BPS Equations from (2,0) Nonlinear Sigma Models

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with Stefan Groot Nibbelink: arXiv:1203.6827, accepted by JHEP

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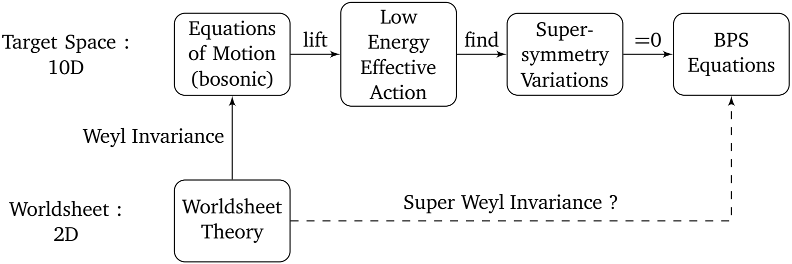
Motivation



$$
\left.\begin{aligned}
0 &= \delta\psi_M = \tilde{\nabla}_M \eta \\
0 &= \delta\chi = F_{MN}\Gamma^{MN}\eta
\end{aligned}\right\} \implies \left\{\begin{array}{c}
\text{Complex Structure} \\
\text{Hermitean Yang–Mills} \\
G^{a\underline{a}} F_{a\underline{a}} = 0
\end{array}\right.
$$

(2,0) NLSM

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- (2,0) Susy Algebra : $\{\overline{Q}_+, Q_+\} = -2i\,\partial_L$
- (2,0) Superspace : $\{\bar{\theta}^+, \theta^+\} = 0$
- Supercovariant derivatives: $\{\overline{D}_+, D_+\} = +2i\,\partial_L$
- (Fermi–) Chiral superfields : $\overline{D}_+\phi = \overline{D}_+\Lambda = 0$

$$S_\phi = \frac{i}{4}\int \mathrm{d}^2\sigma\,\mathrm{d}^2\theta^+ \left\{\overline{K}_{\underline{a}}\,\partial_R\,\bar{\phi}^{\underline{a}} - K_a\,\partial_R\,\phi^a\right\}$$

$$S_\Lambda = -\frac{1}{2}\int \mathrm{d}^2\sigma\,\mathrm{d}^2\theta^+ \left\{\overline{\Lambda}^{\underline{\alpha}} N_{\underline{\alpha}\beta}\,\Lambda^\beta + \frac{1}{2}\Lambda^\alpha M_{\alpha\beta}\,\Lambda^\beta + \frac{1}{2}\overline{\Lambda}^{\underline{\alpha}}\,\overline{M}_{\underline{\alpha}\,\underline{\beta}}\,\overline{\Lambda}^{\underline{\beta}}\right\}$$

Target Space Interpretations

- Hermitean Metric : $G_{\underline{a}b} = \frac{1}{2}(\overline{K}_{\underline{a},b} + K_{b,\underline{a}})$
- H-Field : $H_{ab\underline{c}} = K_{a,b\underline{c}} - K_{b,a\underline{c}}$
- Torsion Connection : $\Gamma_{+}{}^{b}{}_{cd} = G^{b\underline{a}}\left(G_{\underline{a}c,d} + \frac{1}{2}H_{cd\underline{a}}\right)$
- Gauge Connection : $[A_c]^{\beta}{}_{\alpha} = N^{\beta\underline{\gamma}}N_{\underline{\gamma}\alpha,c}$
- Field Strength :

$$[F_{\underline{c}c}]_{\underline{\alpha}\beta} = N_{\underline{\alpha}\beta,\underline{c}c} - N_{\underline{\alpha}\gamma,\underline{c}}\, N^{\gamma\underline{\gamma}}\, N_{\underline{\gamma}\beta,c}$$

Super Feynman Rules

- Super Propagators :

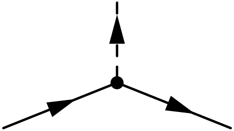
1  2 $= \left(\frac{N^{\beta\underline{\alpha}}\ \overline{D}_+ D_+}{\partial_L} \right)_2 \delta_{12}$

1  2 $= \left(\frac{-iG^{b\underline{a}}\ \overline{D}_+ D_+}{\partial_L \partial_R} \right)_2 \delta_{12}$

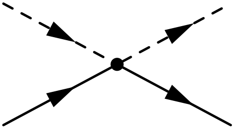
- Super Delta Function :

$$\delta_{12} = \delta^2(\sigma_1 - \sigma_2)\ (\theta_1^+ - \theta_2^+)\ (\bar{\theta}_1^+ - \bar{\theta}_2^+)$$

- Super Vertices:



$-\frac{i}{2} N_{\underline{\alpha}\alpha,a}$



$-\frac{i}{2} N_{\underline{\alpha}\alpha,\underline{a}a}$

Renormalization I

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- Dimensional Regularization :

  $$\int \mathrm{d}^2 p \longrightarrow \int \mathrm{d}^D p \;\; \mu^{2-D}$$

- Renormalization:

  $$S_R = S_0 - \Delta S$$

- Super Beta Function :

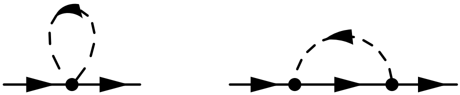
  $$\beta(N_R) = \; \mu \frac{\partial}{\partial \mu} \, N_R$$

- Divergent Momentum Integrals for $D = 2 - 2\epsilon$:

  $$\int \frac{\mathrm{d}^D p}{(2\pi)^D \mu^{D-2}} \frac{1}{p^2 + m^2} = \frac{i}{4\pi} \left[\frac{1}{\epsilon} + \ln \frac{\mu^2}{m^2} + \ldots \right]$$

One Loop Graphs – Fermi Renormalization

- Divergent Fermi Loops ($\overline{\Lambda}^{\underline{\alpha}} N_{\underline{\alpha}\beta} \Lambda^{\beta}$) :



$$\beta(N)_{\underline{\alpha}\beta} = G^{c\underline{c}} \left[N_{\underline{\alpha}\beta, c\underline{c}} \right] - G^{c\underline{c}} \left[N_{\underline{\alpha}\gamma,\underline{c}} \, N^{\gamma\underline{\gamma}} \, N_{\underline{\gamma}\beta,c} \right]$$

- Super Beta Function :

$$\beta(N)_{\underline{\alpha}\beta} = G^{c\underline{c}} \, \left[F_{c\underline{c}} \right]_{\underline{\alpha}\beta}$$

Finiteness

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- Finiteness: β-functions vanish up to
  - reparameterizations:

$$\int d^2\sigma d\bar{\theta}^+ d\theta^+ \; K(\phi,\bar{\phi})_a \partial_{_R} \phi^a \,,$$

$$K(\phi,\bar{\phi}) \to K(\phi,\bar{\phi}) + k(\phi)$$

  - diffeomorphisms:

$$\phi \;\; \to \;\; f(\phi)$$

  - and gauge transformations:

$$\Lambda \;\; \to \;\; g(\phi)\Lambda$$

- Finiteness for N :

$$\beta(N) = \bar{g}(\bar{\phi})N \; + \; N g(\phi) \; + \; N_{,a} f^a(\phi) \; + \; N_{,\underline{a}} \bar{f}^{\underline{a}}(\bar{\phi})$$

Super Weyl transformations

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- Weyl transformations vs. Super Weyl transformations[1]
  (Recall $\sqrt{g} = \varepsilon$)

Weyl transformations	Super Weyl transformations
$\delta_s\, \varepsilon = \quad s\, \varepsilon$	$\delta_S\, \mathscr{E} = \quad S\, \mathscr{E}$
$\delta_s \partial_R \phi = -s\, \partial_R \phi$	$\delta_S \partial_R \phi = -S\, \partial_R \phi$
$\delta_s\, \Lambda = -s\, \frac{1}{2}\, \Lambda$	$\delta_S\, \Lambda = -S\, \frac{1}{2} \Lambda$

[1] Due to Gates, Evans, Ovrut and Howe

Super Weyl anomaly for N

- Super Weyl Anomaly :

$$0 \overset{!}{=} \delta_S \; S_0$$

- Super Weyl Anomaly for N :

$$0 = \frac{1}{2} \int \mathrm{d}^2\sigma \; \mathrm{d}^2\theta^+ \; S \left(\overline{\Lambda}^{\underline{\alpha}} \; \beta(N)_{\underline{\alpha}\beta} \; \Lambda^{\beta} \right)$$

- Conditions on $\beta(N)$:

$$0 = \beta(N)_{\underline{\alpha}\beta} = G^{c\underline{c}} \; \left[F_{\underline{c}c} \right]_{\underline{\alpha}\beta}$$

Summary

1. We determined the super beta functions for K and N at one loop.
2. We established the conditions for finiteness.
3. We used super Weyl invariance to fix them.
   - We found the gaugino BPS condition, the Hermitean Yang–Mills equation :

     $$G^{c\underline{c}} F_{c\underline{c}} = 0$$

   - and a condition on the background geometry :

     $$\frac{1}{16\pi} \Gamma_{+}{}^{b}{}_{ab} \; + \; \Psi_{,a} \; = \; 0$$

Outlook

1. The formalism lends itself to higher loop calculations, which would give α' corrections to BPS equations. (Future work)

2. We would also like to find the modified Bianchi identity for H at one loop. (Future work)

Appendix: BPS Equation for the Dilatino

- The dilatino BPS Equation reads :

$$0 = \delta\zeta = -\frac{1}{2}\,\partial\!\!\!/\,\Psi\,\eta + \frac{1}{4}H_{MNP}\Gamma^{MNP}\,\eta$$

- Together with the Complex structure one finds :

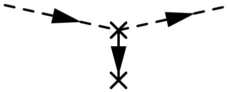
$$||\Omega||^2 = \mathrm{e}^{-4(\Psi+\Psi_0)}$$

The condition for a conformally balanced geometry. In terms of J, the fundamental form, it says :

$$\mathrm{d}\Big(\,\mathrm{e}^{-2\Psi}\,J \wedge J\Big) = 0$$

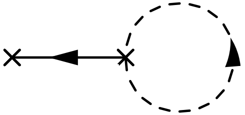
One Loop Graphs – Chiral Renormalization

- Chiral super vertex :



$$\frac{1}{4} G_{\underline{a}c} \Gamma_{+\,ba}^{\;\;c}$$

- Divergent chiral tadpole (renormalizes $K_a \partial_R \phi^a$) :



$$\beta(K)_a \quad = \quad -\frac{1}{4\pi}\, \Gamma_{+\,ab}^{\;\;b}$$

Super Weyl anomaly for K

- Coupling the Dilaton :

$$S_\Psi = \int d^2\sigma\, d^2\theta^+\, \mathscr{E}\, \Psi\, G_R\,, \qquad R = \tfrac{1}{2}[D_+,\overline{D}_+]G_R$$

- Super Weyl Anomaly for K :

$$\begin{aligned} 0 \;&= \frac{i}{4}\int d^2\sigma\, d^2\theta^+\; S\Big[\, \beta(\overline{K})_{\underline{a}}\; \partial_R \bar{\phi}^{\underline{a}} \;-\; \beta(K)_a\; \partial_R \phi^a \Big] \\ &\quad \int d^2\sigma\, d^2\theta^+\; U\Big[\; \Psi_{,\underline{a}} \;\; \partial_R \bar{\phi}^{\underline{a}} \;+\; \Psi_{,a} \;\; \partial_R \phi^a \Big] \end{aligned}$$

- Conditions on K and Ψ (using $D_+\, S = i D_+\, U$) :

$$0 = \tfrac{1}{4}\beta(K)_a \;-\; \Psi_{,a} \;=\; \frac{1}{16\pi}\,\Gamma_{+\,ab}^{\;\;\,b} \;+\; \Psi_{,a}$$

Appendix: Super Weyl

- Weyl transformation of the Ricci scalar :

  $$\sqrt{g'}R' = \sqrt{g}(R - 2\partial_L \partial_R s)$$

  infinitesimally :

  $$\delta_s R = -sR + 2\partial_L \partial_R s$$

- Super Weyl transformation of the super Ricci scalar :

  $$\delta_S G_R = -S G_R + \partial_R U$$

  $$\supset \delta_s R = sR + 2\partial_L \partial_R s$$

- Super Weyl multiplet (S, U). They are related.

  S and U are the real and imaginary part of a chiral field:

  $$0 = D_+ \bar{\chi} = D_+ (S - iU) \quad \Rightarrow \quad D_+ S = i D_+ U$$

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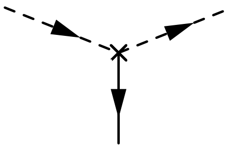
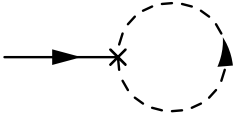
Appendix: Quantum Tadpole

- Quantum chiral super vertex :



$$\frac{1}{8} H_{ba\underline{a}}$$

- Divergent tadpole :



$$T_{(1)}(Y)_a = -\frac{1}{8\pi} H_{ab\underline{b}} G^{b\underline{b}} \ ,$$