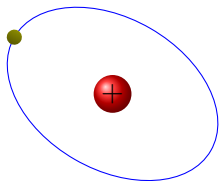


Quantum Dissipation in Periodically Driven Systems

Falk Haßler

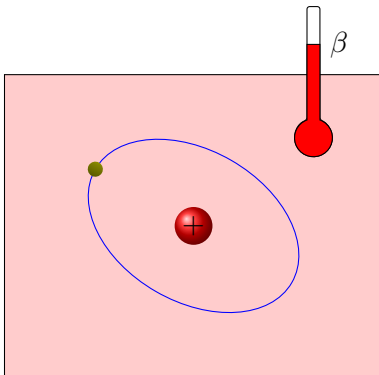
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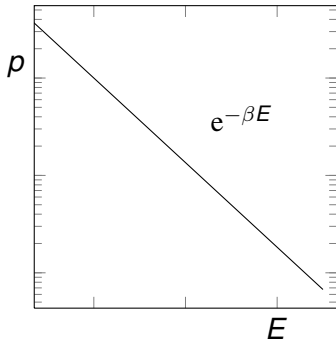
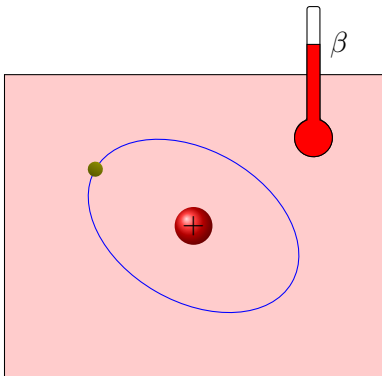
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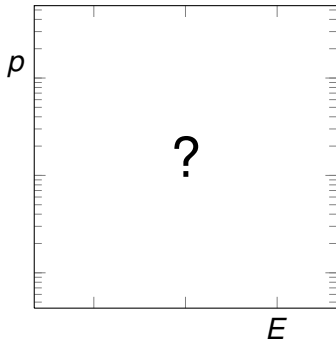
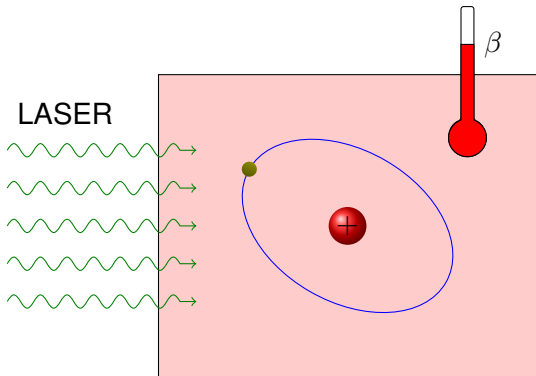
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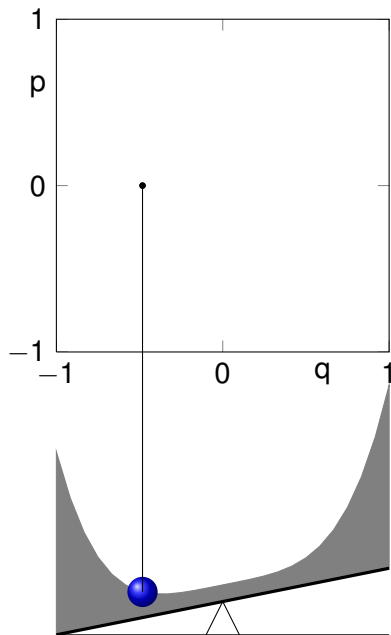
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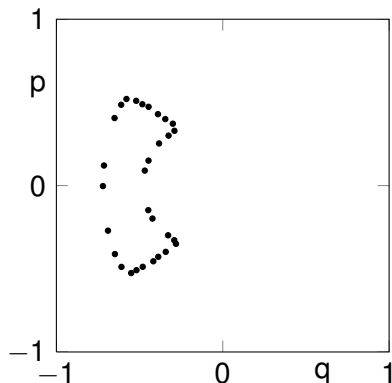
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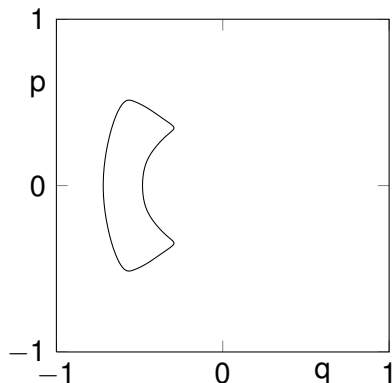


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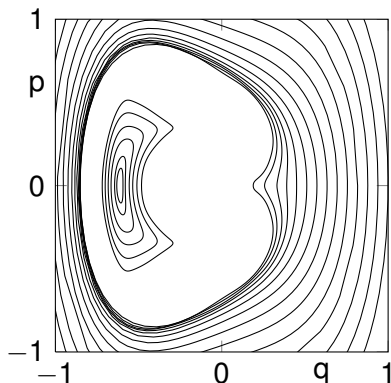


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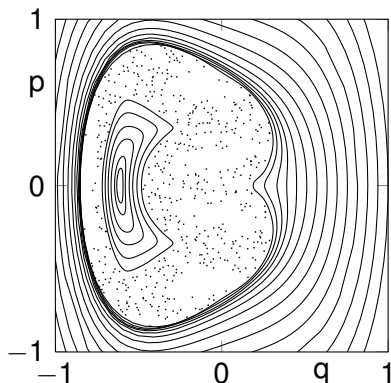


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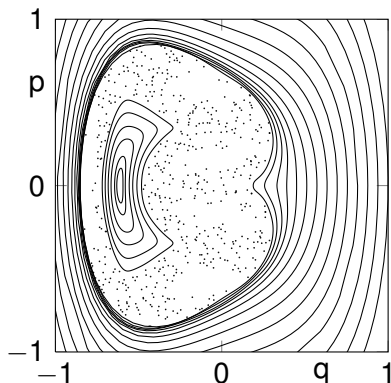
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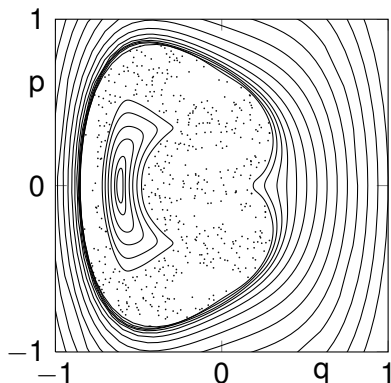
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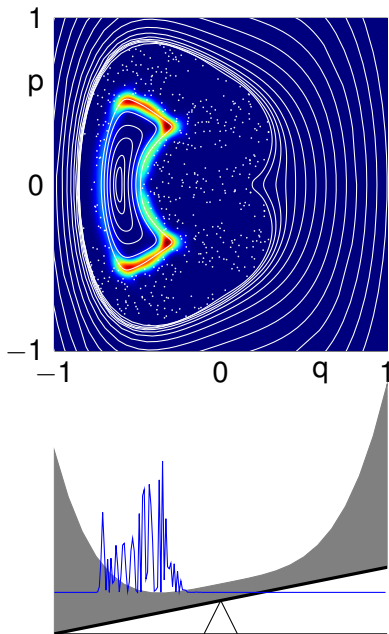
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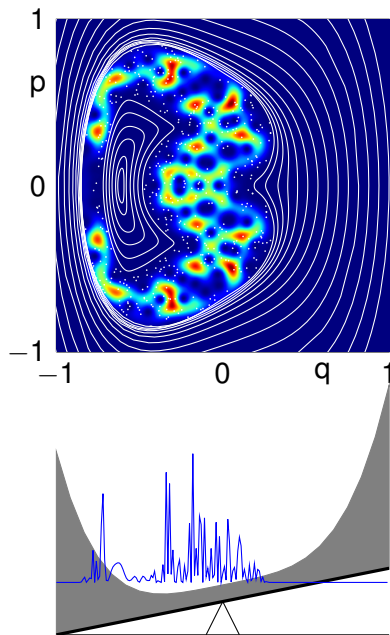
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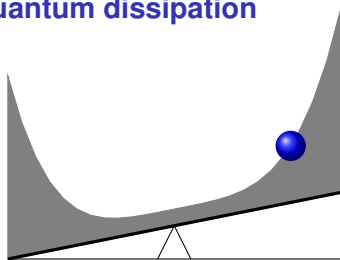
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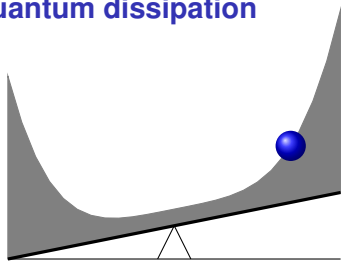
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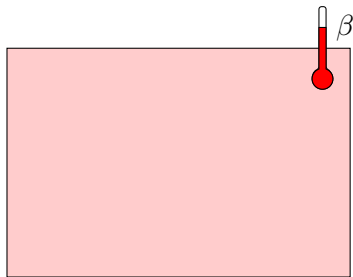


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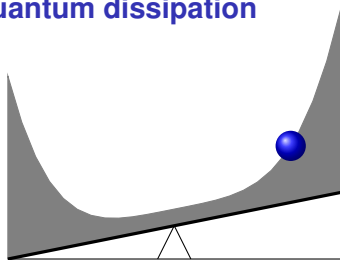
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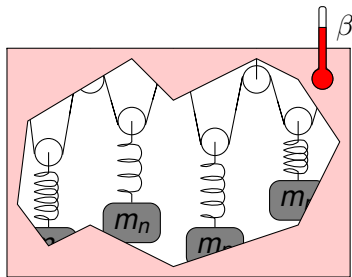
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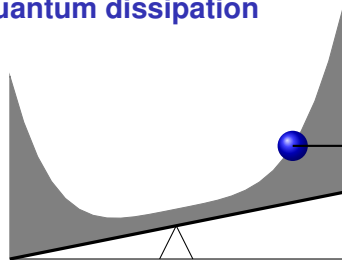


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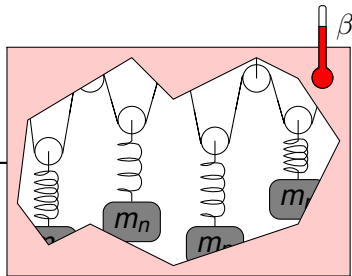
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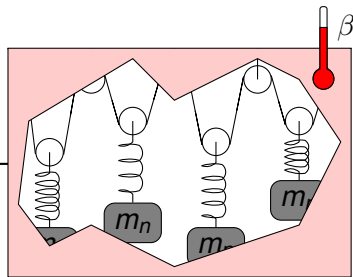
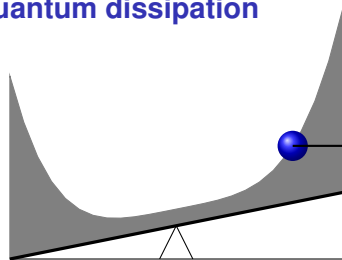
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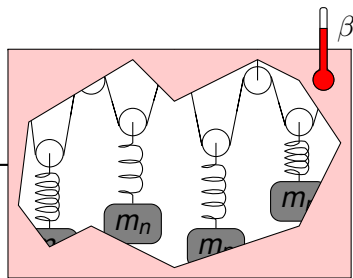
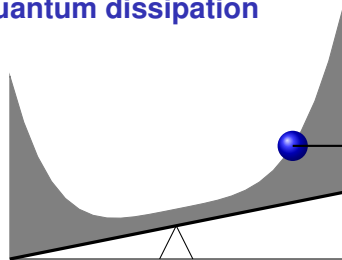
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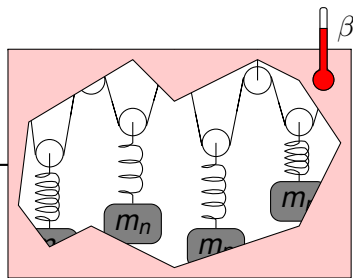
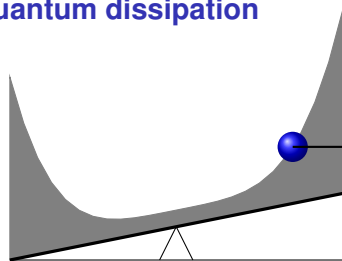


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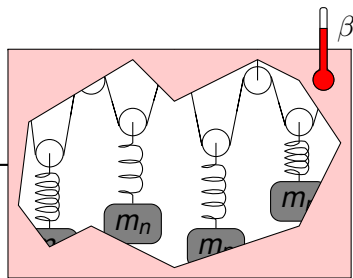
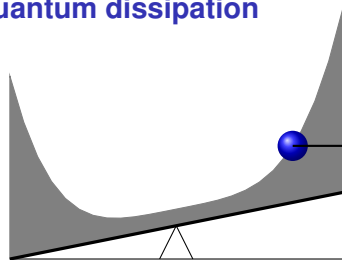
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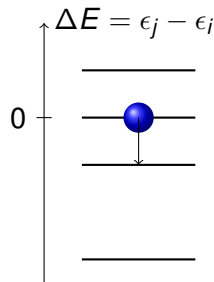


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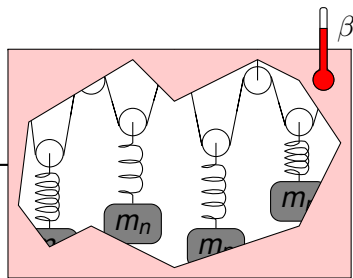
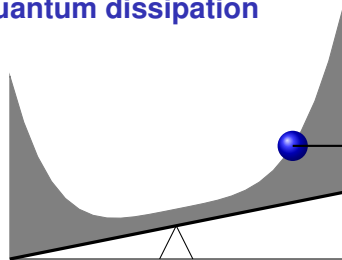
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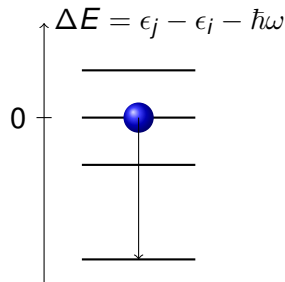


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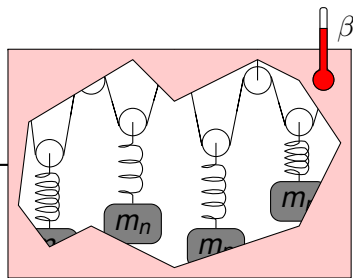
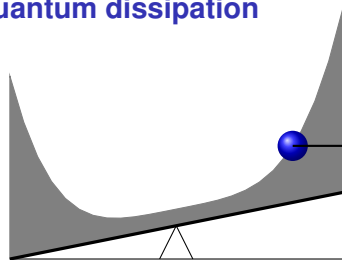
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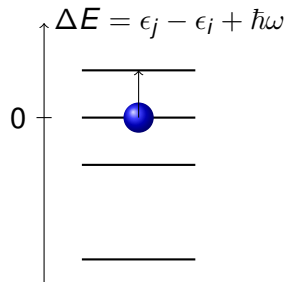


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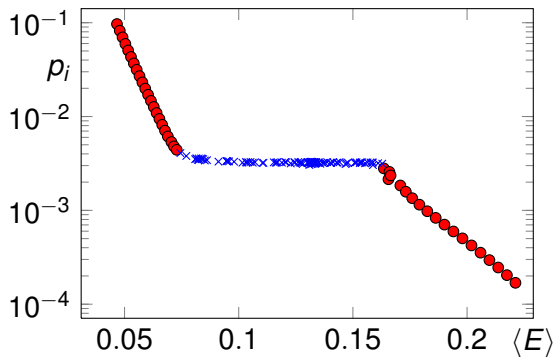
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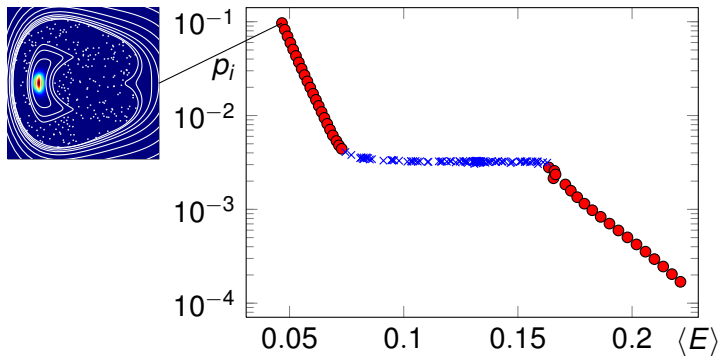


Asymptotic occupation probabilities of the Floquet states

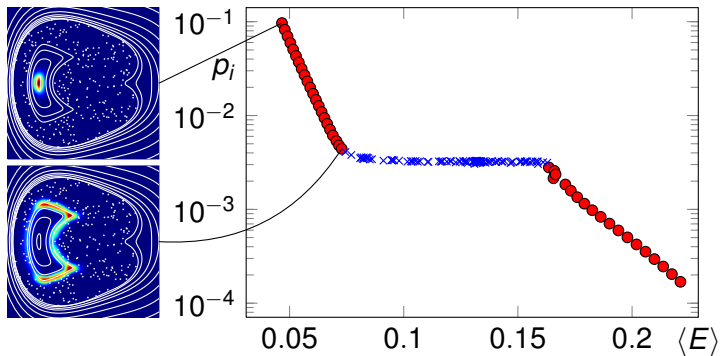
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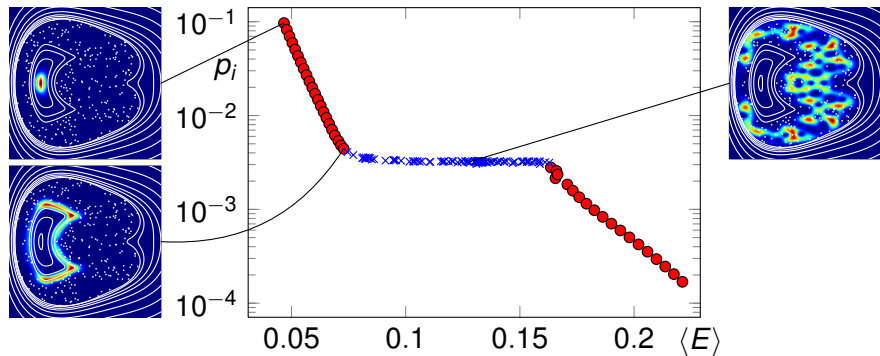
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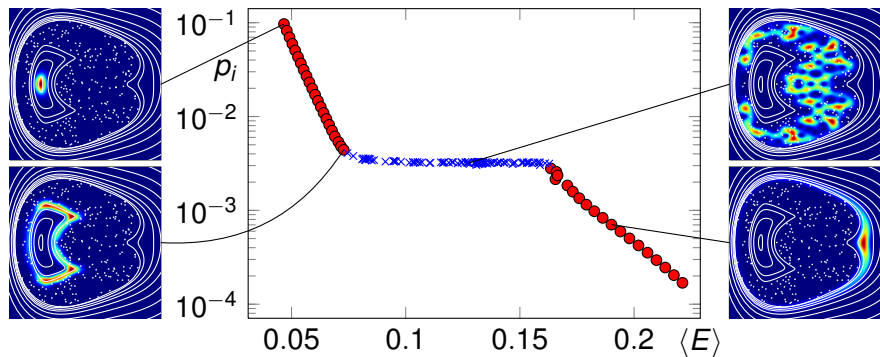
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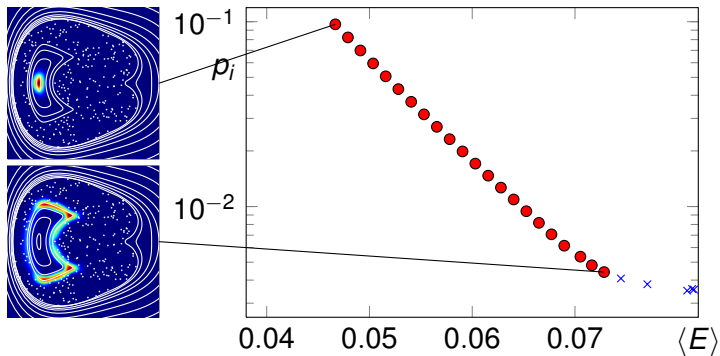
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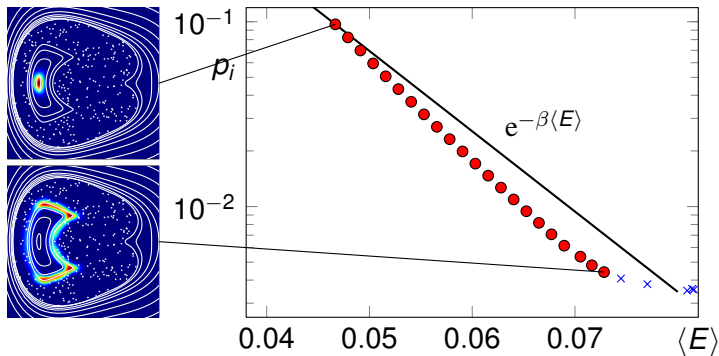
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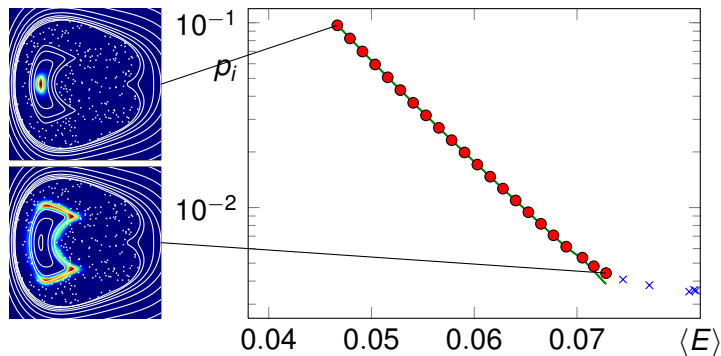
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$$R_{ij}^{\text{sc}} = \frac{\gamma^2}{\hbar} \sum_K \left| A_{ij}^{\text{sc}}(K) \right|^2 \left(\frac{2}{\beta \hbar} + \omega_{ij} + \omega K \right) \quad \omega_{ij} = \omega \nu_{\frac{i+j}{2}} (i - j)$$

$$A_{ij}^{\text{sc}}(K) = \lim_{N \rightarrow \infty} \frac{1}{2NT} \int_{-NT}^{NT} dt \tilde{A}_{\frac{i+j}{2}}(t) \exp[-i(\omega_{ij} + \omega K)t]$$