Semiclassical strings in AdS/CFT

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Gauge/gravity duality

Equivalence between two theories:

QFT	$\longleftrightarrow$	Gravity theory
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CFT		AdS gravity theory
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Conjecture but can be based on physical arguments + significant evidence that it is correct!

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- Interactions renormalize Δ. It gets quantum corrections.

$$\Delta = \Delta_0 + \Delta_{q.c}$$

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$$ds^2 = \left(1 + \frac{R^4}{r^4}\right)^{-1/2} (\eta_{ij} dx^i dx^j) + \left(1 + \frac{R^4}{r^4}\right)^{1/2} (dr^2 + r^2 d\Omega_5^2)$$

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$r \rightarrow \infty \Rightarrow ds^2 \rightarrow \text{Minkowski}_{10}$ $\qquad$ $r \rightarrow 0 \Rightarrow ds^2 \rightarrow ds_{AdS_5}^2 + R^2 d\Omega_5^2$

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$\mathcal{N}=4$ $SU(N)$ SYM		Type IIB superstring theory
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In particular

Δ scaling dimension of operator	$\longleftrightarrow$	E energy of string state

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  Strings on curved backgrounds described by weakly coupled QFT.

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$\Rightarrow$ Check AdS/CFT by comparing $E(\lambda, J)$ to $\Delta(\lambda, J)$

Semiclassical strings on on $AdS_5 \times S^5$

Check the AdS/CFT correspondence:

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✓ Verified for specific cases at order λ, λ^2, λ^3.

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Danke!

Thank you very much for your attention.

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