T-duality covariant formulations of supergravities: Double Field Theory & Generalized Geometry

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Strings in toroidal backgrounds

- compactified string coordinates: $x^i = (x^\mu, x^a)$, $\quad x^a \sim x^a + 2\pi$
- quantized momenta: $\quad p_i = (k_\mu, p_a)$
- additional winding modes: w^a
  $\rightarrow$ conjugated coordinates: $\tilde{x}_i = (0, \tilde{x}_a)$, $\quad \tilde{x}_a \sim \tilde{x}_a + 2\pi$

possibility of two sets of coordinates $x, \tilde{x}$

T-duality

review: Giveon et al., hep-th/9401139

- string theory experiences T-duality in toroidal backgrounds
- T-duality exchanges momenta p and winding w
  $\rightarrow$ "dual" sets of coordinates $x, \tilde{x}$
- equivalence of string theories on dual backgrounds with different geometry
  - example: torus + B-field $\sim$ twisted torus
- Buscher rules transform backgrounds with isometries
  $\rightarrow$ mix metric g and B-field b
- extension to the T-duality group $O(d,d)$
  $\tilde{E} = h(E) = (aE + b)(cE + d)^{-1}, \; E = g + B, \; h \in O(d,d)$

Non-geometry

Hull, hep-th/0406102

- T-duality transf. can lead to non-geometric backgrounds
  - i.e. patching backgrounds with T-duality transition functions

  $\rightarrow$ leave standard differential geometry (diffeomorphisms)
- geometric picture arises from doubling the number of dimensions (use both sets of coordinates $x, \tilde{x}$)
  - examples: T-fold, doubled geometry
- 4D SUGRA: "non-geometric" flux terms in the potential
  $\rightarrow$ interesting phenomenology (de Sitter solutions)

New formalisms provide better control on 4D SUGRA models!

Gen. metric formulation of double field theory (DFT)

Zwiebach et al., hep-th/0904.4664
Hohm et al., hep-th/1006.4823

- origin in massless subsector of string field theory
- fields of DFT: gen. metric $\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik}b_{kj} \\ b_{ik}g^{kj} & g_{ij} - b_{ik}g^{kl}b_{lj} \end{pmatrix}$

  and gen. dilaton d,
- $O(D,D)$-notation: $X^M = \begin{pmatrix} \tilde{x}_i \\ x^i \end{pmatrix}$, $\partial_M = \begin{pmatrix} \tilde{\partial}^i \\ \partial_i \end{pmatrix}$, $\eta_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
- T-duality transf.: $\mathcal{H}'(X') = h\mathcal{H}(X)h^t$,
- coordinate transf.: $X' = hX, \qquad h \in O(D,D)$
- DFT action: $S_{DFT} = \int dxd\tilde{x}\ \mathcal{L}_{DFT}(\mathcal{H}, d)$

$$\mathcal{L}_{DFT}(\mathcal{H}, d) \sim \mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL}, \mathcal{H}^{MN}\partial_M d\partial_N d, \dots$$

$\rightarrow$ manifestly T-duality covariant

Gen. Lie derivative, C-bracket & strong constraint

- gauge transf. governed by a generalized Lie derivative $\hat{\mathcal{L}}$ and the C-bracket $[,]_C$ on double fields
  $\rightarrow$ encode diffeomorphisms and B-field transformations
- strong constraint:

$$\eta^{MN}\partial_M\partial_N A = \partial^M A \partial_M B = 0$$

  $\rightarrow$ fields depend only on half of the coordinates $x, \tilde{x}$

A geometric framework for DFT

Can we find a geometric framework for DFT?

- i.e. mimic general relativity on double space $x, \tilde{x}$
  - use $O(D,D)$ instead of diffeomorphisms
  - unique Levi-Civita connection Γ ($\nabla g = 0$ and torsion-free)
  - torsion tensor $T(v,w) = \nabla_v w - \nabla_w v - [v,w]$
  - Riemannian tensor $R(u,v)w = [\nabla_u, \nabla_v]w - \nabla_{[u,v]}w$
  - Einstein-Hilbert like action $S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}R$

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It is not as straightforward as it seems!

Siegel's formalism & Projection-compatible geometry

Siegel, hep-th/9302036
Hohm, hep-th/1011.4101

- introduce a conn. Ω for tan. space structure $GL(D) \times GL(D)$

mimic the construction of Levi-Civita connection Γ in GR

- generalized tangent metric compatibility
- generalized torsion constraint
- treat the gen. dilaton

gen. analogue Ω of the Levi-Civita connection Γ is not unique

- another approach: projection-compatible geometry uses unique, semi-covariant derivative

Park et al., hep-th/1105.6294

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Similarities to generalized geometry, so curvature in a minute!

Generalized Geometry (GG)

Hitchin, math/0209099
Gualtieri, math/0401221

- generalized tangent bundle E over d-dimensional manifold M

$$0 \to T^*M \to E \to TM \to 0$$

- generalized sections of E mix one-forms and vectors
  $\to$ doubling of dimensions only in the fiber
- natural action of one-forms on vectors
  $\to$ yields $O(D, D)$-metric η
- introduce generalized frame fields $\{E_A = E^M{}_A \partial_M\}$
- structure of E is governed by a generalized Lie derivative $\hat{\mathcal{L}}$ and the Courant-bracket $[,]_{Courant}$ on gen. sections

Gen. analogue of Levi-Civita connection in GG

Waldram et al., hep-th/1107.1733

- define a gen. covariant derivative $\mathcal{D}$ on sections of E by embedding a standard covariant derivative ∇

mimic the construction of Levi-Civita connection Γ in GR

- gen. metric compatibility: embed Levi-Civita connection Γ
- generalized torsion-freeness

gen. analogue $\mathcal{D}^{\Gamma}$ of the Levi-Civita conn. Γ is not unique

Gen. curvature and the link between DFT & GG

Can we yet define unique generalized curvature objects?

- not possible to find a unique gen. Riemann tensor
- but unique gen. Ricci tensor and scalar do exist
$$\mathcal{R}_{Siegel} = e_a\Omega^a + \tfrac{1}{2}\Omega_a^2 - \tfrac{1}{4}\Omega_{ab\bar{c}}{}^2 - \tfrac{1}{12}\Omega_{[abc]}{}^2$$
- Einstein-Hilbert like action: $S_{DFT} \equiv \int dx d\tilde{x} e^{-2d}\mathcal{R}_{Siegel}$

Hohm et al., hep-th/1011.4101

- equivalence of gen. curvature objects of DFT and GG
$$\mathcal{R}_{Siegel}|_{\tilde{\partial}=0} \equiv \mathcal{R}_{GG}$$

Double Field Theory and Generalized Geometry are locally equivalent for vanishing dual coordinates

- possible reduction to NS-NS sector of supergravities of type II
$$S^* = \int dx\sqrt{-g}e^{-2\phi}(R + 4\Box\phi - 4(\partial\phi)^2 - \tfrac{1}{2}H^2)$$

Conclusion & Outlook

DFT & GG are T-duality covariant formulations of supergravities with an underlying geometric description!

- use new formalisms for studying non-geometry
  - non-geometry in generalized geometry
    $\rightarrow$ bivector β instead of 2-form B
  - better understanding of non-geometric fluxes in 10D SUGRA
    Andriot, Larfors, Lüst, Patalong, hep-th/1204.1979
  - compactification in the context of DFT and GG
    $\rightarrow$ phenomenological models with non-geometric fluxes in 4D
- generalization to other dualities or stringy symmetries
  $\rightarrow$ U-duality and exceptional groups $E_{D(D)}$
  Waldram et al., hep-th/1112.3989