Numerical Evaluation of Multi-loop Integrals

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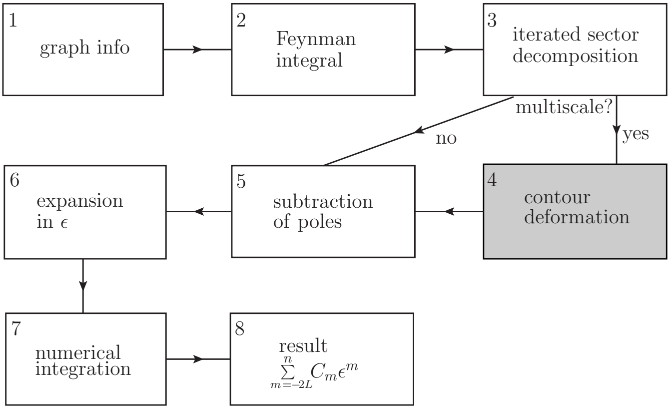
http://projects.hepforge.org/secdec

Making Predictions in the LHC Era

- Precise predictions are desirable for the upcoming wealth of LHC data
- Higher orders in perturbation theory are needed
- Multi-dimensional parameter integrals need to be evaluated which can contain UV, soft and collinear singularities
- Publicly available SecDec 1.0 [1] program does automated factorization and the numerical integration for Euclidean kinematics
- SecDec 2.0 calculates diagrams with arbitrary kinematics

Operational Sequence of the SecDec 2.0 Program



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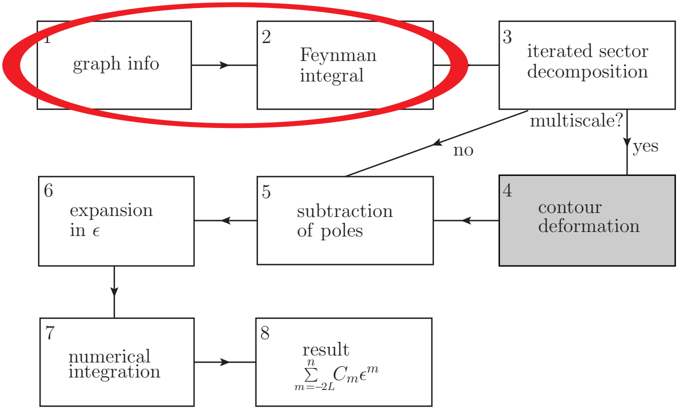

General Feynman Integral

- Graph infos are converted into tensorial **Feynman integral** $G^{\mu_1 \cdots \mu_R}$ in D dimensions at L loops with N propagators to power ν_j of rank R
- After loop momentum integration, a generic scalar **Feynman integral**

$$G = \frac{(-1)^{N_\nu}}{\prod_{j=1}^{N} \Gamma(\nu_j)} \Gamma(N_\nu - LD/2) \int_0^\infty \prod_{j=1}^{N} \mathrm{d}x_j \; x_j^{\nu_j - 1} \, \delta(1 - \sum_{l=1}^{N} x_l) \frac{\mathcal{U}^{N_\nu - (L+1)D/2}(\vec{x})}{\mathcal{F}^{N_\nu - LD/2}(\vec{x})}$$

where $N_\nu = \sum_{j=1}^{N} \nu_j$ and where $\mathcal{F}$ and $\mathcal{U}$ can be constructed via **topological cuts**

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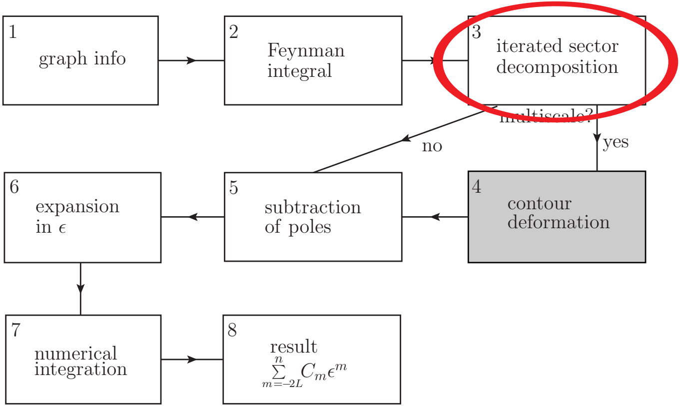
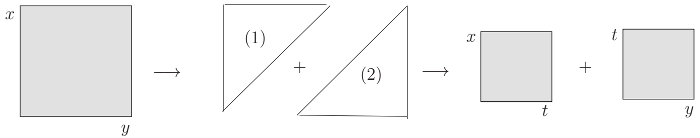


Sector Decomposition

- Overlapping divergences are factorized



$$\int_0^1 dx \int_0^1 dy \, \frac{1}{(x+y)^2} = \int_0^1 dx \int_0^1 dt \, \frac{1}{x\,(1+t)^2} + \int_0^1 dt \int_0^1 dy \, \frac{1}{y\,(1+t)^2}$$

- Iterated **sector decomposition** [2, 3] is done, where dimensionally regulated soft, collinear and UV singularities are factored out

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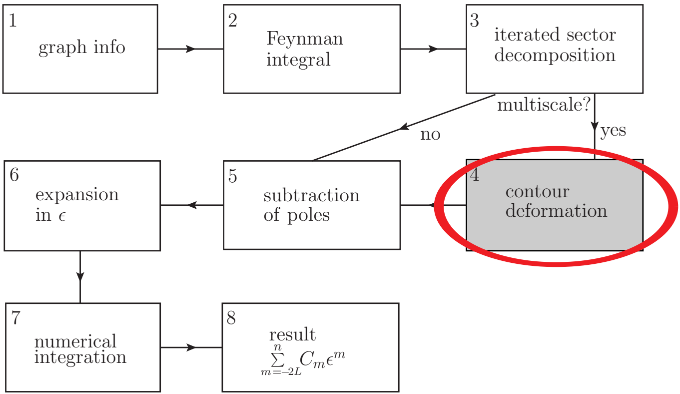
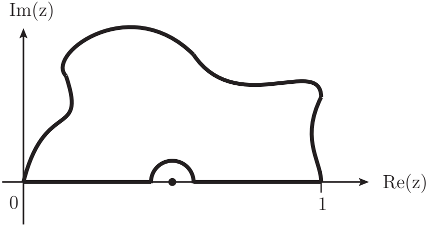


Contour Deformation I

- For kinematics in the physical region, $\mathcal{F}$ can still vanish

$$\mathcal{F}_{example} = -s_{12} - s_{23}\, t_1 t_2 - i\delta$$

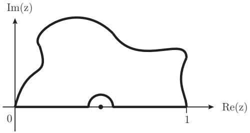
but a deformation of the integration contour



and Cauchy's theorem can help

$$\oint_c f(z)\mathrm{d}z = \int_0^1 \frac{\partial z(t)}{\partial t} f(z(t))\mathrm{d}t + \int_1^0 f(z)\mathrm{d}z = 0$$

Contour Deformation II



- The integration contour is deformed by

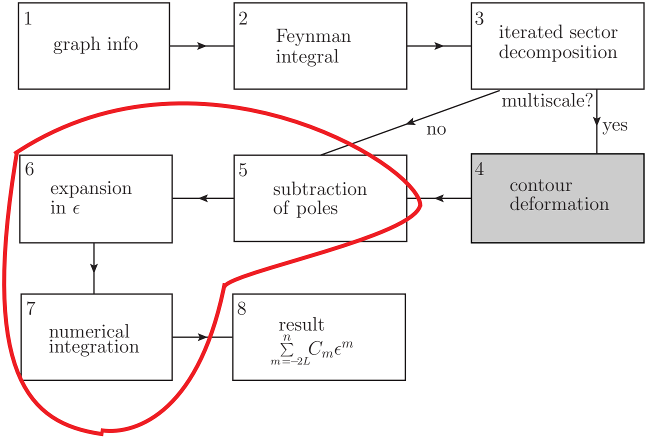
$$\vec{t} \to \vec{z} = \vec{t} + \mathrm{i}\vec{y} \ ,$$

$$y_j(\vec{t}) = -\lambda t_j (1 - t_j) \frac{\partial \mathcal{F}(\vec{t})}{\partial t_j}$$

- Integrand is analytically continued into the complex plane

$$\mathcal{F}(\vec{t}) \to \mathcal{F}(\vec{t} + \mathrm{i}\vec{y}(\vec{t})) = \mathcal{F}(\vec{t}) + \mathrm{i} \sum_j y_j(\vec{t}) \frac{\partial \mathcal{F}(\vec{t})}{\partial t_j} + \mathcal{O}(y(\vec{t})^2)$$

Operational Sequence of the SecDec 2.0 Program



Subtraction, Expansion, Numerical Integration

Subtraction

- The factorized poles in a subsector integrand $\mathcal{I} \propto \mathcal{U}, \mathcal{F}$ are extracted by subtraction (e.g. logarithmic divergence)

$$\int_0^1 dt_j \, t_j^{-1-b_j\epsilon} \mathcal{I}(t_j, \epsilon) = -\frac{\mathcal{I}(0,\epsilon)}{b_j\epsilon} + \int_0^1 dt_j \, t_j^{-1-b_j\epsilon} (\mathcal{I}(t_j,\epsilon) - \mathcal{I}(0,\epsilon))$$

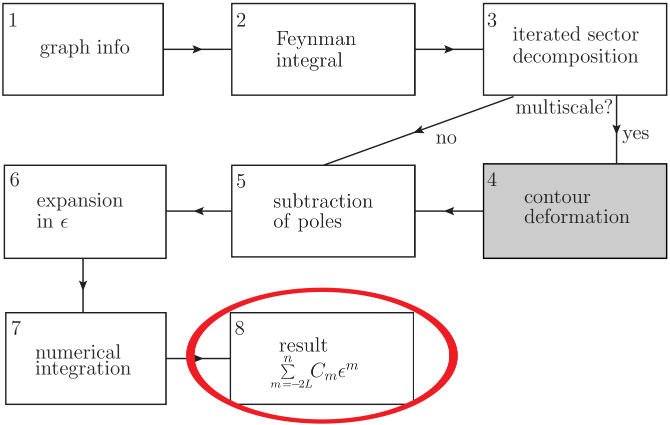
Expansion

- After the extraction of poles, an expansion in the regulator ϵ is done

Numerical Integration

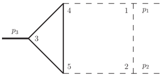
- Monte Carlo integrator programs containted in CUBA library[4, 5] or BASES [6] can be used for numerical integration

Operational Sequence of the SecDec 2.0 Program



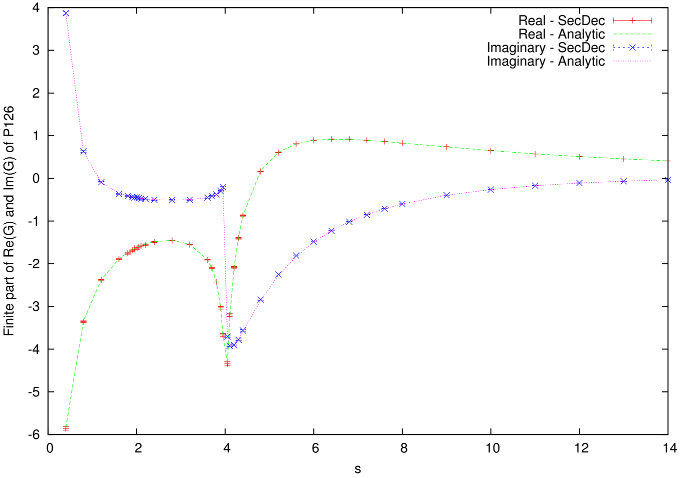
Results

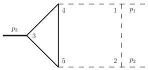
- Successful application of the public **SecDec 1.0** program to massless multi-loop diagrams up to 5-loop 2-point functions and 4-loop 3-point functions for Euclidean kinematics
- Successful application of **SecDec 2.0** to various multi-scale examples, e.g., massive 2-loop vertex graph, planar and non-planar 2-loop massive boxes
- Timings for the 2-loop vertex diagram and a relative accuracy of 1% using the CUBA 3.0 library on an Intel(R) Core i7 CPU at 2.67GHz



s/m^2	timing (finite part)
3.9	13.6 secs
14.0	12.1 secs

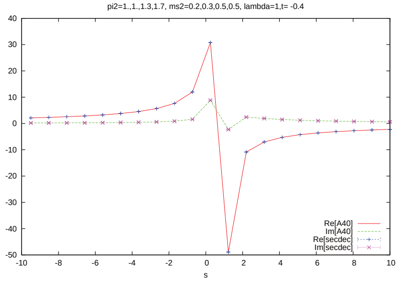
Results II: Massive Two-loop Vertex Graph G

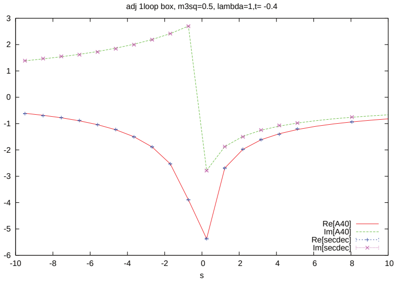




[7, 8, 9]

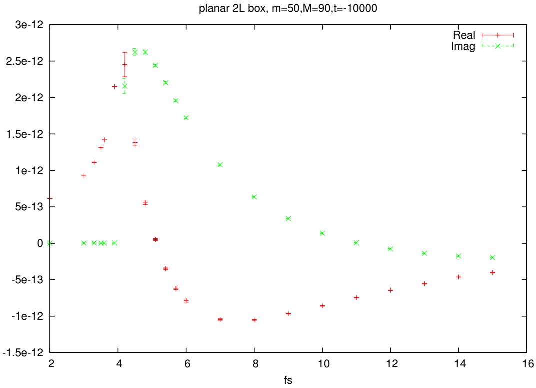
Results III: One-loop Massive Boxes

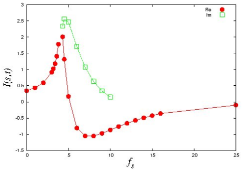




GoSam

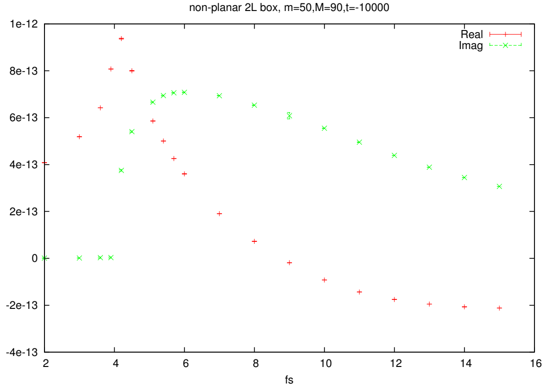
Results IV: Planar Massive Two-loop Box

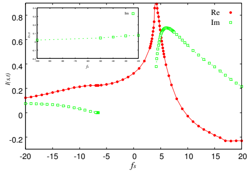




arXiv: 1112.0637 [hep-ph]

Results V: Non-planar Massive Two-loop Box





arXiv: 1112.0637 [hep-ph]

Summary & Outlook

Summary

- With SecDec 2.0 the numerical evaluation of multi-loop integrals is possible for arbitrary kinematics
- Timings close to threshold very similar to timings far from threshold with comparable accuracy

Before SecDec 2.0 is going public

- Testing of more graphs
- Improve timings and numerical robustness

References

[1] J. Carter and G. Heinrich, arXiv:1011.5493 [hep-ph]

[2] T. Binoth and G. Heinrich, arXiv:hep-ph/0004013

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[4] T. Hahn, arXiv:hep-ph/0404043

[5] S. Agrawal, T. Hahn and E. Mirabella, arXiv:1112.0124 [hep-ph]

[6] S. Kawabata, Comput. Phys. Commun. **88** 309-326

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[8] R. Bonciani, P. Mastrolia and E. Remiddi, arXiv:hep-ph/0311145

[9] A. Ferroglia, M. Passera, G. Passarino and S. Uccirati, arXiv:hep-ph/0311186