Cosmic microwave background and large-scale structure

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IMPRS Block course, MPP, March 5 – 8, 2012

Plan...

1. Review: Homogeneous and isotropic universe
2. Inhomogeneities I: cosmological perturbation theory
3. Inhomogeneities II: Boltzmann equation
4. Initial conditions
5. Approximate solutions I: matter density perturbations
6. Approximate solutions II: CMB temperature fluctuations
7. Cosmological parameters from CMB temperature anisotropies

5.11 The matter power spectrum...

- Just as inflation predicts only the fluctuation statistics, we can also only measure the clustering statistics of matter.

- Lowest order: 2-point spectrum (matter power spectrum):

$$\langle \delta_m(\boldsymbol{k},\eta)\,\delta_m(\boldsymbol{k}',\eta)\rangle = (2\pi)^3\,\delta_D^{(3)}(\boldsymbol{k}+\boldsymbol{k}')\,P_\delta(k,\eta)$$

- From theory:

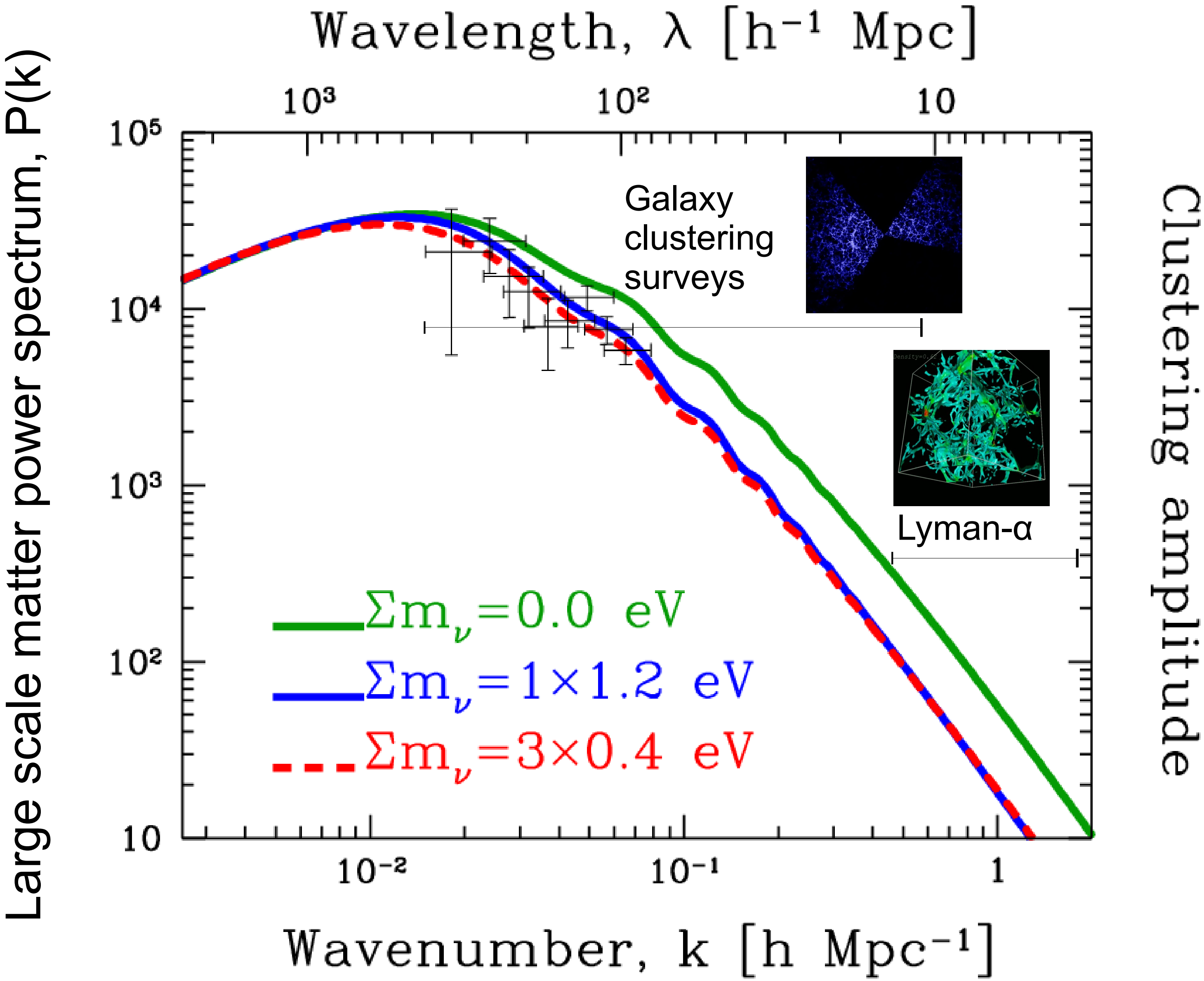
From section 4.4

Einstein equation in the subhorizon limit

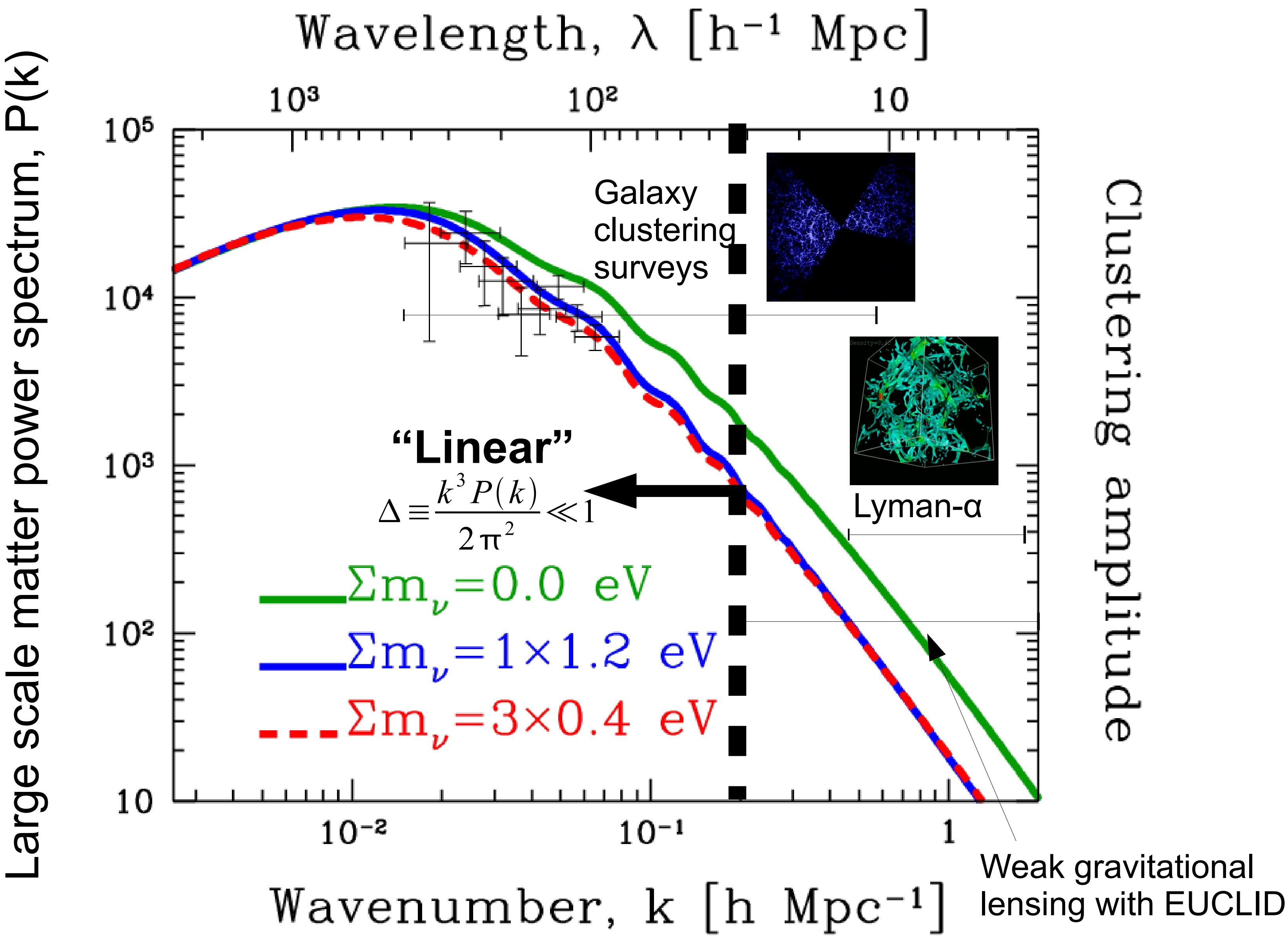
Transfer function

$\propto k^{n_s-4}$

$$\boxed{P_\delta(k,\eta) = \frac{4}{9}\frac{k^4 a^2(\eta)}{\Omega_m^2 H_0^4} P_\Phi(k,\eta) = \frac{4}{9}\frac{k^4 a^2(\eta)}{\Omega_m^2 H_0^4} T^2(k,\eta)\,P_{\Phi_p}(k)}$$



Wavelength, λ [h⁻¹ Mpc]
10³
10²
10
Large scale matter power spectrum, P(k)
10⁵
10⁴
10³
10²
10
Galaxy clustering surveys
Lyman-α
Clustering amplitude
Σm_ν=0.0 eV
Σm_ν=1×1.2 eV
Σm_ν=3×0.4 eV
10⁻²
10⁻¹
1
Wavenumber, k [h Mpc⁻¹]



Wavelength, λ [h⁻¹ Mpc]
10³
10²
10
Large scale matter power spectrum, P(k)
10⁵
10⁴
10³
10²
10
Galaxy clustering surveys
Lyman-α
"Linear"
$\Delta \equiv \frac{k^3 P(k)}{2\pi^2} \ll 1$
$\Sigma m_\nu = 0.0$ eV
$\Sigma m_\nu = 1\times 1.2$ eV
$\Sigma m_\nu = 3\times 0.4$ eV
Clustering amplitude
10⁻²
10⁻¹
1
Wavenumber, k [h Mpc⁻¹]
Weak gravitational lensing with EUCLID

5.11 The matter power spectrum...

- What to do in the nonlinear regime?
  - **Higher order perturbation theory**
    - It's fun, but applicability is limited.
  - **Numerical simulations (N-body)**
    - Discretise fluid into point particles moving under each other's gravity $\rightarrow$ Works for non-interacting matter.
    - Tracking baryons on cluster/galaxy scales (k > 1 Mpc^{-1}) requires hydrodynamics.

5.11 The matter power spectrum: bias...

- We do not observe the actual matter density field.
  - Rather, we observe tracers, and **assume** that their clustering properties follow those of the underlying matter density field.

- For galaxy surveys, this means the assumption:

$$\boxed{\frac{\delta n_{\mathrm{gal}}(k)}{\bar{n}_{\mathrm{gal}}} = b\,\delta_m(k)}$$

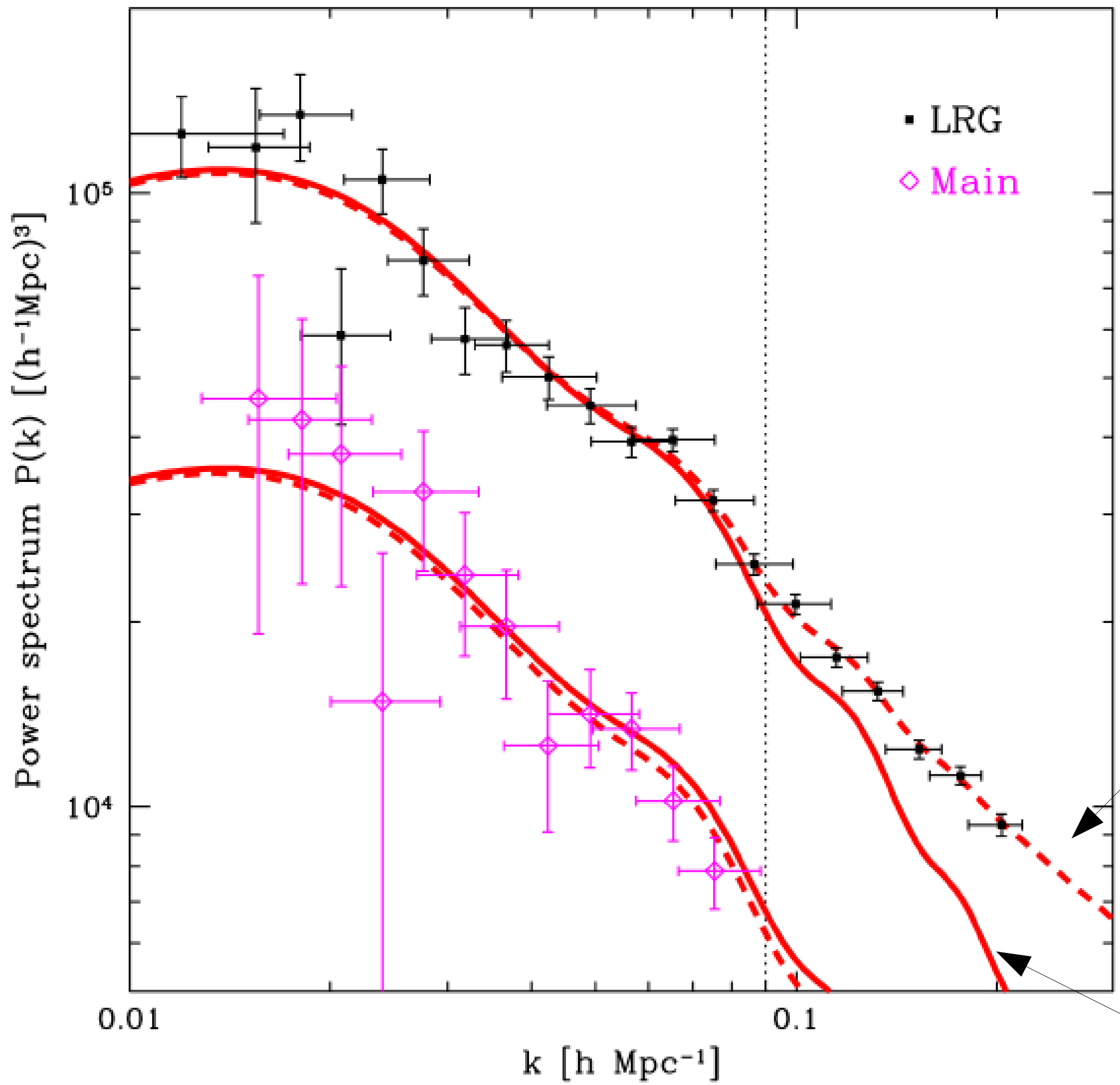
b = bias

  - The bias value depends on the tracers; **cannot** be predicted from first principles...
  - Expected to be constant for small k modes, but certainly becomes scale-dependent for large values of k...

Sloan Digital Sky Survey DR 4

Luminous red galaxies

Main galaxy sample



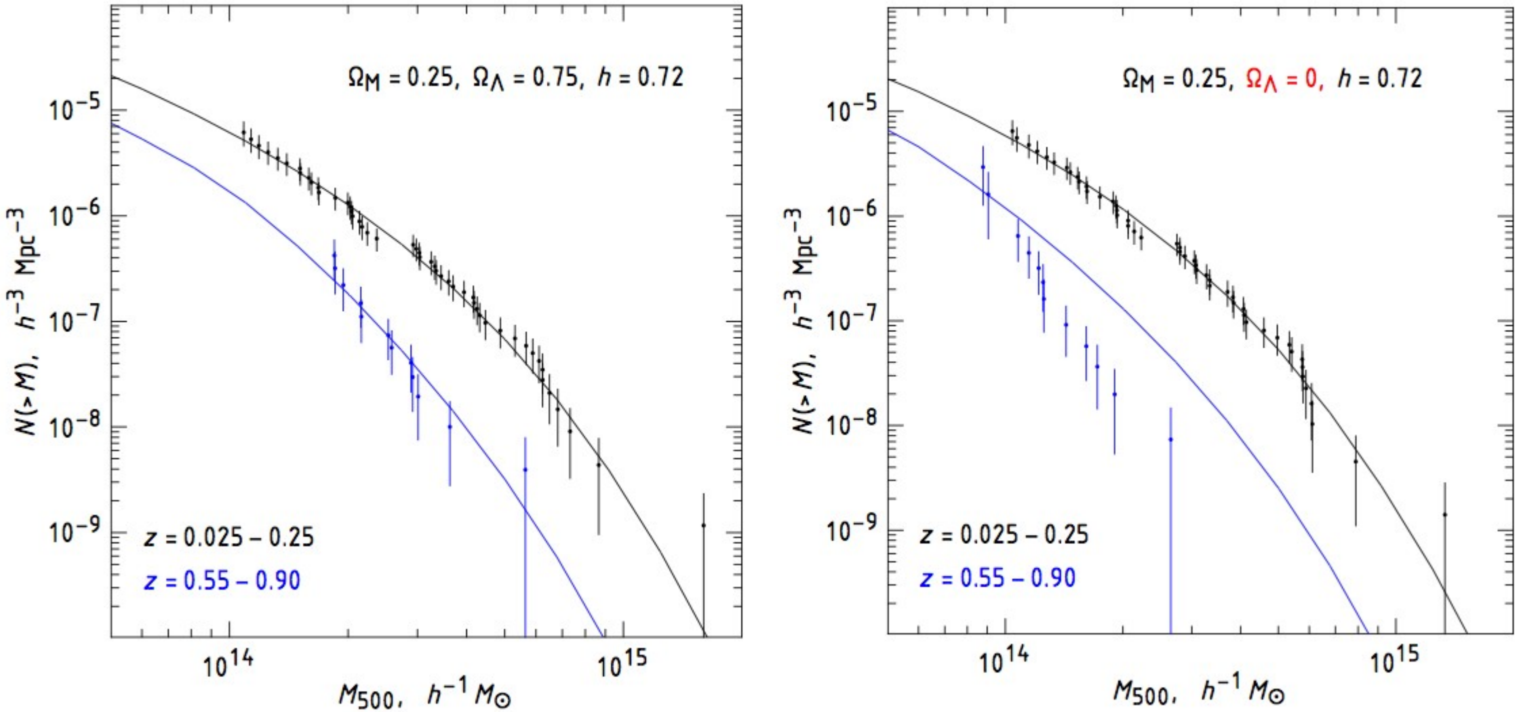
Empirically corrected for scale-dependent bias

Inferred power spectrum for matter density field (up to a constant factor)

Tegmark et al. [SDSS] 2006

5.12 Cluster mass function...

- Another way to probe the large-scale structure distribution.
- CMF = abundance of galaxies/galaxy cluster as a function of mass.



Vikhlinin et al. 2009

5.12 Cluster mass function...

- Not exactly calculable from perturbation theory.

- But there are some fitting functions (calibrated against ΛCDM N-body simulations) using the linear matter power spectrum as input.

  - e.g.,

$$f(M) = 0.315 \exp\left[-\left|\ln \sigma^{-1} + 0.61\right|^{3.8}\right]$$

Jenkins et al. 2000

$$\sigma^2(M) = \frac{1}{2\pi^2} \int dk\, k^2 P_\delta(k) W^2(k, M)$$

- **Warning**: fitting formulae are cosmology-dependent; may not apply if you cosmological model strays too far from standard ΛCDM...

5.13 Section summary...

- **Shape** of matter power spectrum is sensitive to:
  - Scalar spectral index of the primordial curvature perturbation spectrum from inflation.
  - **Location of the turning point** k_{eq} probes comoving Hubble rate at matter-radiation equality.
    - If the radiation energy density is precisely known, this provides a measurement of $\Omega_m h$.
  - Shape, especially at $k >> k_{\mathrm{eq}}$, is also sensitive to the baryon fraction and massive neutrino fraction.

- **Beware** of nonlinearities and scale-dependent bias at large k values!

6. Approximate solutions II: CMB temperature fluctuations...

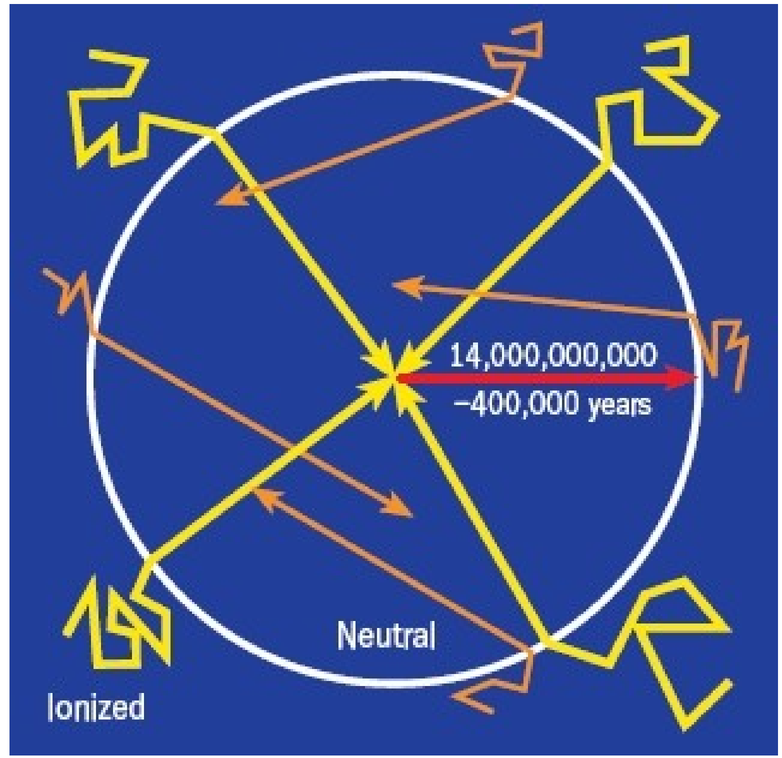
6.1 General remarks...

- The most important event in the photon evolution history is decoupling ($T^* \sim 0.25$ eV).

- In most cosmological models, photon decoupling happens during early matter domination ($z^* \sim 1100$).

  → Evolution of CMB fluctuations can be studied in **two steps**:

  - Evolution up to decoupling
    (super- or subhorizon?)
  - Evolution after decoupling:
    free-streaming



6.2 Superhorizon up to decoupling...

- Scalar Boltzmann equation for photons in the superhorizon limit:

$$\dot{\delta}_\gamma = 4\,\dot{\Theta}_0^{(S)} \simeq 4\,\dot{\Phi}$$

- Supposing:

From section 4.2

Well-founded assumptions!

  - Adiabatic initial conditions: $\Theta_0^{(S)}(k, \eta=0) = -\Phi_p(k)/2$
  - Decoupling happens during MD: $\Psi(k, \eta_*) \simeq \Phi(k, \eta_*) = \dfrac{9}{10}\Phi_p(k)$

From section 5.4

- At decoupling:

$$\boxed{(\Theta_0^{(S)} + \Psi)(k \ll \mathscr{H}_*, \eta_*) = \frac{1}{3}\Phi(k, \eta_*) = -\frac{1}{6}\delta_c(k, \eta_*)}$$

From section 5.4

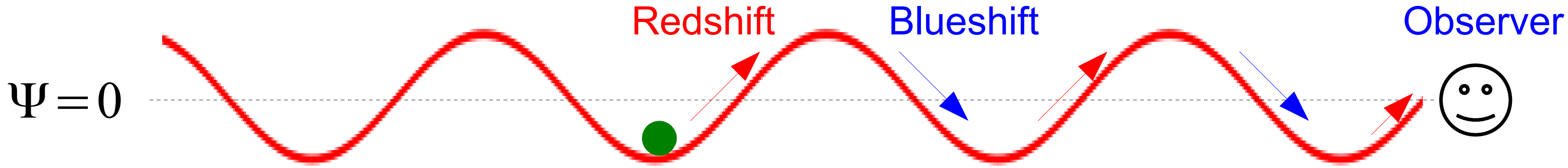
6.2 Superhorizon up to decoupling...

- What does this mean?

$$\left(\Theta_0^{(S)}+\Psi\right)(k,\eta_*)=-\frac{1}{6}\delta_c(k,\eta_*)$$

- $(\Theta^{(S)}{}_0 + \Psi)(\eta_*)$ is the effective temperature at decoupling.

Perturbation of some wavelength



Overdense region = Intrinsically hotter photons; adiabatic initial conditions: $3\Theta^{(S)}{}_0 = \delta_c$

Observed photon energy changed by a factor $(1 + \Psi)$ due to gravitational redshift

$\rightarrow$ An observed photon hot spot corresponds to an **underdense** region.

6.3 Subhorizon @ decoupling: acoustic oscillations...

- For those k modes that are **subhorizon** at photon decoupling, the tightly-coupled limit (between photons and baryons) applies.

- Equations of motion for baryons and photons in this limit:

$$\dot{\delta}_\gamma - \frac{4}{3} k^2 v_\gamma^{(S)} - 4\dot{\Phi} = 0 \qquad \dot{v}_\gamma^{(S)} + \frac{1}{4}\delta_\gamma + \Psi = -\dot{\kappa}\left(v_b^{(S)} - v_\gamma^{(S)}\right) \quad (1)$$

$$\dot{\delta}_b - k^2 v_b^{(S)} - 3\dot{\Phi} = 0 \qquad \dot{v}_b^{(S)} + \mathscr{H} v_b^{(S)} - \Psi = -\frac{\dot{\kappa}}{R}\left(v_b^{(S)} - v_\gamma^{(S)}\right) \quad (2)$$

- Baryon-to-photon ratio: $\boxed{R \equiv \frac{3}{4}\frac{\bar{\rho}_b}{\bar{\rho}_\gamma} = \frac{3}{4}\frac{\Omega_b h^2}{\Omega_\gamma h^2} a}$

6.3 Subhorizon @ decoupling: acoustic oscillations...

- **Tightly-coupled limit** means $\mathscr{H}/\dot{\kappa} \ll 1.$
  - Take (2) and expand to first order in $\mathscr{H}/\dot{\kappa}$:

$$v_b^{(S)} = v_\gamma^{(S)} - \frac{\mathscr{H}}{\dot{\kappa}} R\left[\frac{1}{\mathscr{H}}\dot{\gamma}_\gamma^{(S)} + \frac{1}{\mathscr{H}}\Psi + v_\gamma^{(S)}\right] + O\left(\frac{\mathscr{H}^2}{\dot{\kappa}^2}\right)$$

  - Feed back into (1).

$\delta_\gamma = 4\,\Theta_0^{(S)}$ $\rightarrow$

$$\boxed{\ddot{\Theta}_0^{(S)} + \frac{\dot{R}}{1+R}\dot{\Theta}_0^{(S)} + k^2 c_s^2 \Theta_0^{(S)} = \ddot{\Phi} + \frac{\dot{R}}{1+R}\dot{\Phi} - \frac{k^2}{3}\Psi}$$

- A driven and damped harmonic oscillator with sound speed:

$$c_s^2 \equiv \frac{1}{3(1+R)}$$

The presence of baryons lowers the fluid sound speed

6.3 Subhorizon @ decoupling: acoustic oscillations...

- **Suppose** Φ, Ψ = constant (i.e., deep in MD).

WKB approximation

- For adiabatic initial conditions, the **WKB solution** is: $(k c_s)^2 \gg \dot{R}^2/(1+R)^2$

$$[\Theta_0^{(S)}+\Psi](k,\eta)=[\Theta_0^{(S)}+(1+R)\Psi](k,0)\cos(k r_s)-R\Psi$$ Monopole

$$\Theta_1^{(S)}(k,\eta)=-c_s[\Theta_0^{(S)}+(1+R)\Psi](k,0)\sin(k r_s)$$ Dipole

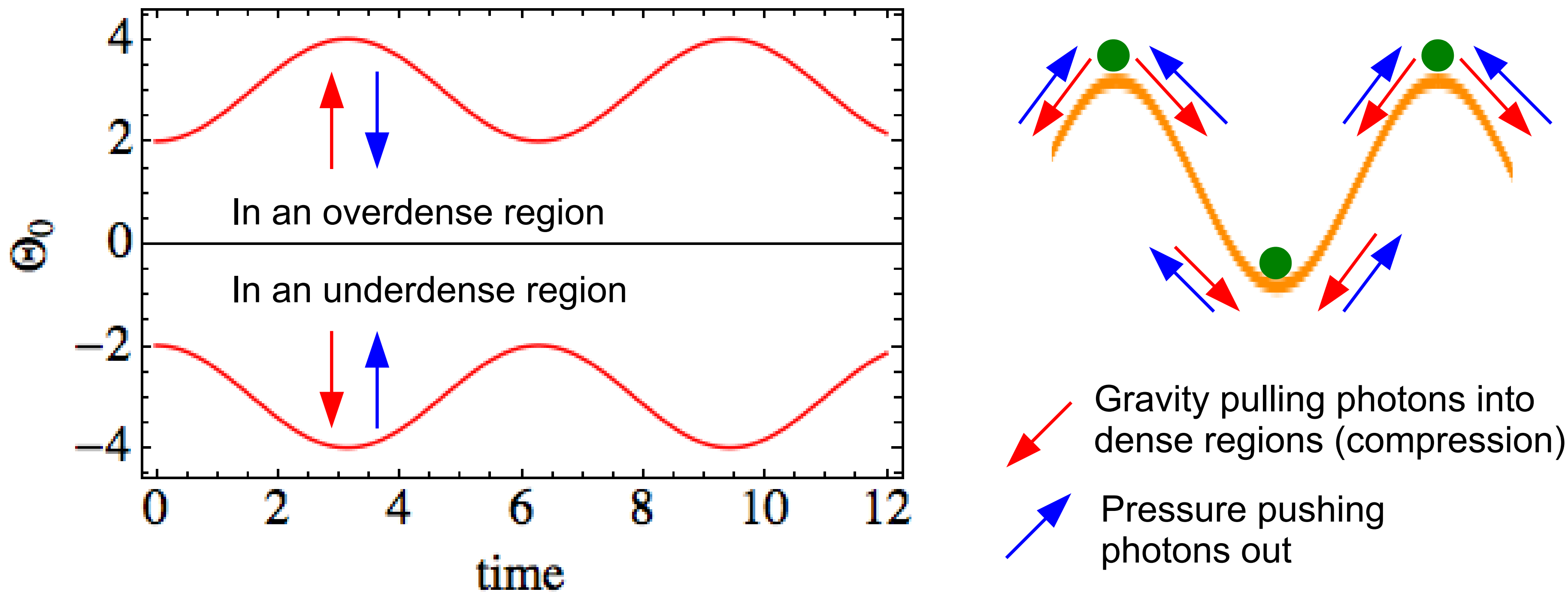
- Sound horizon:

$$\boxed{r_s(\eta)\equiv\int_0^{\eta} d\eta' c_s(\eta')}$$

Coordinate distance travelled by a sound wave since η = 0

6.3 Subhorizon @ decoupling: acoustic oscillations...

- **Suppose baryons are negligible**: R = 0.
- Time evolution for a particular k mode → acoustic oscillations.



6.3 Subhorizon @ decoupling: acoustic oscillations...

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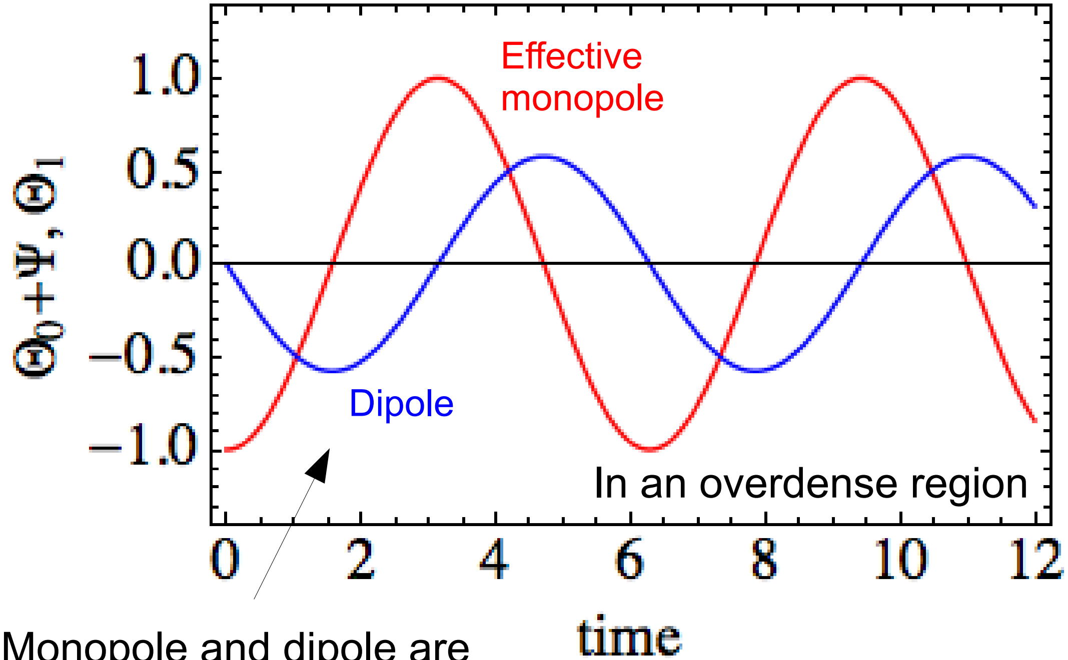
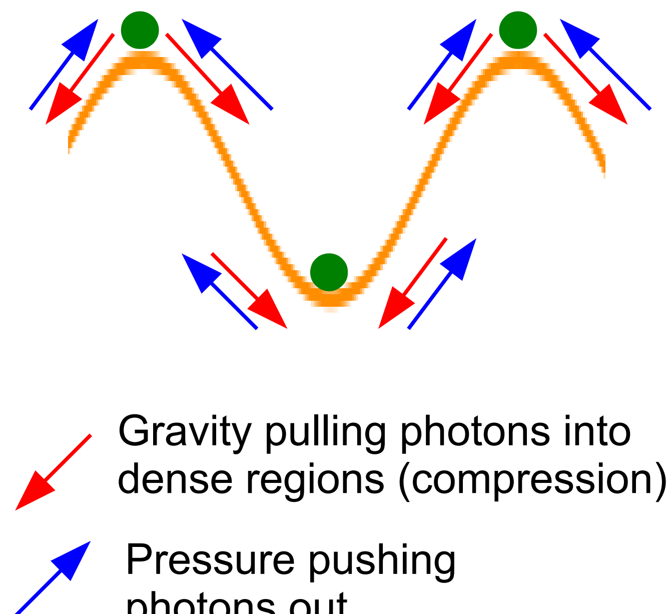


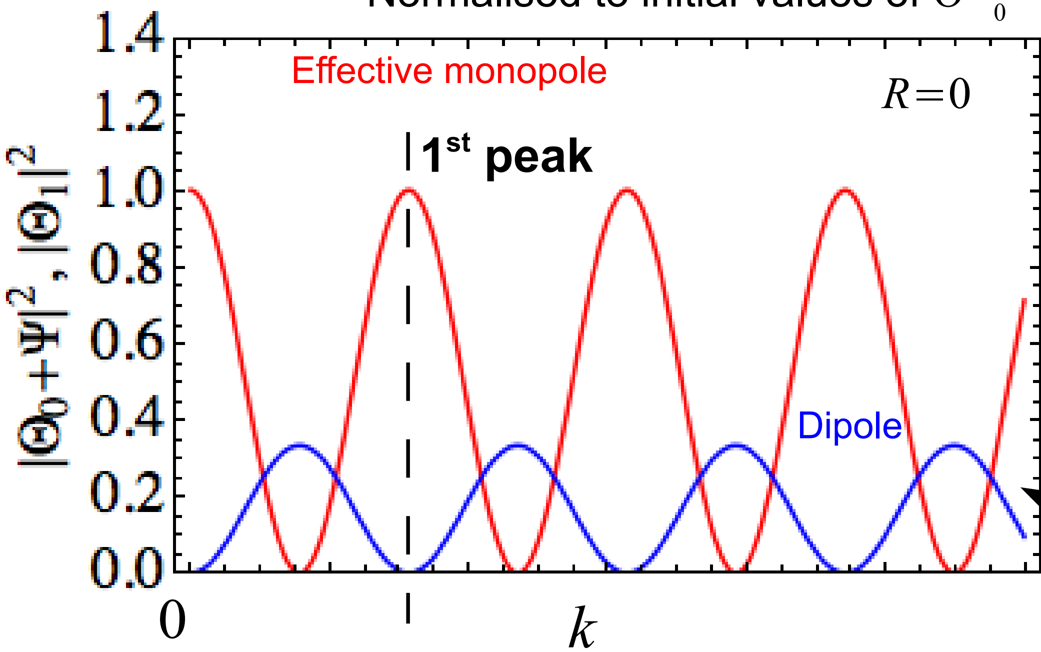
Monopole and dipole are exactly out of phase.



6.3 Subhorizon @ decoupling: acoustic oscillations...

- **Spectrum** at photon decoupling:

Normalised to initial values of $\Theta^{(S)}_0$



Peak positions:

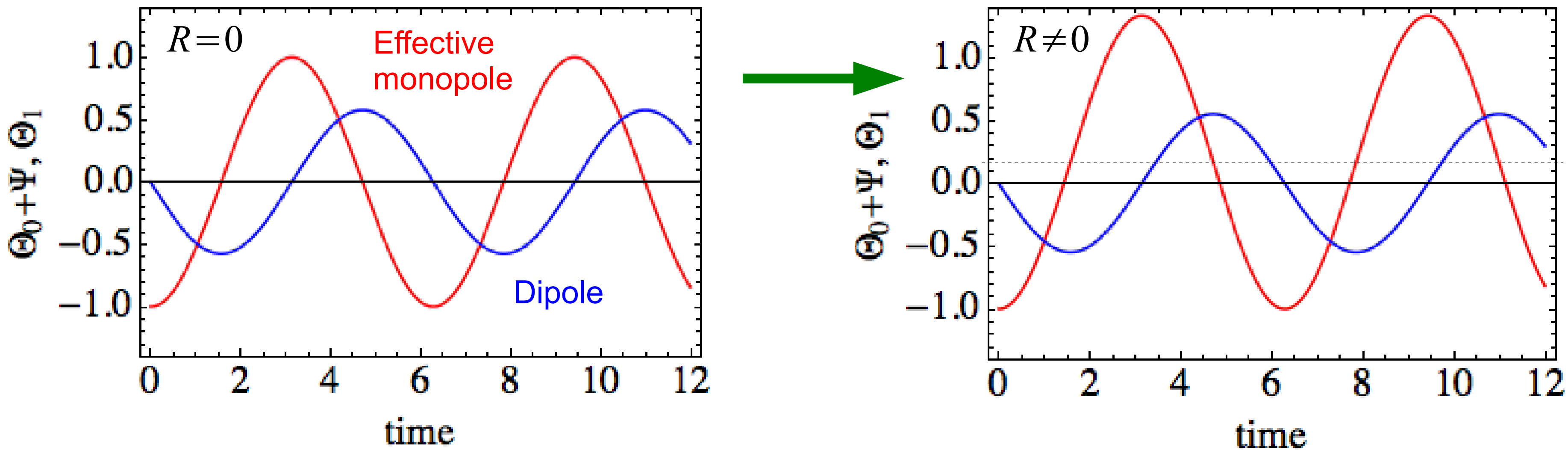
$$\boxed{k_p = \frac{n\pi}{r_s(\eta_*)}, \quad n = 1,2,3,\ldots}$$

Monopole and dipole add incoherently (look at the Legendre series)

- Position of 1^{st} peak corresponds to the k mode that has completed **exactly one** compression at photon decoupling.

6.3 Subhorizon @ decoupling: acoustic oscillations...

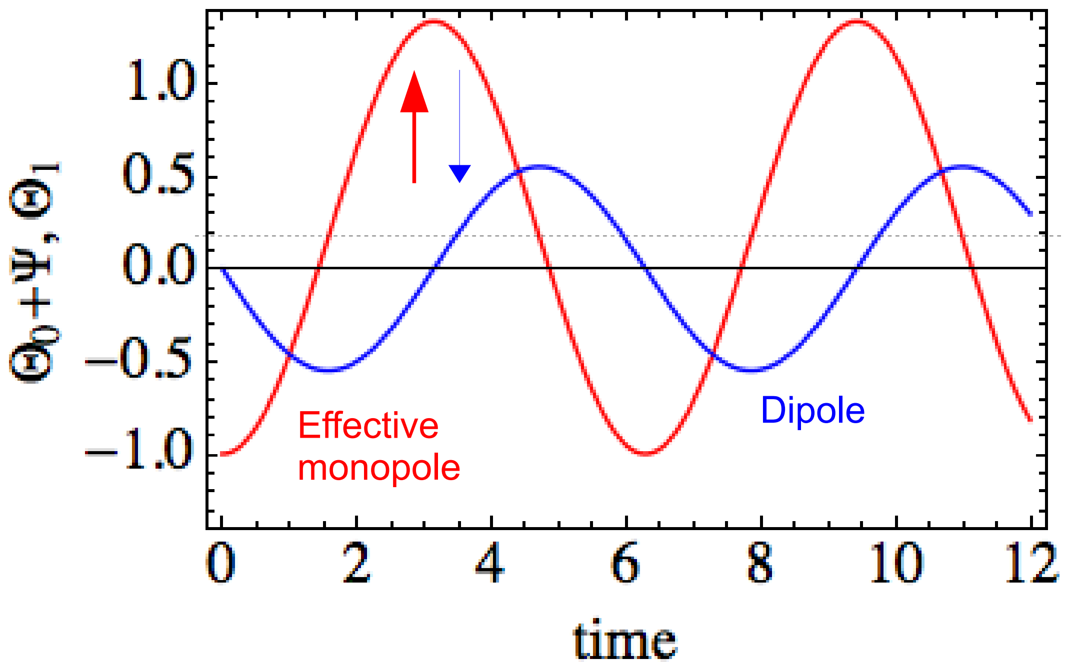
- Now **put the baryons back in**, i.e., $R \neq 0$.

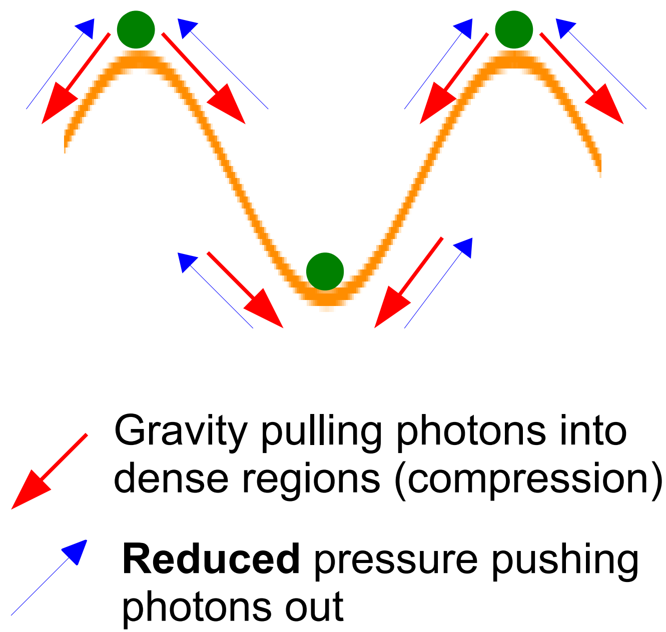


- The presence of baryons offsets the midpoint of oscillations for the **effective monopole,** reduces the **sound horizon**, and alters the oscillation **amplitudes** (monopole and dipole).

6.3 Subhorizon @ decoupling: acoustic oscillations...

- **Physical reason**:



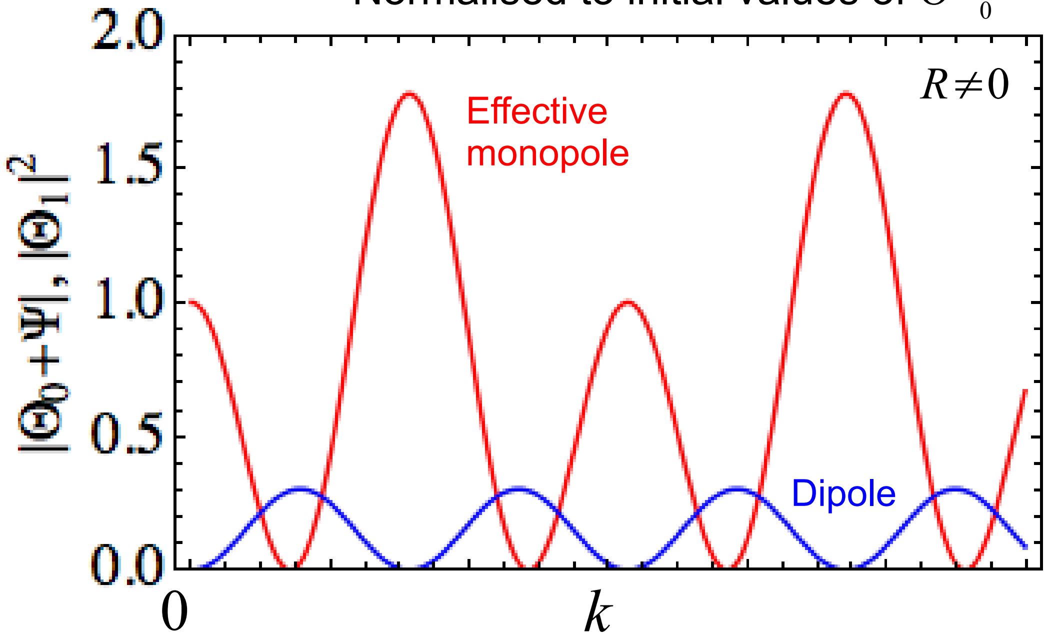


- A **reduced sound speed** due to baryon inertia leads to less pressure resistance → the photons are compressed more and become hotter.

6.3 Subhorizon @ decoupling: acoustic oscillations...

- **Spectrum** at decoupling:

Normalised to initial values of $\Theta^{(S)}_0$



Odd and even peaks now have different heights.

Height ratio depends on the baryon-to-photon ratio R.

- Using time-dependent Φ and Ψ changes the peak heights and positions a little, but the essential features remain.

6.3 Subhorizon @ decoupling: diffusion damping...

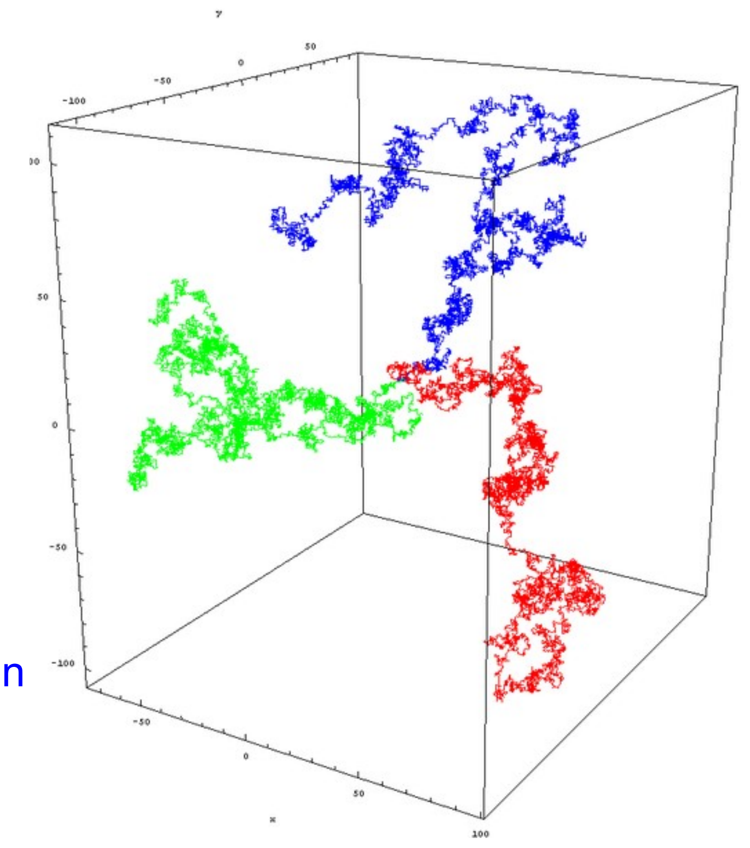
OR Silk damping

- In reality, the motion of the photons and the baryons **cannot** be exactly identical.
  - Photons random walk between Thomson scattering with electrons → diffusion.
  - Diffusion washes out temperature perturbations on scales smaller than the **diffusion length**:

$$\lambda_D = \sqrt{N_{\rm scatter}}\,\lambda_{\rm MFP} \simeq \frac{1}{\sqrt{n_e \sigma_T H}}$$

Photon mean free path

~ a few Mpc at decoupling



6.3 Subhorizon @ decoupling: diffusion damping...

OR Silk damping

- In our Fourier analysis, diffusion damping means an exponential suppression of temperature fluctuations on at $k > k_D$.

$$\Theta_0^{(S)}(k,\eta), \Theta_1^{(S)}(k,\eta) \sim \exp\left(-k^2/k_D^2\right) \times \text{oscillations}$$

- **Diffusion scale**:

(16/15) if including polarisation effects

$$\boxed{\frac{1}{k_D^2(\eta)} \equiv \int_0^{\eta} \frac{d\eta'}{a(\eta')\bar{n}_e \sigma_T}\left[\frac{R^2 + (4/5)(1+R)}{6(1+R)^2}\right]}$$

  - Obtained by keeping $\Theta^{(S)}_2$ in the photon Boltzmann hierarchy in this approximate treatment.

6.4 After decoupling: free-streaming...

- After photon decoupling at T ~ 0.25 eV (z ~ 1100), the universe becomes transparent to photons → photons free-stream.

- To understand the effect of free-streaming on the photon perturbations **today ($\boldsymbol{\eta = \eta_0}$)**, go back to the Boltzmann equation for photons:

$$\partial_\eta \Theta^{(S)} + i k \mu \Theta^{(S)} =$$

$$-ik\mu\Psi + \dot{\Phi} + \dot{\kappa}\left[\Theta_0^{(S)} - \Theta^{(S)} - \frac{1}{2} P_2(\mu)\Theta_2^{(S)} + ik\mu v_b^{(S)}\right]$$

- Formal solution in the $\eta_0 \to \infty$ limit:

$$\Theta^{(S)}(k,\mu,\eta) = \int_0^{\eta_0} d\eta' \, \tilde{S}(k,\mu,\eta) \, e^{ik\mu(\eta'-\eta_0)+\kappa(\eta')}$$

6.4 After decoupling: free-streaming...

- **Decompose** in terms of Legendre polynomial:

$$\Theta_\ell^{(S)}(k,\eta_0)=\int_0^{\eta_0} d\eta\, S(k,\eta)\, j_\ell[k(\eta_0-\eta)]$$

Spherical Bessel functions

- Source function:

$$S(k,\eta)\equiv g(\eta)[\Theta_0^{(S)}+\Psi]-\frac{d}{d\eta}[g(\eta)v_b^{(S)}]$$
$$+e^{\kappa(\eta)}[\dot{\Psi}+\dot{\Phi}]+\frac{1}{4}\left(\frac{3}{k^2}\frac{d^2}{d\eta^2}+1\right)[g(\eta)\Theta_2^{(S)}]$$

Visibility function $g(\eta)\equiv\dot{\kappa}\,e^{\kappa(\eta)}$

6.4 After decoupling: free-streaming...

- Visibility function:

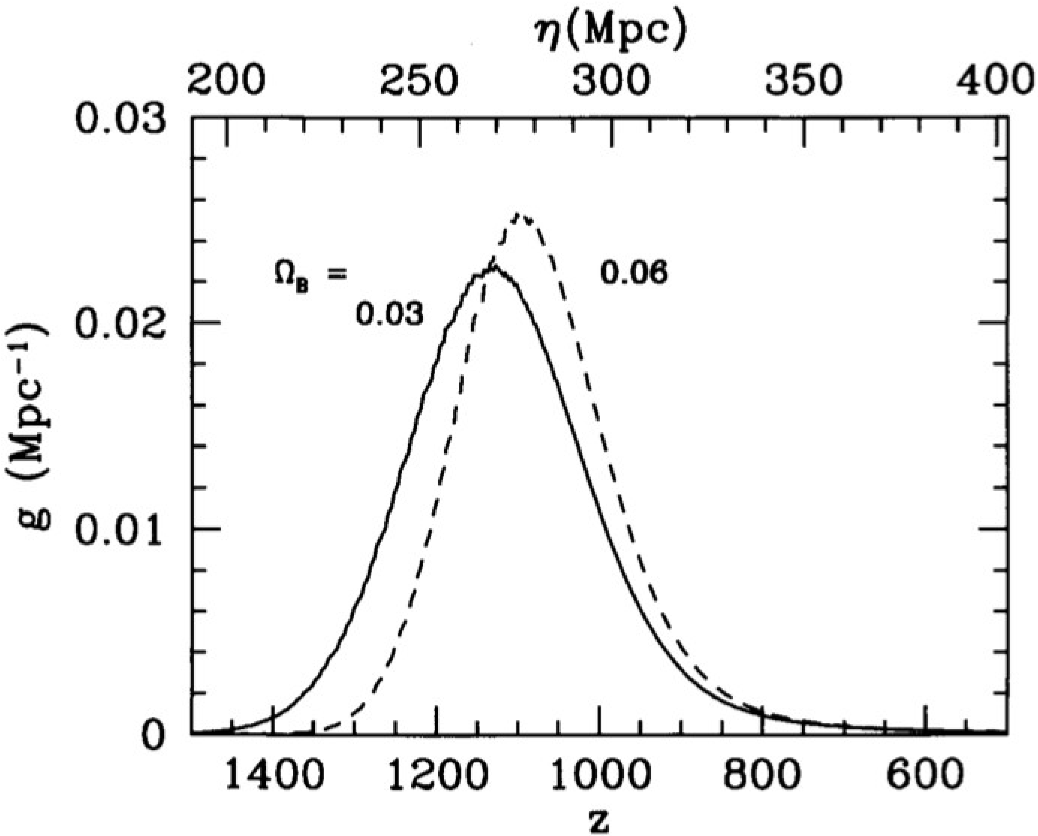
$$\boxed{g(\eta) \equiv \dot{\kappa}\, e^{\kappa(\eta)}}$$

- Normalisation:

$$\int_0^{\eta_0} d\eta' \, g(\eta') = 1$$

→ The visibility function is the **probability** a photon last-scattered at time η.

→ $g(\eta)$ peaks at decoupling (the last scattering surface)



6.4 After decoupling: free-streaming...

- Now we can approximate the **source function**:

$$\Theta_\ell^{(S)}(k,\eta_0)=\int_0^{\eta_0} d\eta\, S(k,\eta)\, j_\ell[k(\eta_0-\eta)]$$

$$S(k,\eta)\equiv g(\eta)[\Theta_0^{(S)}+\Psi]-\frac{d}{d\eta}[g(\eta)v_b^{(S)}]$$
$$+e^{\kappa(\eta)}[\dot{\Psi}+\dot{\Phi}]+\frac{1}{4}\left(\frac{3}{k^2}\frac{d^2}{d\eta^2}+1\right)[g(\eta)\Theta_2^{(S)}]$$

1. $g(\eta)$ peaks at $\eta=\eta_*$ $\rightarrow$ set $g(\eta)=\delta_D(\eta-\eta^*)$.

2. $\Theta^{(S)}{}_2(\eta_*)$ is not generated in a great amount at decoupling compared with $\Theta^{(S)}{}_0(\eta_*)$, $\Theta^{(S)}{}_1(\eta_*)$ $\rightarrow$ set $\Theta^{(S)}{}_2(\eta_*)=0$.

3. Apply the tightly-coupled limit at $\eta=\eta_*$ $\rightarrow$ set $v_b{}^{(S)}(\eta_*)=-(3/k)\,\Theta^{(S)}{}_1(\eta_*)$.

6.4 After decoupling: free-streaming...

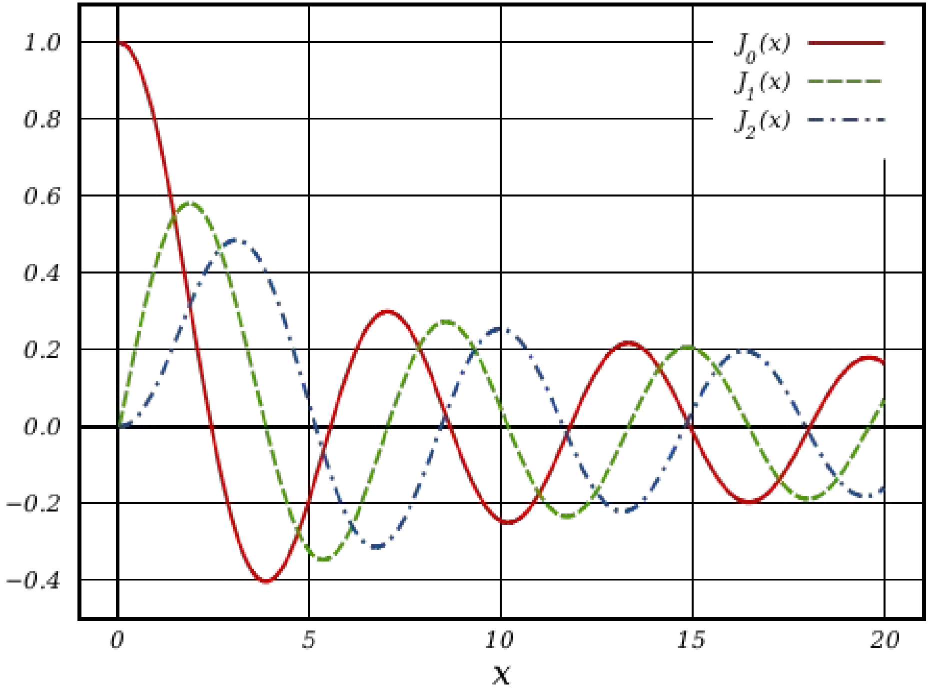
- **Approximate solution**:

$$\Theta_\ell^{(S)}(k,\eta_0) \simeq [\Theta_0^{(S)}(k,\eta_*) + \Psi_0^{(S)}(k,\eta_*)]\, j_\ell[k(\eta_0-\eta_*)]$$
$$- \frac{3}{k}\Theta_1^{(S)}(k,\eta_*)\frac{d}{d\eta} j_\ell[k(\eta_0-\eta_*)]$$
$$+ \int_0^{\eta_0} d\eta\, e^{\kappa(\eta)}[\dot{\Psi}(k,\eta)+\dot{\Phi}(k,\eta)]\, j_\ell[k(\eta_0-\eta)]$$

- Term 1 & term 2: Monopole and dipole at decoupling are primarily responsible for the photon temperature fluctuations observed today .
  - Acoustic oscillations in $\Theta^{(S)}{}_0$ and $\Theta^{(S)}{}_1$ are "spread" to higher multipoles by free-streaming according to the **spherical Bessel functions**.

6.4 After decoupling: free-streaming...

Spherical Bessel functions



- $j_\ell(x)$ peaks at $x \sim \ell$ (not exactly though).

→ $\Theta_\ell(\eta_0)$ gets most contribution from k modes satisfying

$$k \sim \frac{\ell}{\eta_0 - \eta_*}$$

→ We expect the 1st Θ_ℓ peak to occur today at

$$\boxed{\ell_p \sim k_p(\eta_0 - \eta_*) \sim \frac{\pi(\eta_0 - \eta_*)}{r_s(\eta_*)}}$$

From section 6.3

6.4 After decoupling: free-streaming...

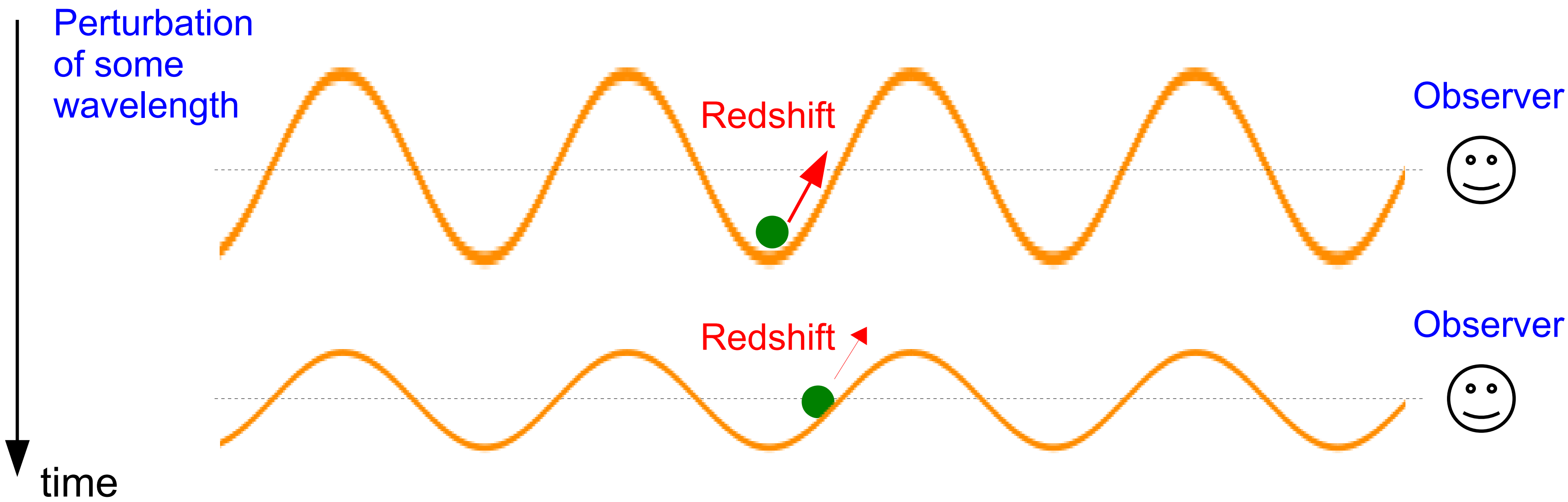
- **Approximate solution**:

$$\Theta_\ell^{(S)}(k,\eta_0) \simeq [\Theta_0^{(S)}(k,\eta_*) + \Psi_0^{(S)}(k,\eta_*)]\, j_\ell[k(\eta_0-\eta_*)]$$
$$- \frac{3}{k}\Theta_1^{(S)}(k,\eta_*)\frac{d}{d\eta} j_\ell[k(\eta_0-\eta_*)]$$
$$+ \int_0^{\eta_0} d\eta\, e^{\kappa(\eta)} [\dot{\Psi}(k,\eta) + \dot{\Phi}(k,\eta)]\, j_\ell[k(\eta_0-\eta)]$$

- Term3: only present if metric perturbations are time-dependent → **Integrated Sachs-Wolfe effect**
  - Important when $|\kappa| << 1$ (i.e., after decoupling)

6.4 After decoupling: free-streaming...

- **Integrated Sachs-Wolfe (ISW) effect**: except deep inside matter domination, subhorizon metric perturbations Φ and Ψ decay.

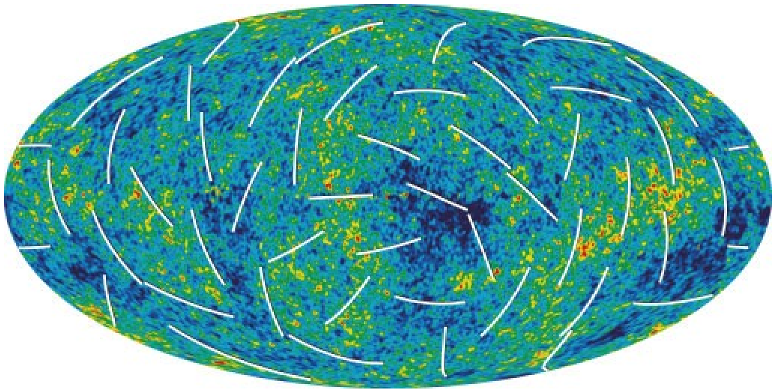


- Photons suffer less gravitational redshift than in the case of constant Φ and Ψ $\rightarrow$ **Larger** observed temperature fluctuations.

6.4 After decoupling: free-streaming...

- There are two ISW effects.
- **Early ISW effect**: photon decoupling occurs quite close to the transition from radiation to matter domination.
  - Residual radiation causes the metric perturbations to decay.
  - Affects most strongly those k modes crossing the horizon at decoupling.
  - Expect strongest contributions close to the first acoustic peak.
- **Late ISW effect**: the transition from matter domination to dark energy domination (i.e., now) also induces metric perturbation decay.
  - Expect contributions on scales close to the present-day horizon.

6.5 Anisotropy power spectrum...



- Recall the photon temperature field is parameterised as:

$$T_\gamma(x^i, n^i, \eta) = \bar{T}_\gamma(\eta)[1 + \Theta(x^i, n^i, \eta)]$$

Direction of 3-momentum

- We can only observe photons here and now → observed temperature fluctuations on a 2D spherical map can be decomposed in terms of spherical harmonics:

$$\Theta(x^i, n^i \eta_0) = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{m=\ell} a_{\ell m}(x^i, \eta_0) Y_{\ell m}(n^i)$$

$$a_{\ell m}(x^i, \eta_0) = \int d\Omega\, Y^*_{\ell m}(n^i) \Theta(x^i, n^i, \eta_0)$$

Fluctuation power spectrum

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell$$

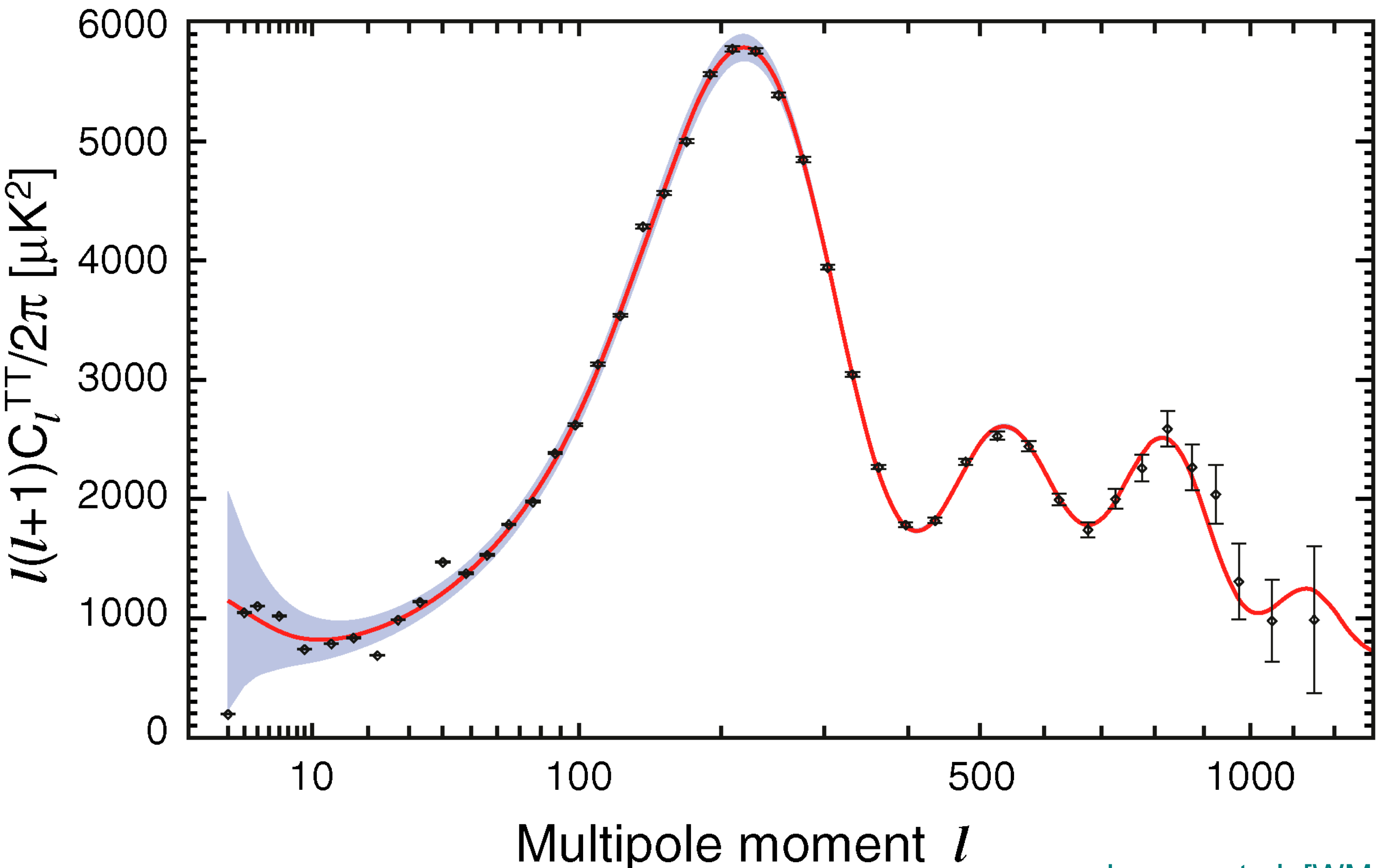
6.5 Anisotropy power spectrum...

- How to get C_ℓ from theory?
- First rewrite $a_{\ell m}(x, \eta_0)$ in terms of $\Theta_\ell(k, \eta_0)$:

$$a_{\ell m}(x^i, \eta_0) = \int d\Omega\, Y^*_{\ell m}(n^i) \int \frac{d^3k}{(2\pi)^3} e^{i k^i x_i} \times \sum_{\ell=0} (-i)^\ell (2\ell+1) P_\ell(k^i n_i / k) \Theta_\ell(k, \eta_0)$$

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell \longrightarrow \boxed{C_\ell = \frac{2}{\pi} \int dk\, k^2 |\Theta_\ell(k, \eta_0)|^2}$$

6.5 Anisotropy power spectrum...



Larson et al. [WMAP7] 2010

6.5 Anisotropy power spectrum...

- Why plot $\ell(\ell+1)C_\ell/2\pi$?

- Suppose the effective monopole at decoupling is given by $[\Theta^{(S)}_0+\Psi](k,\eta_*) = \Phi(k,\eta_*)/3 = (3/10)\Phi_p(k)$ (cf modes outside horizon up to decoupling).

$$\longrightarrow \quad C_\ell = \frac{9}{100}\frac{k_0^3 P_{\Phi_p}(k_0)}{2\pi^2}\left[\underbrace{4\pi\int_0^{k\eta_0\to\infty} dx\, x^{-1} j_\ell^2(x)}_{\frac{2\pi}{\ell(\ell+1)}}\right]$$

Dimensionless power spectrum from inflation at the pivot scale

Assuming a scale-invariant primordial power spectrum $n_S = 1$

$\rightarrow$ A constant $\ell(\ell+1)C_\ell/2\pi$ corresponds to a white-noise spectrum.

6.5 Anisotropy power spectrum: low multipoles...

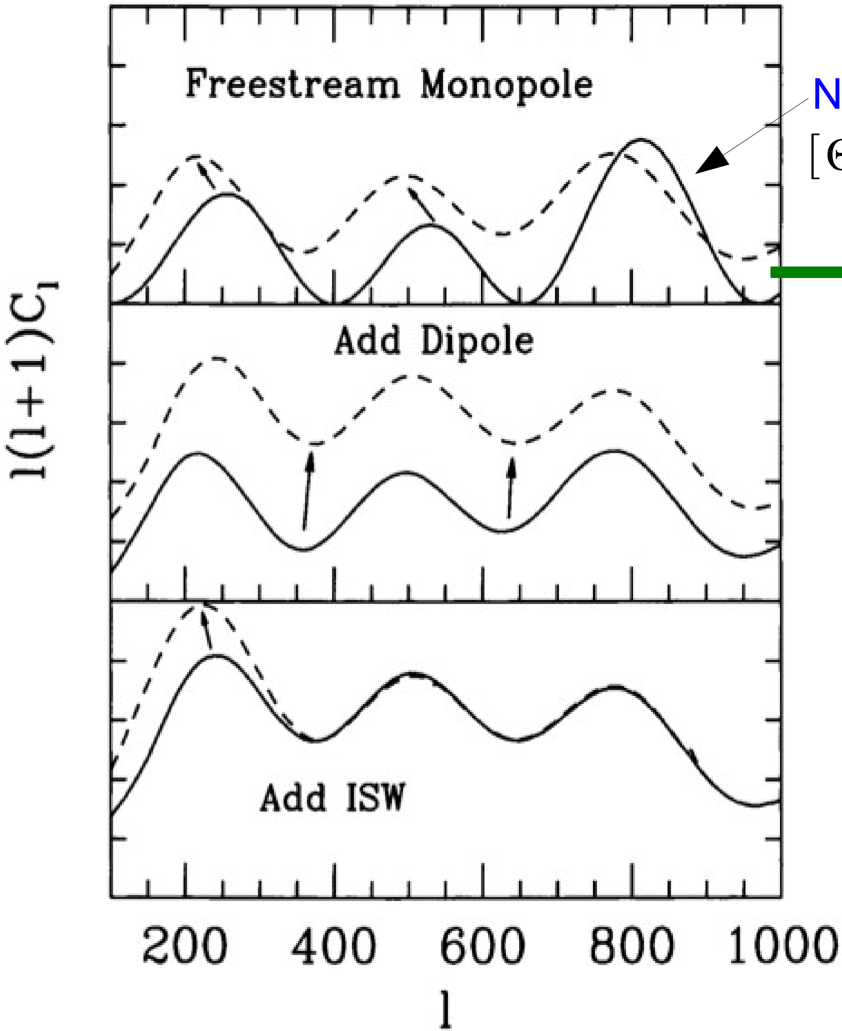
- A more or less flat $\ell(\ell+1)C_\ell/2\pi$ is in fact what we expect to see at **low ℓ multipoles**, where most contributions come from those k modes that were superhorizon at photon decoupling.

  $\rightarrow$ The Sachs-Wolfe plateau.

- A more general expression for an arbitrary scalar spectral index n_s:

$$C_\ell = \frac{9}{100} \frac{P_{\Phi_p}(k_0)}{2\pi(\eta_0 - \eta_*)^{n_S - 1}} \left(\frac{k_0}{2}\right)^{4-n_S} \frac{\Gamma(\ell + n_S/2 - 1/2)\Gamma(3 - n_S)}{\Gamma(\ell + 5/2 - n_S/2)\Gamma^2(2 - n_S/2)}$$

6.5 Anisotropy power spectrum: high multipoles...



Naïve projection

$$[\Theta_0^{(S)}+\Psi](k=\ell/(\eta_0-\eta_*),\eta_*)$$

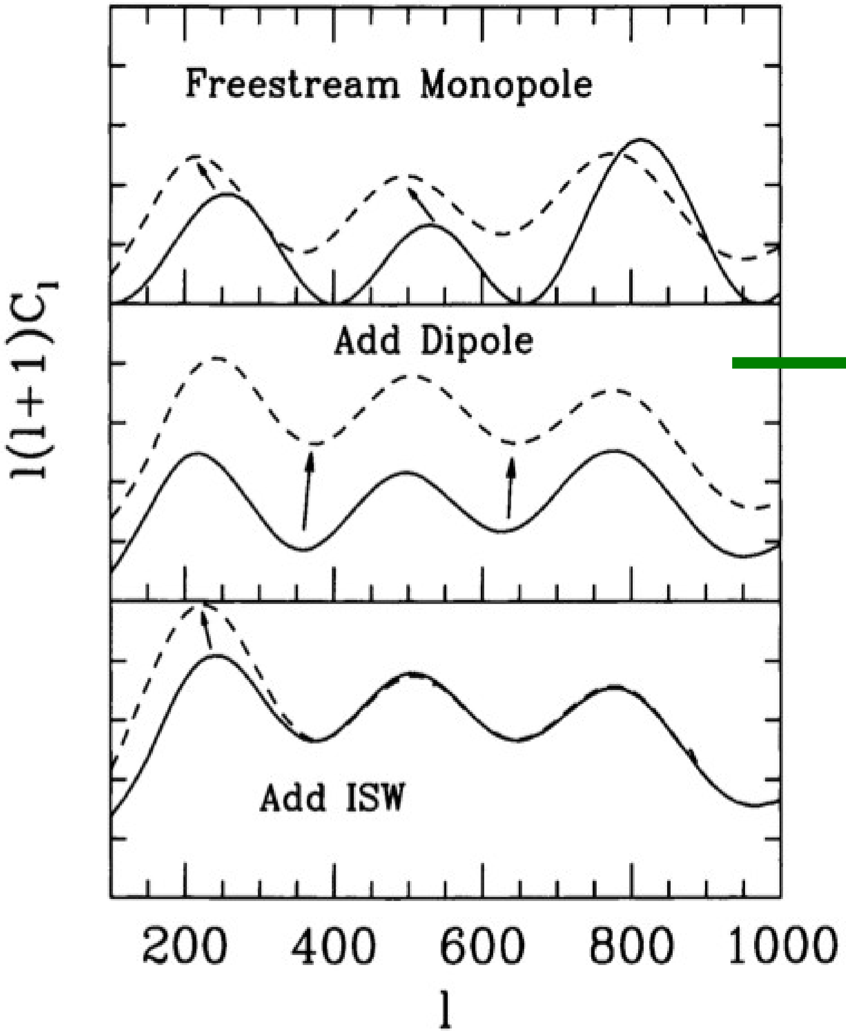
Proper treatment of **free-streaming** shifts peaks a little from their naïvely expected positions.

$$\ell_p \sim \frac{\pi(\eta_0-\eta_*)}{r_s}$$

From section 6.5

$$\Theta_\ell^{(S)}(k,\eta_0) \simeq$$
$$[\Theta_0^{(S)}(k,\eta_*)+\Psi_0^{(S)}(k,\eta_*)]\,j_\ell[k(\eta_0-\eta_*)]$$
$$-\frac{3}{k}\Theta_1^{(S)}(k,\eta_*)\frac{d}{d\eta}j_\ell[k(\eta_0-\eta_*)]$$
$$+\int_0^{\eta_0} d\eta\, e^{\kappa(\eta)}[\dot{\Psi}(k,\eta)+\dot{\Phi}(k,\eta)]\,j_\ell[k(\eta_0-\eta)]$$

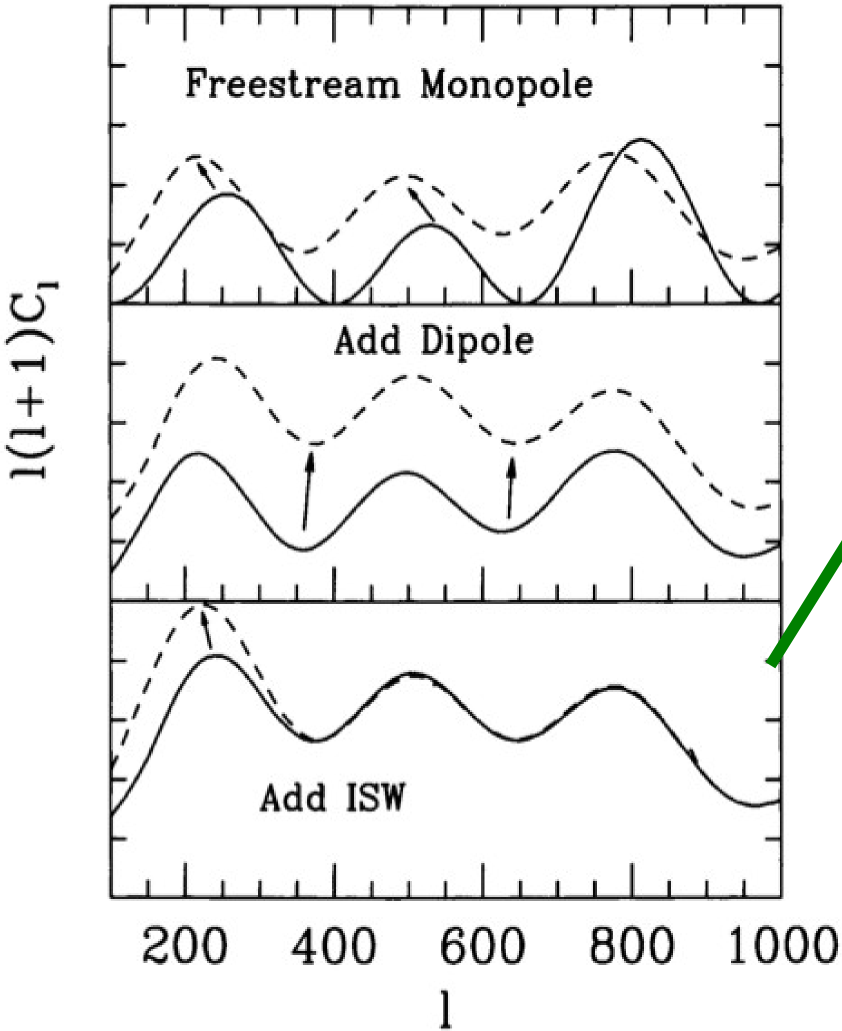
6.5 Anisotropy power spectrum: high multipoles...



Monopole and dipole add incoherently (because of property of spherical Bessel function); adding dipole makes the troughs less prominent.

$$\Theta_\ell^{(S)}(k,\eta_0) \simeq$$
$$[\Theta_0^{(S)}(k,\eta_*)+\Psi_0^{(S)}(k,\eta_*)]\, j_\ell[k(\eta_0-\eta_*)]$$
$$-\frac{3}{k}\Theta_1^{(S)}(k,\eta_*)\frac{d}{d\eta} j_\ell[k(\eta_0-\eta_*)]$$
$$+\int_0^{\eta_0} d\eta\, e^{\kappa(\eta)}[\dot{\Psi}(k,\eta)+\dot{\Phi}(k,\eta)]\, j_\ell[k(\eta_0-\eta)]$$

6.5 Anisotropy power spectrum: high multipoles...



Early ISW effect contributes adds in phase with the monopole

$$\Theta_\ell^{(S)}(k,\eta_0) \simeq \left[\Theta_0^{(S)}(k,\eta_*) + \Psi_0^{(S)}(k,\eta_*)\right] j_\ell[k(\eta_0-\eta_*)] - \frac{3}{k}\Theta_1^{(S)}(k,\eta_*)\frac{d}{d\eta} j_\ell[k(\eta_0-\eta_*)] + \int_0^{\eta_0} d\eta\, e^{\kappa(\eta)}[\dot{\Psi}(k,\eta)+\dot{\Phi}(k,\eta)]\, j_\ell[k(\eta_0-\eta)]$$

7. Cosmological parameters from CMB temperature anisotropies...

7.1 Cosmological parameters...

- Some standard parameters to constrain...
  - **Matter density**: $\Omega_m h^2$
  - **Baryon density**: $\Omega_b h^2$
  - **Hubble parameter**, **spatial curvature**, **dark energy**: h, Ω_K, Ω_Λ
  - **Inflation parameters**: scalar fluctuation amplitude A_S, spectral index n_S
- The CMB temperature anisotropies do not measure these parameters *per se*, rather some combination thereof.

7.2 CMB anisotropies measure z equality...

- The early ISW effect enhances the first peak because of the time-dependence of the metric perturbations when transiting from RD to MD.

  - The **ratio** of the 1st peak to the Sachs-Wolfe plateau, or of the 1st peak to the 3rd peak can establish the early ISW effect.
  - In standard ΛCDM, the only parameter controlling this transition is the **time of matter-radiation equality**.

$$1+z_{\rm eq}=\frac{\Omega_m h^2}{\Omega_\gamma h^2+\Omega_\nu h^2}\simeq\frac{\Omega_m h^2}{\Omega_\gamma h^2}\frac{1}{1+0.2271\,N_\nu}$$

$$\Omega_\gamma h^2=2.47\times10^{-5}$$

- If we **assume** $N_\nu=3$ massless neutrinos, then this constitutes a measurement of $\Omega_m h^2$; no conclusions yet if N_ν is not known.

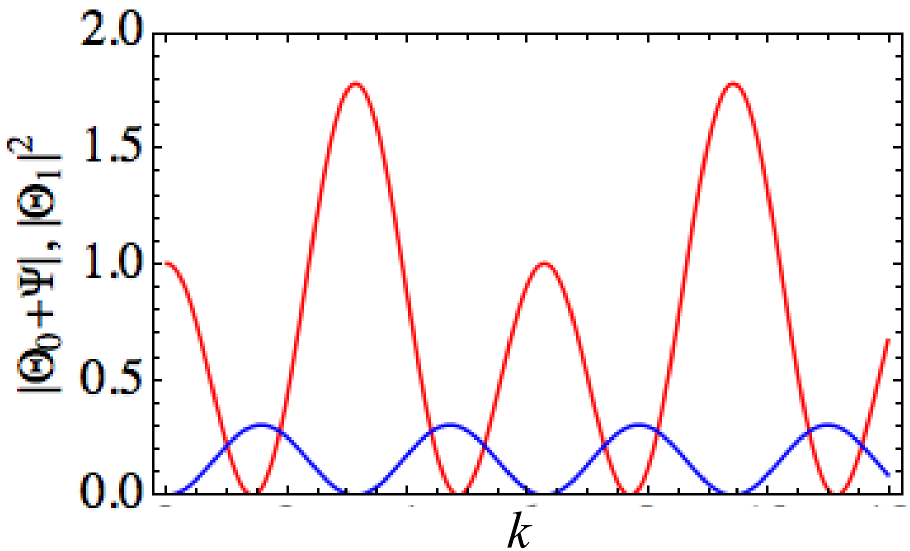
7.3 CMB anisotropies measure baryon-photon ratio...

- **Odd to even acoustic peak ratios** are determined by

$$R \equiv \frac{3}{4}\frac{\bar{\rho}_b}{\bar{\rho}_\gamma} = \frac{3}{4}\frac{\Omega_b h^2}{\Omega_\gamma h^2} a$$

- Since $\Omega_\gamma h^2$ is known, we have a measurement of $\Omega_b h^2$.

- Probably the most robust (i.e. independent of cosmological model) parameter measurement from CMB.



7.4 CMB anisotropies measure angular sound horizon..

- Position of the 1st acoustic peak is given approximately by

$$\ell_p \sim \frac{\pi(\eta_0 - \eta_*)}{r(\eta_*)}$$

Comoving distance to the last scattering surface

$$\eta_0 - \eta_* = \chi(\eta_*)$$

Sound horizon at decoupling

- If we had allowed for spatial curvature:

$$\chi(\eta_*) \to \begin{matrix} \sin\chi(\eta_*) \\ \sinh\chi(\eta_*) \end{matrix}$$

K = +1
K = -1

- A more general expression for the 1st peak position:

Angular sound horizon

$$\boxed{\theta_s \equiv \frac{\pi}{\ell_p} = \frac{a(\eta_*)\, r_s(\eta_*)}{d_A(\eta_*)}}$$

Angular diameter distance to the last scattering surface

7.4 CMB anisotropies measure angular sound horizon..

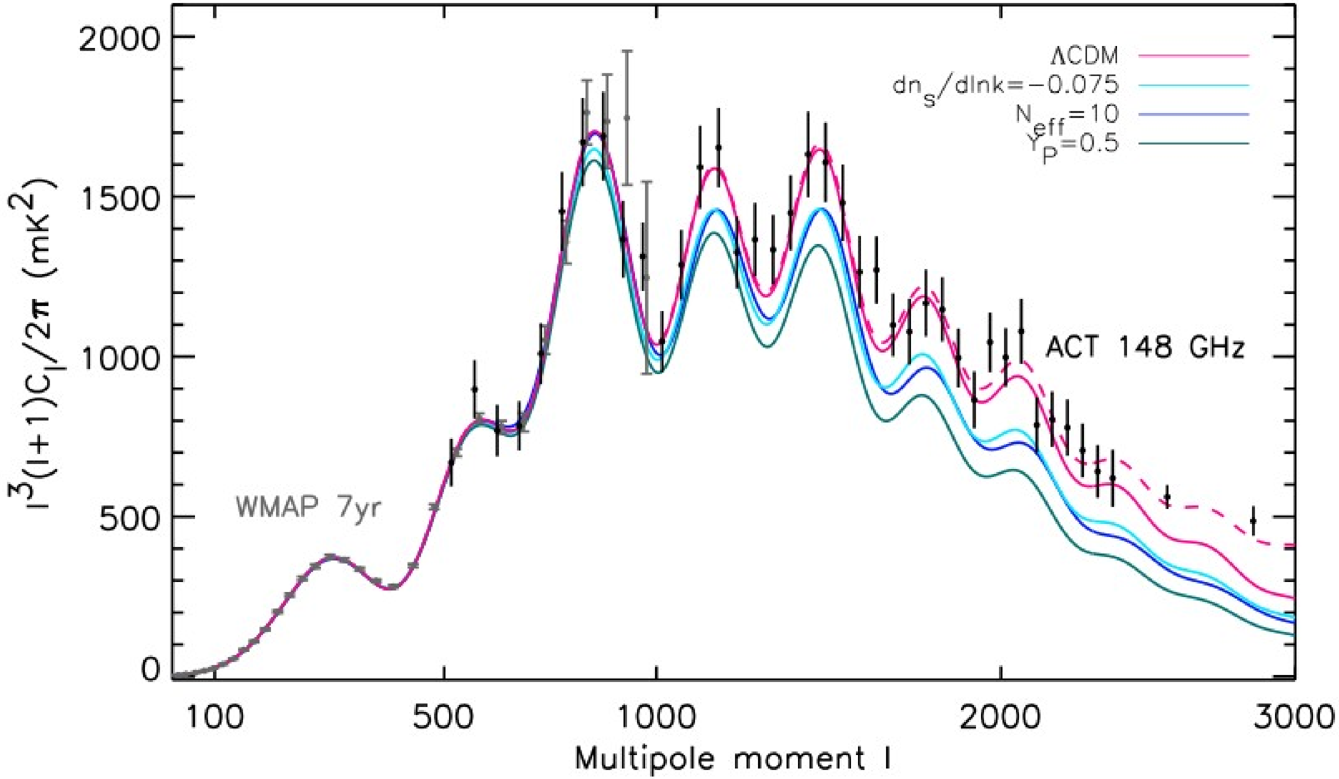
- For fixed z_{eq}, $\Omega_b h^2$, and $a(\eta_*)$, the main parameter dependence of θ_s in **flat ΛCDM** is:

$$\theta_s \propto \frac{\left(\Omega_m h^2\right)^{-1/2}}{\int_{a_{\eta_*}}^{1} \frac{d\,a}{a^2\sqrt{\Omega_m h^2 a^{-3} + \left(h^2 - \Omega_m h^2\right)}}}$$

- **If $\Omega_m h^2$ is known**, then the angular sound horizon provides a measurement of the Hubble parameter h.
- **If $\Omega_m h^2$ is not known** (e.g., because we do not know the exact radiation content), then $\Omega_m h^2$ and h are exactly degenerate parameters.
- More degeneracies if the dark energy has a nontrivial equation of state.

7.5 CMB anisotropies measure the damping scale...

- Only possible with recent measurements from ACT and SPT.



Dunkley et al. [ACT] 2010

7.5 CMB anisotropies measure the damping scale...

- Angular damping scale (with z_{eq}, $\Omega_b h^2$ and $a(\eta_*)$ fixed):

$$\theta_D \equiv \frac{r_D(\eta_*)}{d_A(\eta_*)} \propto \frac{\left(\Omega_m h^2\right)^{-1/4}}{\displaystyle\int_{a_{\eta_*}}^{1} \frac{d\,a}{a^2\sqrt{\Omega_m h^2 a^{-3} + \left(h^2 - \Omega_m h^2\right)}}}$$

Diffusion damping

$$r_D(\eta) \equiv \frac{1}{k_D(\eta)}$$

From section 6.4

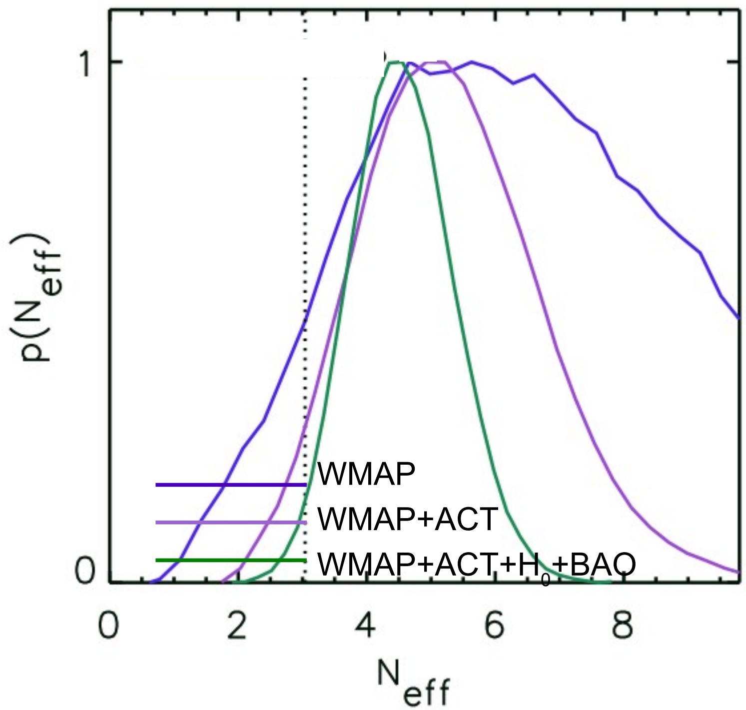
- **Combine** with measurement of angular sound horizon:

$$\frac{\theta_D}{\theta_s} = \frac{r_D(\eta_*)}{r_s(\eta_*)} \propto \left(\Omega_m h^2\right)^{1/4}$$

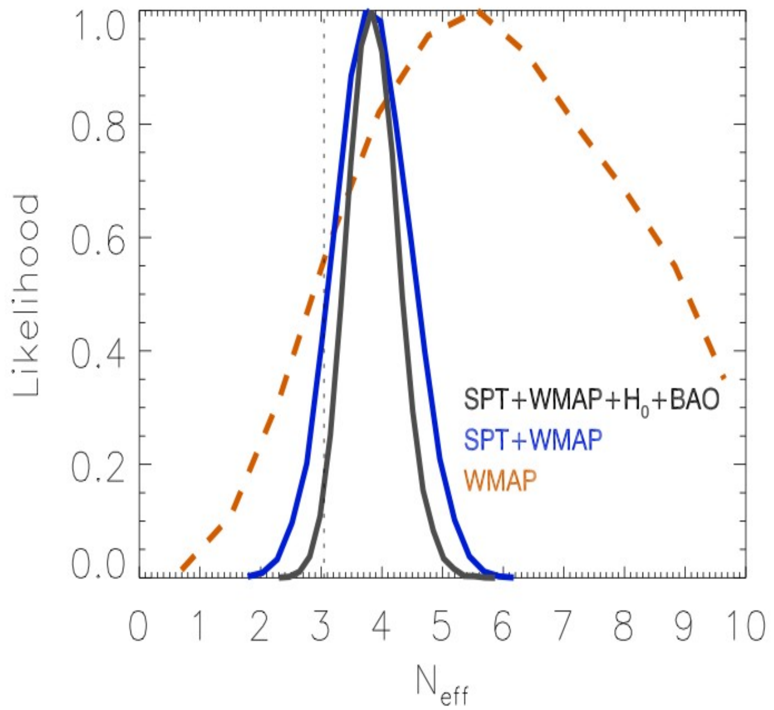
$\rightarrow$ A measurement of the matter density that is independent of the assumptions about **spatial curvature**, **dark energy**, etc. (because the angular diameter distance has now factored out).

7.5 CMB anisotropies measure the damping scale...

- Measurements of the CMB damping tail by ACT and SPT seem to suggest that the number of neutrino species is **larger than 3**!



Dunkley et al. [Atacama Cosmology Telescope] 2010



Keisler et al. [South Pole Telescope] 2011

7.6 Section summary...

- The CMB temperature anisotropies are sensitive to:
  - The redshift of matter-radiation equality (1st to 3rd peak heights, 1st peak height to Sachs-Wolfe plateau).
  - The **baryon-to-photon ratio** (odd to even peak heights)
  - The angular sound horizon (peak positions)
  - The **angular damping scale** (damping tail)
- Combining these measurements, it is possible to constrain the underlying cosmological model parameters.
- Beware of parameter degeneracies!