Effective field theories and cosmology: Majorana neutrinos, Dark Matter and Leptogenesis

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Outline

Standard Model and Beyond

The **Standard Model** of **Particle Physics**

Renormalizable Gauge Field Theory

$$SU(3) \otimes SU(2) \otimes U(1)$$

$$\Downarrow$$

- correct predictions of many observed phenomena
- agreement with most of experimental data

But....

Standard Model and Beyond

Neutrino Physics...

- we observe neutrino oscillation:

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle \Rightarrow P(\nu_\alpha \to \nu_\beta) \propto \sin^2\left(\frac{\Delta m_{ij}^2 L}{2E}\right)$$

- the neutrinos are massless particles: $m_\nu = 0$

Dark Matter...

- we need a **suitable** dark matter candidate in agreement with **cosmological constraints**

$Q_X = 0$ non baryonic, stable $M_X \neq 0$

- the SM particle content consists in quarks, leptons, gauge bosons

Baryon Asymmetry...

- we live in a matter dominated universe: Baryon Asymmetry in the Universe
- the SM CP violation is not sufficient to explain the Baryon Asymmetry in the Universe

Neutrino Oscillation

- the idea was first put forward by B. Pontecorvo (1957)

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle \quad \begin{cases} \alpha = e, \mu, \tau \\ i = 1, 2, 3 \end{cases}$$

- the mixing matrix involve: mass eigenstates $|\nu_i\rangle$ and flavour eigenstates $|\nu_\alpha\rangle$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- the experiments provide (*Super Kamiokande, K2, Minos...*)

$$\Delta m^2_{sol} = 7.65 \times 10^{-5} \text{eV}^2 \quad , \quad \Delta m^2_{atm} = 2.40 \times 10^{-3} \text{eV}^2 \;\rightarrow m_\nu \geq 0.05 \text{eV}$$

$\nu_{e,\mu} \rightarrow \nu_{\mu,\tau} \Rightarrow$ different mass eigenstates $|\nu_i\rangle \Rightarrow \mathbf{m}_\nu$

How can we construct ν masses in the QFT Lagrangian?

Neutrinos masses and See-Saw

- adding in the most conservative way a set of ν_R $\rightarrow$ sterile
- neutrinos are the only **electrically neutral** fundamental particle: $\boldsymbol{\nu = \bar{\nu}}$?

Dirac Mass Term

$$\mathcal{L}_D = -m_D \bar{\nu}_R \nu_L + h.c.$$

$$\Delta L = 0$$

Majorana Mass Term : $\nu^c = \gamma^0 C \nu^*$

$$\mathcal{L}_M = -m_L \bar{\nu}_L^c \nu_L - m_L \bar{\nu}_R^c \nu_R + h.c.$$

$$\Delta L \neq 0$$

- Putting together the two terms $\rightarrow$ the **See-Saw** mechanism

$$\mathcal{L}_{M+D} = -m_D \bar{\nu}_R \nu_L - m_L \bar{\nu}_L^c \nu_L - m_R \bar{\nu}_R^c \nu_R + h.c.$$

$$\mathcal{M} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \quad \text{with} \quad m_R = M >> m_D >> m_L = \mu$$

- mass eigenvalues:

$$m_1 = \left| \mu - \frac{m_D^2}{M} \right|, \quad m_2 = M$$

Extended SM Lagrangian

- mass eigenstates:

$$\nu \sim [\nu_L - \nu_L^c] + \frac{m_D}{M}[\nu_R - \nu_R^c]\ ,\ N \sim [\nu_R + \nu_R^c] + \frac{m_D}{M}[\nu_L + \nu_L^c]$$

- the heavy mass eigenstate is composed by RIGHT neutrinos: $N \simeq \nu_R$
- the light mass eigenstate is composed by LEFT neutrinos: $\nu \simeq \nu_L$

General case: $\nu_\alpha = U_{\alpha i}\,\nu_i + \Theta_{\alpha I} N_I \qquad \Theta_{\alpha I} = \frac{(M^D)_{\alpha I}}{M_I} << 1$

Standard Model Extension

- $\mathcal{N}$ singlet fermions $N_I\ (I = 1, ..., \mathcal{N})$ $\qquad M_{N_1} \leq M_{N_2} ... \leq M_{\mathcal{N}}$

$$Q = 0\,;\quad I_W = 0\,;\quad Y = 0 \rightarrow \text{sterile particles}$$

- renormalizable Lagrangian with Dirac-Majorana mass term

$$\mathcal{L} = \mathcal{L}_{SM} + i\bar{N}_I \partial_\mu \gamma^\mu N_I - \left(F_{\alpha i} \bar{L}_\alpha N_I \tilde{\Phi} + \frac{M_I}{2} \bar{N}_I^c N_I + h.c. \right)$$

Towards Cosmology problems

The **minimal changes** introduced in the SM:

- can explain the neutrino oscillations $M \leq 10^{15}$GeV
- can give small masses, as observed *experimentally* , to ν_e, ν_μ, ν_τ

Adding **new fields** $\rightarrow$ new particles

- heavy neutrinos (lightest one) obtained by See-Saw mechanism

$\Downarrow$

Dark Matter candidate

- lepton number violating terms, Majorana nature of heavy neutrinos

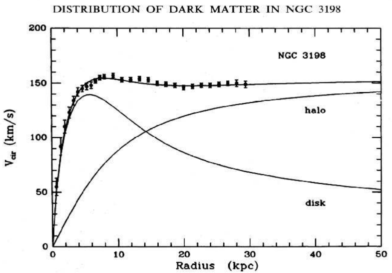
$\Downarrow$

Baryon Asymmetry via Leptogenesis

hep-ph 0604236, Phys. Lett. B 155, 36, Phys. Lett. B 74, 45

Dark Matter

1. Mass estimation by rotation curves: $v = \sqrt{\frac{GM(r)}{r}}$



$\Rightarrow \; M_v(r) \propto r$

far away from the galaxy core

2. Estimation by visible stars and dust $\rightarrow$ electromagnetic radiation (disk)

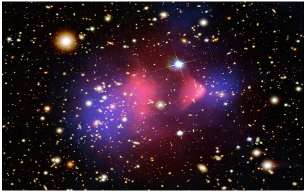
if we assume General Relativity (Newton's laws) being correct

$\Downarrow$

additional matter to explain rotation curves

Dark Matter

- DM is necessary for the galaxies formation and structure
- a **glue to clump** observed celestial objects



DM properties

- does not carry electromagnetic charge
- have to be massive, i.e. gravitationally interacting
- non baryonic

- Dwarf Spheroidal Galaxies: DM dominated objects
- $\rho_{DM} \leq \rho_{Fermi\ gas}$
- present limits: $M_{DM} \geq 400$ eV

SM $\nu \neq$ DM particles

- Several candidates: LSP (lightest supersymmetric particle) neutralino or gravitino, axions and sterile neutrinos, WIMPZillas, solitons (Q-balls, B balls), extra dimension dark matter LKP

Warm dark matter

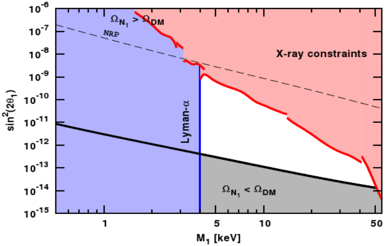
Cold Dark Matter

- Most studied candidates: LSP, in general WIMPS $M_X \sim 100\,\text{GeV}$

  Drawbacks: satellite problem (to many galaxies), does not reproduce galactic and cluster of galaxies observations

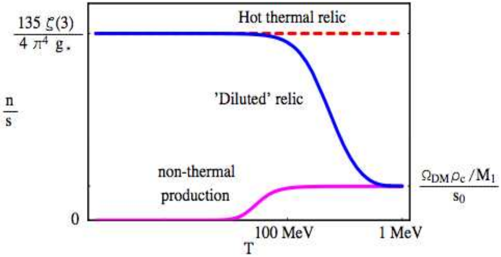
...Warm Dark Matter

- Numerical simulation with $M = \mathcal{O}$ (kev) particles reproduce astronomical observation at all scales hep-ph 0009083 , 1109.3187
- Possible candidate: Sterile Neutrinos hep-ph 9303287



- νMSM
- X ray analysis $N \to \gamma\nu$
- Lyman-α spectrum

Motivation 1: reproduce ρ_{DM}



- N_1 DM candidate
- Thermal Equilibrium

  $\Gamma_i > H$
- Dilution by increasing of **s**

  $N_{2,3} \to a, b, c\ldots$

Thermal approach to the Dark Matter problem

- produced in a hot dense plasma at high temperature
- masses involved: $M_{N_1} = \mathcal{O}(\text{kev})$ $M_{N_{2,3}} \leq 10^{15}\text{GeV}$
- **production** and **decays** of $N_{2,3}$ play an important role

Baryon Asymmetry in the Universe

- excess of matter over anti-matter is observed (CMB, BBN)

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} \sim 6 \times 10^{-10} \Rightarrow n_{\bar{B}} \simeq 0$$

- dynamical generation of η_B in the early hot universe (hot plasma)

Sakharov conditions (1967)

1. baryon number (B) violation
2. C and CP violation
3. processes out of thermal equilibrium

- There must exists a process in which baryon number is violated

$$X \rightarrow Y + B$$

BAU and Sakarov conditions

- If B is violated but C is conserved,

$$\Gamma(X \to Y + B) = \Gamma(\bar{X} \to \bar{Y} + \bar{B}) \Rightarrow B - \bar{B} = 0$$

- If B and C are violated

$$\Gamma(X \to q_L q_L) \neq \Gamma(\bar{X} \to \bar{q}_L \bar{q}_L)$$

  but CP is conserved

$$\Gamma(X \to q_L q_L) + \Gamma(X \to q_R q_R) = \Gamma(\bar{X} \to \bar{q}_L \bar{q}_L) + \Gamma(\bar{X} \to \bar{q}_R \bar{q}_R)$$

$$B - \bar{B} = 0$$

- CPT theorem: $m_X = m_{\bar{X}}$ and in thermal equilibrium particles follow Bose-Einstein or Fermi-Dirac distribution $\Rightarrow$ *density depend only on masses* $\Rightarrow$ **no $B - \bar{B}$ can be generated**

Leptogenesis

CP violation in quark sector is not enough:

SPHALERONS : **Baryons** $\leftrightarrows$ **Leptons**

- B and L well **conserved** at **classical level** (low temperature regime)
- $T > T_{EW}$ transition between vacua of non Abelian Gauge Theory ($SU(2)$)

$$\Delta B = \Delta L = n_f \Delta N_v$$

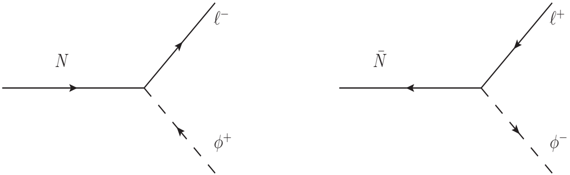
- $100\,\text{GeV} \leq T \leq 10^{12}\,\text{GeV}$: sphaleron transitions activated
- due to sphalerons properties a Baryon Asymmetry can be generated if

$$\eta_B = \frac{\alpha_{sph}}{\alpha_{sph} - 1}\,\eta_L$$

With sterile neutrinos $\rightarrow$ PMSN matrix for ν

1. we have additional CP violation in leptonic sector
2. processes that may violate lepton number

Motivation 2: reproduce BAU (η_B)



Thermal approach to the Leptogenesis problem

- temperature of the process: $T_{lept} \simeq 10^9$ GeV
- masses involved: $M_{N_1} = \mathcal{O}(\text{kev})$ $\quad M_{N_{2,3}} \leq 10^{15}$GeV

Possible hierarchy scale: $M >> T$ $\rightarrow$ Effective field theory?

Setting up the tools

Dealing with problems involving **more than one energy scale**:

Effective Field Theories

1. a hierarchy of energy scales: separation of the scales, e.g. $m << M$
2. identify which is the scale you are interested in, e.g. m
3. organize an expansion of the operators in terms of

$$\frac{m}{M} \rightarrow \text{power counting}$$

4. dimensional analysis helps in building the effective Lagrangian

$$\mathcal{L}_{FT} \rightarrow \mathcal{L}_{EFT} = \sum_i c_i \frac{\mathcal{O}_i^n}{M^{n-4}}$$

EFT strategy

- identify the symmetries of the low energy Lagrangian
- identify the suitable degrees of freedom, ingredients of your system
- write down the low energy Lagrangian exploiting the hierarchy of the scales

Defining the problem

Our physical system and degrees of freedom

- hot plasma of SM particles at $T \simeq 10^9$ GeV: $m_i \ll T \quad \vec{p}_i \sim T$
- Majorana neutrinos (N, M) are almost not affected by **T**

$\Rightarrow$ small corrections to N dynamics, N is a **NON RELATIVISTIC particle**

- It is possible to build an **EFT** to get **thermal production rate**:

$M \gg T \;\rightarrow$ **hierarchy of scales**

Different approaches:

1. consider directly thermal field theory (hep-ph 1112.1205):

$$\Gamma_N(K) = \frac{1}{k^0} Im\left\{\Pi(K)\right\}$$

2. EFT for heavy Majorana neutrinos (*N. Brambilla, A. Vairo, M. Escobedo*)
   - computation at T=0 via one loop diagrams $M \gg T$
   - thermal effects as correction via **simple tad pole diagrams** suppressed by powers of $\frac{T}{M}$

Comparing FT and EFT: matching

EFT strategy:

1. Galilean invariance, rotational invariance (no preferred direction)
2. Non relativistic Majorana neutrinos ($|\vec{p}| \ll M$)
3. Low energy Lagrangian

$$\mathcal{L}_{EFT} = N^{\dagger}\partial_0 N + \frac{A}{M} N^{\dagger} N \phi^{\dagger}\phi + \frac{B}{M^2} N^{\dagger} N \bar{\psi}\psi + \frac{C}{M^3} N^{\dagger} N F^2 + \ldots$$

where ϕ is the Higgs doublet, ψ are fermions, F gauge bosons (field strength)

Thermal correction of each term through dimensional analysis:

$$\delta\Gamma(N)_{\phi} = \frac{T^2}{M}\,, \quad \delta\Gamma(N)_{\psi} = \frac{T^3}{M^2}\,, \quad \delta\Gamma(N)_{F} = \frac{T^4}{M^3}$$

- A, B, C called *matching coefficients* (FT dependent)
- the power counting + $M \gg T$ $\Rightarrow$ **expansion under control**

Matching (1):

- an effective Lagrangian is not a new alternative theory
- it is a simplified version of the FT in a region of the parameters (hierarchy of scales)

$$\mathcal{L}_{FT} \to \mathcal{L}_{EFT}$$

$$\mathcal{L}_{FT} = \mathcal{L}_{SM} + i\bar{N}_I \partial_\mu \gamma^\mu N_I - \left(F_{\alpha i} \bar{L}_\alpha N_I \tilde{\Phi} + \frac{M_I}{2} \bar{N}_I^c N_I + h.c. \right)$$

$$\Downarrow$$

$$\mathcal{L}_{EFT} = N^\dagger \partial_0 N + \frac{A}{M} N^\dagger N \phi^\dagger \phi + \frac{B}{M^2} N^\dagger N \bar{\psi} \psi + \frac{C}{M^3} N^\dagger N F^2 + \ldots$$

Matching (2)

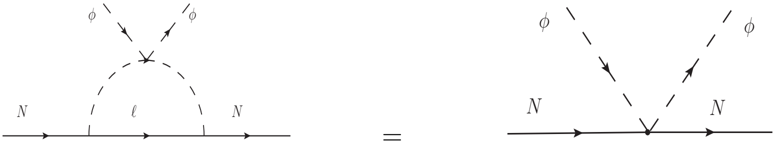
- the EFT must reproduce the FT in the same parameter range
- thermal correction for $T << M \Rightarrow$ **integrate out** the high energy scale M

Example: matching for a coefficient

Matching strategy

- Compute a matrix element in both FT and EFT
- Make the non relativistic expansion for FT ($M \gg T$) or ($\vec{p_N} \ll M$)
- Compare the results to get the expression for the EFT coefficient

$$\langle \Omega | N_i(x)\, \bar{N}_j(y)\, \phi(z)\, \phi^\dagger(t) | \Omega \rangle \to N^\dagger N \phi^\dagger \phi$$



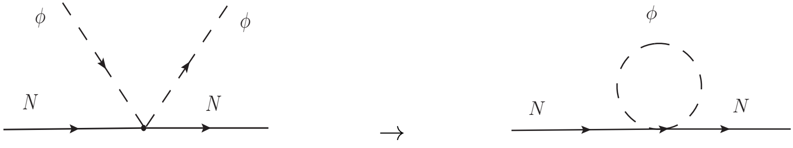
- the **only difference**: **LOOP**, $k^\mu \sim M \Rightarrow$ integrate out $k^\mu \gg T$
- perform the integral at T=0 (Feynman parameters, $m_i \simeq 0$, *imaginary part*)

From T=0 to thermal correction

Matching A coefficient

- We get the result for A as follows

$$A = -i\frac{3}{4}\frac{|F|^2\lambda}{\pi} \;\Rightarrow\; \mathcal{L}_{NN\phi\phi} = -i\frac{3}{4}\frac{|F|^2\lambda}{\pi M}N^\dagger N\phi^\dagger\phi$$



Thermal propagators

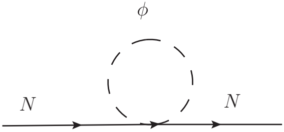
- In a hot plasma particles are thermally excited $\Rightarrow$ propagators affected by

$$i\Delta(x-y) = \int \frac{d^4K}{2\pi^4}\left[\frac{i}{K^2-m^2+i\epsilon} + 2\pi n_B(|k_0|)\delta(K^2-m^2)\right]e^{-iK(x-y)}$$

Thermal production rate at $\mathcal{O}(\frac{T^2}{M^2})$

Final Result

- the first term $\rightarrow$ divergence $\rightarrow$ renormalization
- the second term $\rightarrow$ thermal contribution $\rightarrow$ thermal correction



$$= \frac{|F|^2\lambda}{\pi M}\frac{3}{4}\int \frac{d^4K}{2\pi^4}\, 2\pi n_B(|k_0|)\delta(K^2 - m^2)$$

- T is entering in Bose-Einstein distribution ($m_\phi = 0$)

$$n_B = \frac{1}{e^{k/T}-1} \quad \Rightarrow \quad \int_0^\infty dk \frac{k}{e^{k/T}-1}\,, \quad k = xT$$

- Hence one gets

$$\Gamma_N(T) = \frac{M|F|^2}{8\pi}\left[1 - \lambda\frac{T^2}{M^2} + \mathcal{O}\left(\frac{T}{M}\right)^4\right]$$

Conclusions

Neutrinos

- neutrino oscillation need $m_\nu \neq 0$
- Possible extension: *Sterile Neutrinos*
- See-Saw and Heavy Majorana neutrinos

Dark Matter and Leptogenesis

- *heavy neutrinos* may be a suitable candidate for DM
- *heavy neutrinos* may be a source for Leptogenesis $\rightarrow$ BAU

Effective Field Theory for Heavy Neutrinos

- For both Leptogenesis and DM may be relevant

$$M >> T$$

- effective field theories $\rightarrow$ a **good tool** to get

$$\Gamma_N \equiv \Gamma_N(T, M) \;\rightarrow\; \text{production rate with thermal effects}$$

Outlook

1. compute the other matching coefficients
2. generalization to *non equilibrium*
3. consider other model for R-handed neutrinos (L-R symmetric model....)