is sufficient

Nelson-Seiberg: Proof

- (1) Need a symmetry

- No symmetries

n equations $W_i = 0$

for n unknowns ϕ_i

➔ The eqs. are polynomial ➔ always solution

➔ SUSY unbroken

Nelson-Seiberg: Proof

- (2) Need an R-symmetry

- Non-R symmetry (continuous, global) with l generators
  ➔ W is (holomorphic) function of (n-l) fields

Example: U(1) symmetry

ϕ_i, charge q_i -> $X_i=\phi_i/\phi_n^{qi/qn}$

Superpotential uncharged ➔ ϕ_n drops out!
➔(n-1) X_i

Nelson-Seiberg: Proof

- (2) Need an R-symmetry

- Non-R symmetry (continuous, global) with l generators
  → W is (holomorphic) function of (n-l) fields

(n-l) equations $W_i = 0$
for (n-l) unknowns ϕ_i

→ The eqs. are polynomial → always solution
→ SUSY unbroken

Nelson-Seiberg: Proof

- (3) Spontaneously broken R-symmetry works
- U(1) R symmetry (continuous, global)

ϕ_i, charge q_i -> $X_i=\phi_i/\phi_n^{qi/qn}$

R is spontaneously broken → at least one $\phi_n \neq 0$

Superpotential has charge 2

→ $W=\phi_n^{2/qn} f(X_i)$

For SUSY to be unbroken → (n-1) eq. df/dX_i and $f=0$

→ n equations for (n-1) unkown X_i → no solution

→ ~~SUSY~~

Trouble with R-symmetry

Metastability I

The R-axion

- R-symmetry is an axial symmetry

- Why?

  Consider $W = h\phi_1\phi_2\Phi$ $R(\phi_1)=R(\phi_2)=0$, $R(\Phi)=2$

$$\mathcal{L} \supset h(\psi_1\Phi_B\psi_2 + h.c.) = h\bar{\psi}_R\Phi_B\psi_L + h.c.$$

$$\psi_1 = \bar{\psi}_R, \quad \psi_2 = \psi_L$$

R-charge for ψ_1 and ψ_2 =-1, Φ_B=2

$\Rightarrow$ $\bar{\psi}_R \to \bar{\psi}_R \exp(-i\alpha_R), \qquad \psi_L \to \exp(-i\alpha_R)\psi_L$

The R-axion

- R-symmetry is an axial symmetry
- Why?

  Consider $W = h\phi_1\phi_2\Phi$

$$\mathcal{L} \supset h(\psi_1\Phi_B\psi_2 + h.c.) = h\bar{\psi}_R\Phi_B\psi_L + h.c.$$

$$\psi_1 = \bar{\psi}_R, \quad \psi_2 = \psi_L$$

R(ψ_1)=R(ψ_2)=-1, R(Φ_B)=0, electric charge +1, -1, 0

➔ $\bar{\psi}_R \longrightarrow \bar{\psi}_R \exp(i\alpha_R), \qquad \psi_L \longrightarrow \exp(i\alpha_R)\psi_L$

$$\bar{\psi}_R \longrightarrow \bar{\psi}_R \exp(i\alpha_{el}), \qquad \psi_L \longrightarrow \exp(-i\alpha_{el})\psi_L$$

The R-axion

- Due to the axial anomaly the Goldstone direction a_R of an axial symmetry receives coupling term to two photons

$$\sim \frac{\alpha}{2\pi f_R}\left(\sum_{R-charged} Q^2_{electric}\right) a_R F^{\mu\nu}\tilde{F}_{\mu\nu}$$

Spontaneous R-breaking scale

The R-axion

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$$\sim \frac{\alpha}{2\pi f_R} \left(\sum_{R-charged} Q_R Q^2_{electric} \right) a_R F^{\mu\nu} \tilde{F}_{\mu\nu}$$

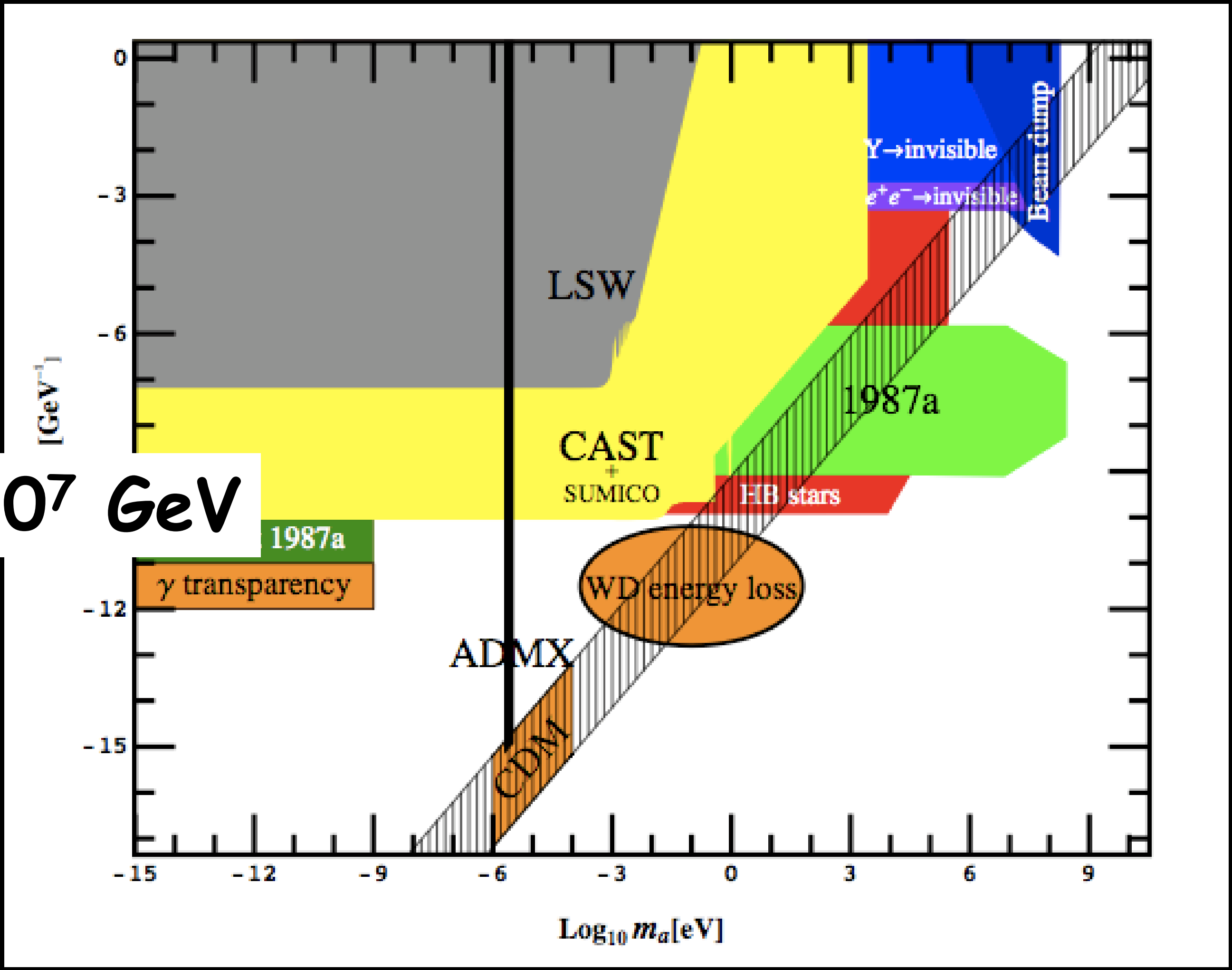
Spontaneous R-breaking scale

Similar interaction for gluons!

➔ Massless particle coupled to two photons/gluons

Strongly constrained

- For example two photon coupling…



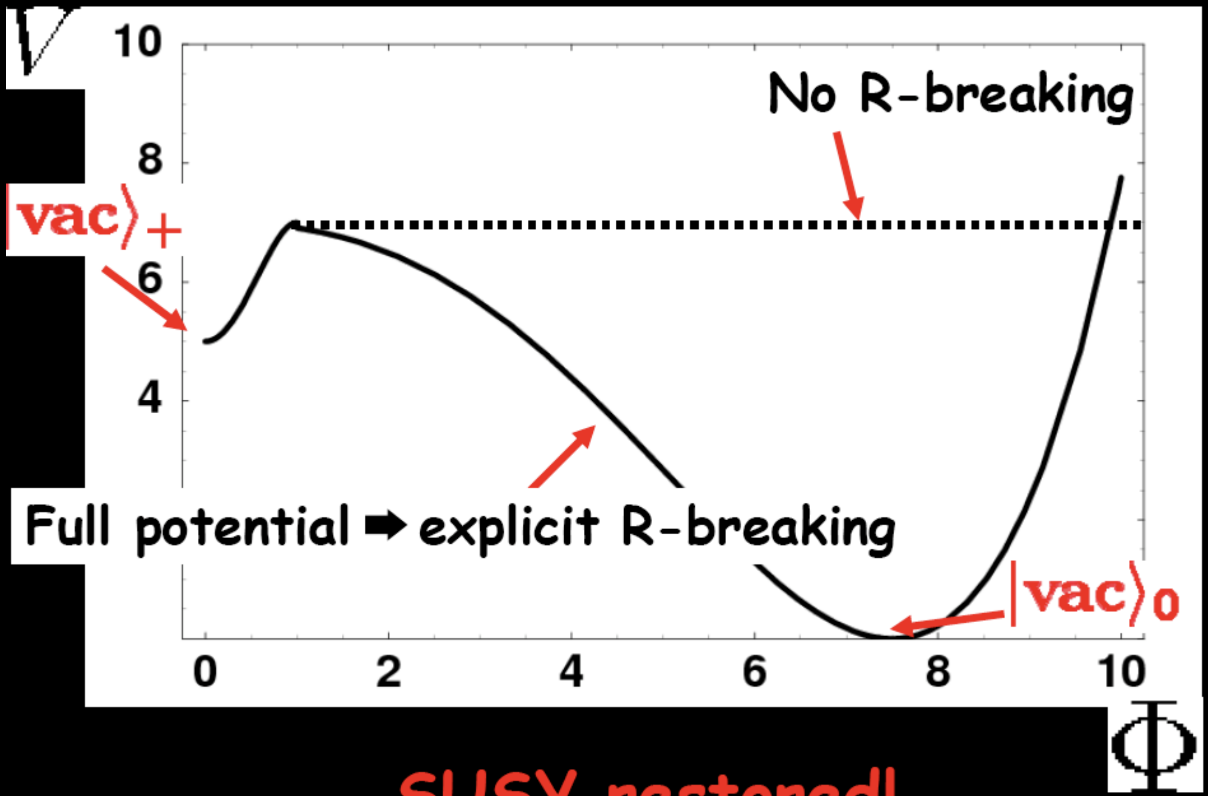
$f_R \sim 10^7$ GeV

More constraints from other couplings…

R-axion strongly constrained

➔ typically need (small) explicit R-breaking

➔ ~~SUSY~~ vacuum will be metastable



SUSY restored!

R-axion strongly constrained

➔ typically need (small) explicit R-breaking

➔ ~~SUSY~~ vacuum will be metastable

Alternative: No spontaneous R-breaking (does not work ➔ tomorrow)

R-axion strongly constrained

→ typically need (small) explicit R-breaking

→ ~~SUSY~~ vacuum will be metastable

Alternative: Gravity corrections (works → Bagger, Randall, Poppitz)

Summary II

Summary II

- O'Raifeartaigh model is a simple realization of SUSY
- STr(M^2)=0 even after ~~SUSY~~ (at tree-level)
  - ➔ Pheno problems
  - ➔ Need Loop level ~~SUSY~~ effects
- R-symmetry
  - transforms bosons and fermions differently
  - Needed for F-term SUSY breaking (Nelson-Seiberg)

  ➔ R-axion or metastability

~~SUSY~~ in the MSSM

The MSSM

in a nutshell

The MSSM

the matter bit

The minimal SUSY Standard Model

- The matter fields

	LHχSF	spin 0	spin $\frac{1}{2}$	$(SU(3), SU(2), U_Y(1))$
squarks and quarks	Q	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$(3, 2, \frac{1}{6})$
	U	$\tilde{u}_R^\dagger$	$u_R^\dagger$	$(\bar{3}, 1, -\frac{2}{3})$
	D	$\tilde{d}_R^\dagger$	$d_R^\dagger$	$(\bar{3}, 1, \frac{1}{3})$
sleptons and leptons	L	$(\tilde{\nu}, \tilde{e}_L)$	(ν, e_L)	$(1, 2, -\frac{1}{2})$
	E	$\tilde{e}_R^\dagger$	$e_R^\dagger$	$(1, 1, 1)$
higgs and higgsinos	H_u	(h_u^+, h_u^0)	$(\tilde{h}_u^+, \tilde{h}_u^0)$	$(1, 2, \frac{1}{2})$
	H_d	(h_d^0, h_d^-)	$(\tilde{h}_d^0, \tilde{h}_d^-)$	$(1, 2, -\frac{1}{2})$

Two Higgs fields

Why two Higgs fields?

- We want masses for up and down quarks

$$W_{\mathrm{MSSM}} = y_u\, U\, Q\, H_u - y_d\, D\, Q\, H_d$$

Y=-2/3 1/6 1/3 1/6

1/2 $\neq$ -1/2

Why two Higgs fields?

- We want masses for up and down quarks

$$W_{\rm MSSM} = y_u\, U\, Q\, H_u - y_d\, D\, Q\, H_d$$

Y=-2/3 1/6 1/3 1/6

1/2 $\neq$ -1/2

(holomorphicity forbids $H^\dagger$ in SUSY!)

- Also anomaly cancellation!

➔ Need two separate Higgs fields!

The superpotential

- Having this we obtain the MSSM superpotential

$$W_{\mathrm{MSSM}} = y_u\, U\, Q\, H_u - y_d\, D\, Q\, H_d - y_e\, E\, L\, H_d + \mu\, H_u\, H_d$$

so far no new parameters!!!

What we don't want…

- The SM symmetries allow more terms

$$W_{\not R} = \frac{1}{2}\lambda\, E\, L\, L + \lambda'\, D\, L\, Q + \mu'\, L\, H_u + \frac{1}{2}\lambda''\, U\, D\, D$$

violate Lepton #

violates Baryon #

➔ Evil proton decay

- Forbid using R-parity

$$R \equiv (-1)^{3B+L+2s}$$

The MSSM

the gauge bit

Gauge interactions

- Vector gauge superfield

$$V_{\mathrm{WZ}}(x,\theta,\bar\theta) = \theta\sigma^\mu\bar\theta\, v_\mu(x) + i(\theta\theta)\,\bar\theta\bar\lambda(x) - i(\bar\theta\bar\theta)\,\theta\lambda(x) + \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)\,D(x)$$

Gauge field　　gauginos　　D-term

- The super-derivative is a χSF

$$W_\alpha = -\frac{1}{4}(\bar D\bar D)\,D_\alpha V$$

Gauge interactions

- Vector gauge superfield

$$V_{\mathrm{WZ}}(x,\theta,\bar\theta) = \theta\sigma^\mu\bar\theta\, v_\mu(x) + i(\theta\theta)\,\bar\theta\bar\lambda(x) - i(\bar\theta\bar\theta)\,\theta\lambda(x) + \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)\,D(x)$$

Gauge field　　gauginos　　D-term

- The super-derivative is a χSF

$$W_\alpha = -\frac{1}{4}(\bar D\bar D)\,D_\alpha V$$

$$= -i\,\lambda_\alpha(y) - \theta\theta\,\sigma^\nu_{\alpha\dot\beta}\,\partial_\nu\bar\lambda^{\dot\beta}(y) - \frac{i}{2}\,\theta_\beta\,(\sigma^\mu\bar\sigma^\nu)_\alpha{}^\beta F_{\mu\nu}(y) + \theta_\alpha\,D(y)$$

The SUSY gauge Lagrangian

- W_α χSF

$$\mathcal{L} \supset \int d^2\theta \left[-\frac{1}{4} W^\alpha W_\alpha \right] + h.c.$$

$$= -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{i}{2} (\partial_\mu \lambda) \sigma^\mu \bar{\lambda} + \frac{i}{2} \lambda \sigma^\mu (\partial_\mu \bar{\lambda}) + \frac{1}{2} D^2$$

- V vector SF

$$\mathcal{L} \supset \int d^2\theta d^2\bar{\theta} \left[\phi_i^\dagger \exp(2gV)_{ij} \phi_j \right]$$

$$= g \phi_i^\dagger T^a_{ij} \phi_j D^a + \text{terms without } D$$

Integrate out D

- D has no kinetic term → integrate out

→ D-term scalar potential contribution

$$V_D = \frac{1}{2}\sum_a (D^a)^2 = \frac{1}{2}\sum_a \left[\sum_{i,j} g\phi_i^\dagger T^a_{ij}\phi_j\right]^2$$

A note on gaugino masses

- So far no gaugino mass term

  no surprise: SUSY partner of massless gauge bosons

- We could do Higgs mechanism

- But: Photino needs mass, too.

  → Need ~~SUSY~~ mass term

$$\sim \frac{1}{2} m \lambda \lambda$$

(Majorana) Gaugino masses break R-sym

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- ~~SUSY~~ Majorana gaugino mass

$$\sim \frac{1}{2} m \lambda \lambda$$

- W_α has R-charge 1

$$\mathcal{L} \supset \int d^2\theta \left[-\frac{1}{4} W^\alpha W_\alpha \right] + h.c.$$

- $W_\alpha \sim \lambda + \ldots$ ➔ λ has R-charge 1

➔ Gaugino masses need R-symmetry breaking (to Z_2 or less)

see yesterday

"The Mess"

soft ~~SUSY~~ terms in the MSSM

105 new parameters

- So far no additional parameters
- But here they come: soft-~~SUSY~~ terms

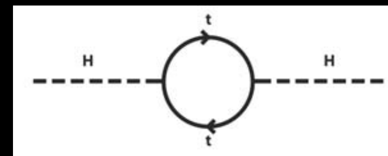
$$
\begin{aligned}
\mathcal{L}_{\rm soft} &= -\frac{1}{2}\left(M_1\,\widetilde{B}\widetilde{B} + M_2\,\widetilde{W}\widetilde{W} + M_3\,\tilde{g}\tilde{g}\right) + \text{h.c.} \\
&- m^2_{H_u}\,h_u^\dagger\,h_u - m^2_{H_d}\,h_d^\dagger\,h_d - (b\,h_u\,h_d + \text{h.c.}) \\
&- \left(a_u\tilde{u}_R\,\tilde{q}\,h_u - a_d\,\tilde{d}_R\,\tilde{q}\,h_d - a_e\,\tilde{e}_R\,\tilde{l}\,h_u\right) + \text{h.c.} \\
&- m^2_Q\,\tilde{q}^\dagger\,\tilde{q} - m^2_L\,\tilde{l}^\dagger\,\tilde{l} - m^2_u\,\tilde{u}^\dagger_R\,\tilde{u}_R - m^2_d\,\tilde{d}^\dagger_R\,\tilde{d}_R - m^2_e\,\tilde{e}^\dagger_R\,\tilde{e}_R
\end{aligned}
$$

3x3 matrices

What is soft?

- **Without SUSY**

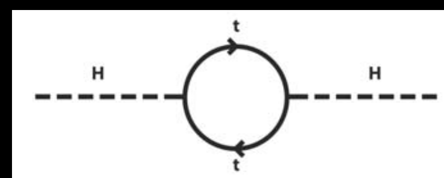
$$v_{EW}^2 \sim m_{H,tree}^2 +$$



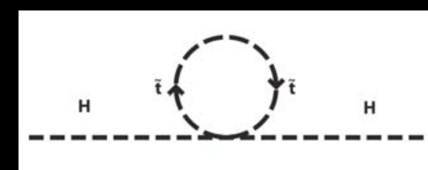
$$\sim \Lambda_{\mathrm{UV}}^2 \sim M_P^2$$

- **Unbroken SUSY**

$$v_{EW}^2 \sim m_{H,tree}^2 +$$



+



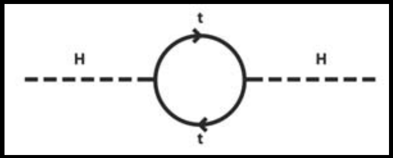
$$\sim -y^2 \left(\int d^4k \frac{1}{k^2 + m_F^2} - \int d^4k \frac{1}{k^2 + m_B^2} \right) \sim 0$$

➔ **Quadratic divergence cancels!**

What is soft?

- **Without SUSY**

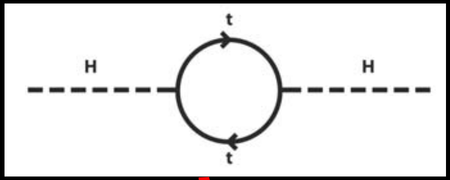
$$v_{EW}^2 \sim m_{H,tree}^2 +$$



$$\sim \Lambda_{UV}^2 \sim M_P^2$$

- **Unbroken SUSY**

$$v_{EW}^2 \sim m_{H,tree}^2 +$$



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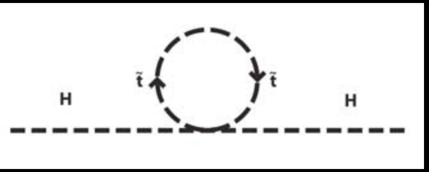
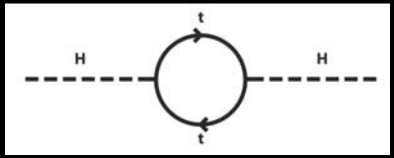


$$\sim -y^2 \left(\int d^4k \frac{1}{k^2+m_F^2} - \int d^4k \frac{1}{k^2+m_B^2} \right) \sim 0$$

- **Soft ~~SUSY~~**

$$v_{EW}^2 \sim m_{H,tree}^2 +$$



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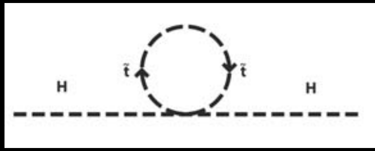


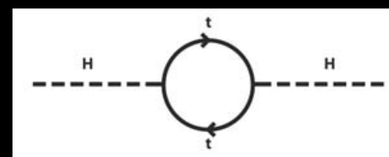
$$\sim -y^2 \left(\int d^4k \frac{1}{k^2+m_F^2} - \int d^4k \frac{1}{k^2+m_B^2} \right) \sim m_B^2 - m_F^2 \sim m_{soft}^2$$

→ Quadratic divergence still cancels

What is soft?

- Hard ~~SUSY~~

$$v_{EW}^2 \sim m_{H,tree}^2 +$$



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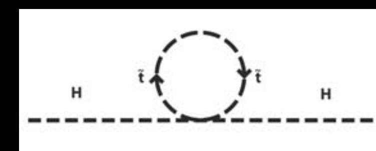

$$\sim -\left(y_F^2 \int d^4k \frac{1}{k^2 + m_F^2} - y_B^2 \int d^4k \frac{1}{k^2 + m_B^2} \right) \sim -(y_F^2 - y_B^2)\Lambda_{\rm UV}^2$$

Breaking in dimensionless coupling
➔ Quadratic divergence is back

What is soft

- In general one finds:

  Terms with positive mass dimension break SUSY softly.

Explaining “The Mess”

Mediating ~~SUSY~~ to the MSSM

105 new parameters – connecting to a model

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- Plenty soft-~~SUSY~~ terms

$$
\begin{aligned}
\mathcal{L}_{\text{soft}} &= -\frac{1}{2}\left(M_1\,\widetilde{B}\widetilde{B} + M_2\,\widetilde{W}\widetilde{W} + M_3\,\tilde{g}\tilde{g}\right) + \text{h.c.} \\
&- m_{H_u}^2\,h_u^\dagger\,h_u - m_{H_d}^2\,h_d^\dagger\,h_d - (b\,h_u\,h_d + \text{h.c.}) \\
&- \left(a_u\tilde{u}_R\,\tilde{q}\,h_u - a_d\,\tilde{d}_R\,\tilde{q}\,h_d - a_e\,\tilde{e}_R\,\tilde{l}\,h_u\right) + \text{h.c.} \\
&- m_Q^2\,\tilde{q}^\dagger\,\tilde{q} - m_L^2\,\tilde{l}^\dagger\,\tilde{l} - m_u^2\,\tilde{u}_R^\dagger\,\tilde{u}_R - m_d^2\,\tilde{d}_R^\dagger\,\tilde{d}_R - m_e^2\,\tilde{e}_R^\dagger\,\tilde{e}_R
\end{aligned}
$$

- But writing them down in not an explanation

- Want to connect to a proper ~~SUSY~~ model

Remember from yesterday: Supertrace

- $\mathrm{STr} M^2 = 0$ for each "charge" multiplet

- Example:
  - electric charge =-1/3
  - Triplett under $SU(3)_c$

$$m^2_{\tilde{d}} + m^2_{\tilde{s}} + m^2_{\tilde{b}} = 2(m^2_d + m^2_s + m^2_b)$$
$$= 2(5\mathrm{GeV})^2$$

This is RULED OUT!

Evading the Supertrace problem

$$\mathrm{STr}\, M^2 = 0$$ Tree-Level!!!

→ Generate soft-SUSY terms in the MSSM at loop-level

Evading the Supertrace problem

$$\mathrm{STr}M^2 = 0$$

Tree-Level!!!

→ Generate soft-SUSY terms in the MSSM at loop-level and only at loop level

Otherwise:
larger tree-level bit
fulfills $\mathrm{STr}M^2=0$
→ Some $m^2<0$ → vev → ~~color~~

Evading the Supertrace problem

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$$\mathrm{STr}\,M^2 = 0$$ Tree-Level!!!

➔ Generate soft-SUSY terms in the MSSM at loop-level and only at loop level

Otherwise:
larger tree-level bit
fulfills $\mathrm{STr}M^2=0$
➔ Some $m^2<0$ ➔ vev ➔ ~~color~~

Assumes perturbative!

Hidden sector ~~SUSY~~

Hidden SUSY-breaking sector

Interactions (messengers)

SUSY SM sector (MSSM)

Soft SUSY breaking terms in MSSM

Gauge mediation

Why gauge mediation?

Hidden SUSY-breaking sector

Flavour blind

$\longleftrightarrow$

Interactions (messengers)

SUSY SM sector (MSSM)

$\Rightarrow$ Soft SUSY breaking terms in MSSM

Gauge mediation

Hidden SUSY-breaking sector

Flavour blind

Interactions (messengers)

SUSY SM sector (MSSM)

Coupled via SM gauge interactions

Gauge mediation

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Hidden SUSY-breaking sector

Flavour blind

Interactions (messengers)

SUSY SM sector (MSSM)

Coupled via SM gauge interactions

Carry gauge charges!

Gauge mediation

University of Durham

Hidden SUSY-breaking sector

Flavour blind

Interactions (messengers)

SUSY SM sector (MSSM)

Couples (only) to messengers

Carry gauge charges!

Coupled via SM gauge interactions

Realizing gauge mediation

- Need ~~SUSY~~ model (hidden sector)

$$W(\Phi)_{\text{SUSYbreaking}} = -F\Phi$$

Realizing gauge mediation

- Need ~~SUSY~~ model (hidden sector)

$$W(\Phi)_{\text{SUSY breaking}} = -F\Phi$$

- Need messenger fields (gauged under SM group)

  ➔ Use two SU(5) fields $\tilde{f}, \; f$

Complete SU(5) mutiplets do not spoil unification!
(check as homework)

Realizing gauge mediation

- Need ~~SUSY~~ model (hidden sector)

$$W(\Phi)_{\text{SUSY breaking}} = -F\Phi$$

- Need messenger fields (gauged under SM group)

  ➔ Use two SU(5) fields $\tilde{f},\ f$

- Couple messenger to hidden sector ~~SUSY~~

$$W_{\text{couple}}(\Phi, \tilde{f}, f) = \lambda\Phi\tilde{f}f + M\tilde{f}f$$

SUSY breaking…

- Without coupling everything is great

$$\frac{\partial W_{\text{SUSY breaking}}(\Phi)}{\partial \Phi} = -F \neq 0$$

- R(Φ)=2 (has R-symmetry)

SUSY breaking… or not

- Without coupling everything is great

$$\frac{\partial W_{\text{SUSY breaking}}(\Phi)}{\partial \Phi} = -F \neq 0$$

- R(Φ)=2 (has R-symmetry)

- But $W_{\text{couple}}(\Phi, \tilde{f}, f) = \lambda \Phi \tilde{f} f + M \tilde{f} f$ breaks R explicitly

➔ Consider

$$\frac{\partial W_{\text{total}}(\Phi, \tilde{f}, f)}{\partial \Phi} = -F + \lambda \tilde{f} f, \qquad \frac{\partial W_{\text{total}}(\Phi, \tilde{f}, f)}{\partial \tilde{f}} = (M - \lambda\Phi) f$$

- Both vanish for

$$\Phi = \frac{M}{\lambda}, \qquad \tilde{f} = f = \sqrt{\frac{F}{\lambda}}$$

➔ SUSY vacuum exists

SUSY breaking… or not

➔ This problem is very common

Indeed if we want $m_{gaugino} \sim m_{sfermion}$ metastability is unavoidable (Komargodski + Shih)

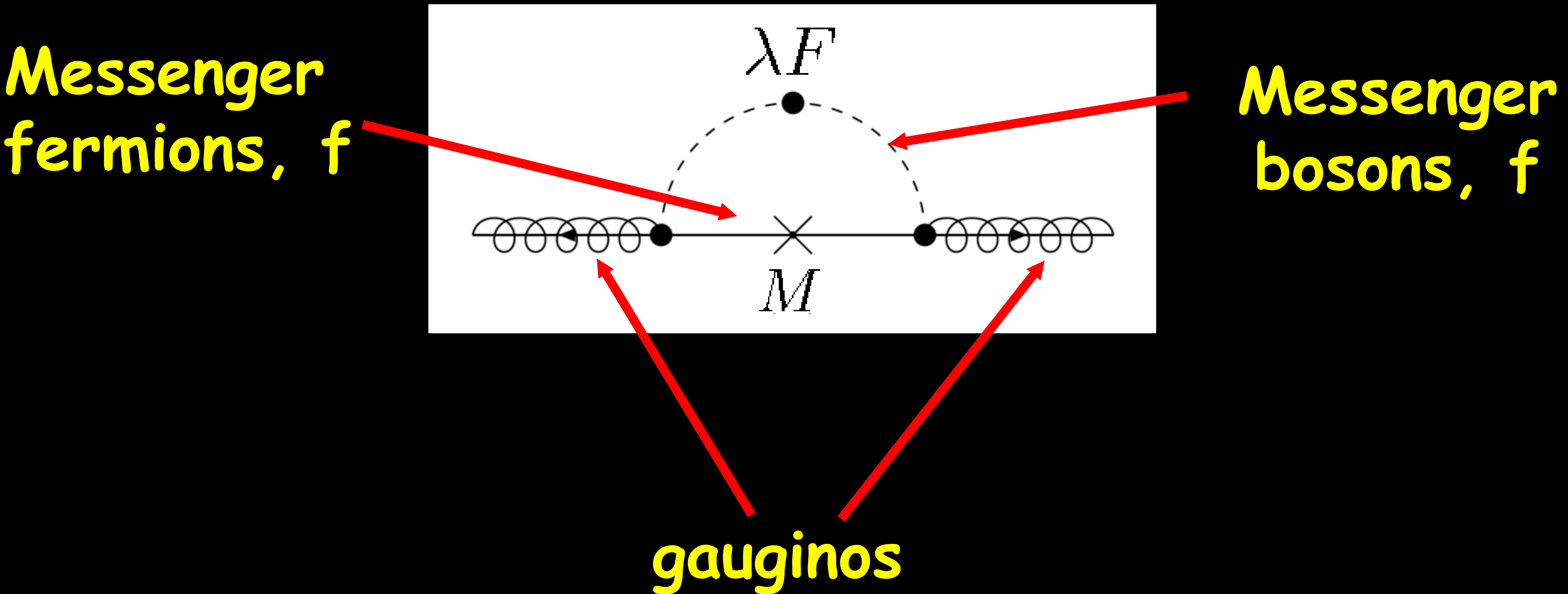
But it might be interesting to consider $m_{gaugino} << m_{sfermion}$!

➔ Accept meta-stability for now.

Gaugino masses

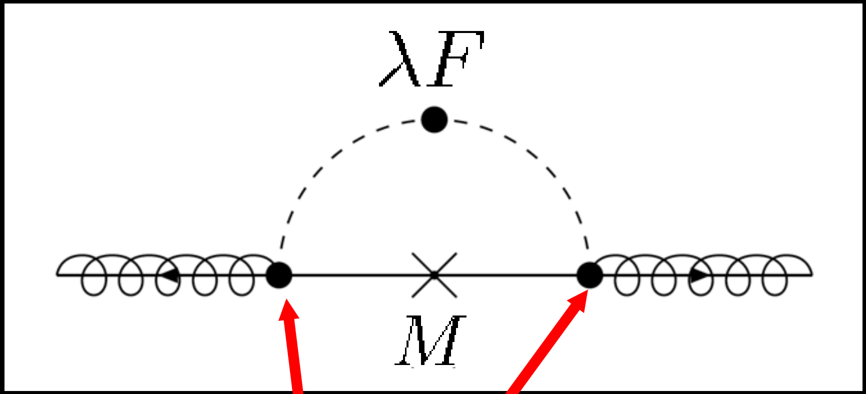
Generating gaugino masses

- Gaugino masses arise now from one loop diagrams



Generating gaugino masses

- Gaugino masses arise now from one loop diagrams

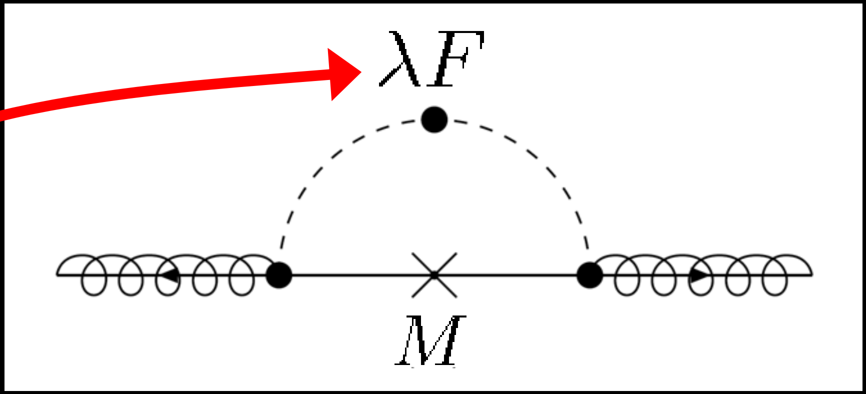


Messenger bosons, f

Gauge couplings

Generating gaugino masses

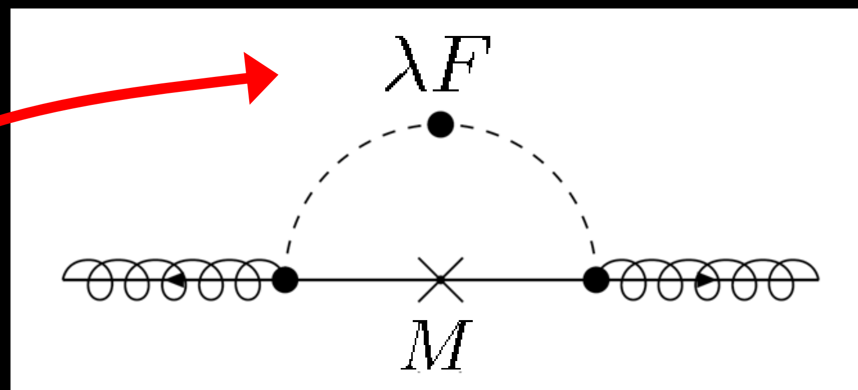
- Gaugino masses arise now from one loop diagrams



SUSY breaking changes boson masses

Generating gaugino masses

- Gaugino masses arise now from one loop diagrams



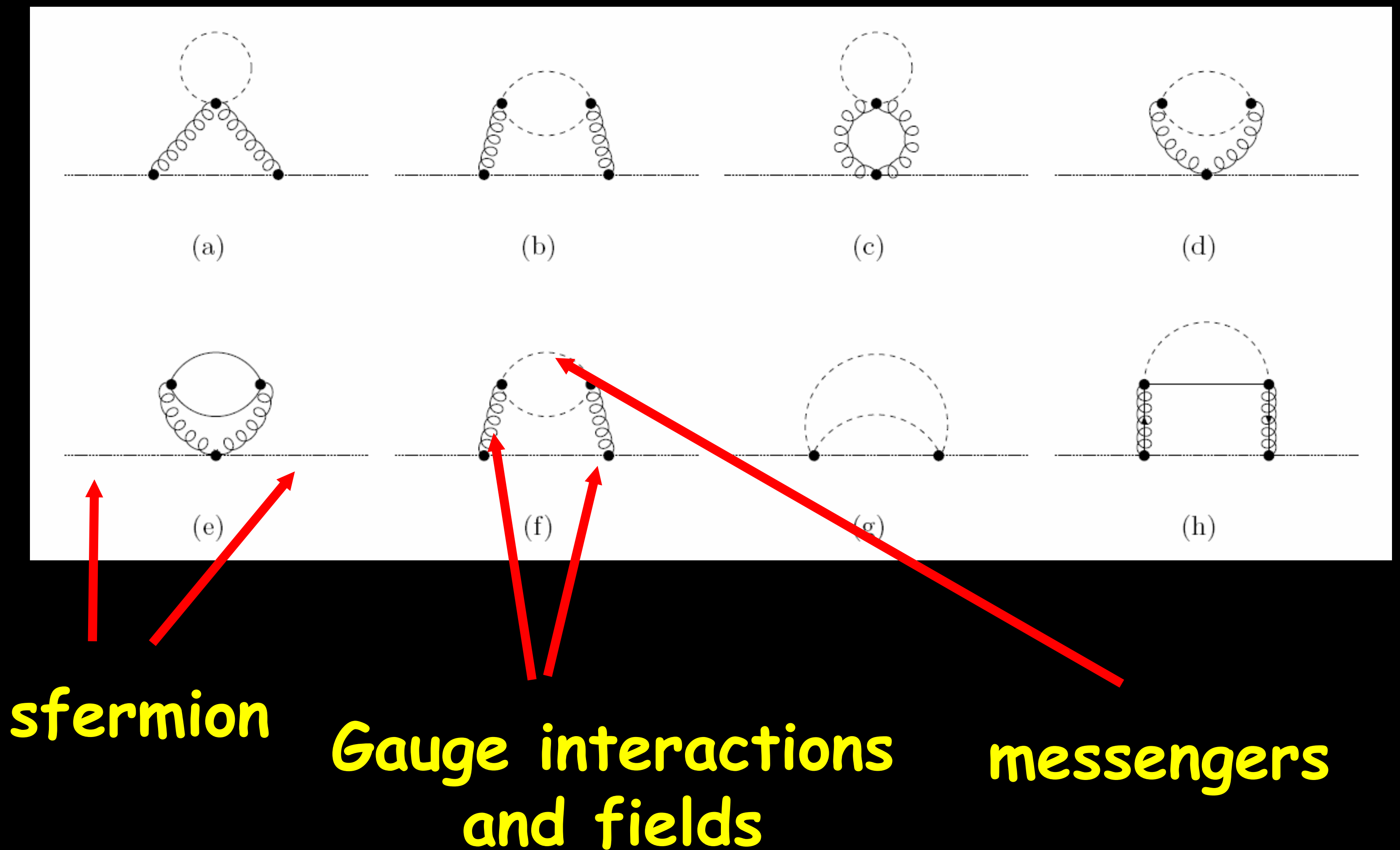
$$\rightarrow \quad m_{\lambda,r} = \frac{k_r \alpha_r}{4\pi} \frac{\lambda F}{M} \left[1 + \frac{1}{6} \frac{(\lambda F)^2}{M^4} + \ldots \right]$$

SUSY breaking

sfermion masses

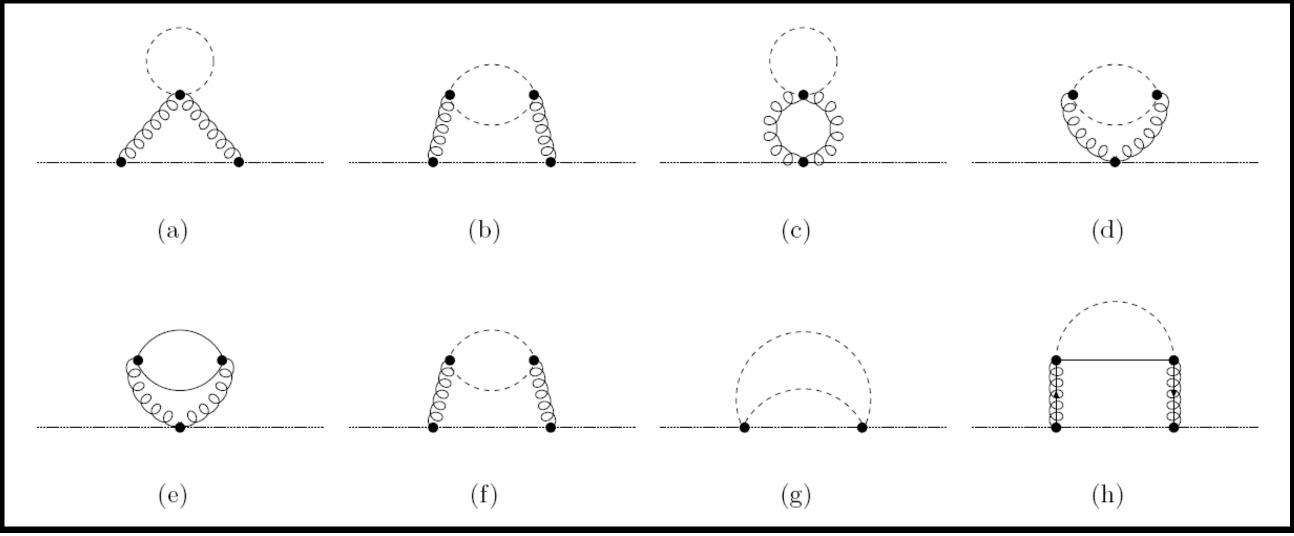
Generating sfermion masses

- sfermion masses arise now from two loop diagrams



Generating sfermion masses

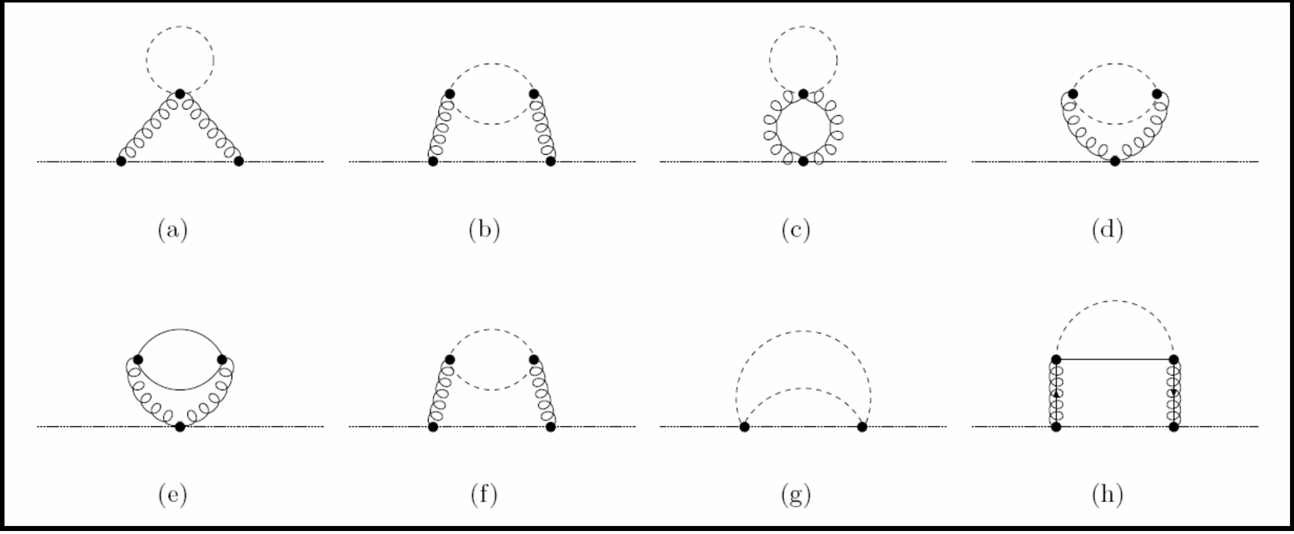
- sfermion masses arise now from two loop diagrams



$$m^2_{\text{sfermion}} = \sum_r C_r^{\text{sfermion}} k_r \frac{\alpha_r^2}{(4\pi)^2} \frac{\lambda F^2}{M^2} [1 + \ldots]$$

Generating sfermion masses

- sfermion masses arise now from two loop diagrams



$$m^2_{\mathrm{sfermion}} = \sum_r C_r^{\mathrm{sfermion}} k_r \frac{\alpha_r^2}{(4\pi)^2} \frac{(\lambda F)^2}{M^2} [1 + \ldots]$$

Mass squared

Two loop factor

Gaugino vs sfermion masses

- One-loop vs two loop
- Mass vs mass squared

➔ $$m_\lambda \sim m_{\mathrm{sfermion}} \sim \frac{\alpha}{4\pi}\frac{F}{M}$$

Gaugino and sfermion masses are of the same order
(in this simple model!!)

Mass patterns? Yes

- gaugino masses are related by gauge couplings

$$\frac{m_{\lambda,1}}{m_{\lambda,3}} = \frac{\alpha_1}{\alpha_3}$$

- Sfermion masses related by gauge coupling

$$\frac{m_{\tilde{e}_R}}{m_{\tilde{q}_R}} \sim \frac{\alpha_1}{\alpha_3}$$

Other soft terms?

All other soft terms

- For example a-terms

$$a_u^{ij} H_u Q^i \bar{u}^j + a_d^{ij} H_d Q^i \bar{d}^j + a_L^{ij} H_d L^i \bar{E}^j \quad ,$$

➔ Predicted to be small (at scale M) (arise only at higher loop level!)

Predicted all soft-terms?

- That's the nice thing about gauge mediation!

Gravity mediation

(in less than a nutshell)

Gravity mediation

- Soft terms arise from non-renormalizable Planck suppressed terms

$$
\begin{aligned}
\mathcal{L}_{\mathrm{NR}} \;=\; & -\frac{1}{M_{\mathrm{P}}} F \left(\frac{1}{2} f_a \lambda^a \lambda^a + \frac{1}{6} y'^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} \mu'^{ij} \phi_i \phi_j \right) + \text{c.c.} \\
& -\frac{1}{M_{\mathrm{P}}^2} F F^* \, k^i_j \phi_i \phi^{*j}
\end{aligned}
$$

Gravity mediation

- Soft terms arise from non-renormalizable Planck suppressed terms

$$\mathcal{L}_{\rm NR} = -\frac{1}{M_{\rm P}} F \left(\frac{1}{2} f_a \lambda^a \lambda^a + \frac{1}{6} y'^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} \mu'^{ij} \phi_i \phi_j \right) + \text{c.c.} \\ - \frac{1}{M_{\rm P}^2} F F^* \, k^i_j \phi_i \phi^{*j}$$

Gaugino masses

Sfermion masses

Gravity mediation

- Soft terms arise from non-renormalizable Planck suppressed terms

$$\begin{aligned}\mathcal{L}_{\rm NR} \;=\; & -\frac{1}{M_{\rm P}}F\left(\frac{1}{2}f_a\lambda^a\lambda^a+\frac{1}{6}y'^{ijk}\phi_i\phi_j\phi_k+\frac{1}{2}\mu'^{ij}\phi_i\phi_j\right)+\text{c.c.}\\ & -\frac{1}{M_{\rm P}^2}FF^*\,k^i_j\phi_i\phi^{*j}\end{aligned}$$

A-terms

Gravity mediation

- Soft terms arise from non-renormalizable Planck suppressed terms

$$\mathcal{L}_{\rm NR} = -\frac{1}{M_{\rm P}} F \left(\frac{1}{2} f_a \lambda^a \lambda^a + \frac{1}{6} y'^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} \mu'^{ij} \phi_i \phi_j \right) + \text{c.c.}$$
$$- \frac{1}{M_{\rm P}^2} F F^* \, k^i_j \phi_i \phi^{*j}$$

Gauge symmetry forbids for all but Higgses (comment tomorrow)

Gravity mediation

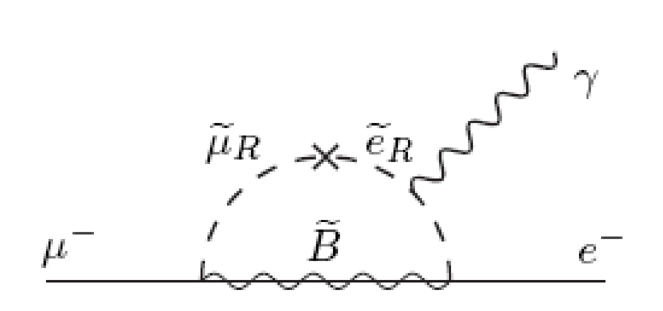
- Soft terms arise from non-renormalizable Planck suppressed terms

$$\mathcal{L}_{\rm NR} = -\frac{1}{M_{\rm P}} F \left(\frac{1}{2} f_a \lambda^a \lambda^a + \frac{1}{6} y'^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} \mu'^{ij} \phi_i \phi_j \right) + \text{c.c.} - \frac{1}{M_{\rm P}^2} F F^* \, k^i_j \phi_i \phi^{*j}$$

No reason to be diagonal in flavor space (gravity violated global symmetries)

→ New flavour changing processes

→ very EVIL!!!



Gravity mediation

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- Soft terms arise from non-renormalizable Planck suppressed terms

$$\mathcal{L}_{\rm NR} = -\frac{1}{M_{\rm P}} F \left(\frac{1}{2} f_a \lambda^a \lambda^a + \frac{1}{6} y'^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} \mu'^{ij} \phi_i \phi_j \right) + \text{c.c.}$$
$$-\frac{1}{M_{\rm P}^2} F F^* \, k^i_j \phi_i \phi^{*j}$$

Assume everything to be nicely diagonal
→ CMSSM, "minimal SuGra"

$$m_{1/2} = f \frac{\langle F \rangle}{M_{\rm P}}, \qquad m_0^2 = k \frac{|\langle F \rangle|^2}{M_{\rm P}^2}, \qquad A_0 = \alpha \frac{\langle F \rangle}{M_{\rm P}}, \qquad B_0 = \beta \frac{\langle F \rangle}{M_{\rm P}}$$

→ All soft-terms "predicted"

~~SUSY~~ scale

~~SUSY~~ scale

- In gauge mediation

$$m_\lambda \sim m_{\text{sfermion}} \sim \frac{\alpha}{4\pi}\frac{\lambda F}{M}$$

10^{-2}-10^{-3}

- Stability requires λ F>M²

~~SUSY~~ scale

- In gauge mediation

$$m_\lambda \sim m_{\text{sfermion}} \sim \frac{\alpha}{4\pi}\frac{\lambda F}{M} \gtrsim \text{few} \times 100\text{GeV}$$

10^{-2}-10^{-3}

- Stability requires $\lambda F > M^2$

➔ $$\frac{\lambda F}{M} \gtrsim 10^5\text{GeV}, \qquad M \gtrsim 10^5\,\text{GeV}$$

➔ ~~SUSY~~ @ New High scale >> TeV

~~SUSY~~ scale

- In gravity mediation

$$m_\lambda \sim m_{\text{sfermion}} \sim \frac{F}{M_P}$$

10^{19} GeV

➔ Even higher scale of ~~SUSY~~

$$F \sim (10^{11}\,\text{GeV})^2$$

Summary III

Summary III

- SUSY gauge interactions arise from Vector SF
- Gaugino masses need ~~R~~ + ~~SUSY~~

- Soft-SUSY breaking in the MSSM
  ~~SUSY~~ terms with positive mass dimension
  add 105 new parameters!!!!

- $\mathrm{STr}M^2=0$ → Hidden sector ~~SUSY~~

→ Need to mediate ~~SUSY~~
  - Gauge mediation → Calculate 105 parameters ✓
  - Gravity mediation…

~~SUSY~~ Phenomenology

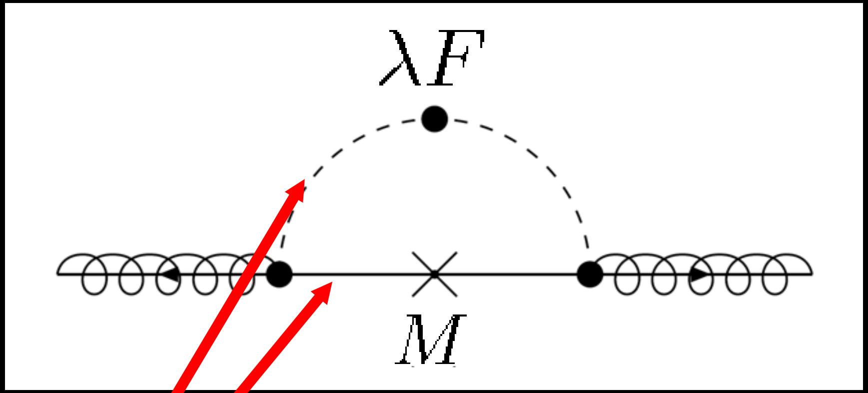
RG evolution

connecting to the TeV scale

Not at the TeV scale

- All soft terms predicted at some high scale M

- Why?

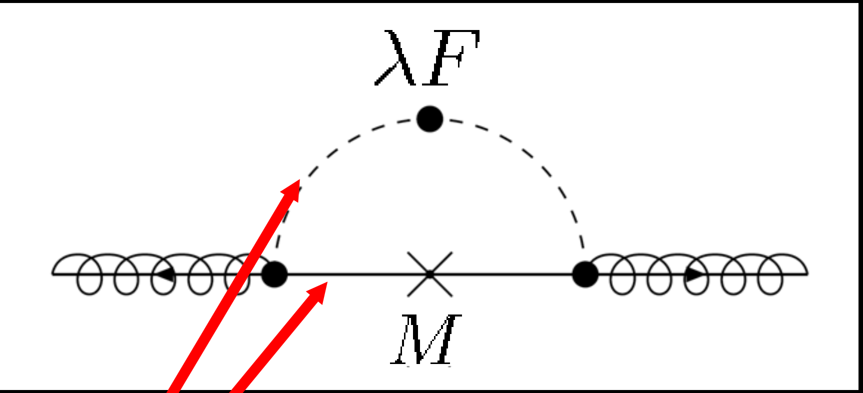


Fields in the loop have mass M

Not at the TeV scale

- All soft terms predicted at some high scale M

- Why?

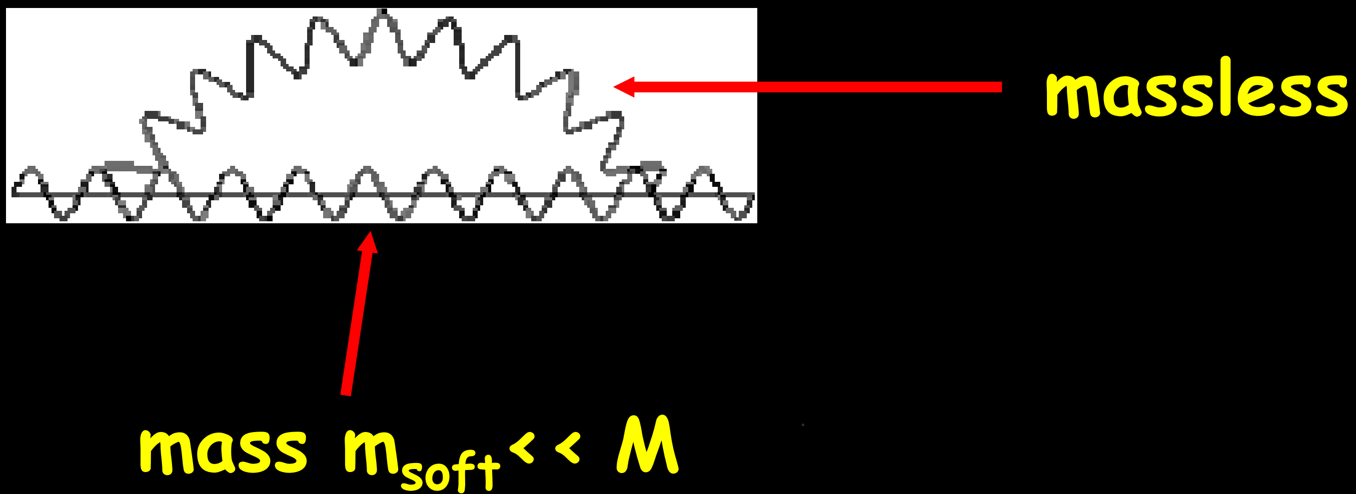


Fields in the loop have mass M

We have seen M> 10^5 GeV yesterday
even ~M_P in gravity mediation!!!

Need to do RG evolution

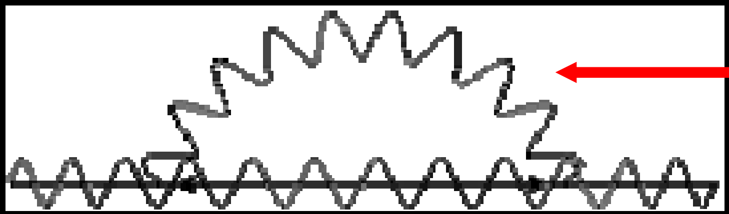
- Loop corrections



$$\beta_{m_{\lambda,a}} \equiv \frac{d}{dt} m_{\lambda,a} = \frac{1}{16\pi^2} g_a^2 \Big[2 \sum_n I_a(n) - 6 C_a(G) \Big] m_{\lambda,a}$$

Need to do RG evolution

- Loop corrections



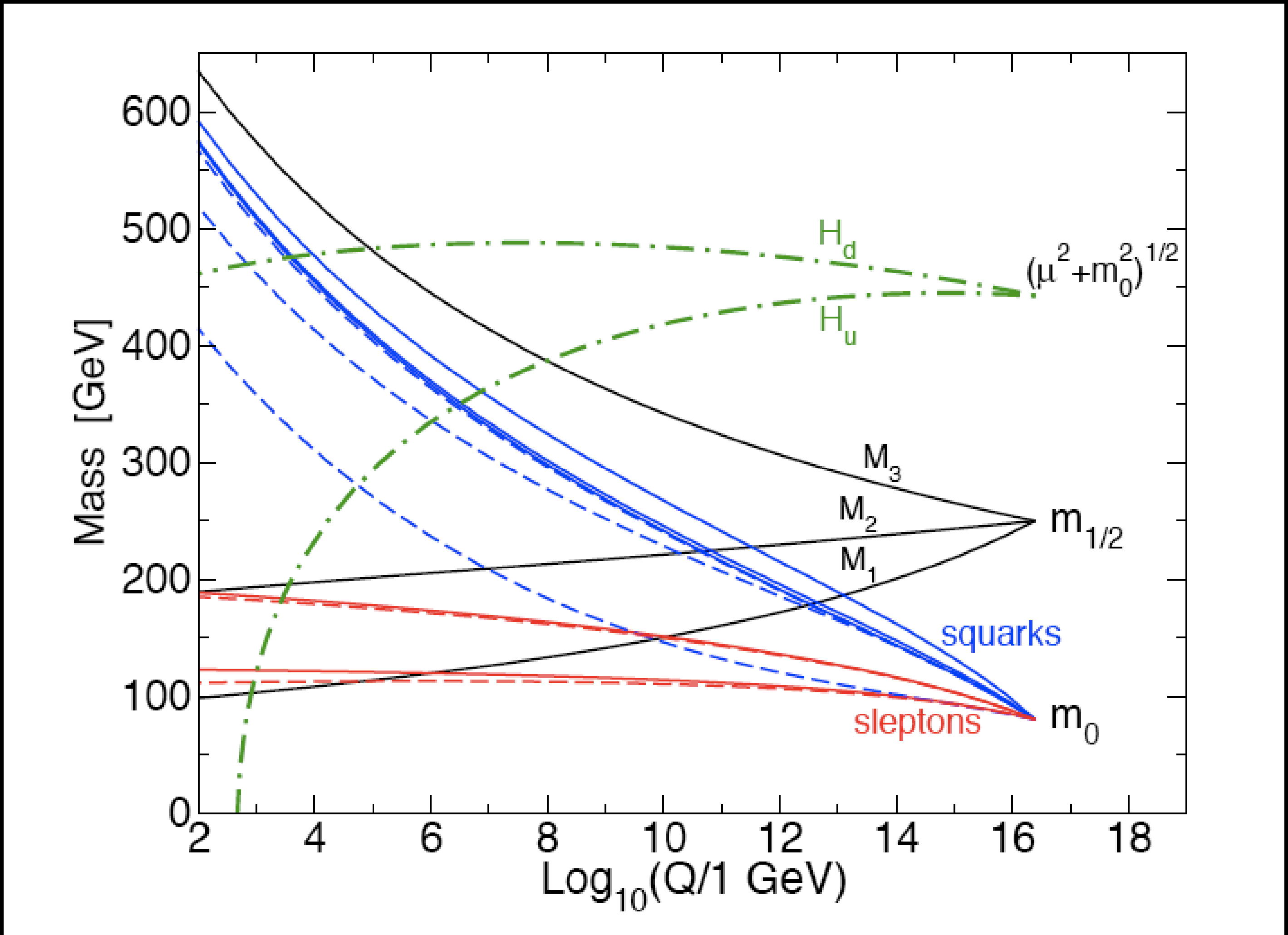
massless

mass $m_{soft} << M$

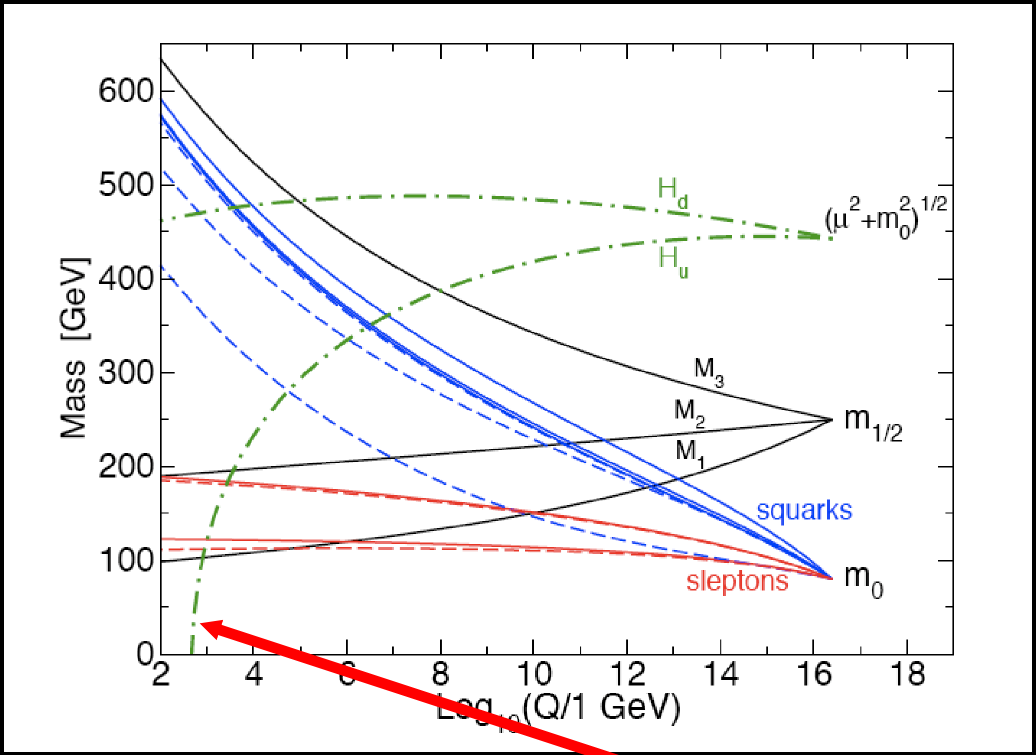
$$\beta_{m_{\lambda,a}} \equiv \frac{d}{dt} m_{\lambda,a} = \frac{1}{16\pi^2} g_a^2 \left[2 \sum_n I_a(n) - 6 C_a(G)\right] m_{\lambda,a}$$

Similar for all soft terms!

RG evolution is crucial



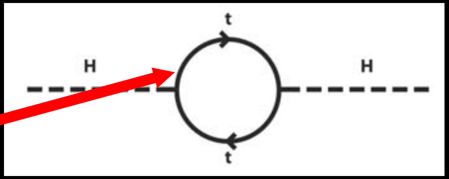
RG evolution breaks EW symmetry



(Up-type) Higgs mass turns negative
→ EW symmetry breaking!

Wins:

$m_t < m_{stop}$



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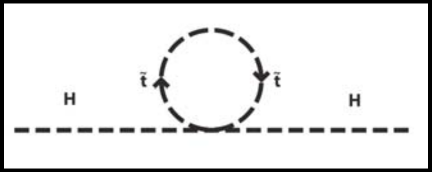

>0

One vev is not enough…

- $m^2_{H_u}<0$ → vev for h_u

$$W_{\mathrm{MSSM}} = y_u\, U\, Q\, H_u - y_d\, D\, Q\, H_d - y_e\, E\, L\, H_d + \mu\, H_u\, H_d$$

mass

- No mass for down type particles…

B_μ needed + tan(β)

- B_μ gives vev to h_d

$$m_u^2|H_u|^2 + m_d^2|H_d|^2 + (B_\mu H_u H_d + c.c.) \; ,$$

→ $$m^2_{H,real} = \begin{pmatrix} m_u^2 & B_\mu \\ B_\mu & m_d^2 \end{pmatrix}$$

h_u and h_d

→ Vev also for h_d

→ $$\tan(\beta) = \frac{v_u}{v_d}$$

Tan(β) determines down-Yukawas

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- Down-type masses

$\sim y_d\, v_d$

$\sim y_d\, v_u/\tan(\beta)=\text{fixed}$

➔ $$y_d \sim \tan(\beta)$$

➔ large tan(β) ➔ large down Yukawas!

An example

Pure GGM

(sorry for the bias)

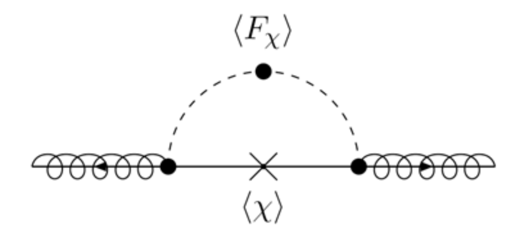
Pure General Gauge Mediation

- Want to construct simple but theoretically well motivated setup to study phenomenology of gauge mediation
  - Few parameters
  - Capturing essential features of a large class of models

- Construct Gauge Mediation analog of CMSSM!

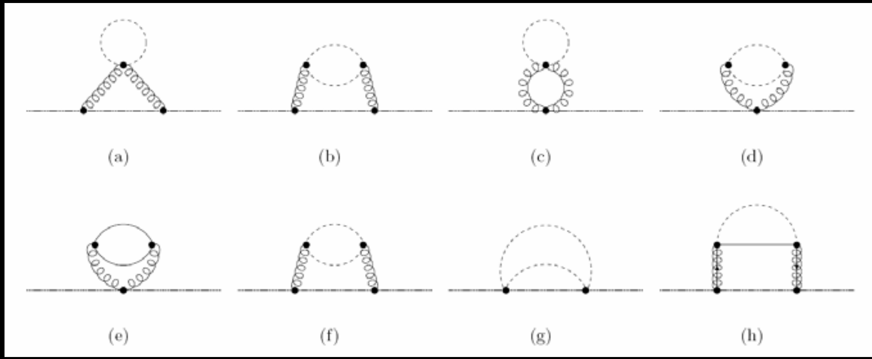
Pure General Gauge Mediation

In general gauge mediation we have



$$M_{\tilde{\lambda}_i}(M_{mess}) = k_i \frac{\alpha_i(M_{mess})}{4\pi} \Lambda_{G,i}$$

$$m^2_{\tilde{f}}(M_{mess}) = 2 \sum_{i=1}^{3} C_i k_i \frac{\alpha_i^2(M_{mess})}{(4\pi)^2} \Lambda^2_{S,i}$$



Pure General Gauge Mediation

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In general gauge mediation we have

$$M_{\tilde{\lambda}_i}(M_{mess}) = k_i \frac{\alpha_i(M_{mess})}{4\pi} \Lambda_G$$

$$m^2_{\tilde{f}}(M_{mess}) = 2 \sum_{i=1}^{3} C_i k_i \frac{\alpha_i^2(M_{mess})}{(4\pi)^2} \Lambda_S^2$$

Assume unification and unsplit multiplets

Pure General Gauge Mediation

- In general gauge mediation we have

$$M_{\tilde{\lambda}_i}(M_{mess}) = k_i \frac{\alpha_i(M_{mess})}{4\pi} \Lambda_G$$

$$m^2_{\tilde{f}}(M_{mess}) = 2 \sum_{i=1}^{3} C_i k_i \frac{\alpha_i^2(M_{mess})}{(4\pi)^2} \Lambda_S^2$$

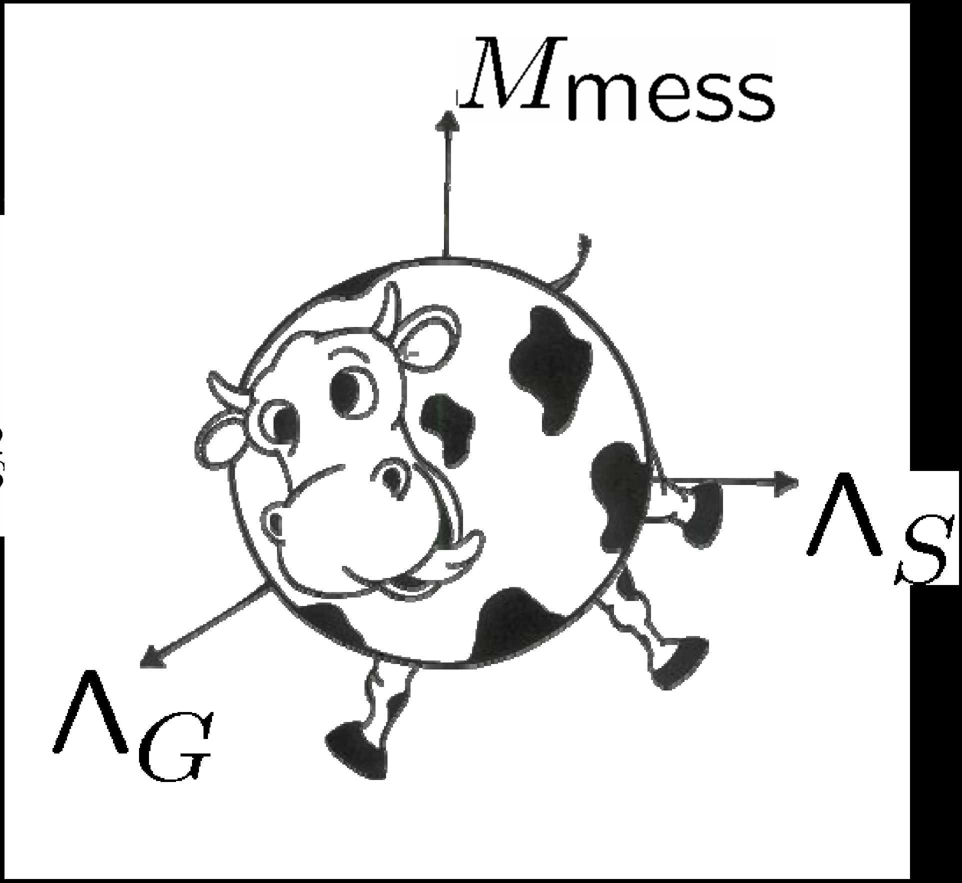
But keep $\Lambda_G \neq \Lambda_S$ as
predicted in many explicit models

Pure General Gauge Mediation

➔ We have a simple setup with three parameters, Λ_G, Λ_S, M_{mess}

$$M_{\tilde{\lambda}_i}(M_{mess}) = k_i \frac{\alpha_i(M_{mess})}{4\pi} \Lambda_G$$

$$m^2_{\tilde{f}}(M_{mess}) = 2 \sum_{i=1}^{3} C_i k_i \frac{\alpha_i^2(M_{mess})}{(4\pi)^2} \Lambda_S^2$$



What about a-terms?

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Soft terms include:

$$a_u^{ij} h_u Q^i \bar{u}^j + a_d^{ij} h_d Q^i \bar{d}^j + a_L^{ij} h_d L^i \bar{E}^j \quad ,$$

Are predicted in GGM to be small at M_{mess}

The (soft)Higgs sector and B_μ

$$m_u^2|h_u|^2 + m_d^2|h_d|^2 + (B_\mu h_u h_d + c.c.) \ ,$$

$$= m_L^2 + \mu^2$$

$$= 0$$

@M_{mess}

The (soft)Higgs sector and B_μ

$$m_u^2|h_u|^2 + m_d^2|h_d|^2 + (B_\mu h_u h_d + c.c.) \ ,$$

$$= m_L^2 + \mu^2 \qquad\qquad = 0 \quad @M_{mess}$$

Need Higgs vev, i.e. $m^2_u<0$

B_μ needed to give vev to H_d
(and masses to down-type particles)???

The (soft)Higgs sector and B_μ

$$m_u^2|h_u|^2 + m_d^2|h_d|^2 + (B_\mu h_u h_d + c.c.) \ ,$$

< 0 $\qquad\qquad$ $\neq 0$ $\qquad$ @M_{EW}

$B_\mu \neq 0$ generated by RG evolution to M_{EW}

B_μ typically remains small → Large $\tan\beta$

What about μ?

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- SUSY Higgs term:

$$\mathcal{L}_{eff} \supset \int d^2\theta \; \mu \, H_u H_d$$

→ Not necessarily connected to SUSY breaking

→ Determine from EW symmetry breaking

→ Accept finetuning

Importance of

The (N)LSP

The LSP determines pheno

If R-parity is exact

- The lightest super-particle (LSP) is stable
- Everything decays eventually to LSP!

In gauge mediation:

goldstino=gravitino is LSP

Gravity gives mass to

(more precisely the goldstino is “eaten” by the gravitino and becomes the longitudinal component of the gravitino)

The Next-to-LSP determines pheno

- Gravitino=goldstino has suppressed interactions

$$\mathcal{L} = \frac{1}{F_0}\left((m_f^2 - m_{\tilde{f}}^2)\bar{f}_L\tilde{f} + \frac{M_{\tilde{\lambda}_i}}{4\sqrt{2}}\bar{\tilde{\lambda}}_i\sigma^{\mu\nu}F^i_{\mu\nu}\right)\tilde{G} + h.c.$$

SUSY F-term (scale of symm. Breaking)

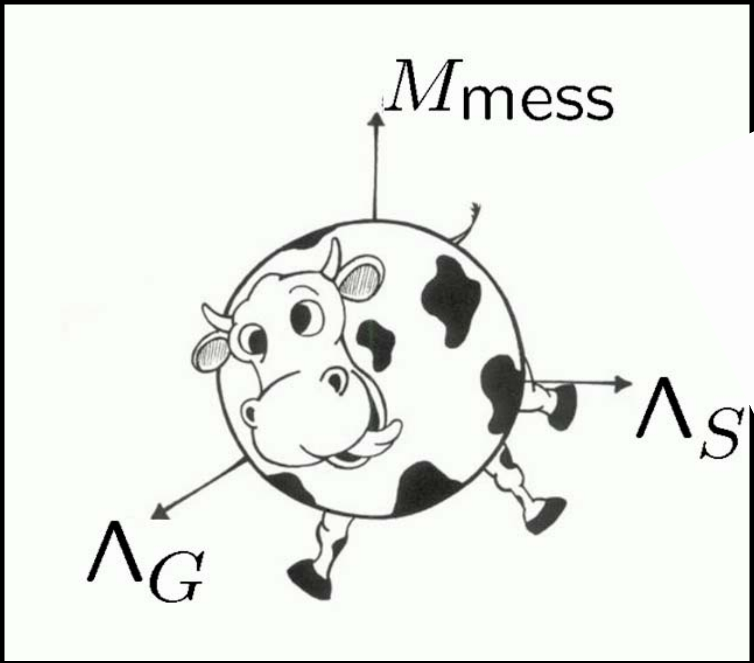
➔ higher F ➔ weaker gravitino interactions

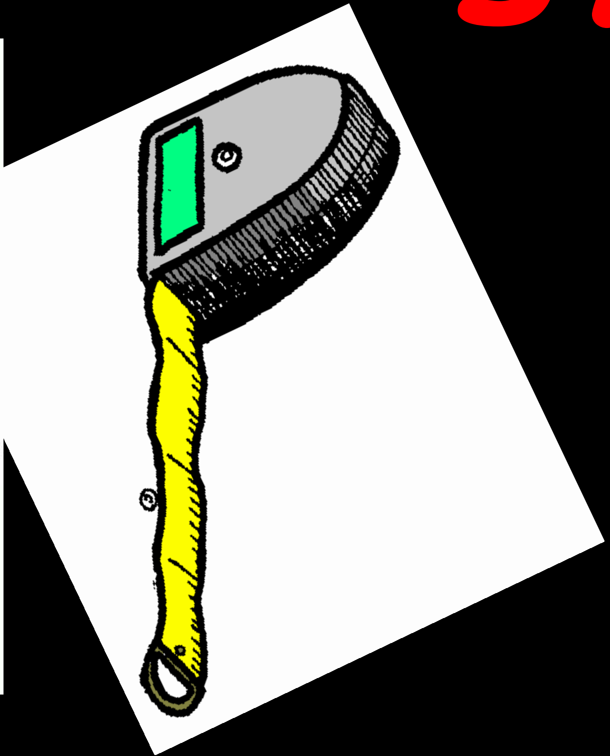
➔ NLSP->gravitino(=LSP)+X decay slow

➔ NLSP has longish livetime

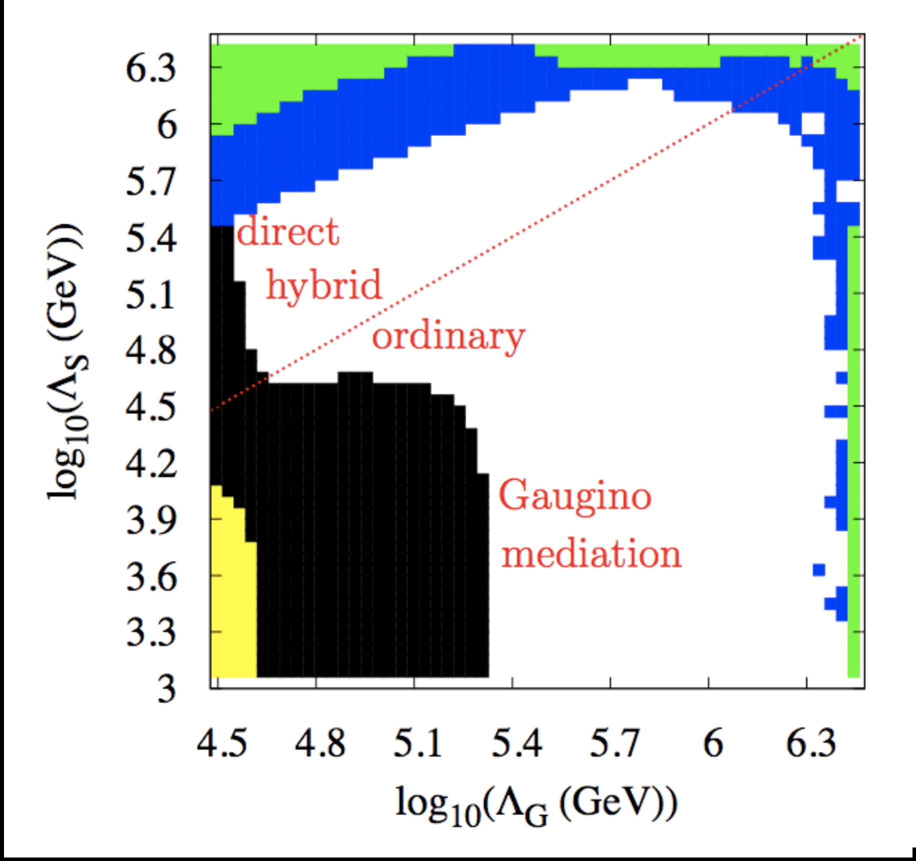
➔ All decays pass through NLSP to LSP

Pre LHC Phenomenology

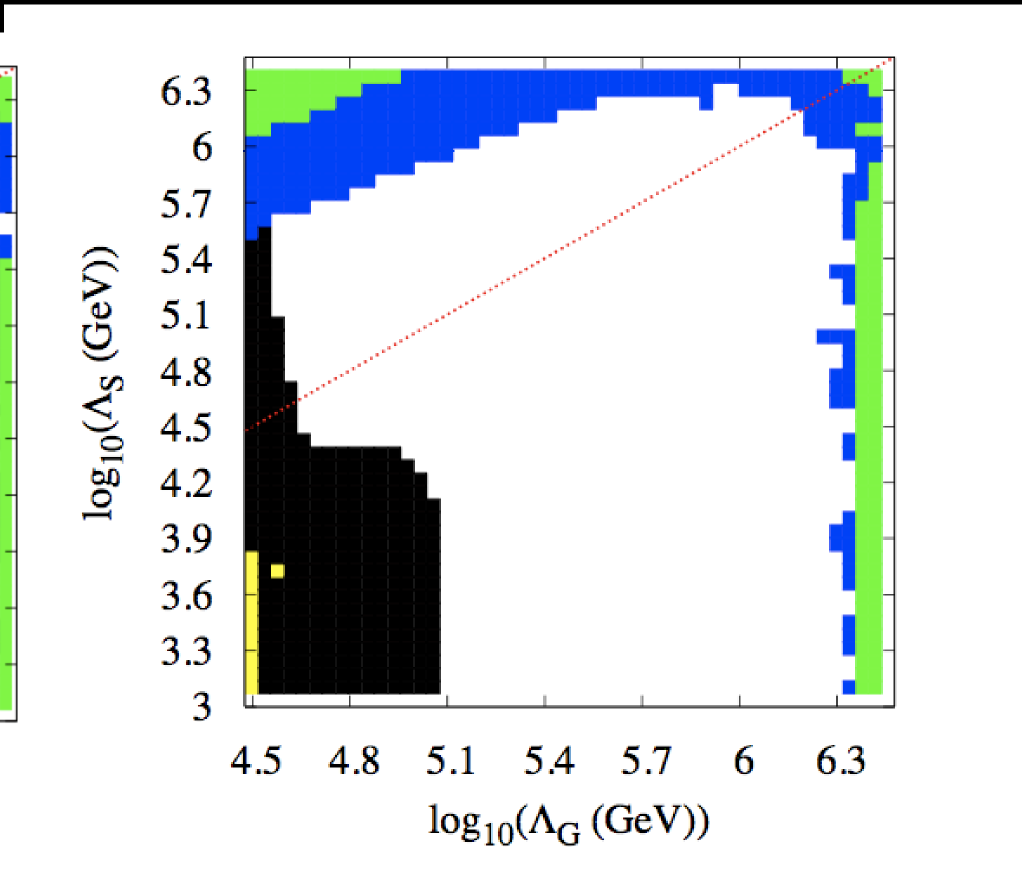




The pure GGM parameter space

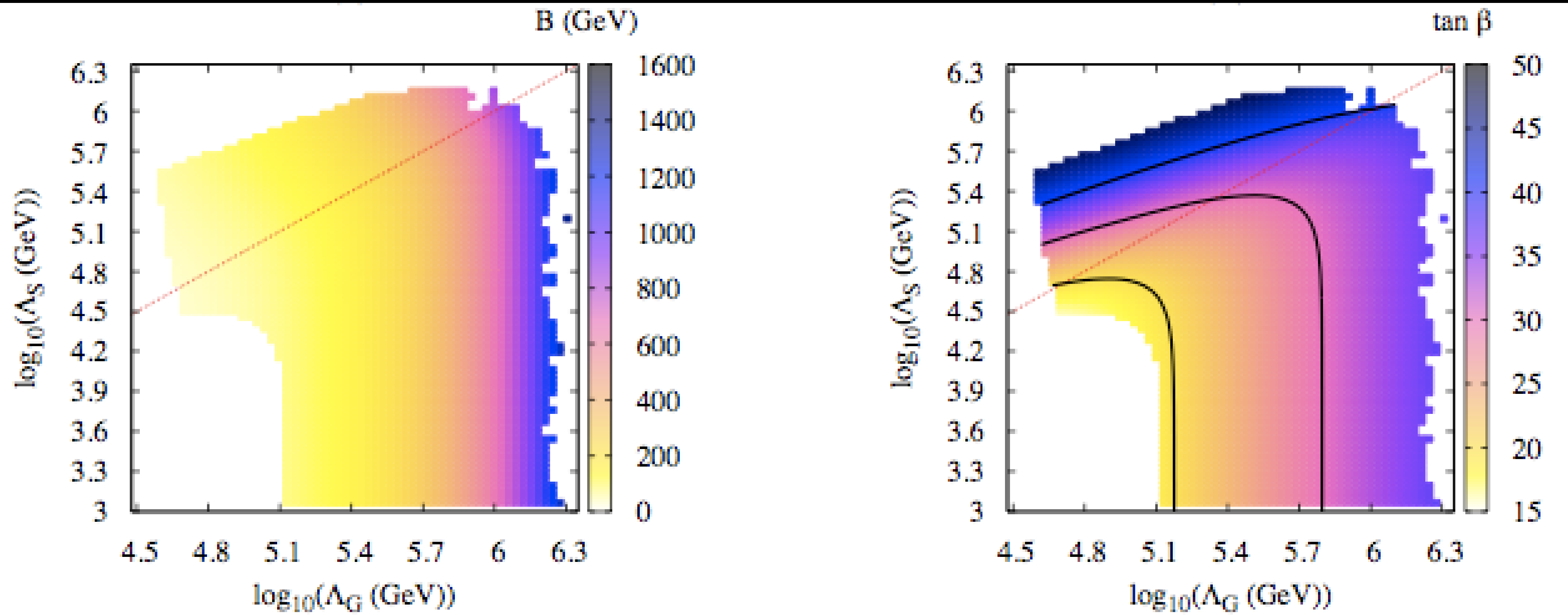


$M_{mess}=10^{10}$ GeV



$M_{mess}=10^{14}$ GeV

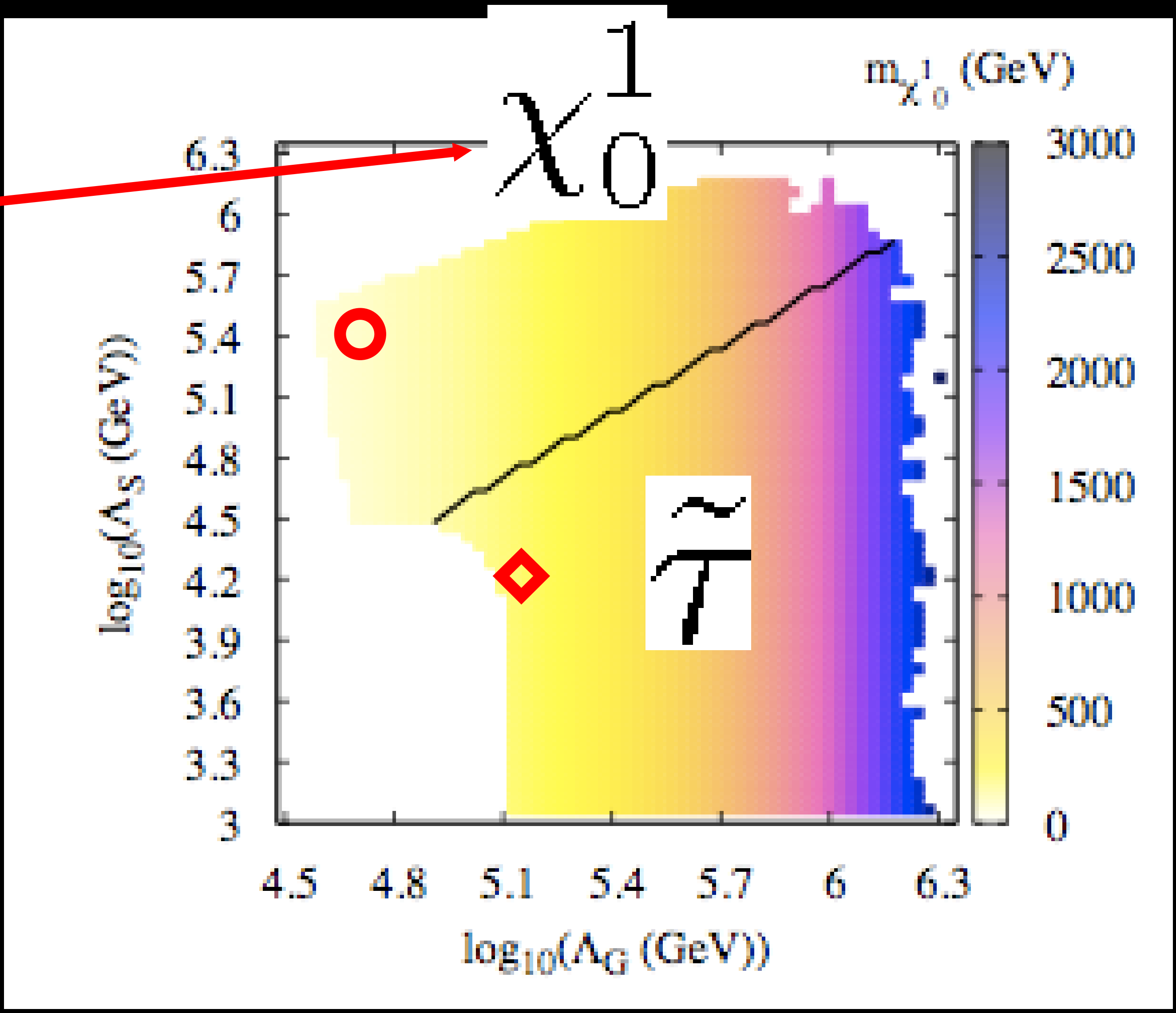
Large(ish) tanβ



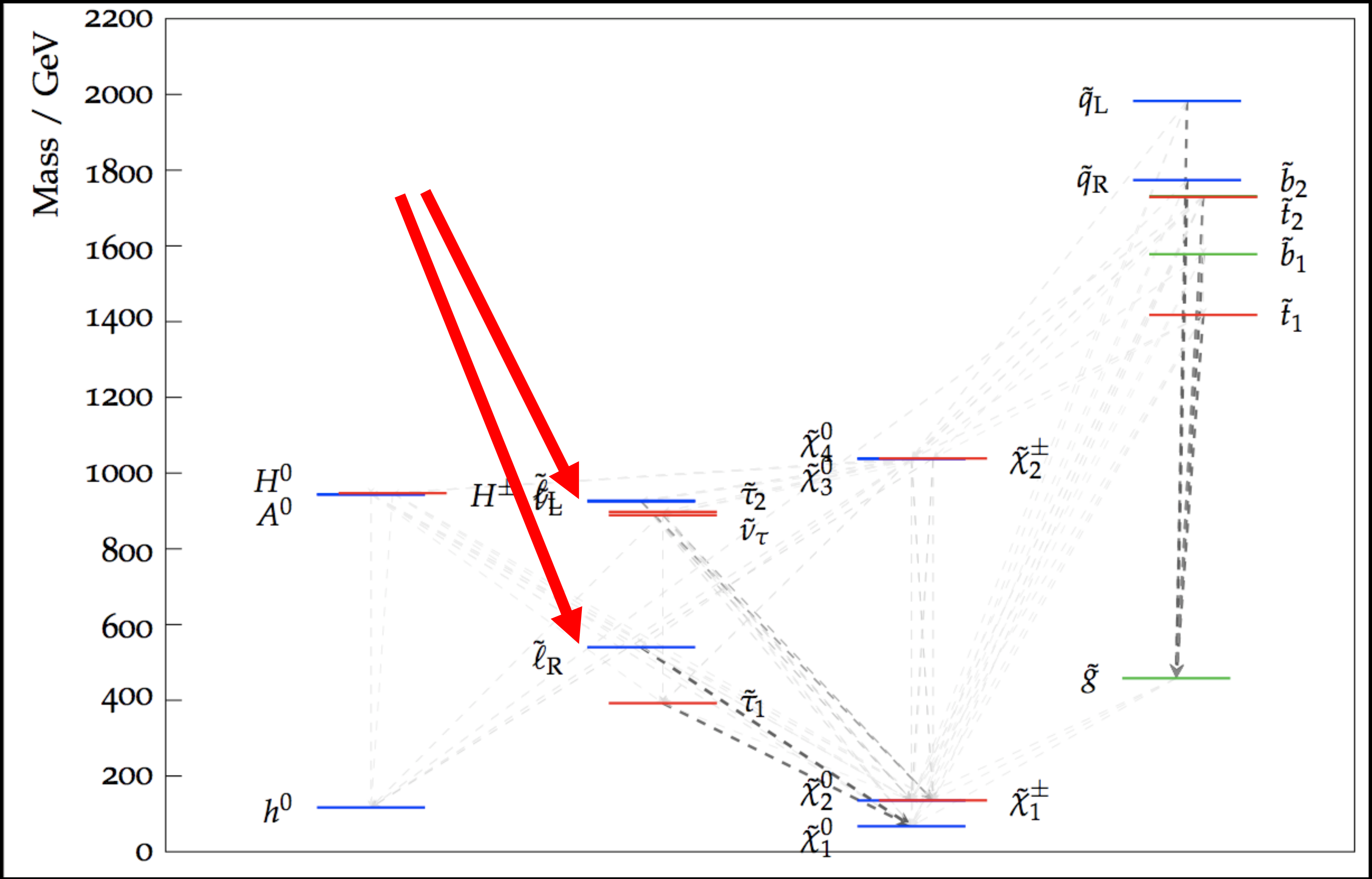
$M_{mess}=10^{14}$ GeV

NLSP

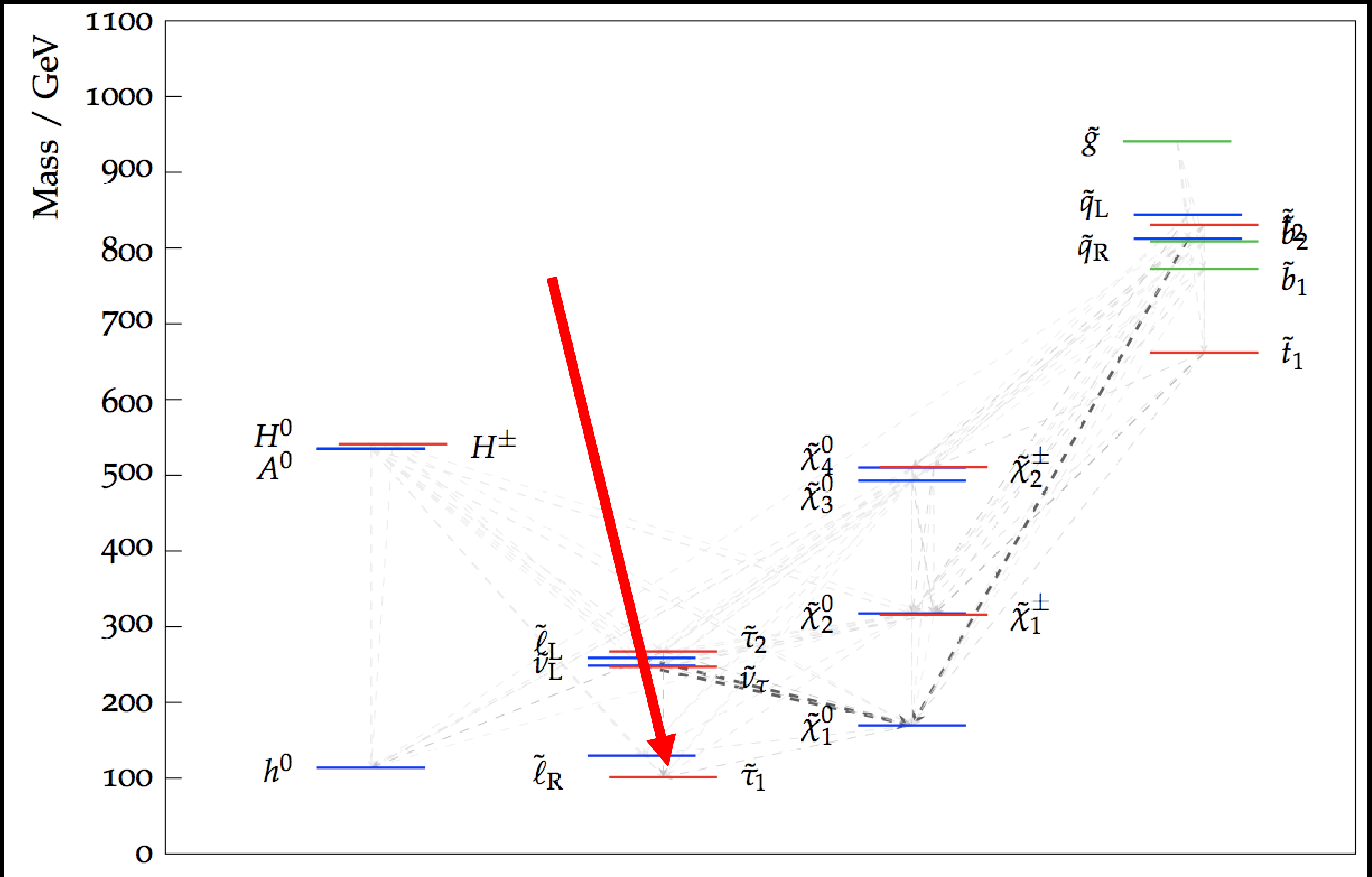
neutral combination of bino, zino, higgsino



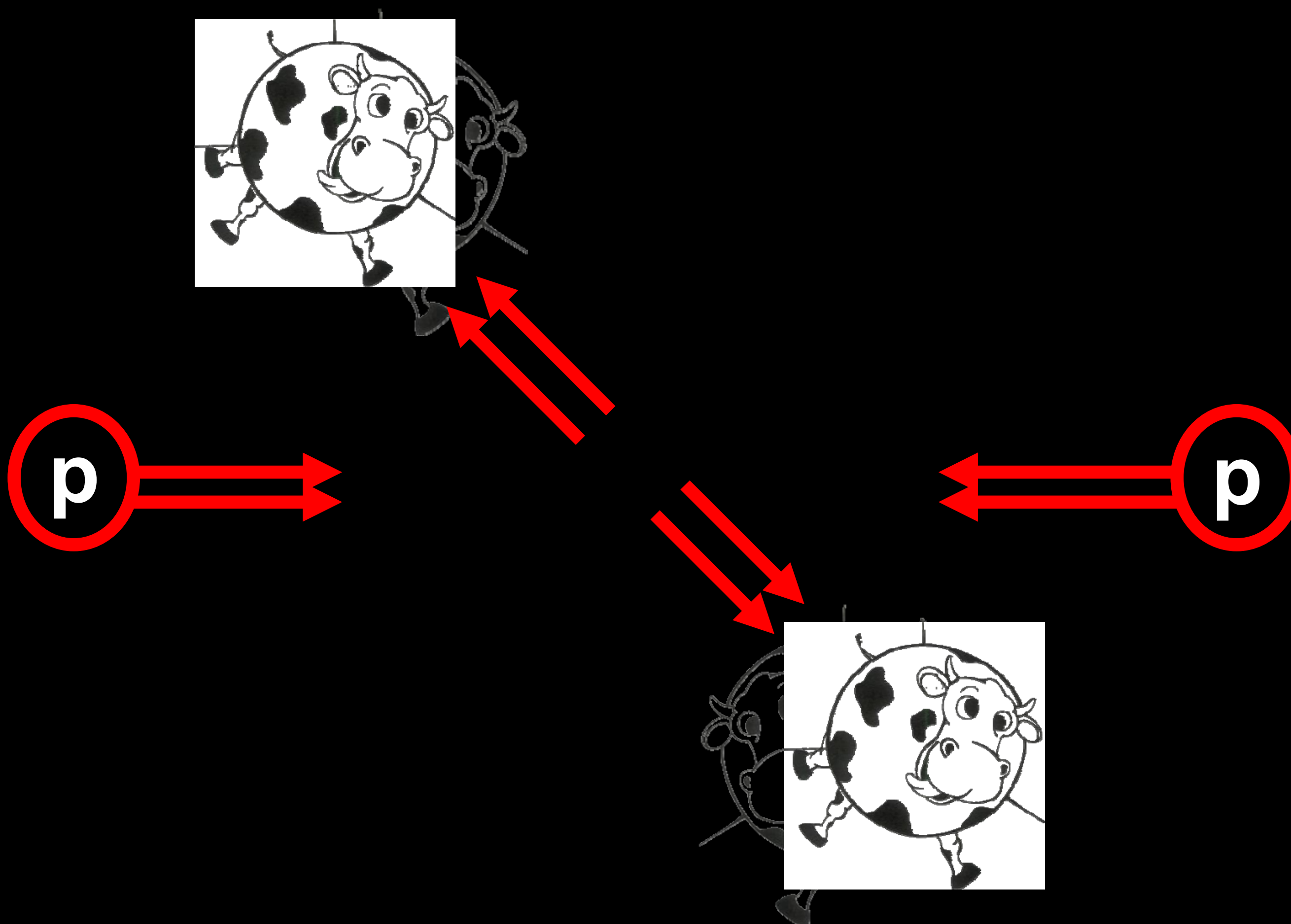
Typical spectrum



Typical spectrum



LHC
First Data



What kind of SUSY signal

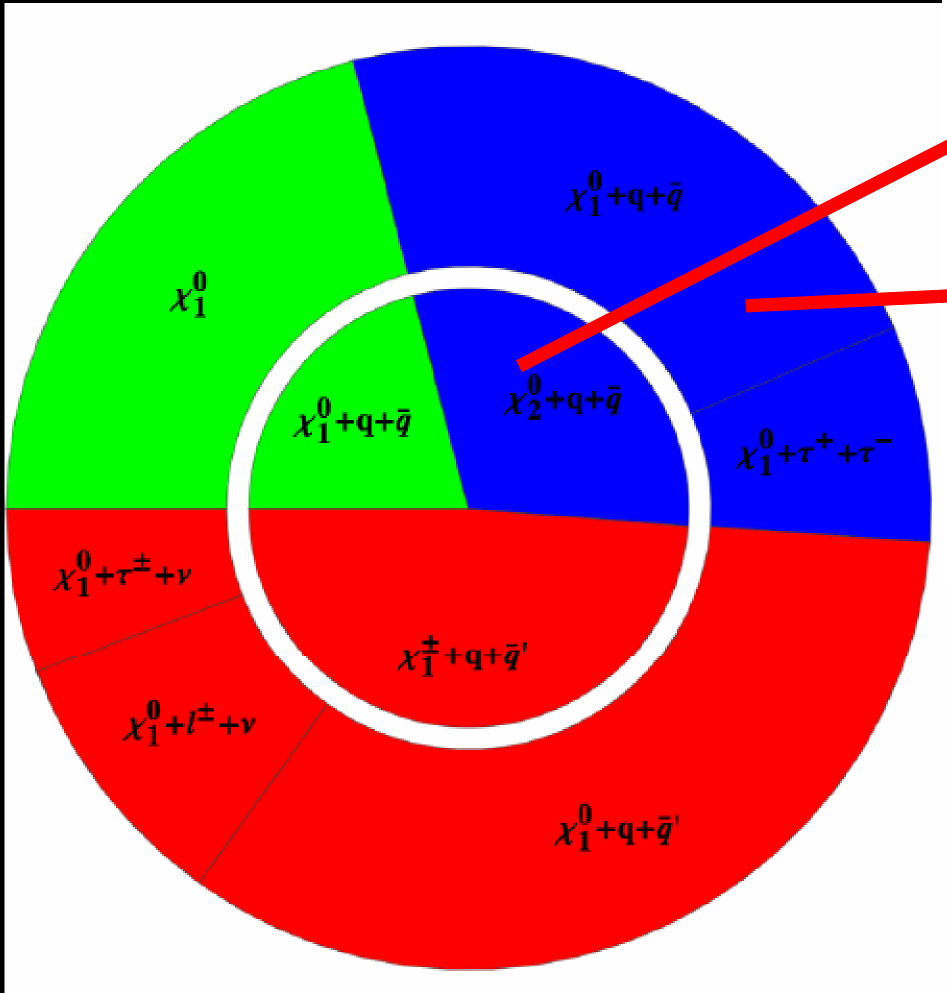
- At LHC strongly interacting stuff has by far highest cross sections

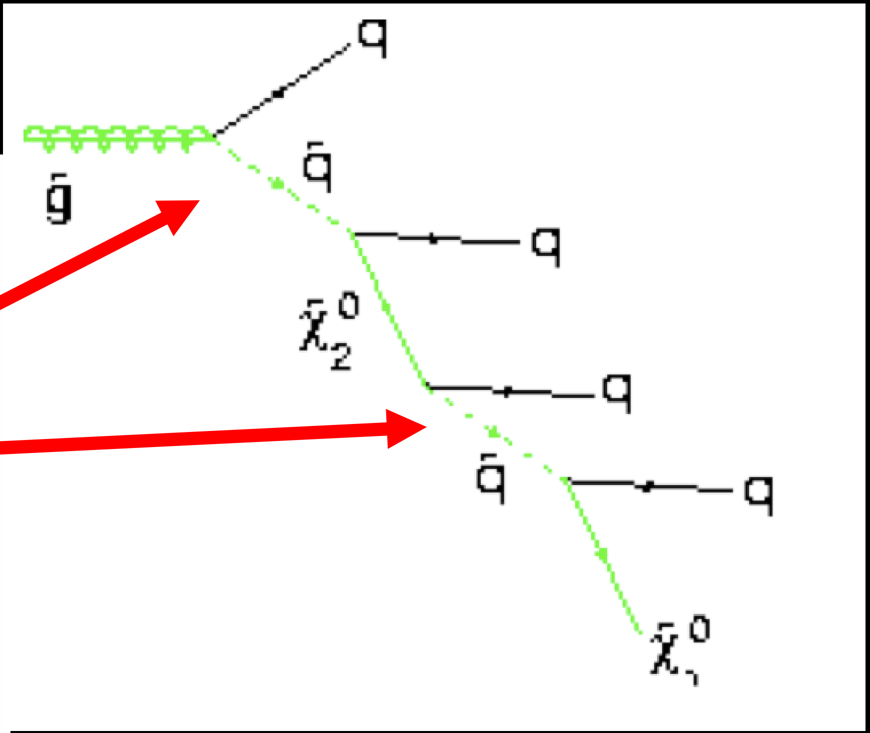
  (proton collider)

→ Look for strongly interacting sparticles!!!

What kind of Signal?

Gluino decay modes



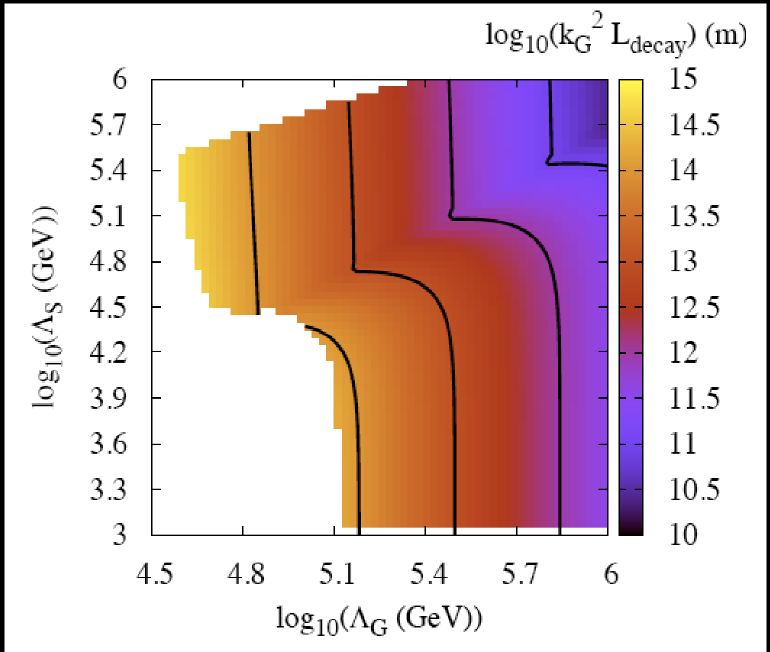


Jets
+
Missing Energy

A word about the NLSP

- Pure GGM requires high messenger scale (to generate sufficient tanβ)

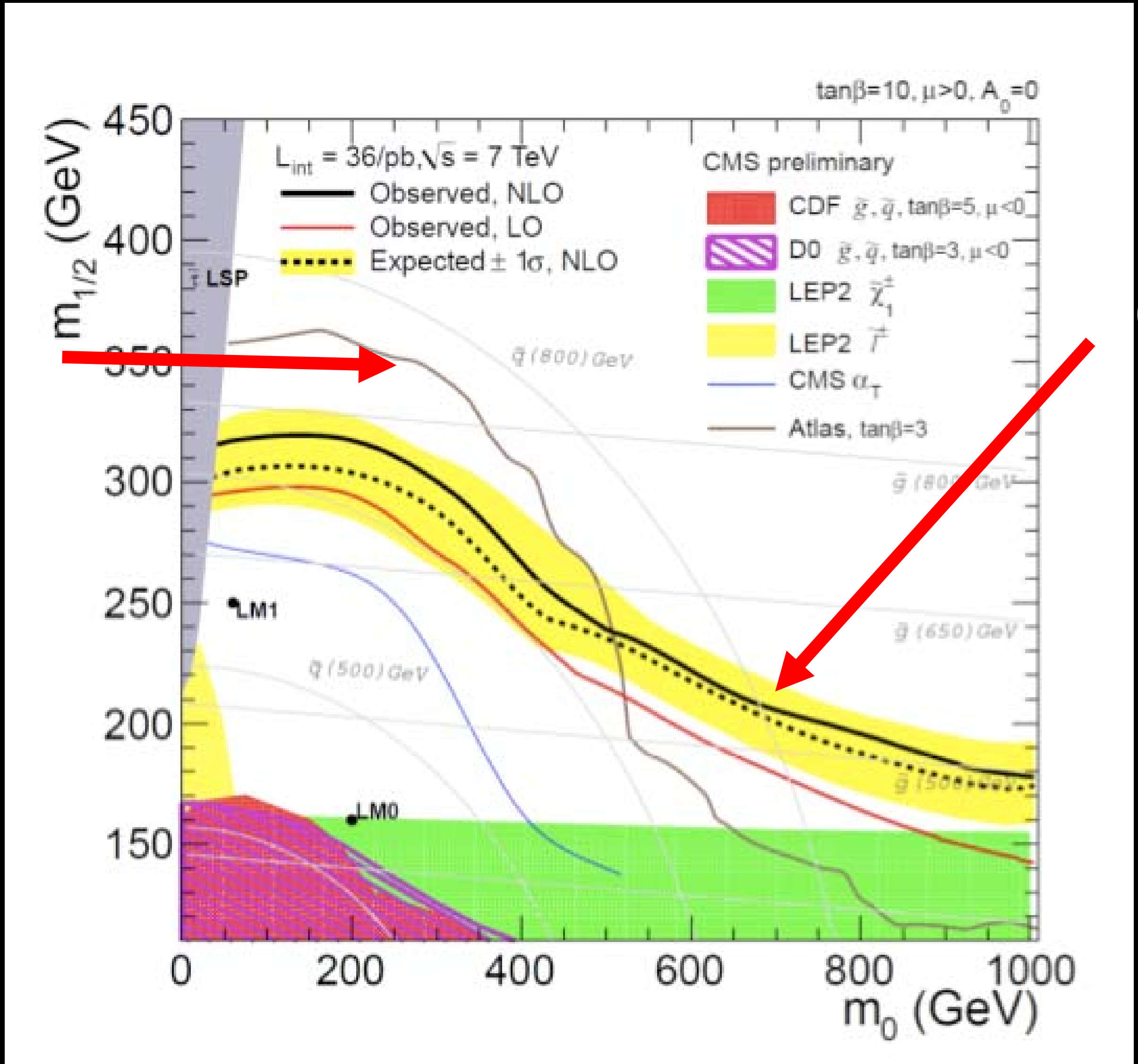
⇒ NLSP stable on collider scales



CMS + ATLAS

Have searched for jets + missing energy

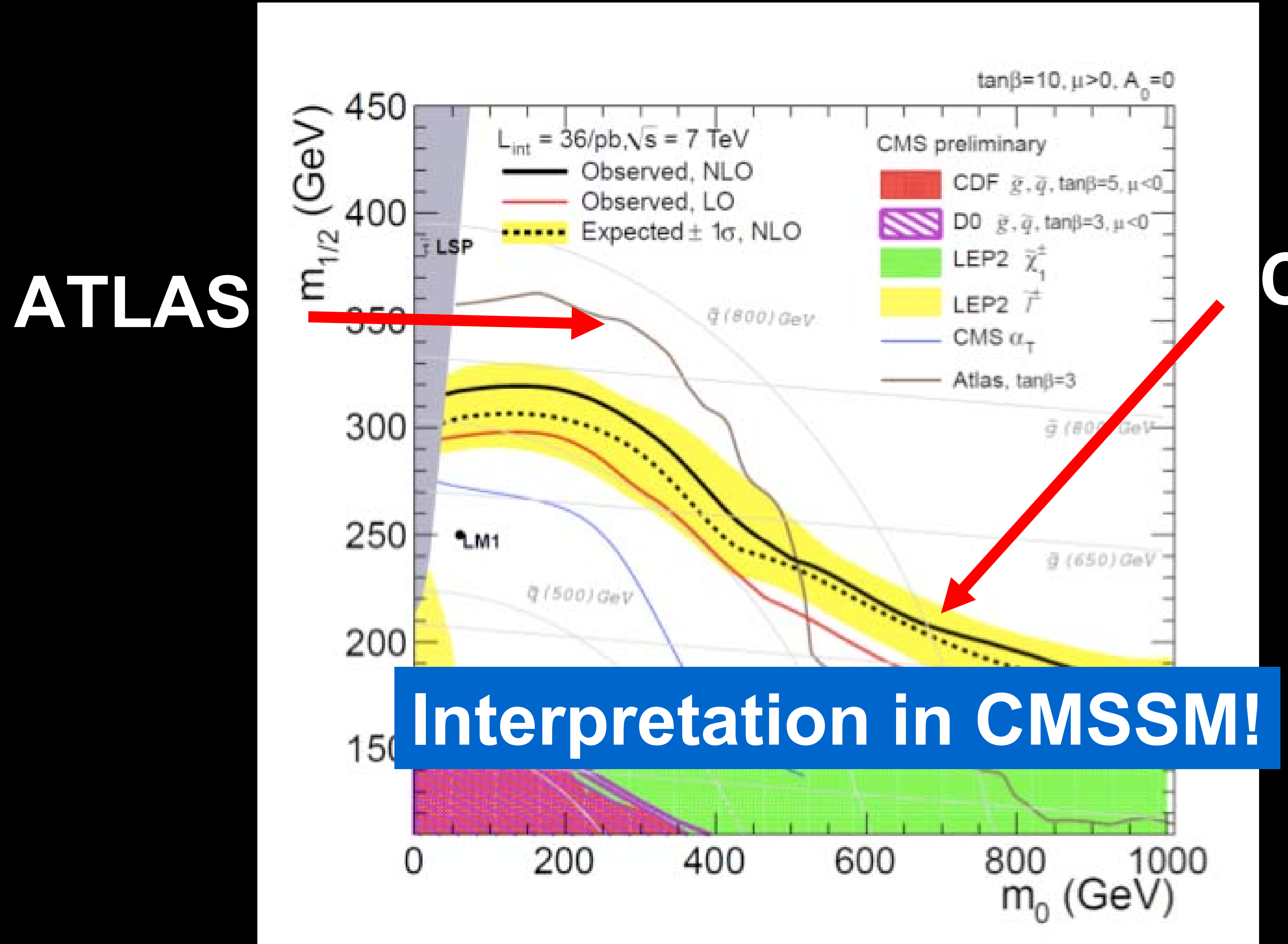
ATLAS



CMS

CMS + ATLAS

Have searched for jets + missing energy



Interpretation in CMSSM!

Focus on ATLAS search with $35pb^{-1}$

- Interpret the ATLAS results for jets plus missing energy in terms of pure GGM

What is measured?

Maximal cross section after cuts are applied

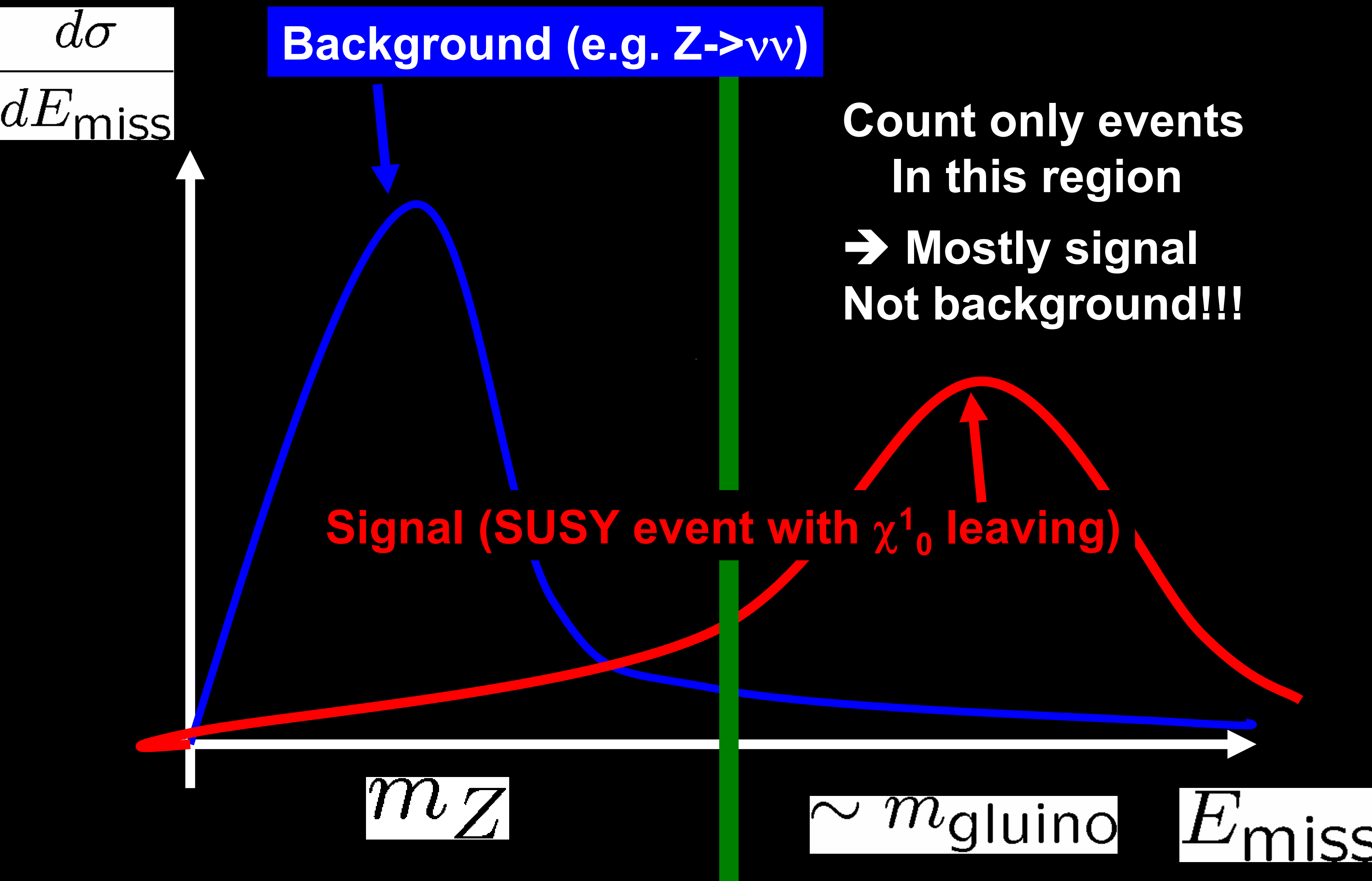
		A	B	C	D
Pre-selection	Number of required jets	≥ 2	≥ 2	≥ 3	≥ 3
	Leading jet p_T [GeV]	> 120	> 120	> 120	> 120
	Other jet(s) p_T [GeV]	> 40	> 40	> 40	> 40
	E_T^{miss} [GeV]	> 100	> 100	> 100	> 100
Final selection	$\Delta\phi(\text{jet}, \vec{P}_T^{miss})_{min}$	> 0.4	> 0.4	> 0.4	> 0.4
	E_T^{miss}/m_{eff}	> 0.3	–	> 0.25	> 0.25
	m_{eff} [GeV]	> 500	–	> 500	> 1000
	m_{T2} [GeV]	–	> 300	–	–

$\sigma(\text{after cuts}) \leq$ 1.3 0.35 1.1 0.11 pb

designed for $\tilde{q}\tilde{q}$ $\tilde{q}\tilde{q}$ $\tilde{g}\tilde{g}$ $\tilde{q}\tilde{g}$

heavy

What are cuts good for?



From models to LHC

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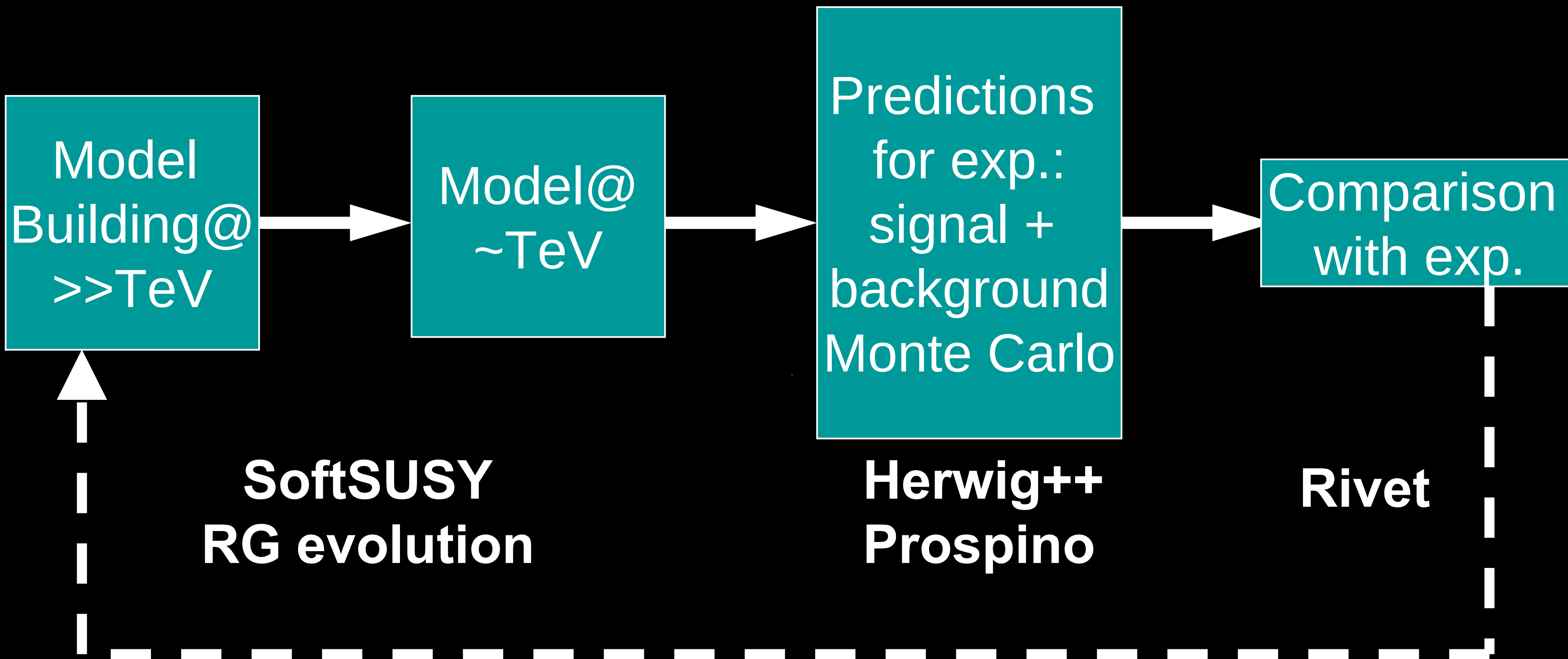
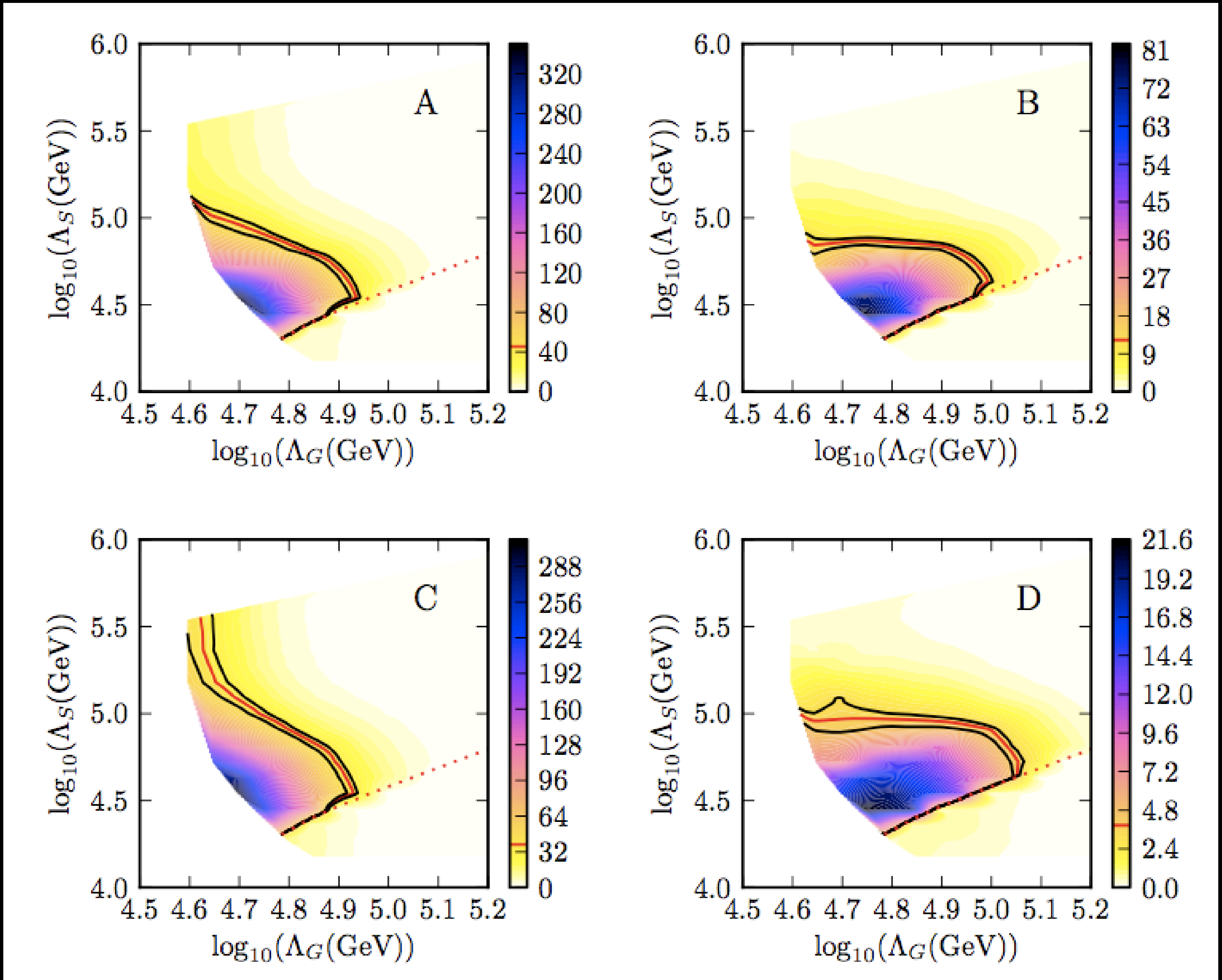


Calculate σ
(with kinematics)

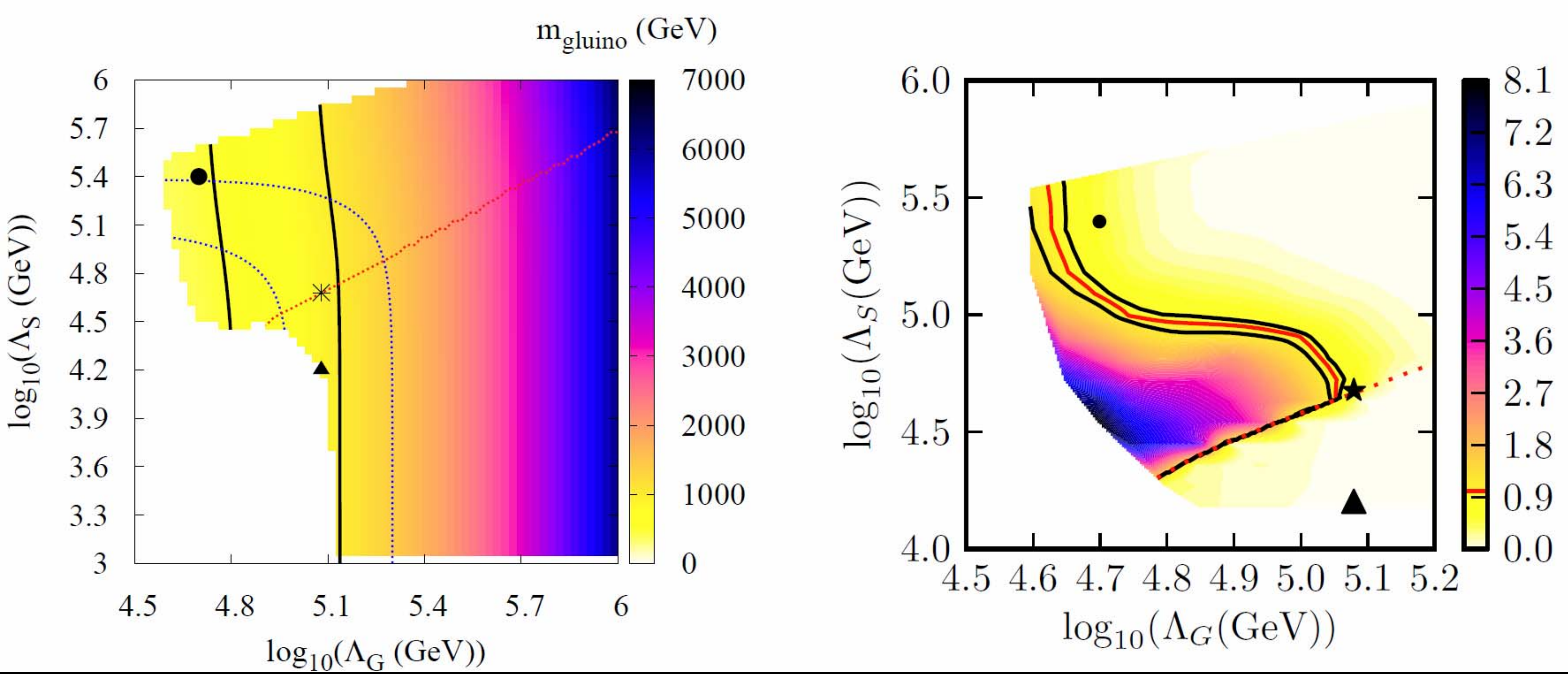
apply
cuts

ATLAS constraints on pure GGM



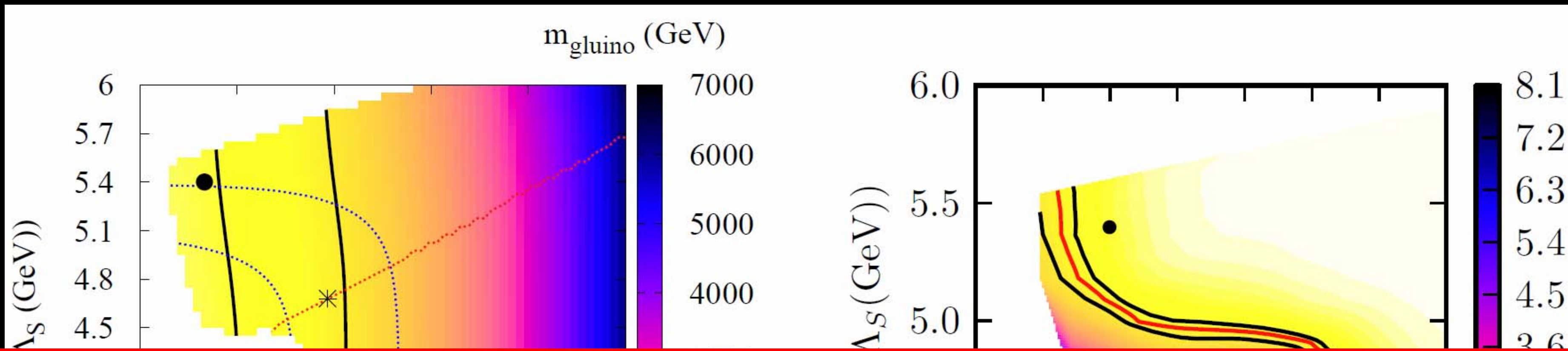
$M_{mess} = 10^{14}$ GeV

Everything combined...



- Big chunk of new parameter space excluded!
- LHC probes new untested region!

Everything combined…

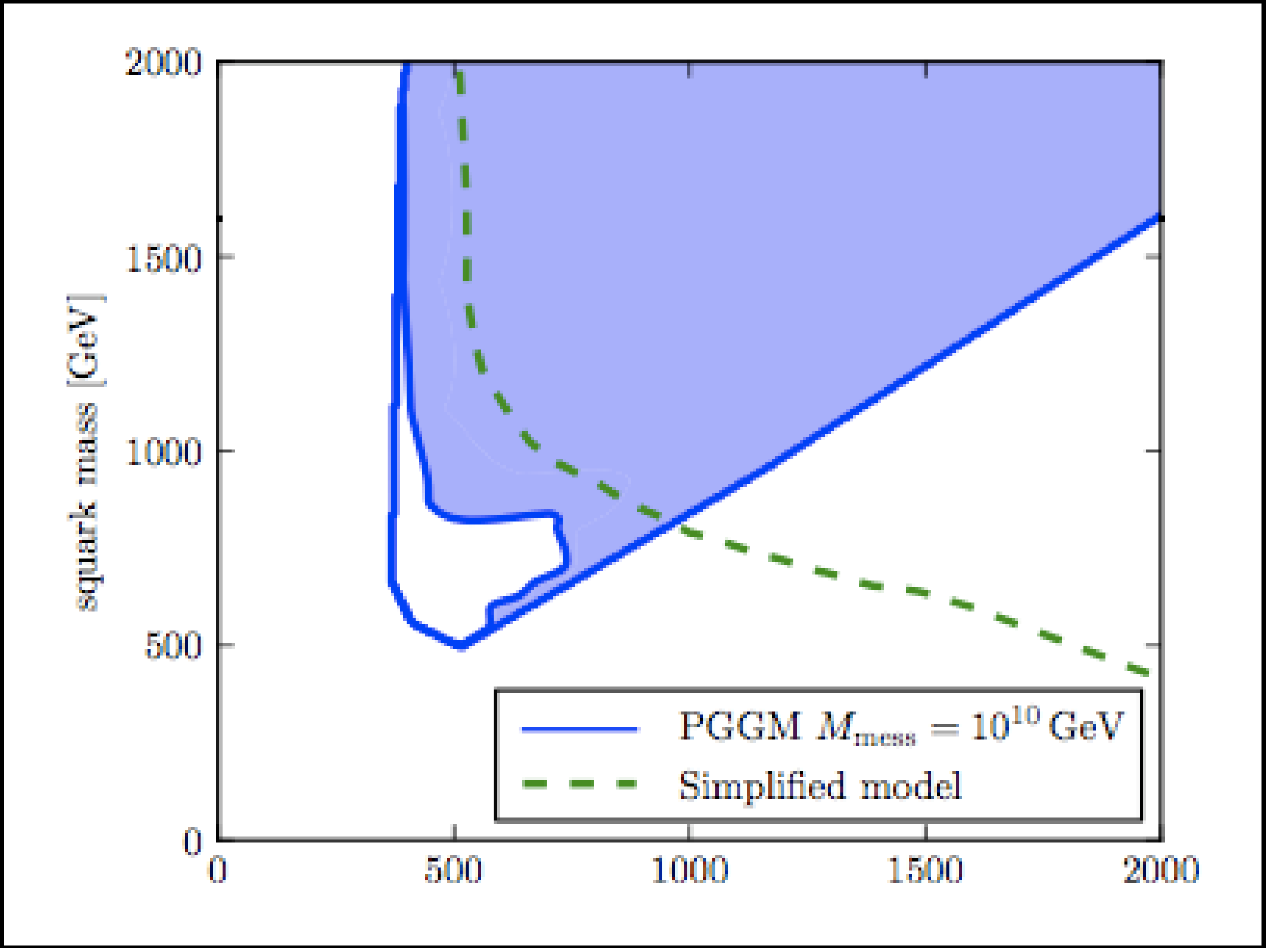


Remember only 35 pb^{-1}
By now >4000 pb^{-1} collected!

- Big chunk of new parameter space excluded!
- LHC probes new untested region!

Comparing different models…

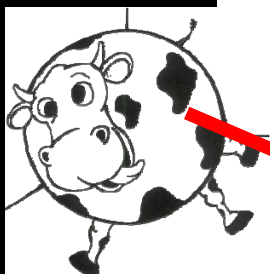
…in the squark-gluino mass plane

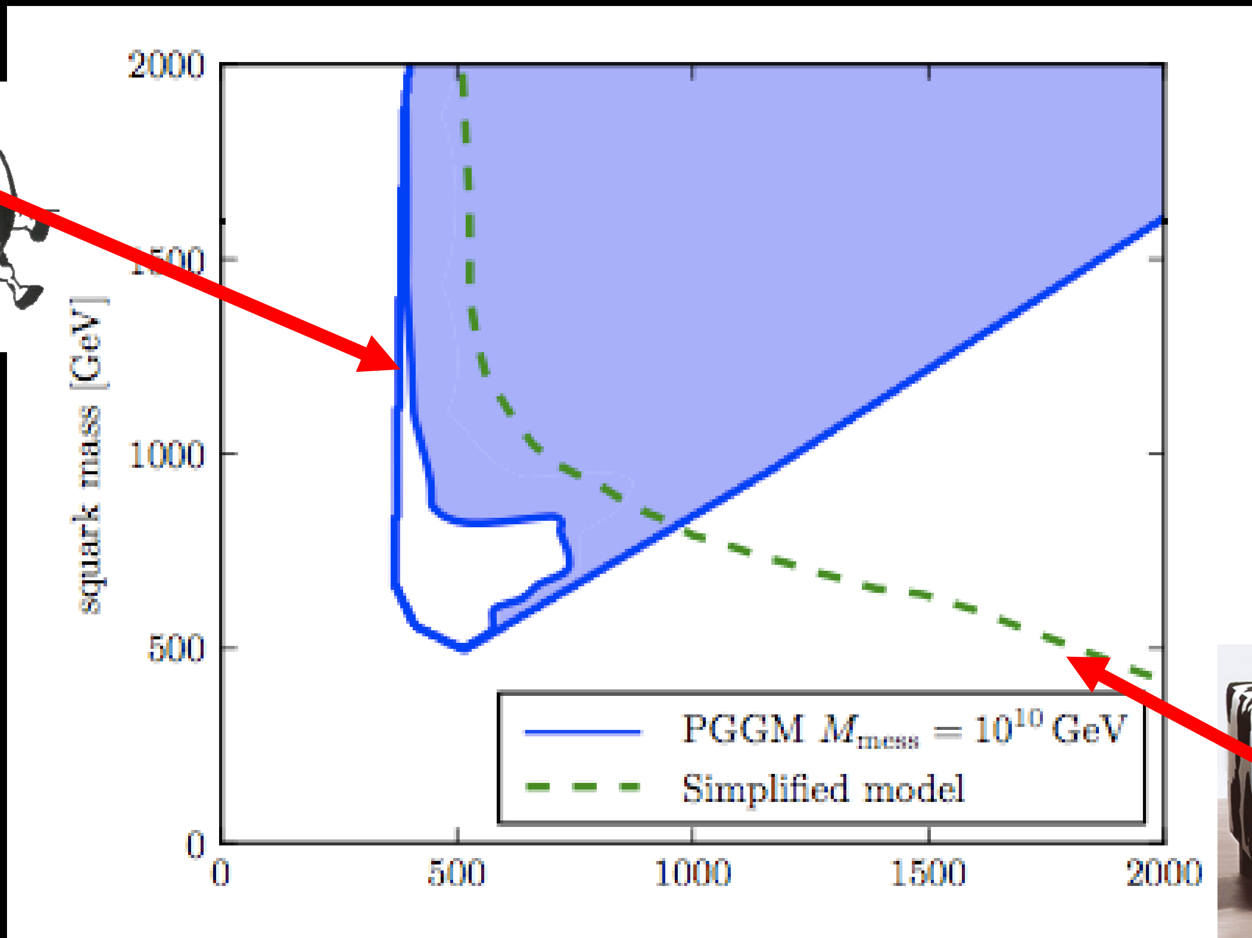


M_{mess} $=10^{10}$ GeV

Comparing different models…

…in the squark-gluino mass plane



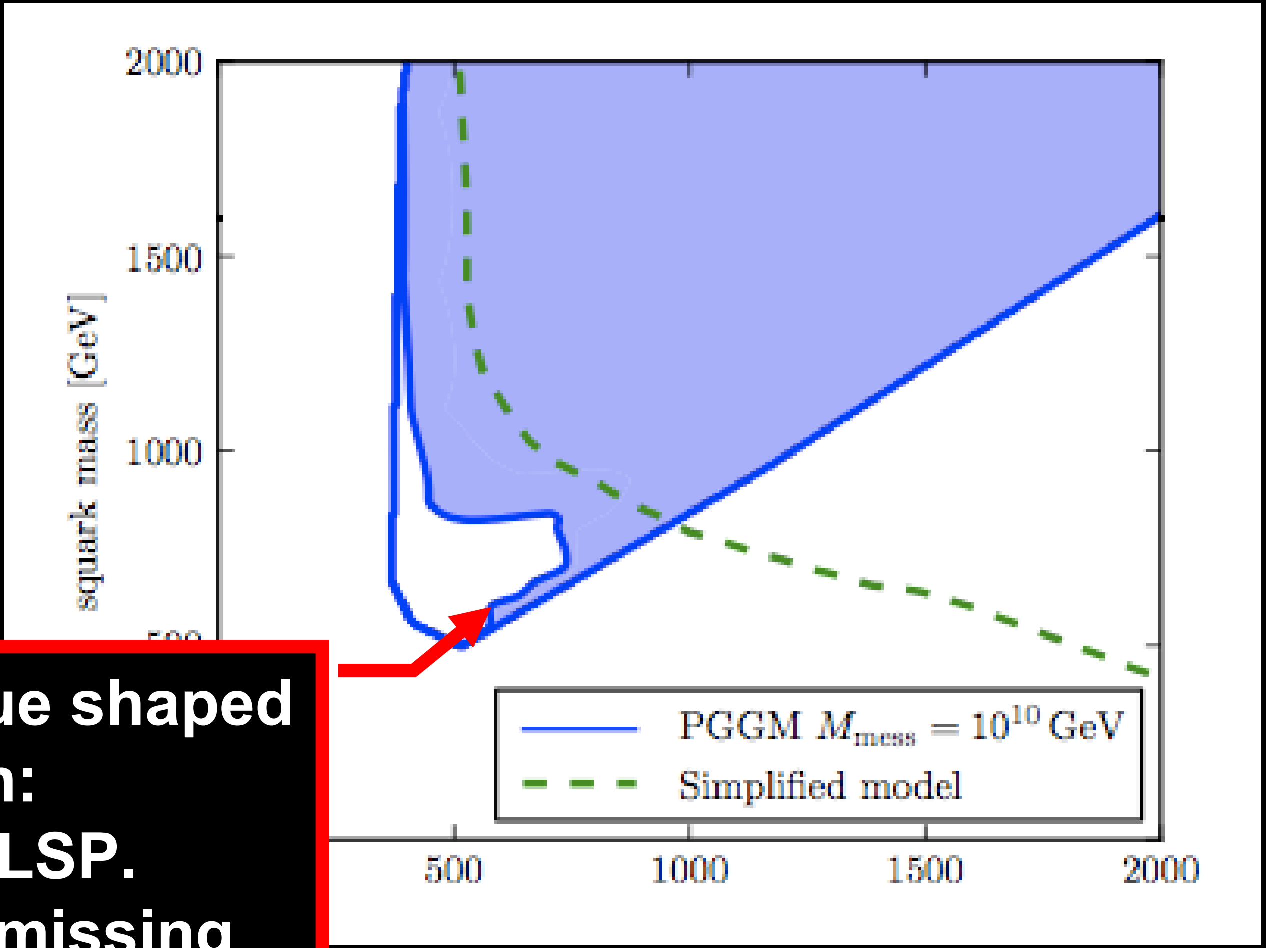


M_{mess}
$=10^{10}$ GeV

Bigger mass reach for simplified model:
neutralino mass=0, no branching ratios➔ " a bit too simple'

Comparing different models…

…in the squark-gluino mass plane

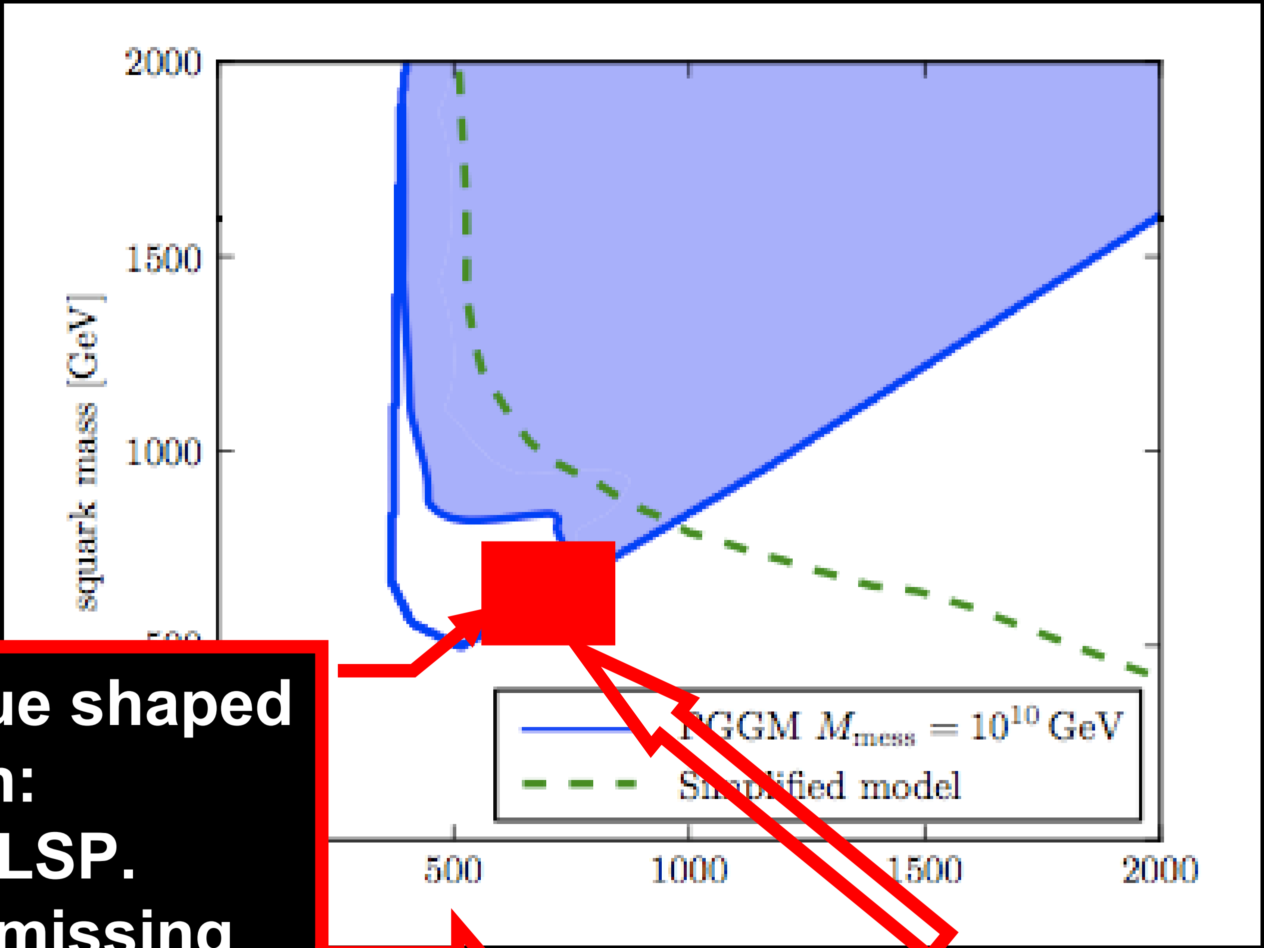


$M_{mess} = 10^{10}$ GeV

Tongue shaped region:
$\tilde{\tau}$ -NLSP.
→ No missing energy

Comparing different models…

…in the squark-gluino mass plane

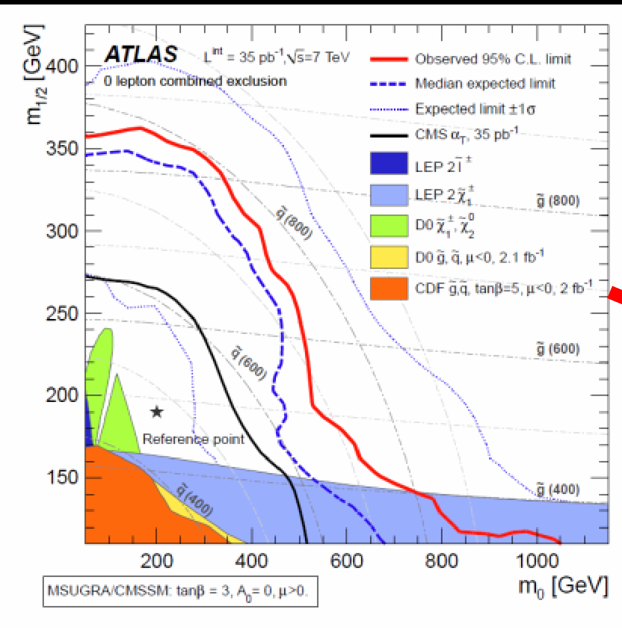


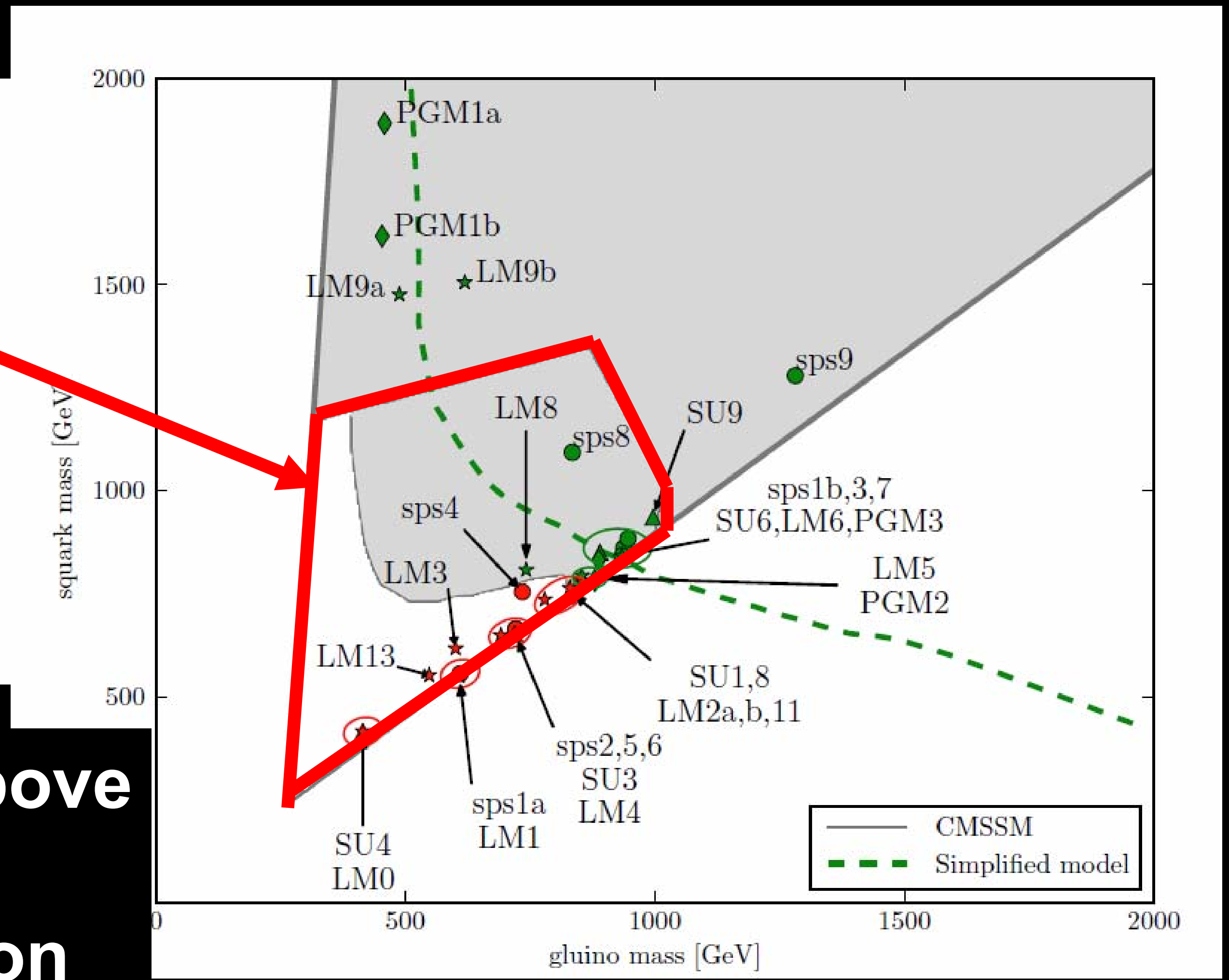
$M_{mess} = 10^{10}$ GeV

Tongue shaped region:
$\tilde{\tau}$-NLSP.
→No missing energy

Search for "stable" charged particles

For Comparison: Another look at the CMSSM





m_0-$m_{1/2}$ region above mapped onto Kite-shaped region

For Comparison: Another look at the CMSSM

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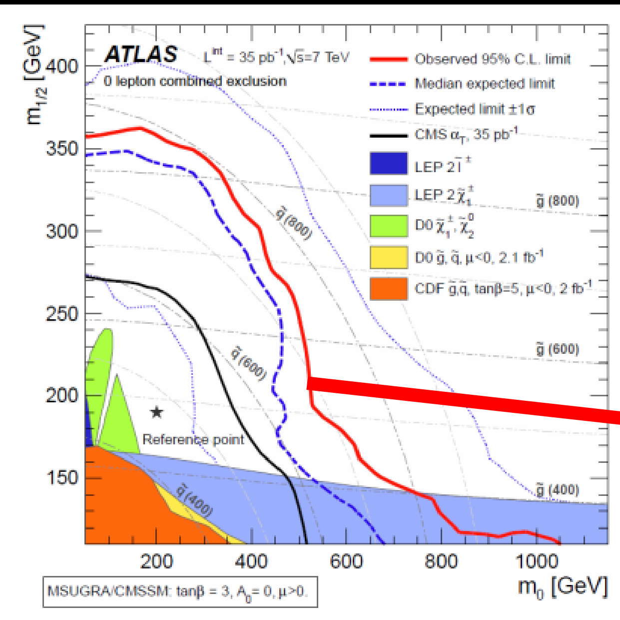
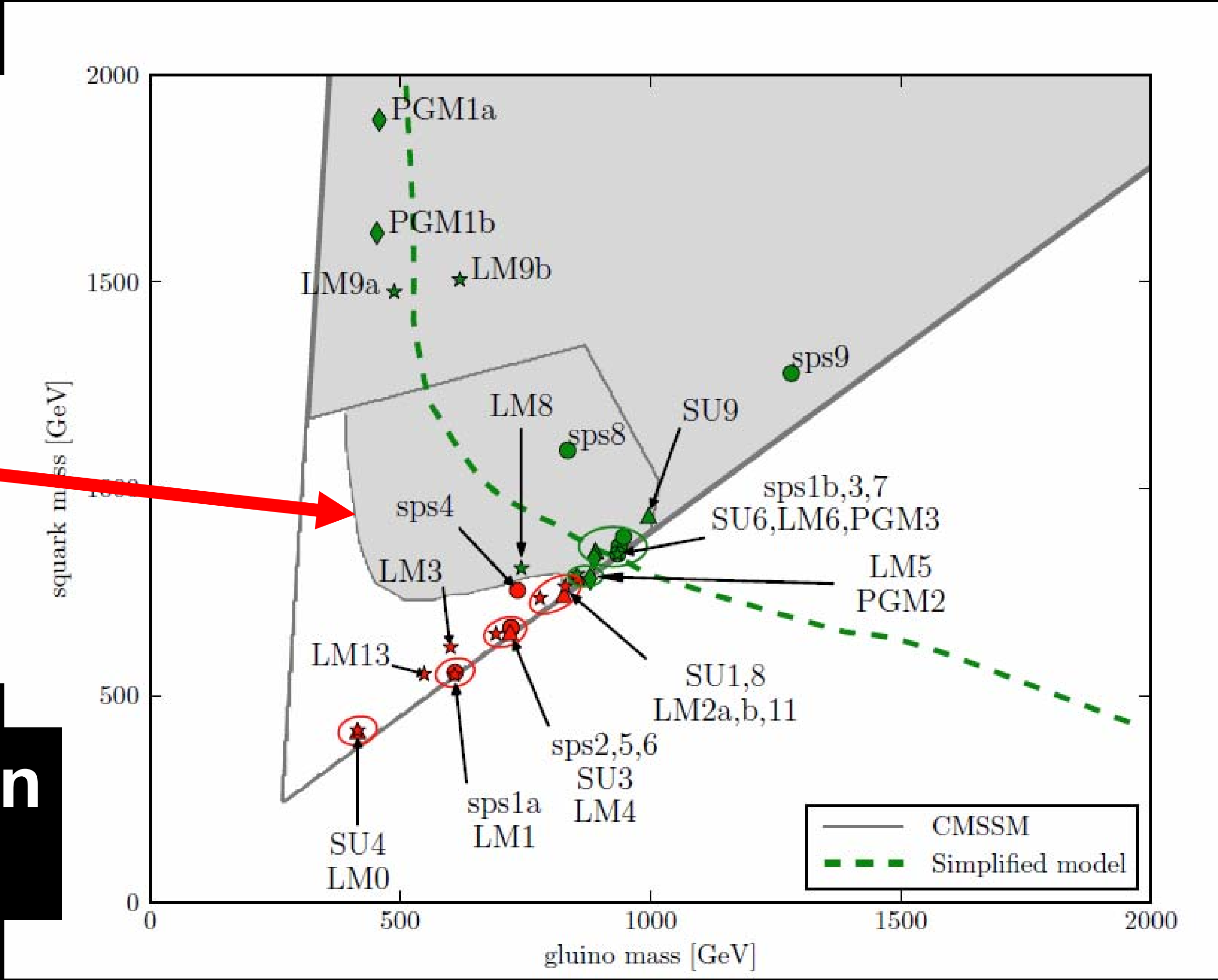
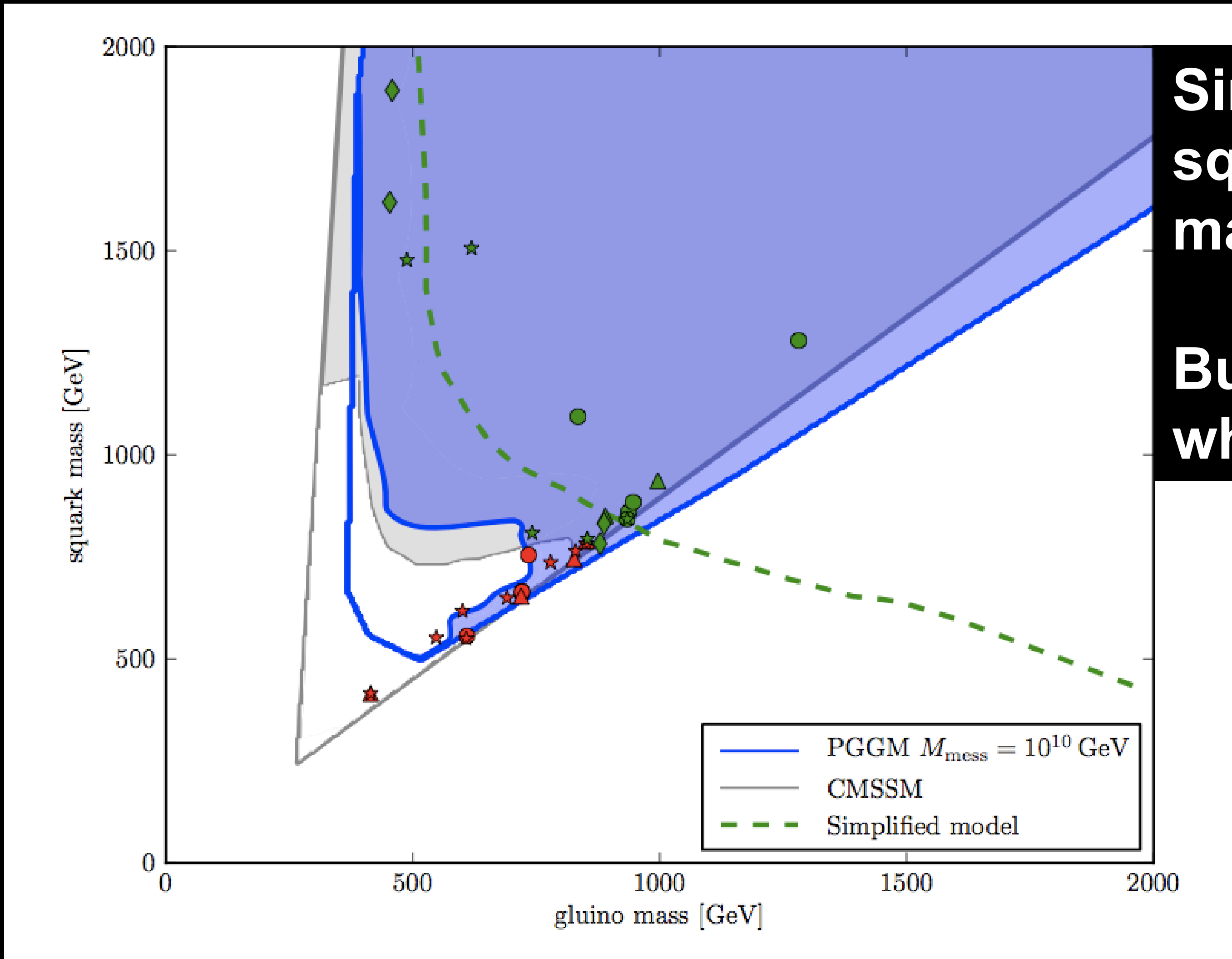




ATLAS exclusion contour

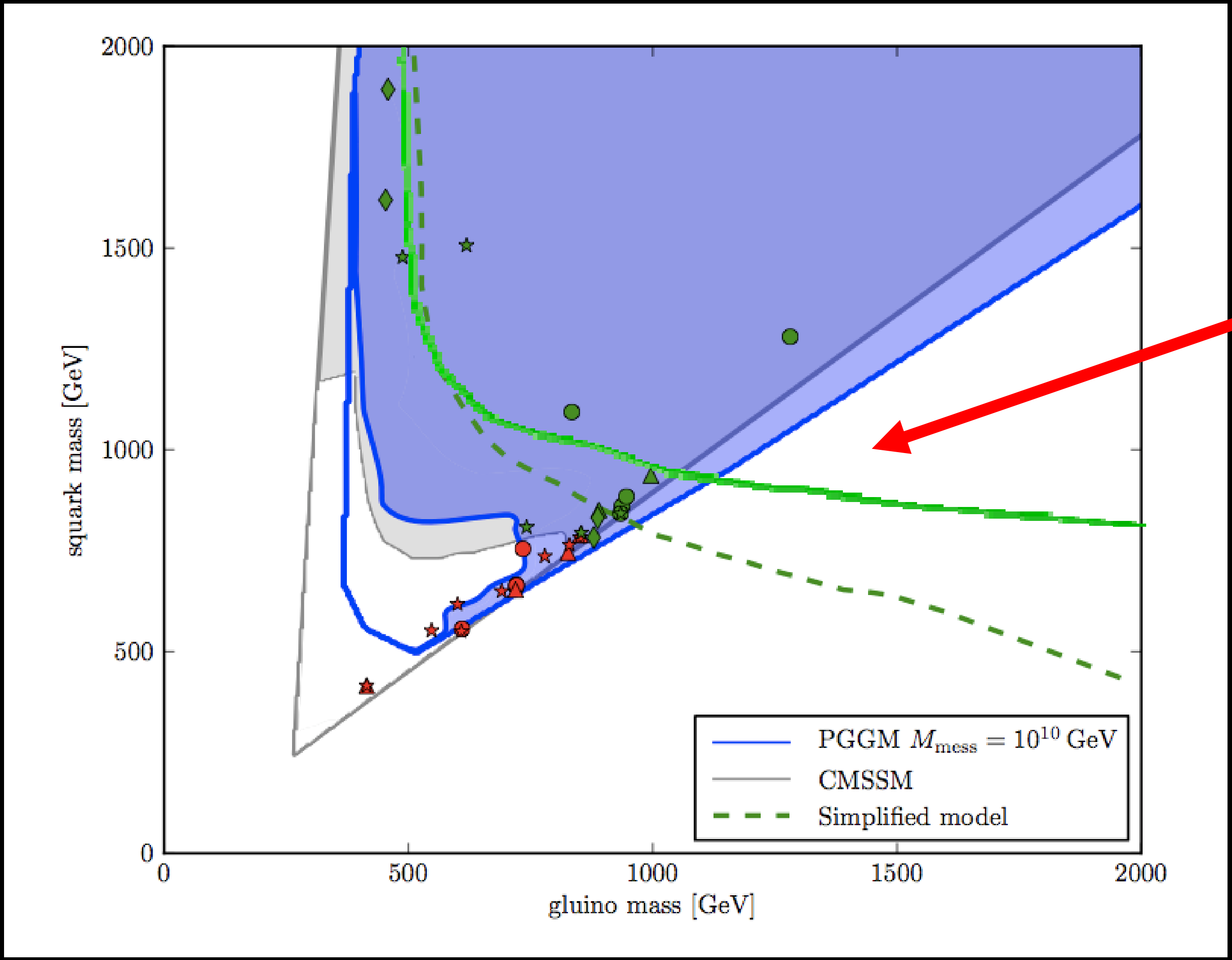
Compare with pure GGM



Similar reach in the squark-gluino mass plane

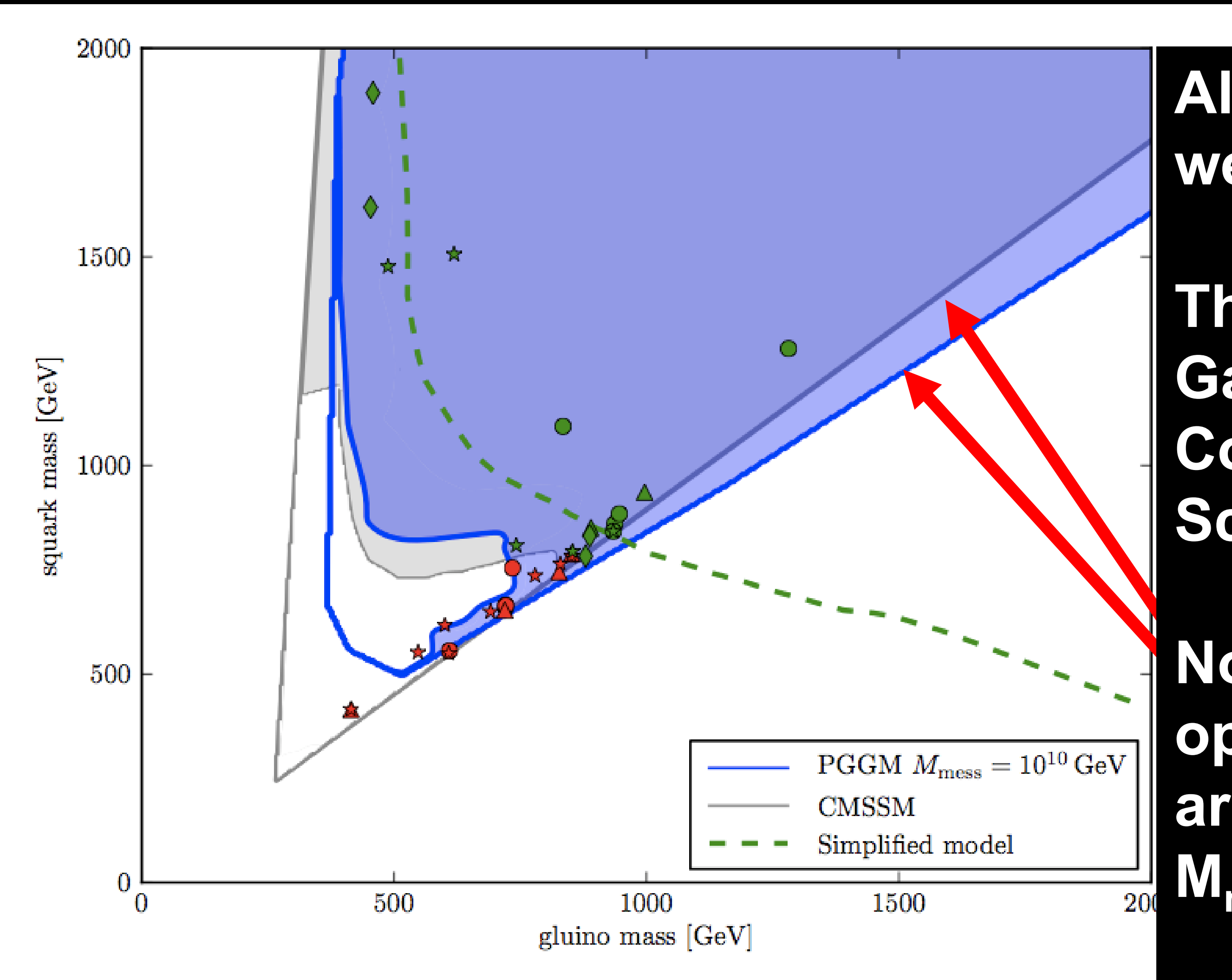
But very different where $\tilde{\tau}$ NLSP

New data...



~1 fb^{-1} reach

General features



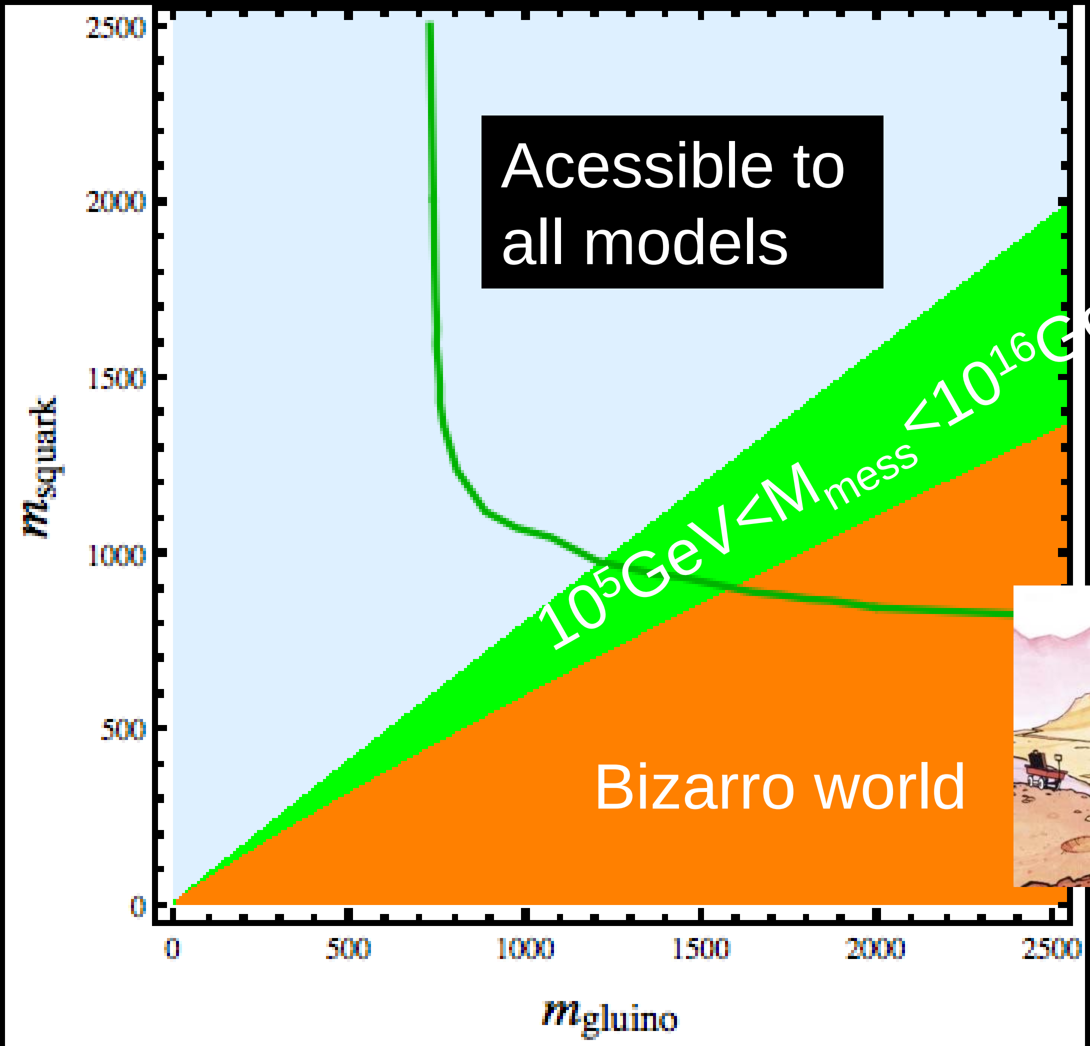
All models "live" in wedge shaped region

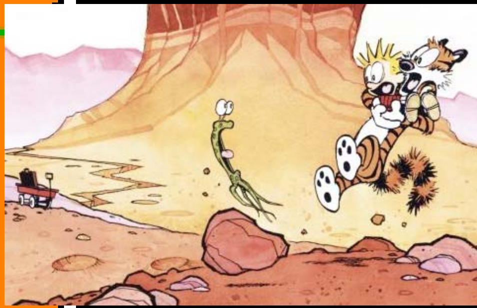
This is an RG effect: Gaugino masses Contribute to running Scalar masses

Note different opening angles arising from lower M_{mess} in GGM!

Learn something about underlying physics

RG running:
Gauginos give positive contribution to sfermion mass





Looking for gauge mediation

Stau a signal for gauge mediation

- In gauge mediation
  stau is often NLSP and long-lived

- Stau is charged → leaves track → ☺

- (In gravity med. stau typically short lived)

Stau a signal of gauge mediation

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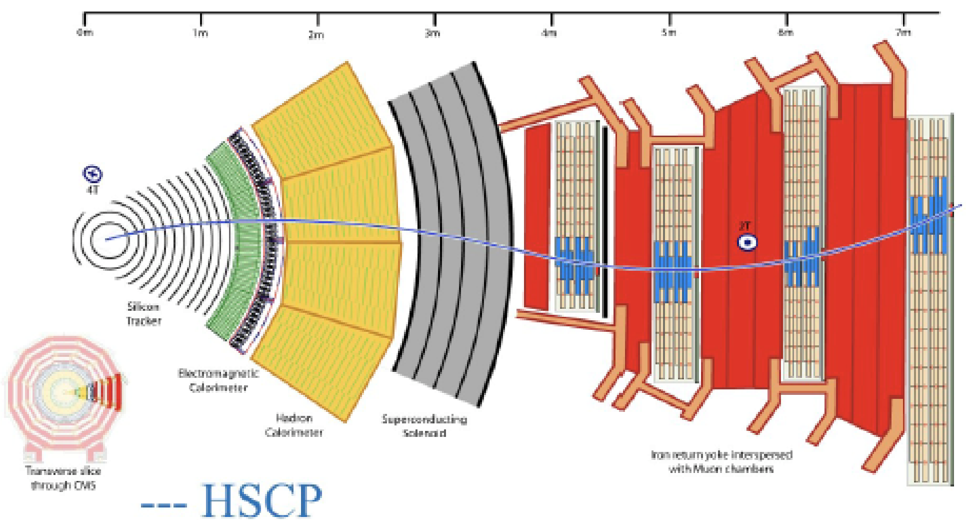
- s-tau can be long lived (superpartner of the τ-lepton)

- The stau is charged!!!
  → Search for long lived charged particles



← HSCP makes it through CMS (looks like a muon)

But v<c

→ measure time of flight

$m_{stau} > 221$ GeV

Neutralino decay

- Can have long-lived neutralino decaying

$$\chi_1^0 \longrightarrow \tilde{G} + \gamma$$

- Lifetime ~ns, decay length ~meter
  ➔ displaced vertices

~~SUSY~~ and the Higgs

The quartic coupling is determined

- In the MSSM
  quartic Higgs couplings from D-terms
  determined by electroweak gauge couplings

$$\rightarrow \quad \lambda \sim g^2$$

$$\rightarrow \quad m_h \sim \sqrt{\lambda} v = g v \sim m_Z$$

Indeed one finds for the lightest Higgs:

$$m_{h_0} < m_Z \cos(2\beta)$$

The lightest Higgs

$$m_{h_0} < m_Z \cos(2\beta)$$

- RULED OUT!!!

The lightest Higgs

$$m_{h_0} < m_Z \cos(2\beta)$$

The lightest Higgs

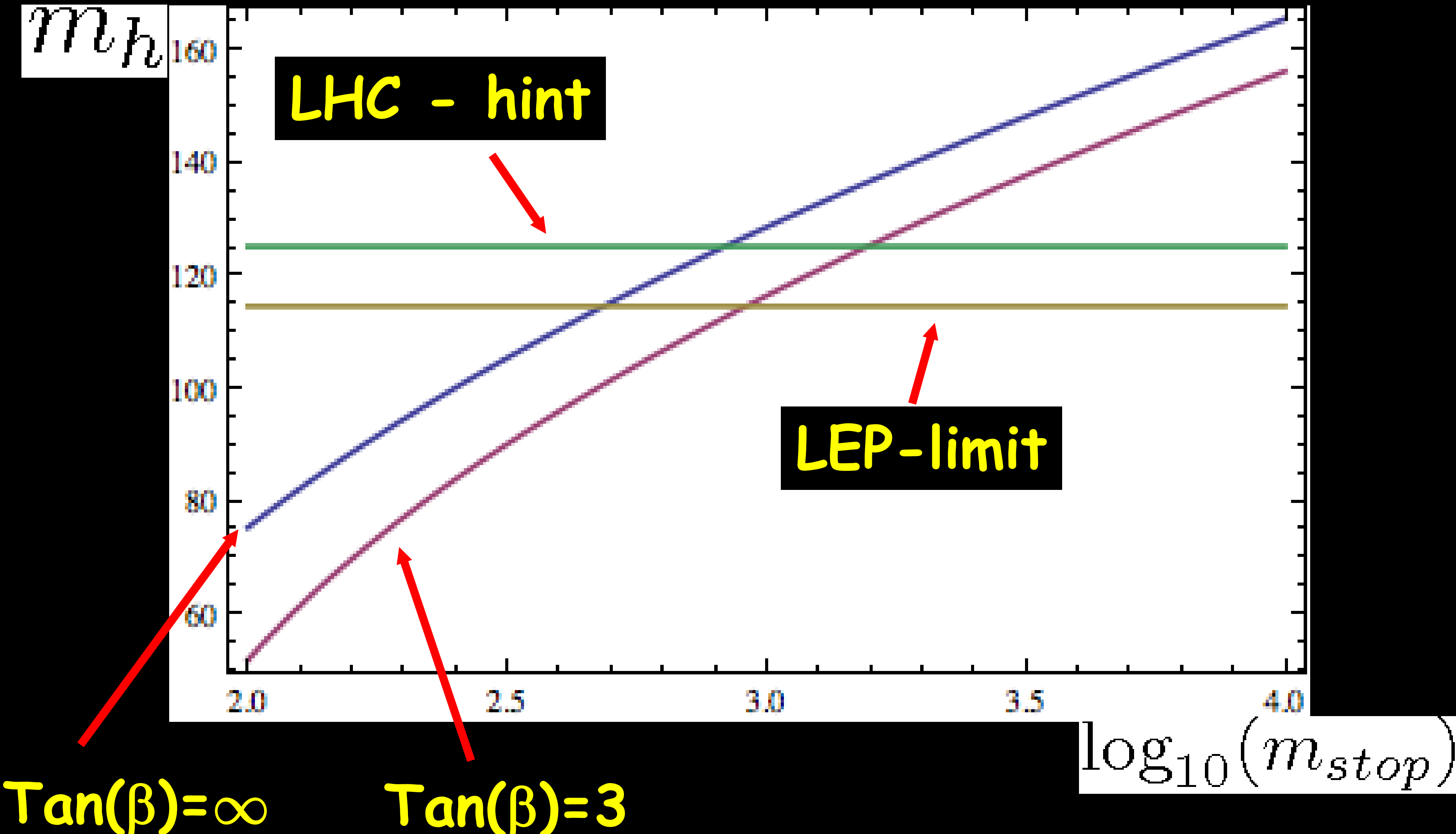
$$m_{h_0} < m_Z \cos(2\beta)$$ @ tree-level

+ Loop contributions from stop-loop

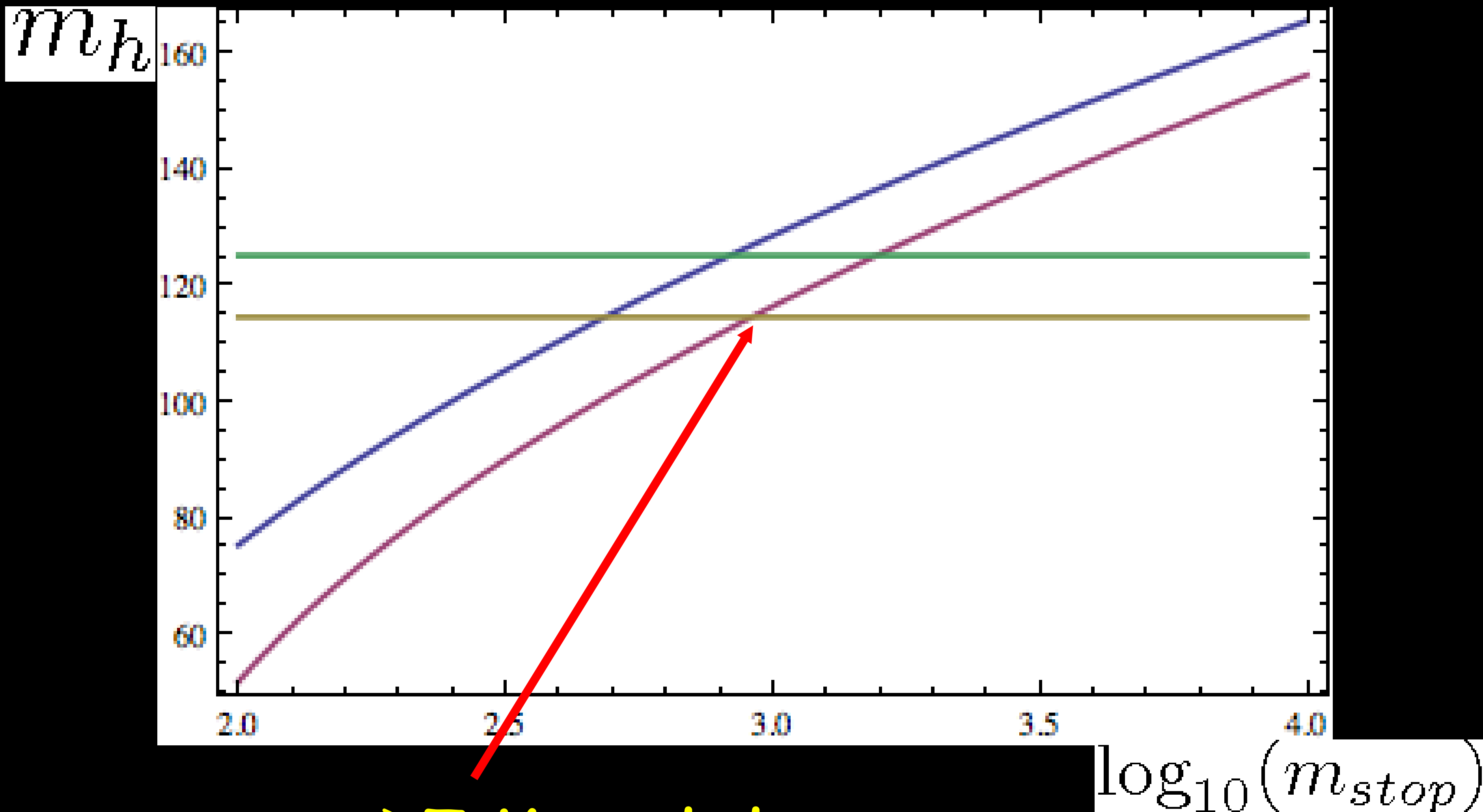
$$\Delta(m^2_{h^0}) = \frac{3}{4\pi^2}\cos^2\alpha \; y_t^2 m_t^2 \ln\left(m_{\tilde{t}_1} m_{\tilde{t}_2}/m_t^2\right)$$

(nearly) max scenario

The lightest Higgs



The lightest Higgs



$m_{stop} \gtrsim$ TeV needed

Gauge mediation models

- To explain 125 GeV Higgs

- Often need $m_{stop} \gtrsim$ 10 TeV

➔ Consider $m_{gluino} << m_{stop} \sim m_{squark}$

Can still observe gluino

Predicted in models with less metastability

Summary IV

Summary IV

- In hidden sector, i.e. high scale ~~SUSY~~ models
  - RG evolution to EW scale is important!
  - Certain regions in parameter space not allowed
- First SUSY searches → Jets + missing Energy
- (N)LSP important for phenomenology
- Gauge mediation smoking guns…
  - Long lived staus
  - Displaced vertices from NLSP decay
- Higgs mass tells us a lot about ~~SUSY~~ scale

Concluding remarks

should have colled lecture

“~~SUSY~~ is hard…

…but doable” ;-)

Concluding remarks

- ~~SUSY~~ not simple ← many theory constraints

- ~~SUSY~~ not simple ← many pheno constraints

- Hidden sector ~~SUSY~~ seems an OK option
  - Attempt to predict 105 soft parameters
  - Gauge (and gravity mediation) can do it

- Higgs can tell us a lot!

  → thing to watch in near future

What I wanted to tell you…

- … but didn't have time to do
- ~~SUSY~~ and gravity
- Witten index theorem
- Dynamical SUSY breaking
- Loop corrections to SUSY potentials (Coleman-Weinberg potential)
- More metastability
- …

Some literature

Basics:

- Stephen Martin, hep-ph/9709356
  "A supersymmetry primer"

- Adrian Signer, 0905.4630 [hep-ph]
  "ABC of SUSY"

Gauge mediation:

- Giudice, Rattazzi, hep-ph/9801271
  "Theories with gauge-mediated supersymmetry breaking"