Computation of Master Integrals at Higher Orders

IMPRS
EPP

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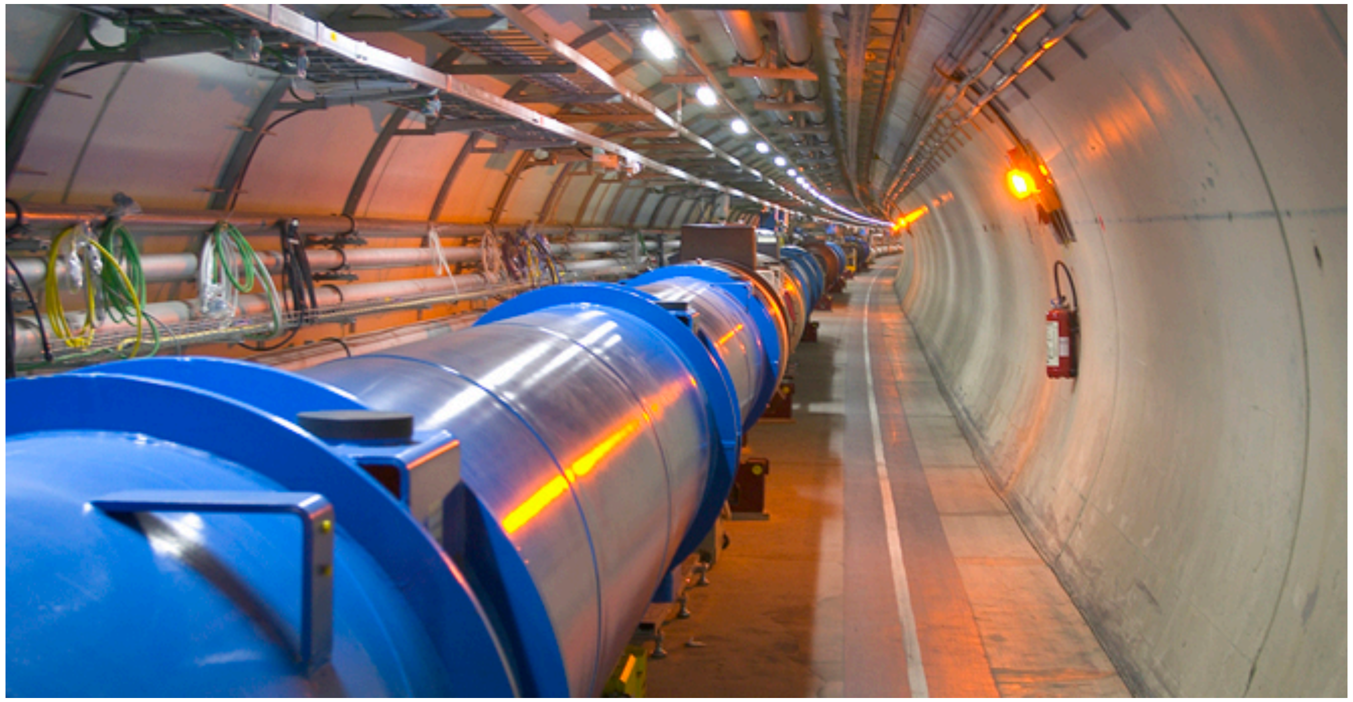
Young Scientists Workshop 2012
Ringberg, July 24th 2012

http://secdec.hepforge.org

Outline

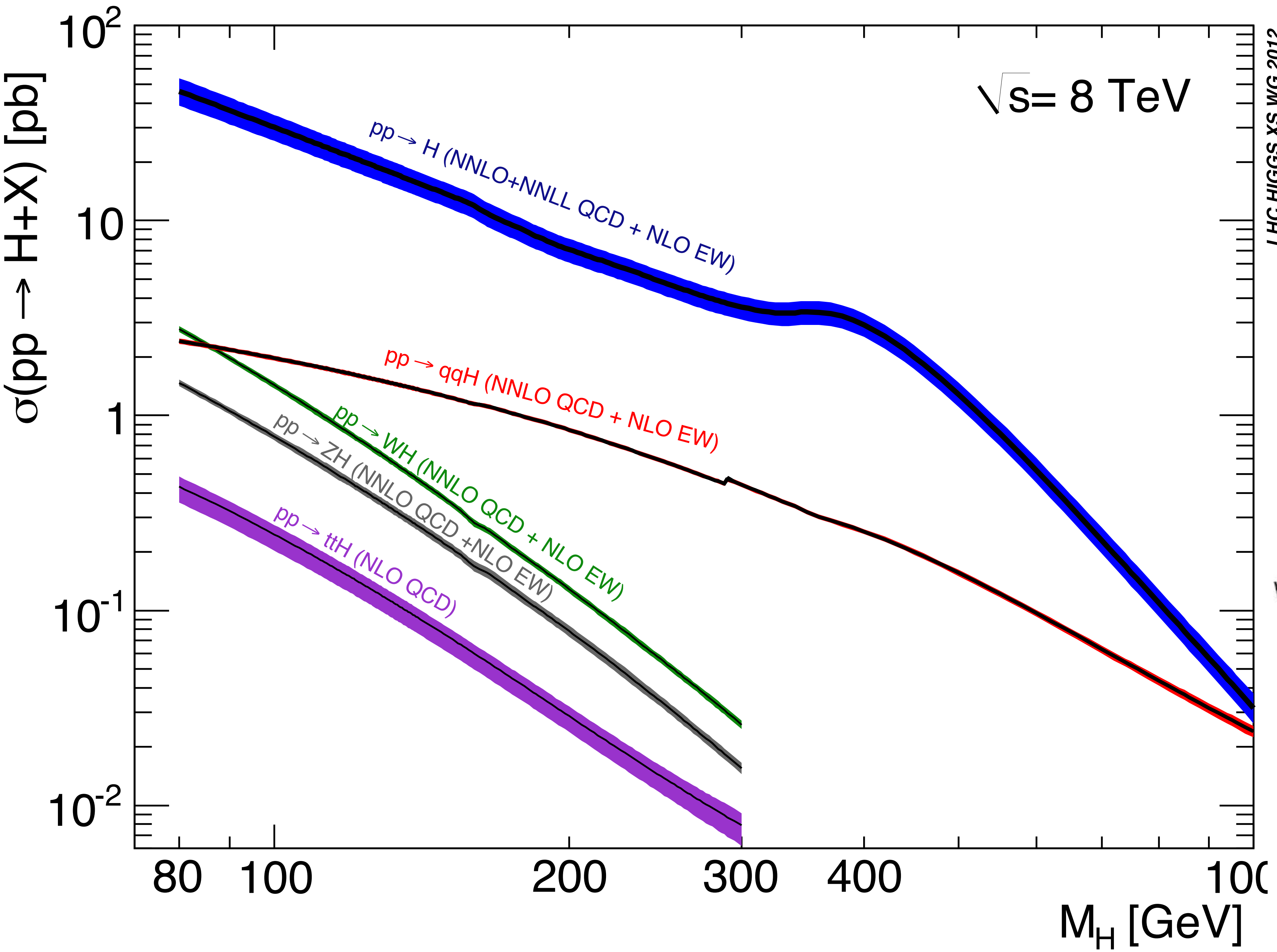
- Why are higher order corrections necessary?
- Constructing loop amplitudes from diagrams
- Analyzing divergences
- Analytic vs numerical approach
- SecDec program

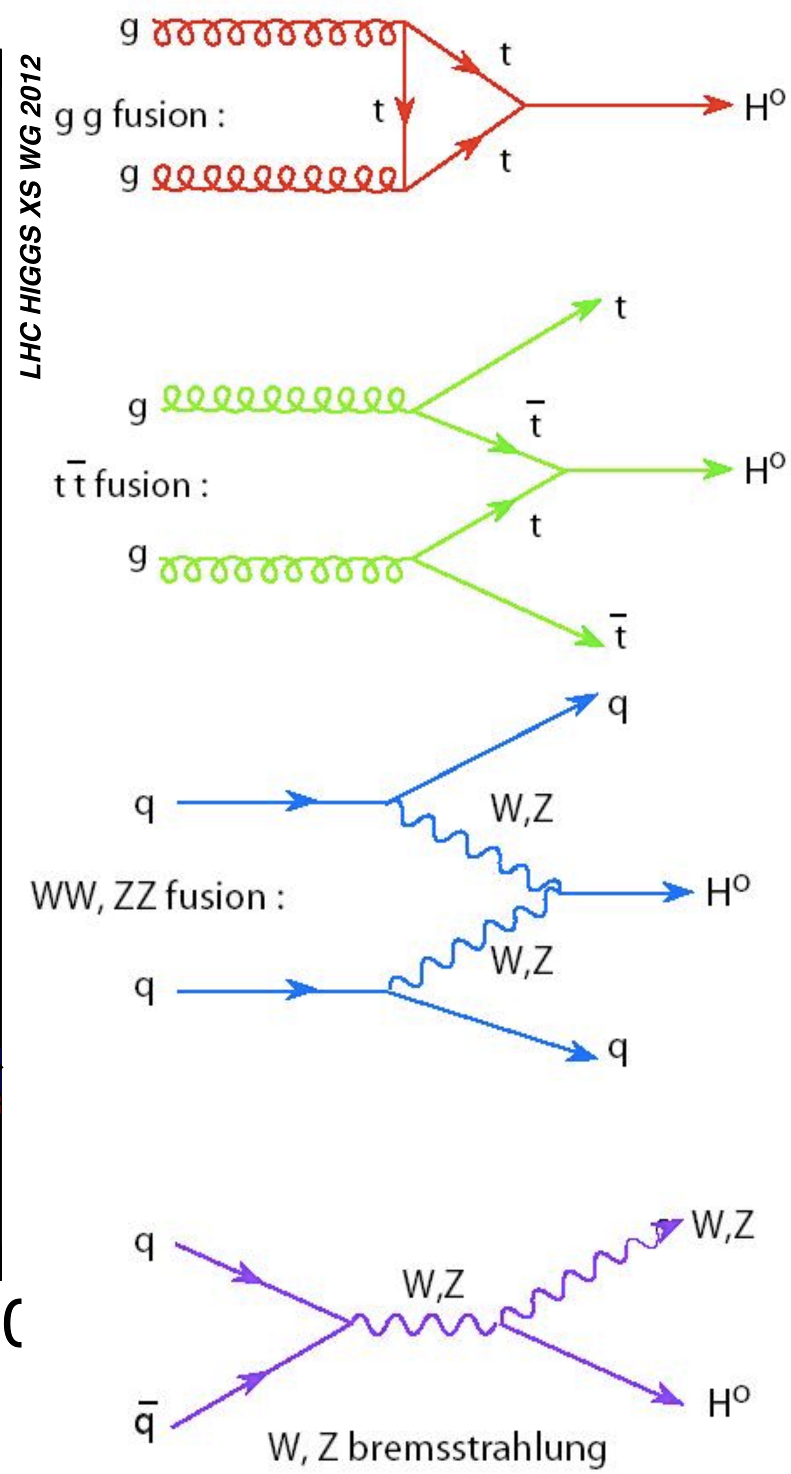
The LHC Era has begun...



- We are probing energies which have never been reached at colliders before
- High experimental precision is possible due to high luminosities
- Precise theoretical predictions become necessary

Higgs Production Channels

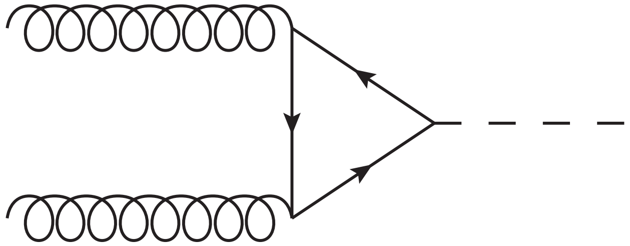
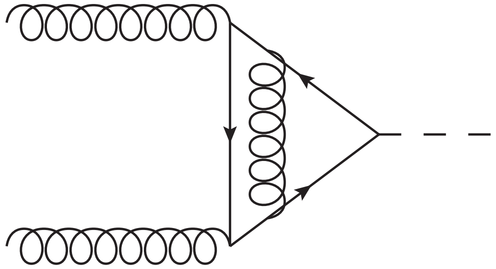
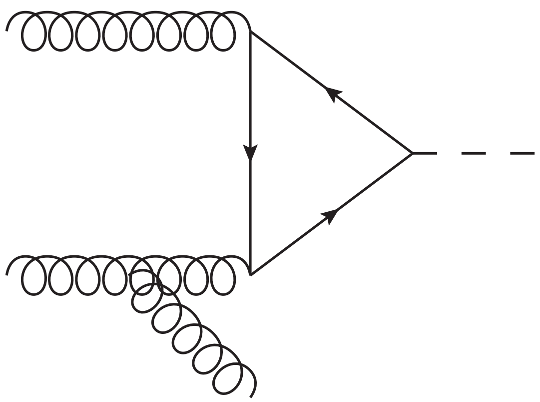
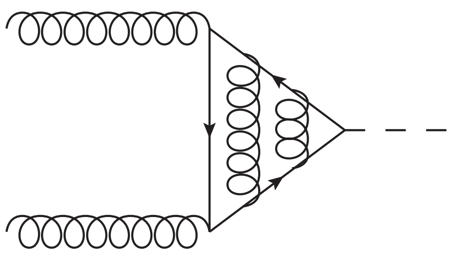
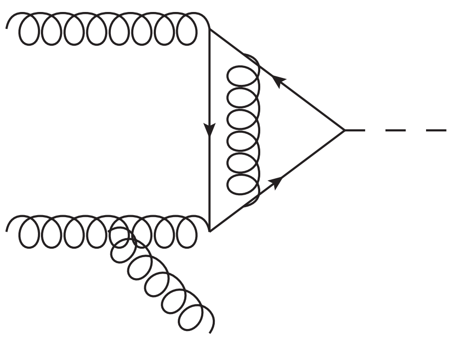
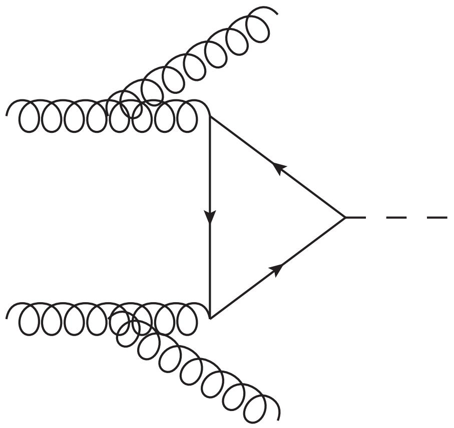




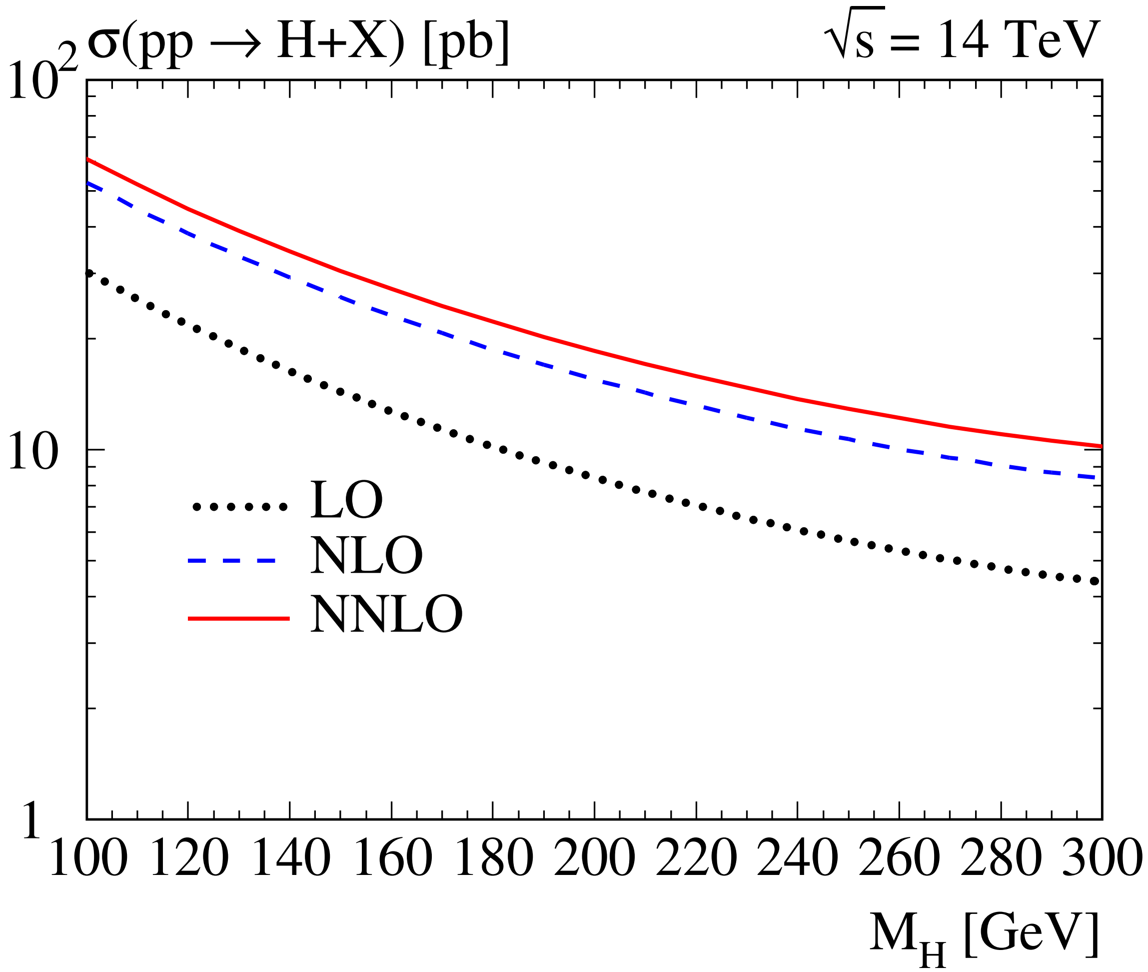
http://www.hep.ph.ic.ac.uk

Higgs Production in Gluon Fusion

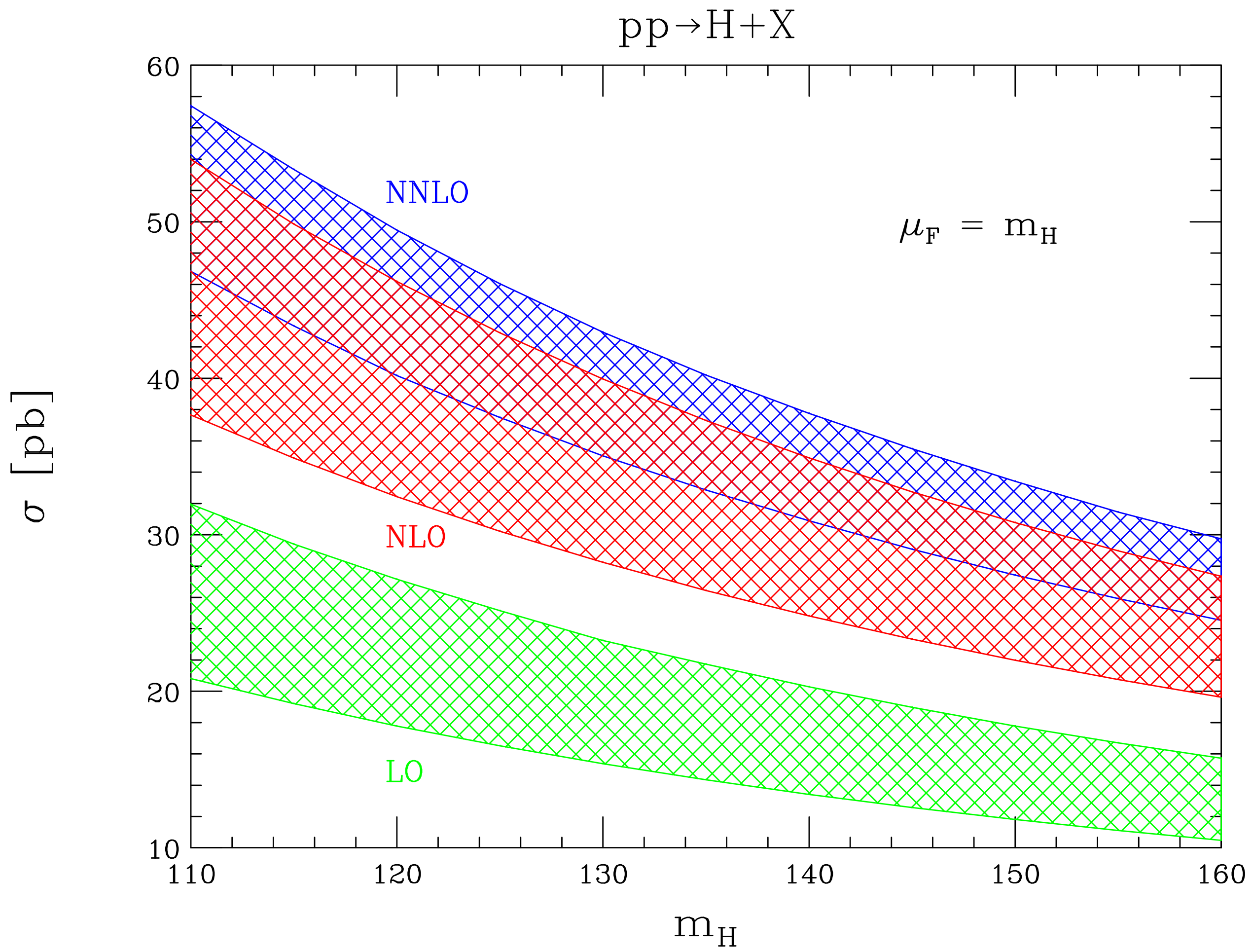
Multi-dimensional parameter integrals need to be evaluated which can contain UV, soft and collinear singularities

- LO  (NLO in other processes)
- NLO  
- NNLO   

Higher Order Corrections to the Higgs Production



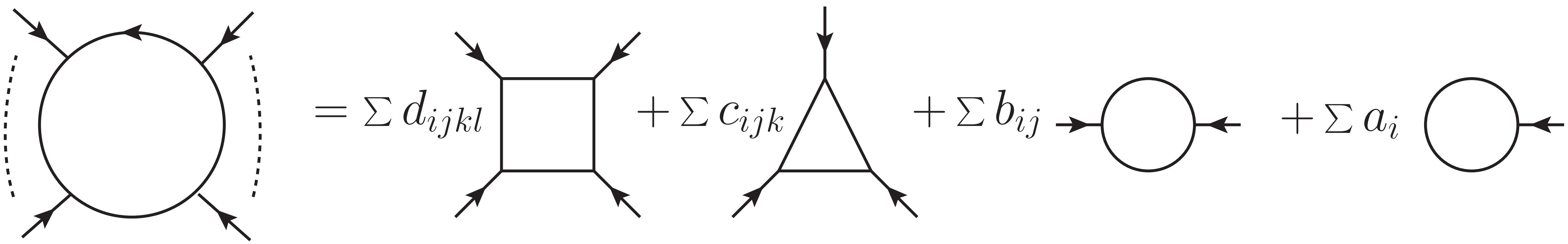
Harlander & Kilgore '02



Anastasiou, Melnikov, Petriello '05

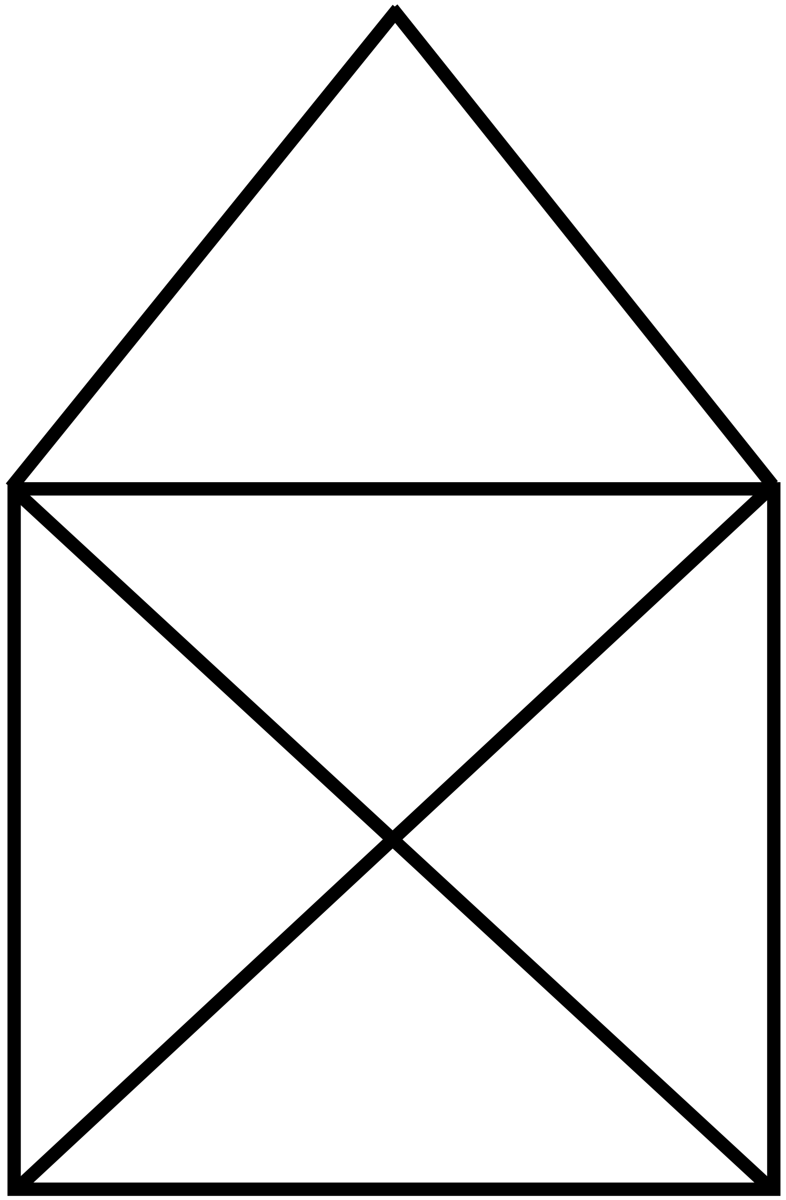
➔ In some cases, higher order corrections can make a huge difference!

- Tiziano's talk: @1-loop all master integrals are known

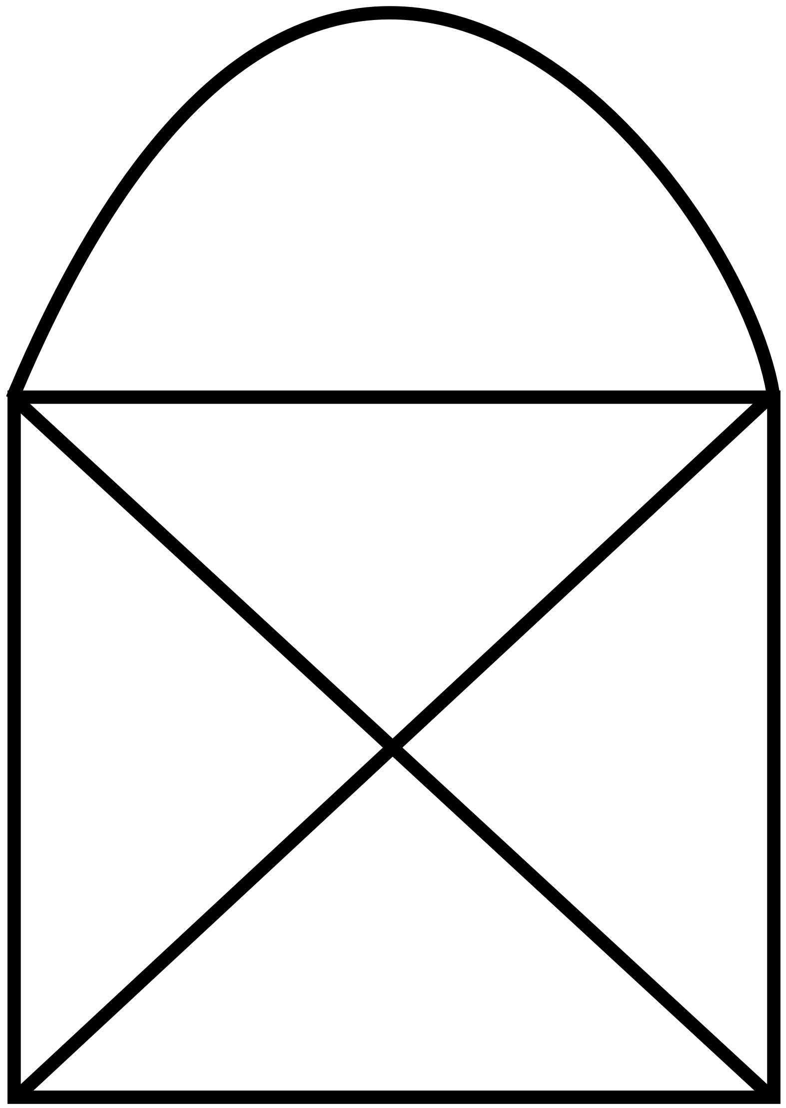


- Two and more loops: master integrals need to be found!

Let's Build a House



Let's Build a House



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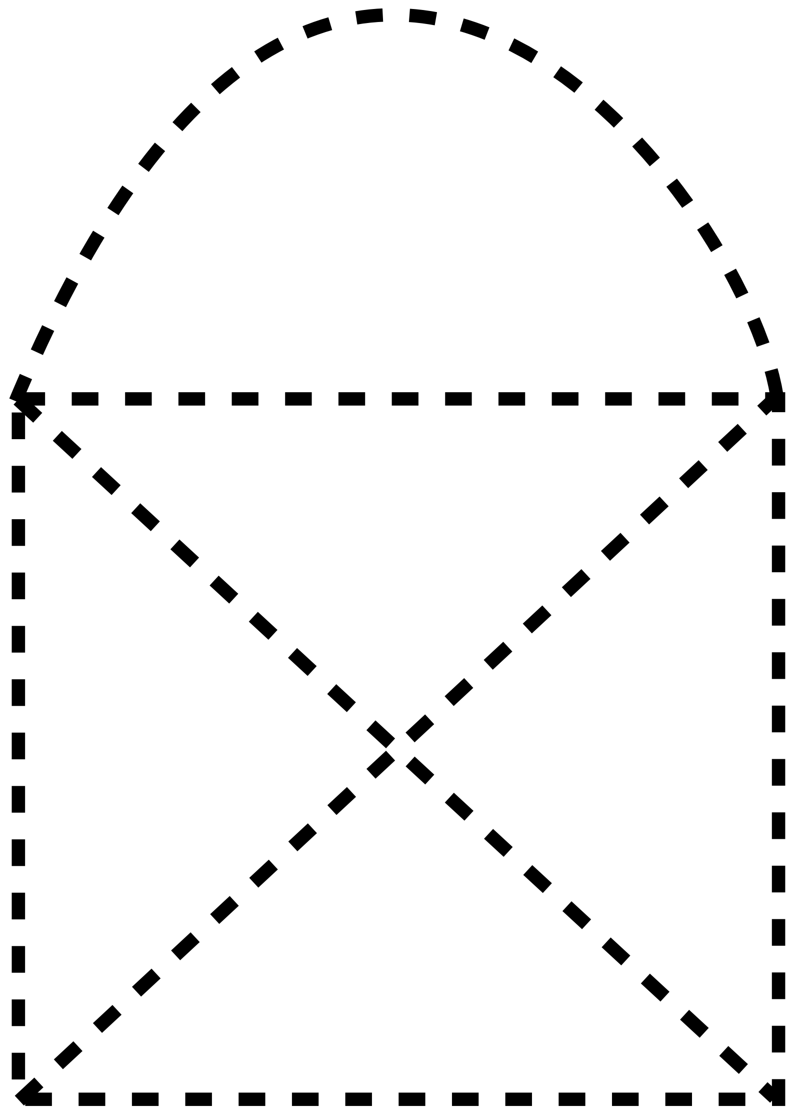
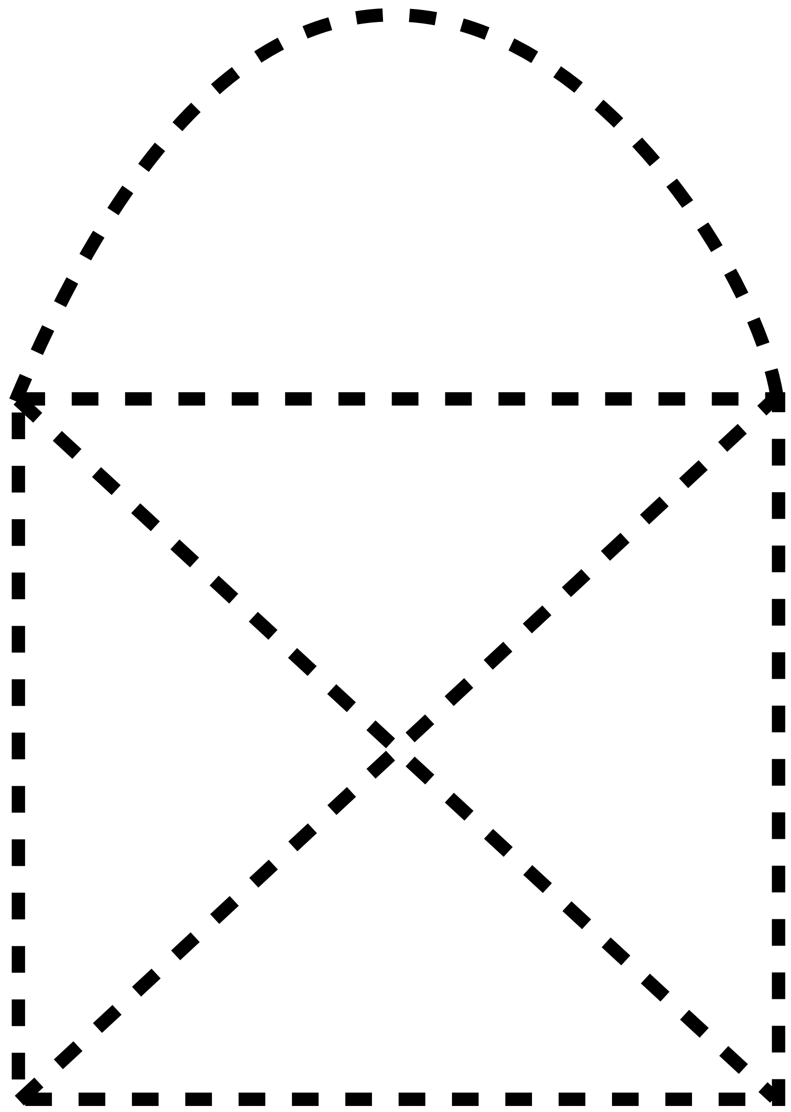


And make all its columns massless

Let's Build a House



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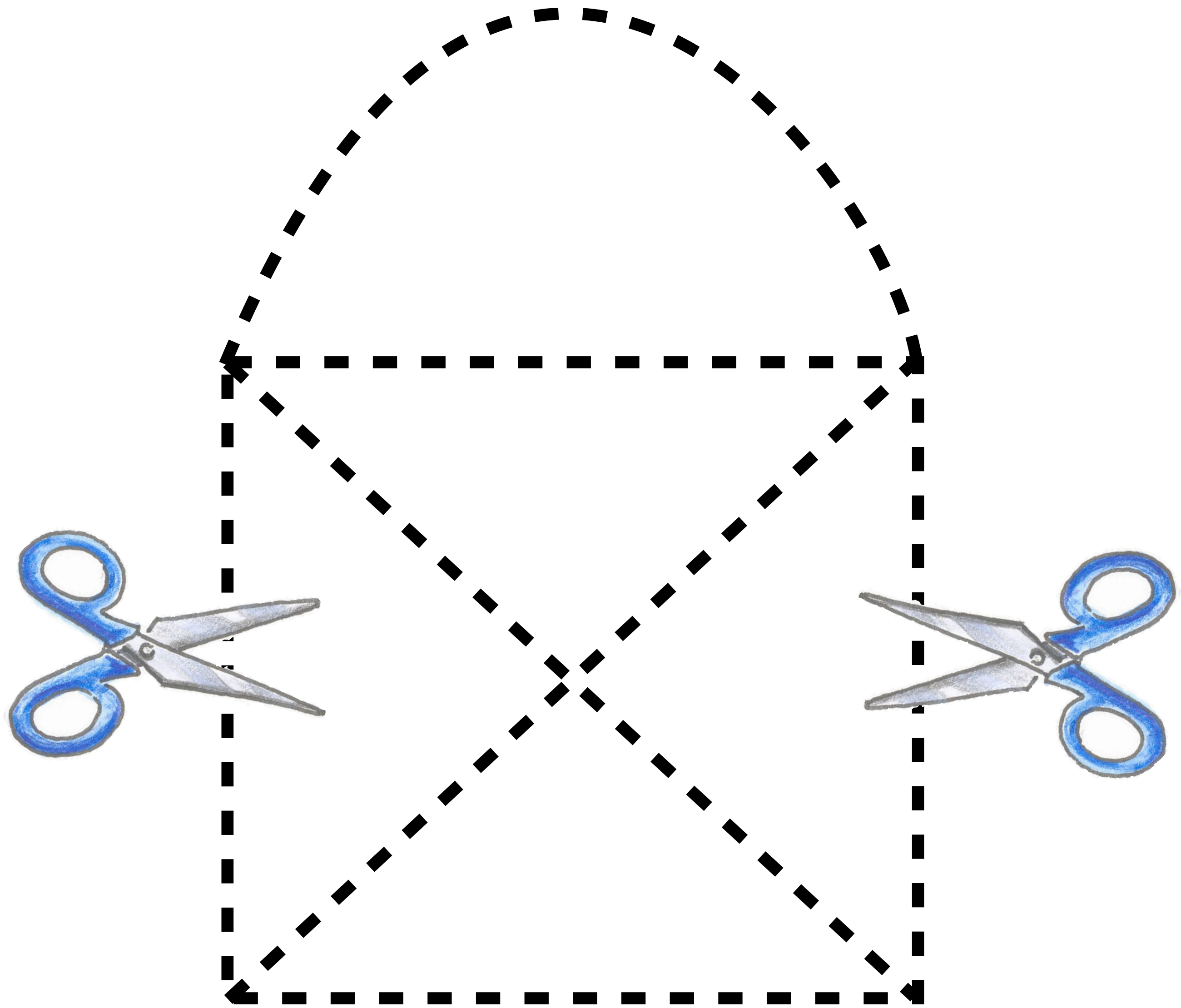

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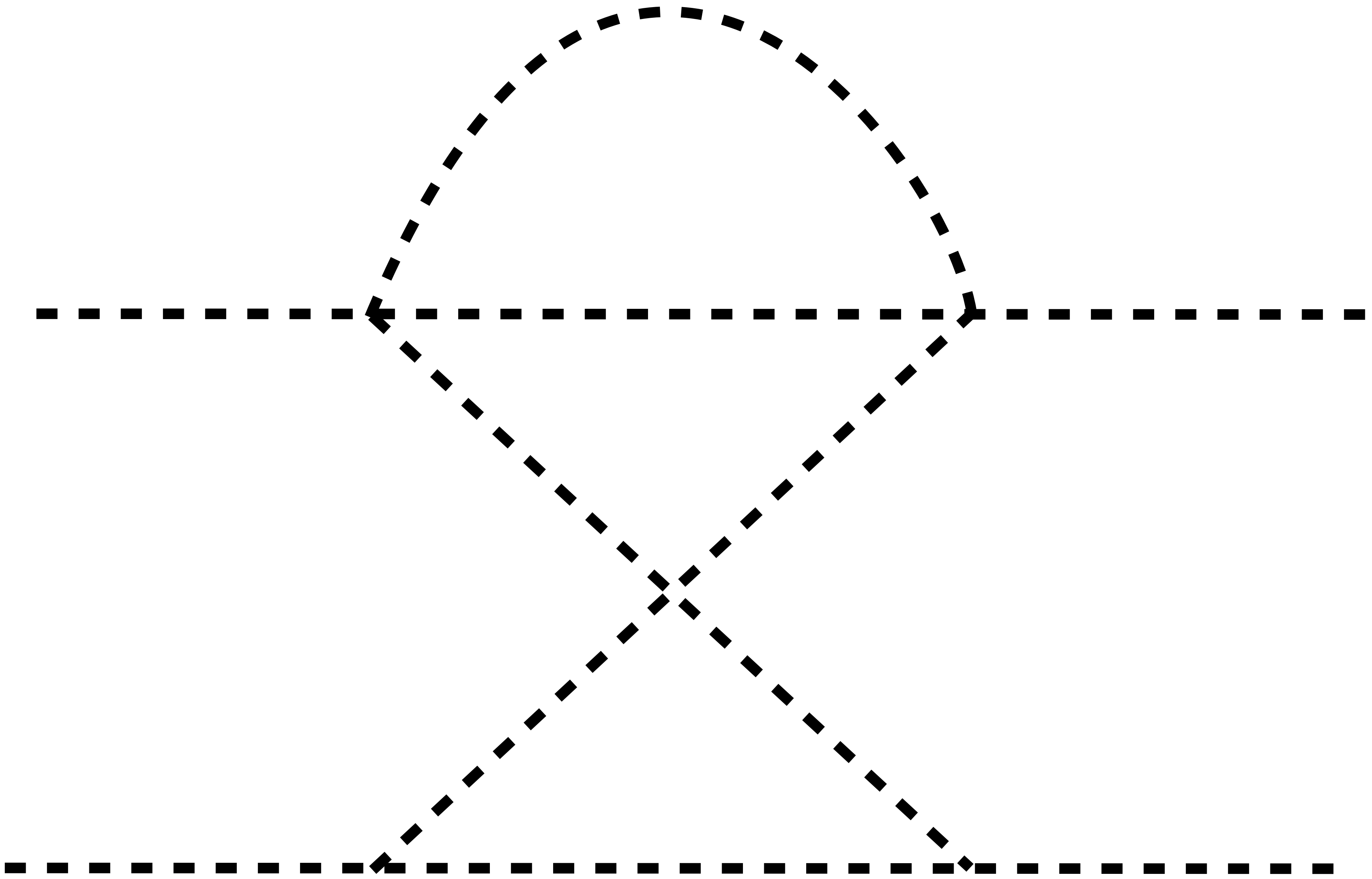

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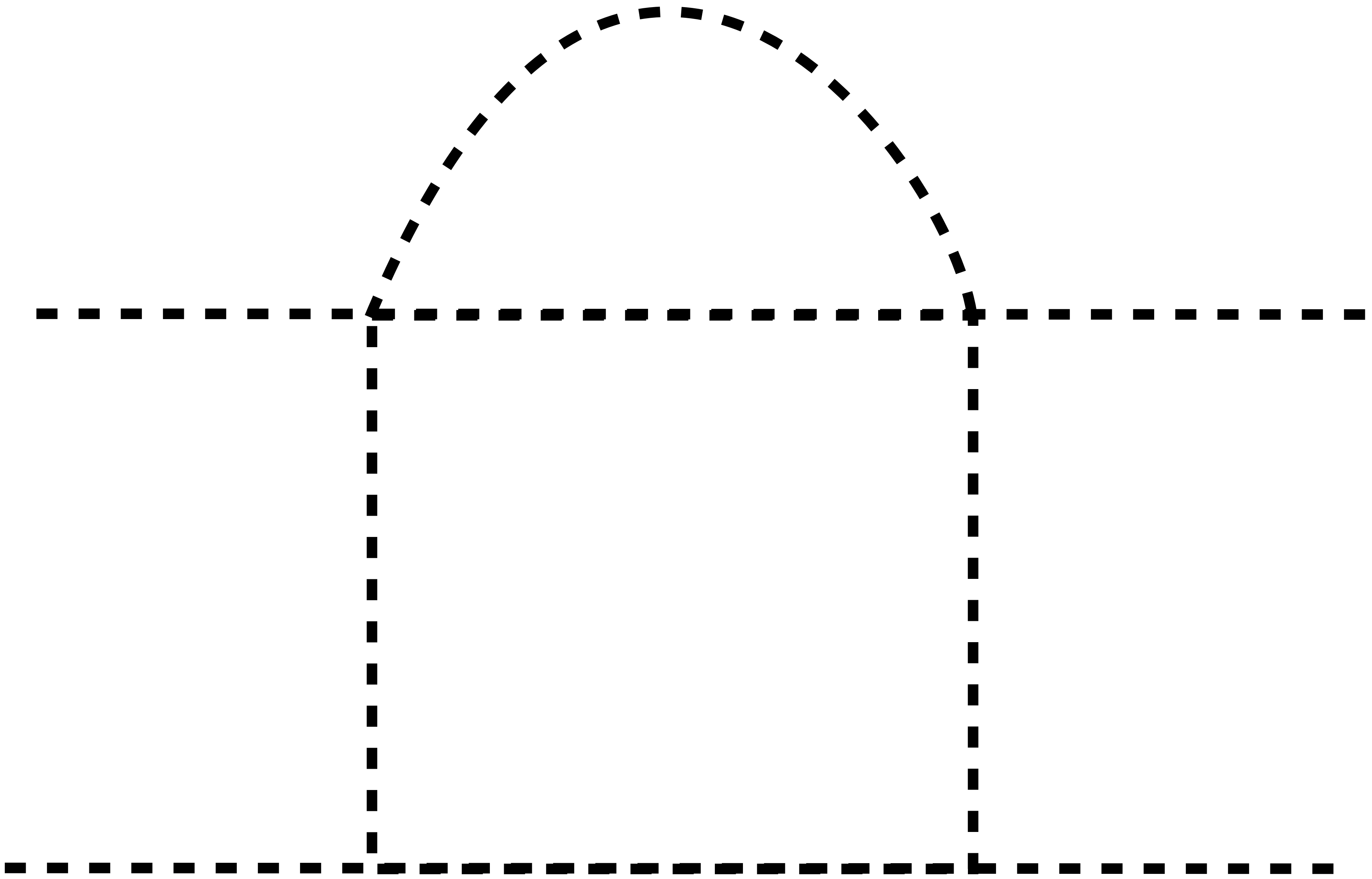
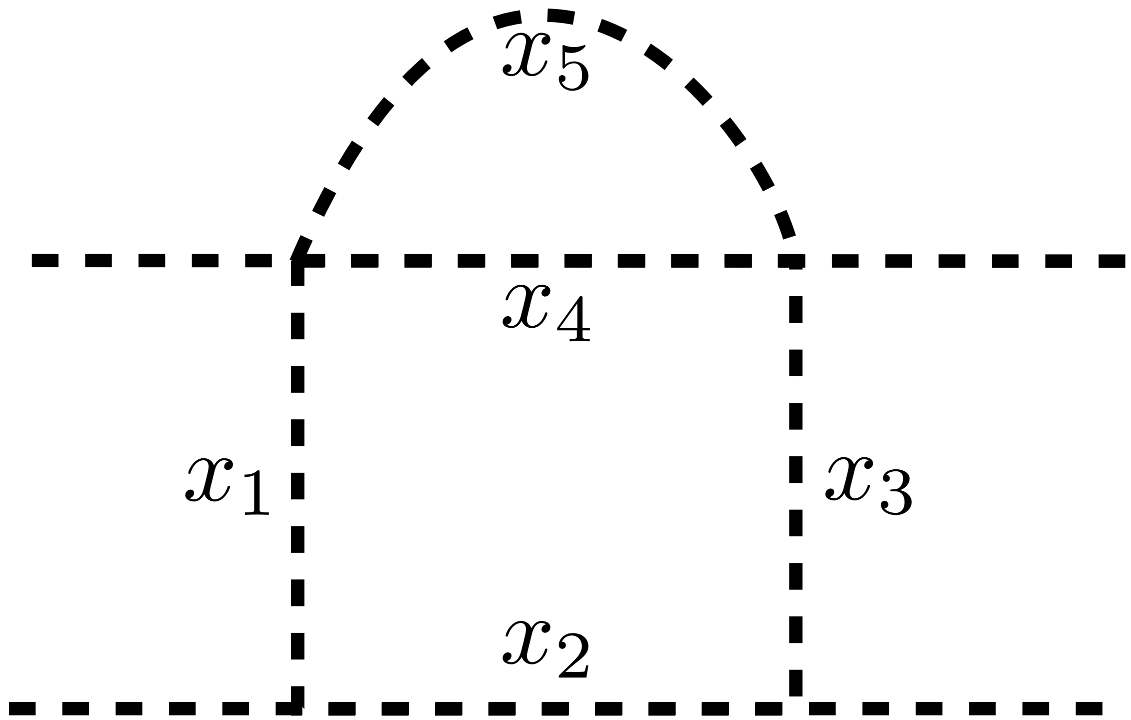
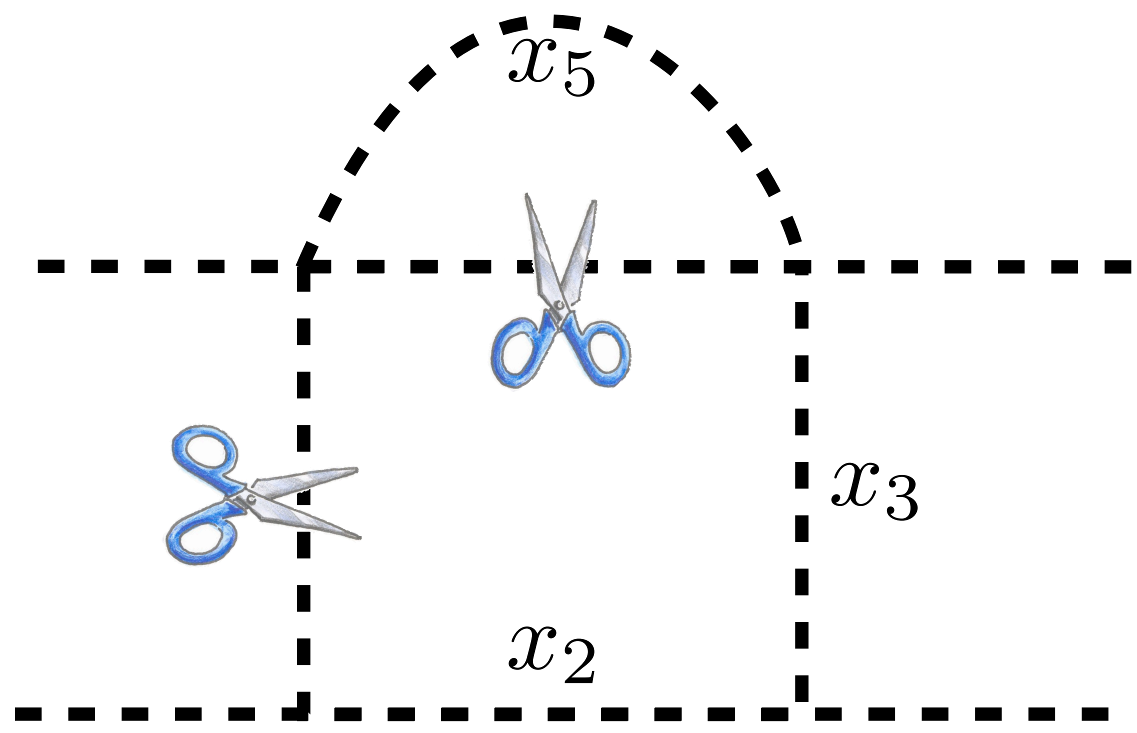


Construct the Integrand - U



- Find all connected 1-tree graphs by cutting L lines, where L is the number of loops

Construct the Integrand - U



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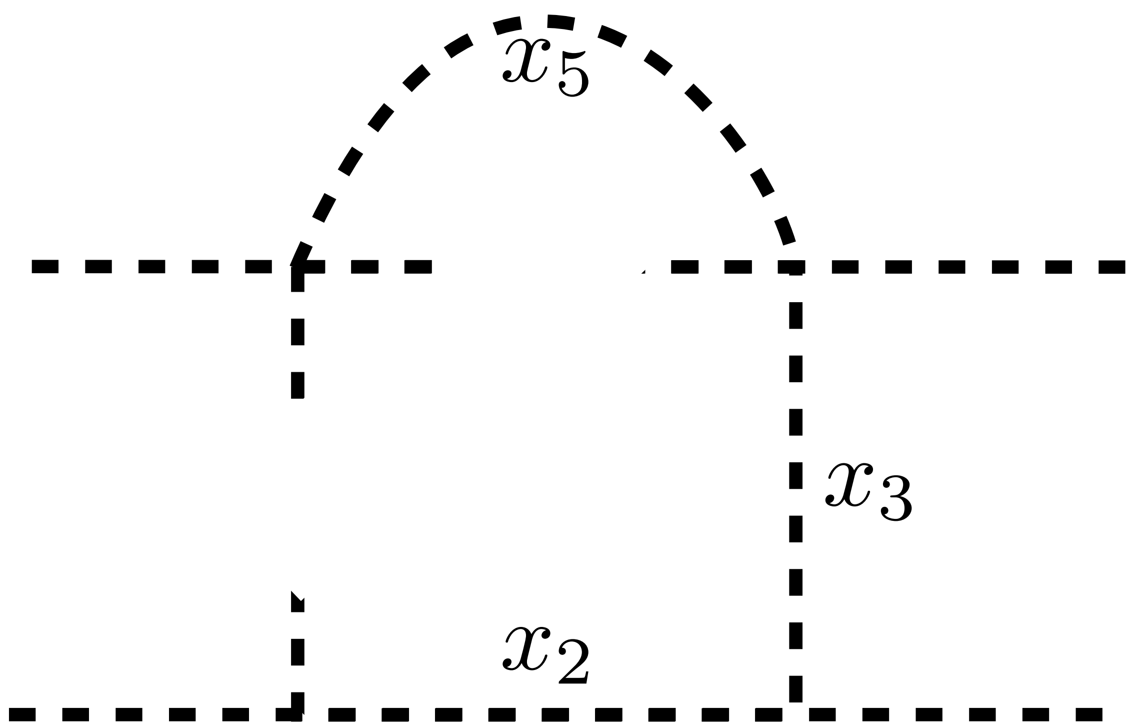
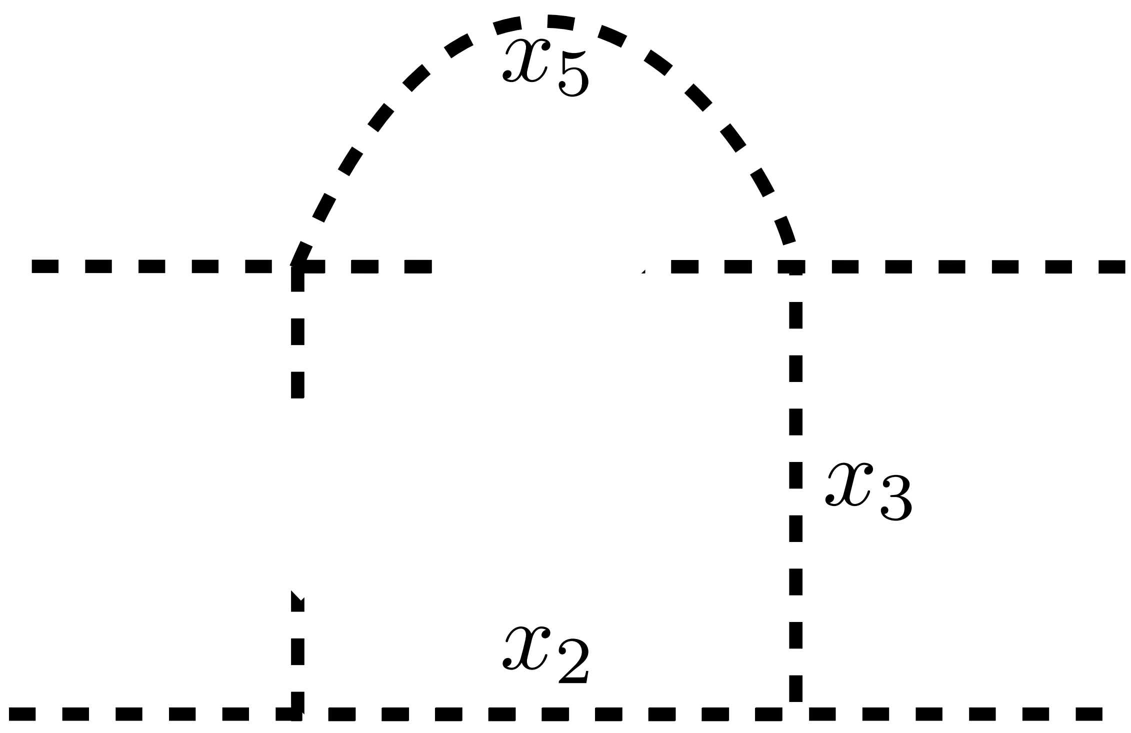
Construct the Integrand - U



$$\mathcal{U} = x_1 x_4 +$$

- Find all connected 1-tree graphs by cutting L lines, where L is the number of loops

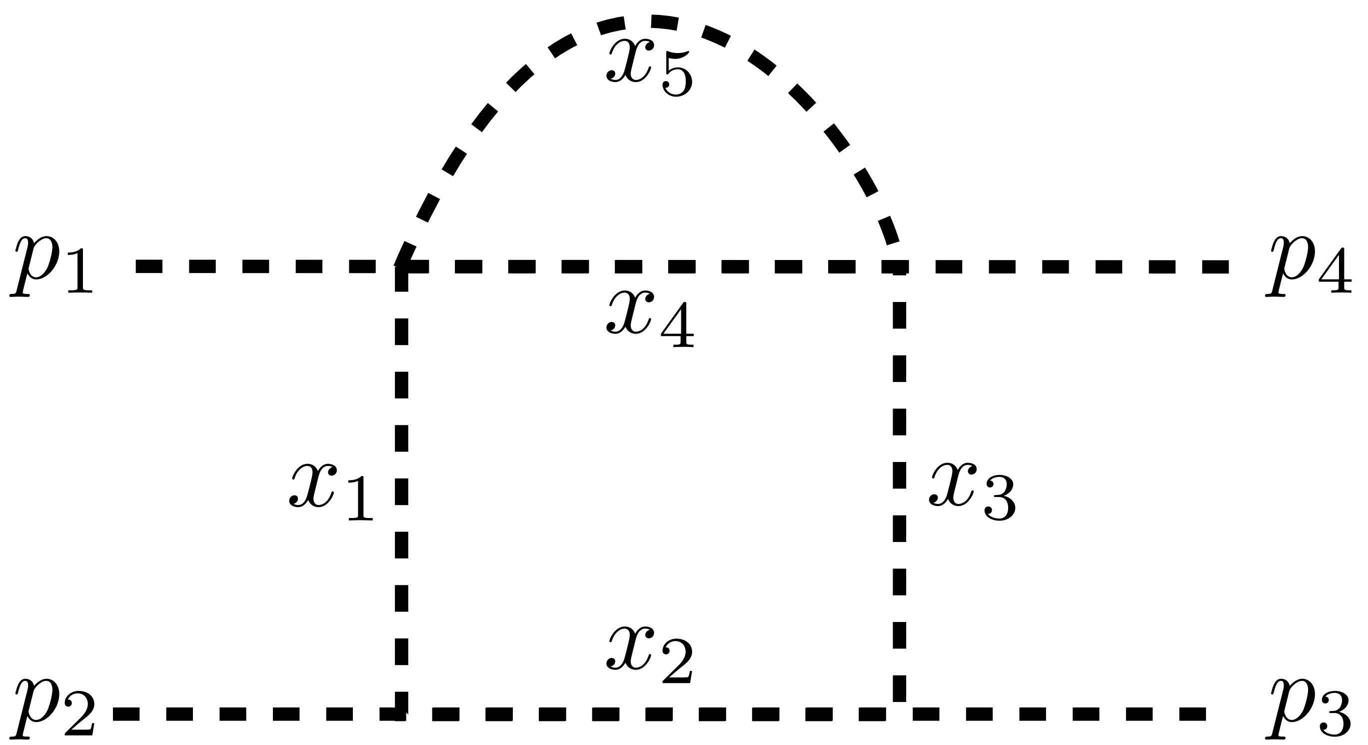
Construct the Integrand - U



$$\mathcal{U} = x_1x_4 + x_1x_5 + x_2x_4 + x_2x_5 + x_3x_4 + x_3x_5 + x_4x_5$$

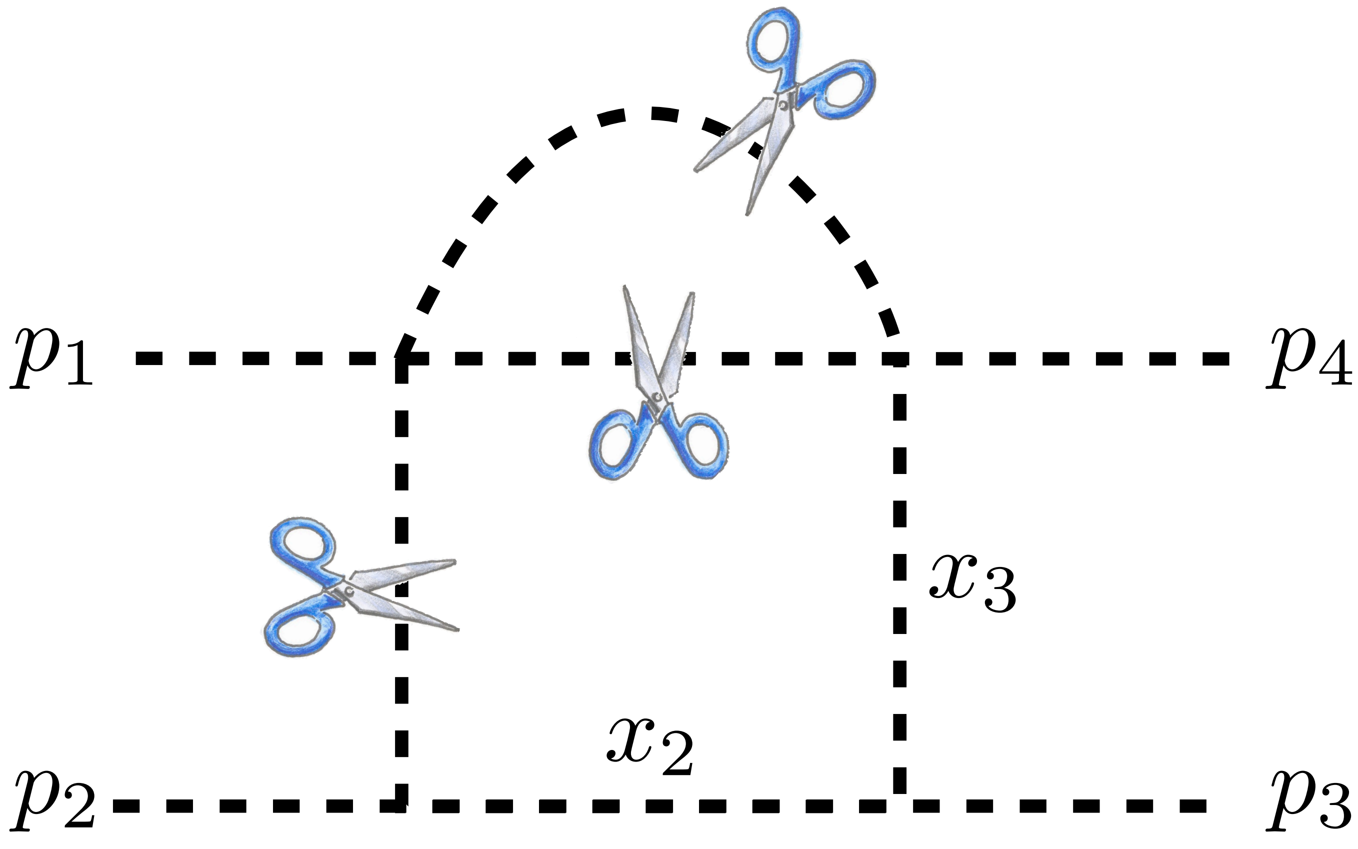
- Find all connected 1-tree graphs by cutting L lines, where L is the number of loops

Construct the Integrand - F



- Find all 2-tree graphs by cutting L+1 lines of the graph and multiplying all Feynman parameters, which correspond to the cut propagators, with the incoming momentum flow

Construct the Integrand - F



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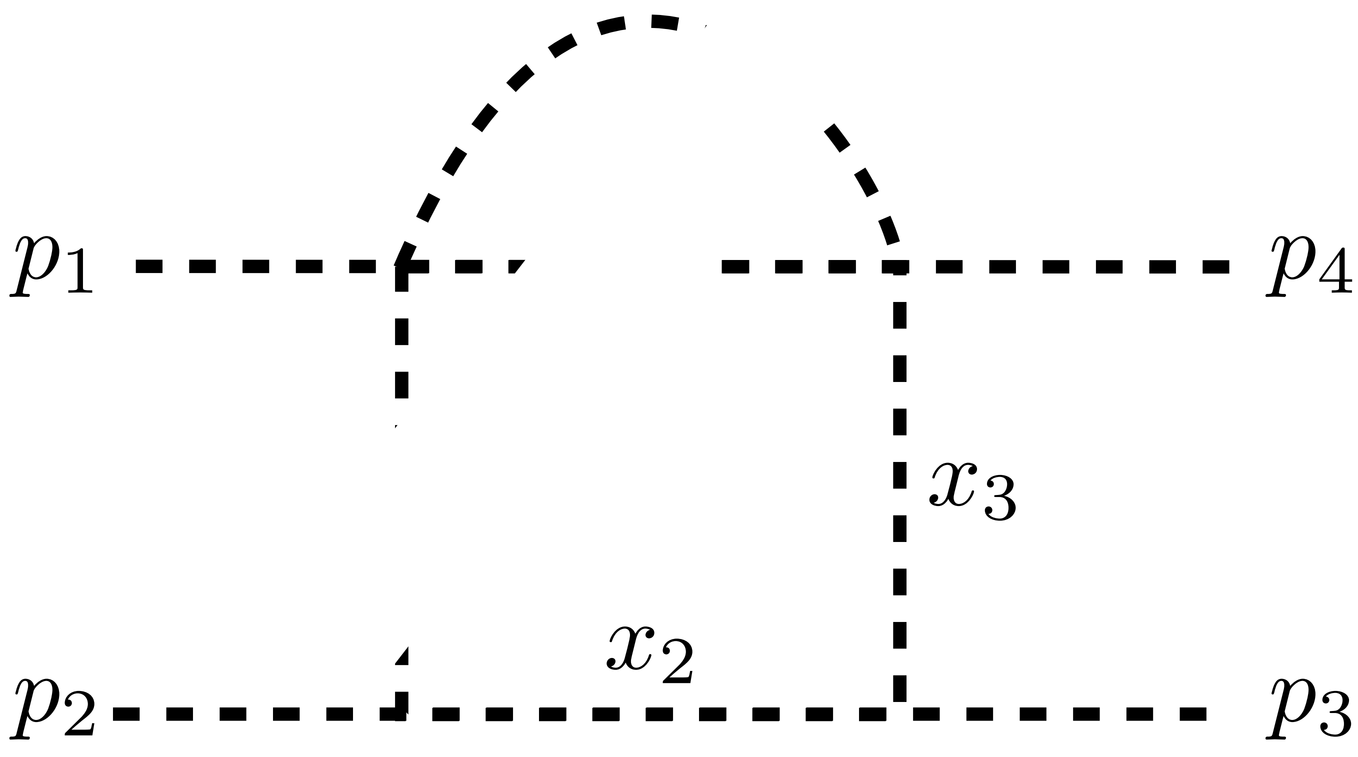
Construct the Integrand - F

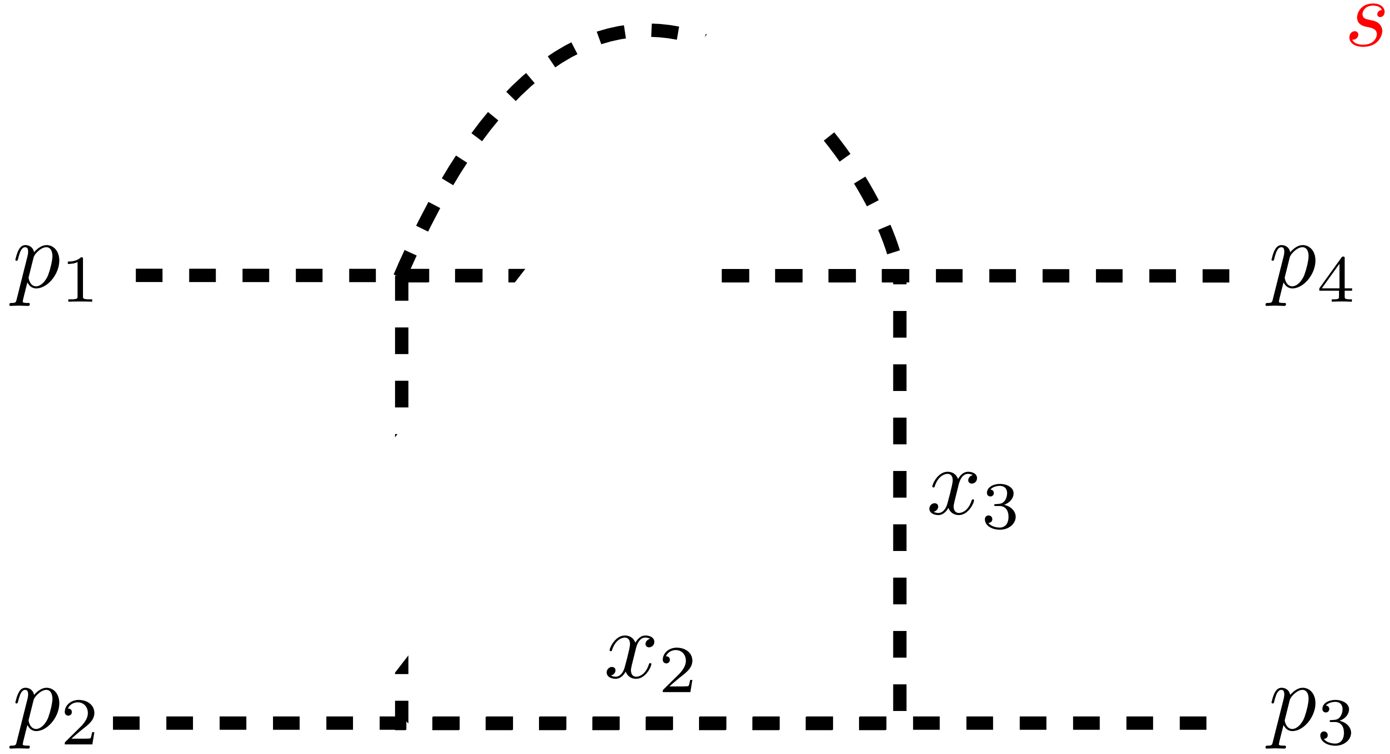


$$-\mathcal{F} = p_1^2 x_1 x_4 x_5 +$$

- Find all 2-tree graphs by cutting L+1 lines of the graph and multiplying all Feynman parameters, which correspond to the cut propagators, with the incoming momentum flow

Construct the Integrand - F

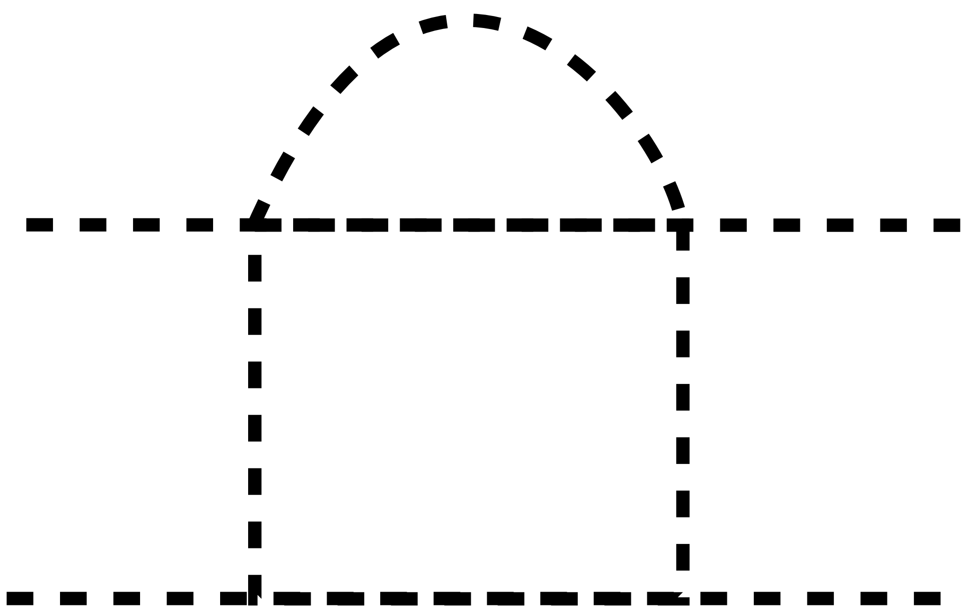
$s_{ij} = (p_i + p_j)^2$



$$-\mathcal{F} = p_1^2 x_1 x_4 x_5 + p_2^2 x_1 x_2 x_4 + p_2^2 x_1 x_2 x_5 + p_3^2 x_2 x_3 x_4 + p_3^2 x_2 x_3 x_5 + p_4^2 x_3 x_4 x_5 + s_{12} x_2 x_4 x_5 + s_{23} x_1 x_3 x_4 + s_{23} x_1 x_3 x_5$$

- Find all 2-tree graphs by cutting L+1 lines of the graph and multiplying all Feynman parameters, which correspond to the cut propagators, with the incoming momentum flow

The Full Integrand

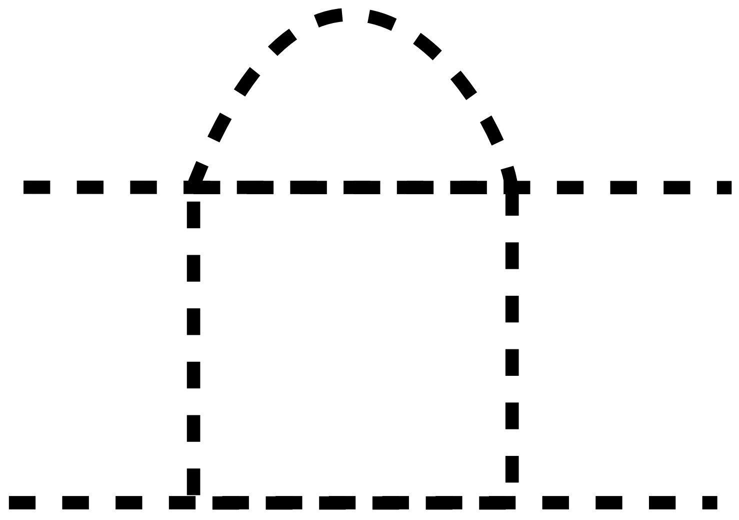


- The full integrand G after loop momentum integration in D dimensions with N propagators to power ν_j

$$G = \frac{(-1)^{N_\nu}}{\prod_{j=1}^{N}\Gamma(\nu_j)}\Gamma(N_\nu - LD/2)\int_0^\infty \prod_{j=1}^{N} \mathrm{d}x_j \; x_j^{\nu_j-1}\,\delta(1-\sum_{l=1}^{N} x_l)\frac{\mathcal{U}^{N_\nu-(L+1)D/2}(\vec{x})}{\mathcal{F}^{N_\nu-LD/2}(\vec{x})}$$

and $N_\nu = \sum_{j=1}^{N}\nu_j$

Any Divergences?



UV sub-divergence

$$G = \frac{(-1)^{N_\nu}}{\prod_{j=1}^{N}\Gamma(\nu_j)}\Gamma(N_\nu - LD/2)\int_0^\infty \prod_{j=1}^{N} \mathrm{d}x_j \; x_j^{\nu_j-1}\,\delta(1-\sum_{l=1}^{N} x_l)\frac{\mathcal{U}^{N_\nu-(L+1)D/2}(\vec{x})}{\mathcal{F}^{N_\nu-LD/2}(\vec{x})}$$

overall UV singularity

IR divergence

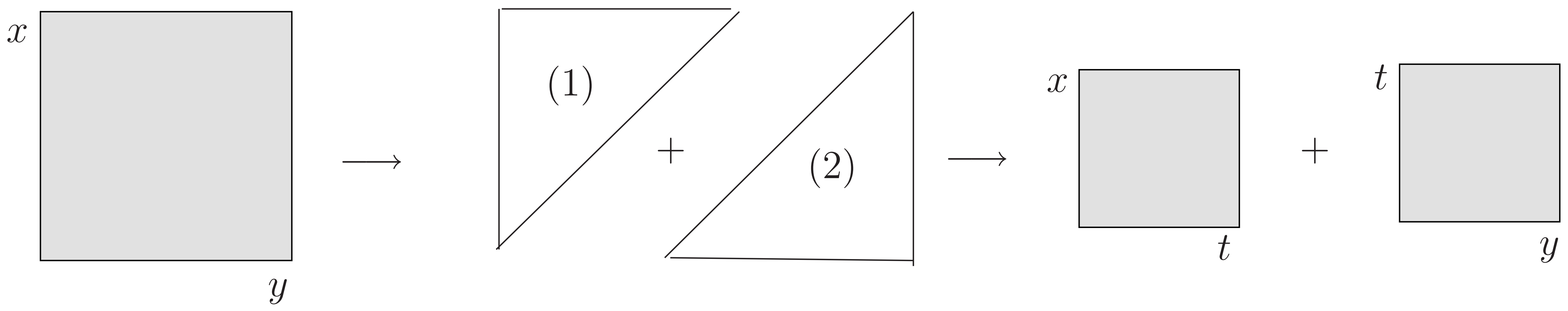
- Problem: Divergences can overlap!

$$\int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y \ \frac{1}{(x+y)^{2+\epsilon}} =$$

Sector Decomposition

- Problem: Divergences can overlap!

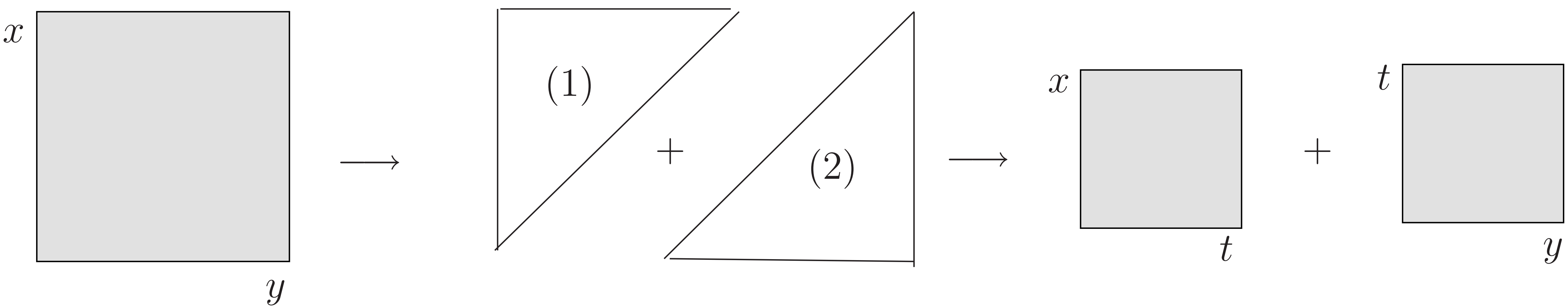
$$\int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y \, \frac{1}{(x+y)^{2+\epsilon}} = \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}t \, \frac{1}{x^{1+\epsilon}(1+t)^{2+\epsilon}} + \int_0^1 \mathrm{d}t \int_0^1 \mathrm{d}y \, \frac{1}{y^{1+\epsilon}(1+t)^{2+\epsilon}}$$



Sector Decomposition

- Problem: Divergences can overlap!

$$\int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y \; \frac{1}{(x+y)^{2+\epsilon}} = \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}t \; \frac{1}{x^{1+\epsilon}(1+t)^{2+\epsilon}} + \int_0^1 \mathrm{d}t \int_0^1 \mathrm{d}y \; \frac{1}{y^{1+\epsilon}(1+t)^{2+\epsilon}}$$



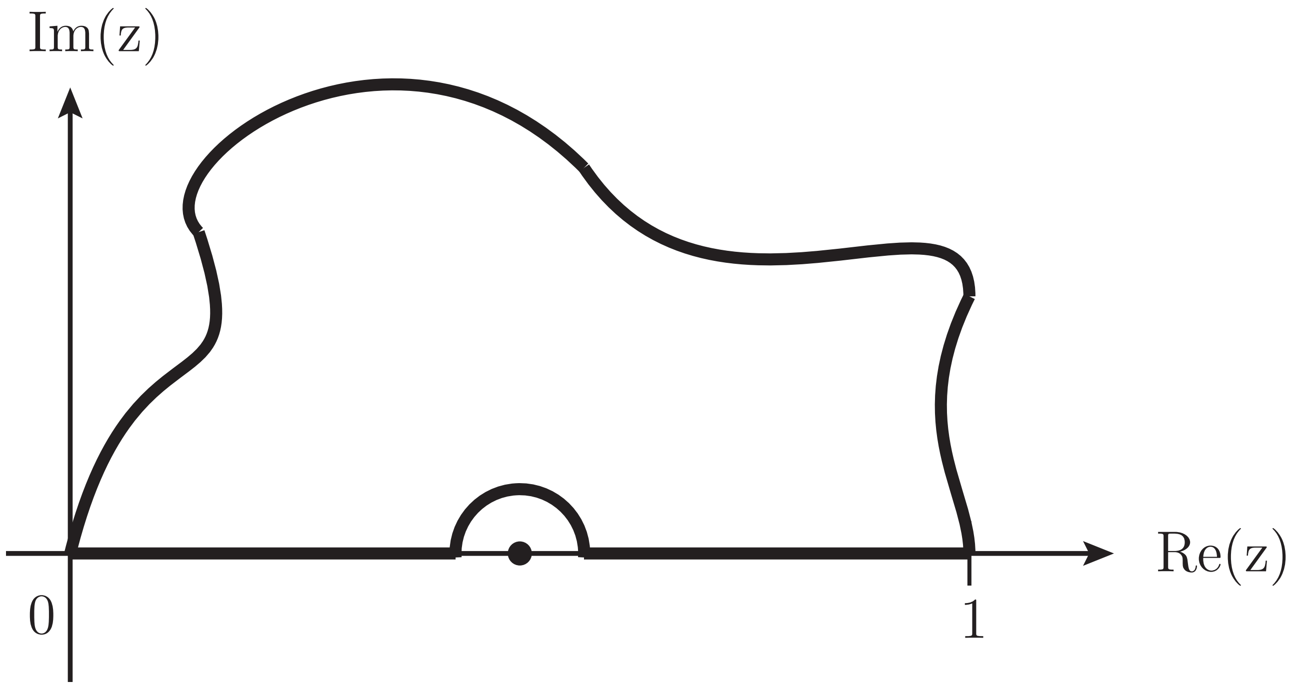
- Result: After iterated sector decomposition procedure, dimensionally regulated soft, collinear and UV singularities are factored out   Hepp '66, Binoth & Heinrich '00

Deformation of the Integration Contour

- When computing diagrams with more than one scale, function $\mathcal{F}$ can still vanish

$$\mathcal{F}_{Bubble} = m^2(1+t_1)^2 - s\,t_1 - i\delta$$

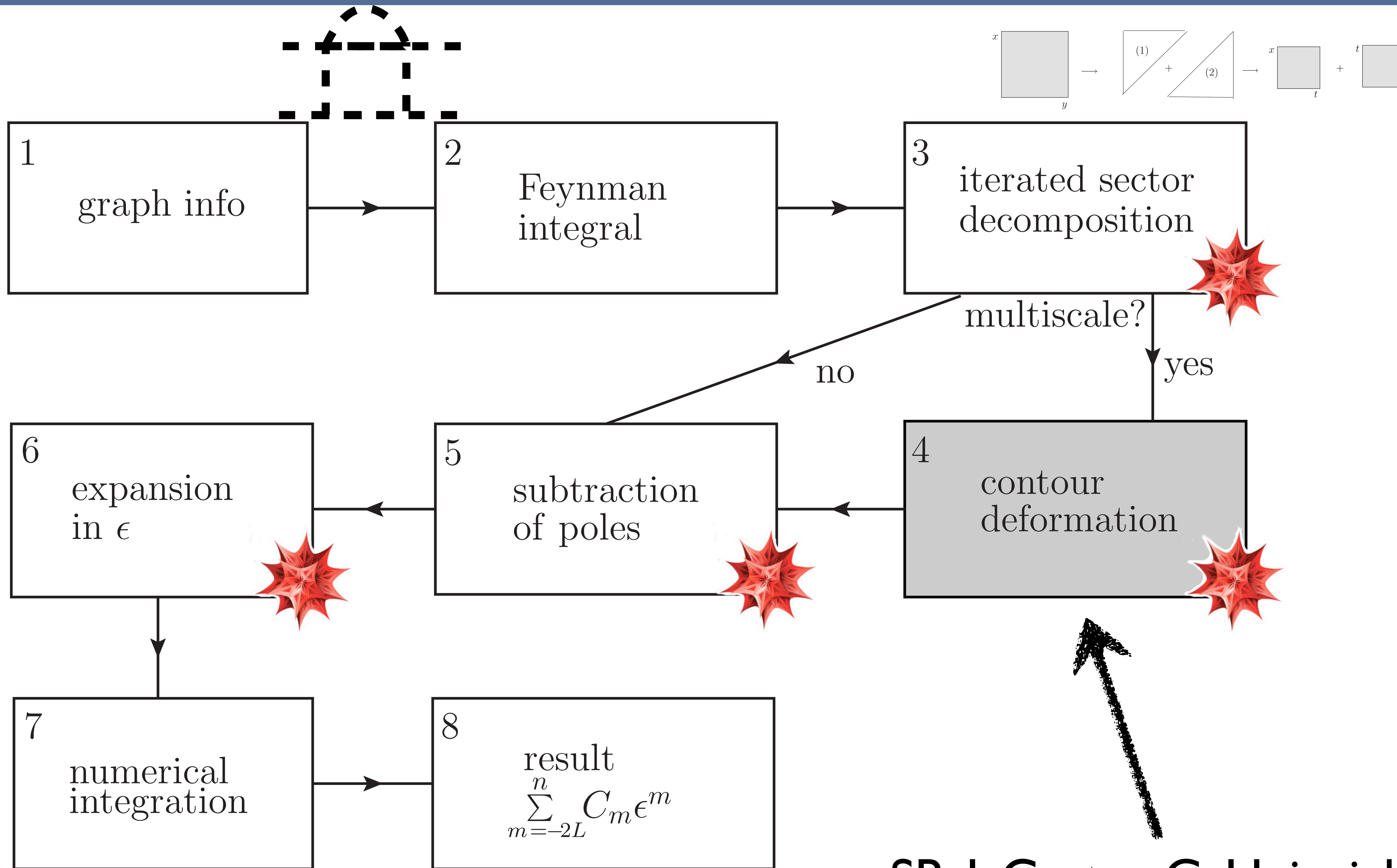
but a deformation of the integration contour



and Cauchy's theorem can help

$$\oint_c f(t)\mathrm{d}t = \int_0^1 f(t)\mathrm{d}t + \int_1^0 \frac{\partial z(t)}{\partial t} f(z(t))\mathrm{d}t = 0$$

Operational Sequence of SecDec 2.0



Cuba 3.0

Hahn et al. '04 '11

SB, J. Carter, G. Heinrich

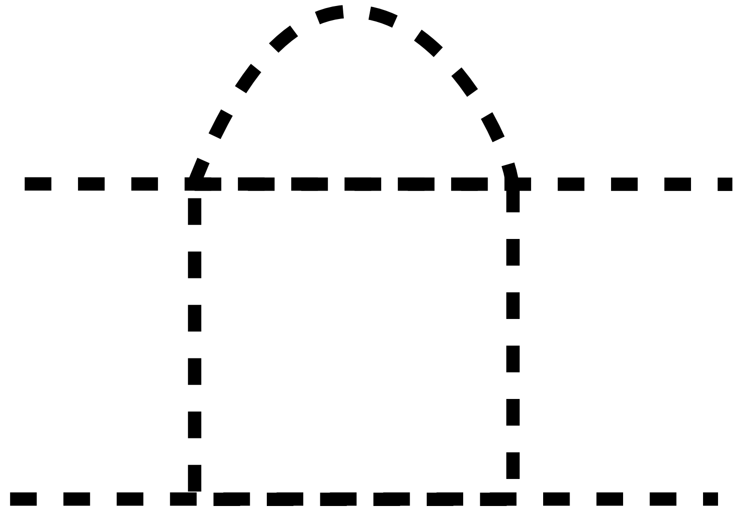
arXiv:1204.4152 [hep-ph]

Analytic vs Numerical Approach

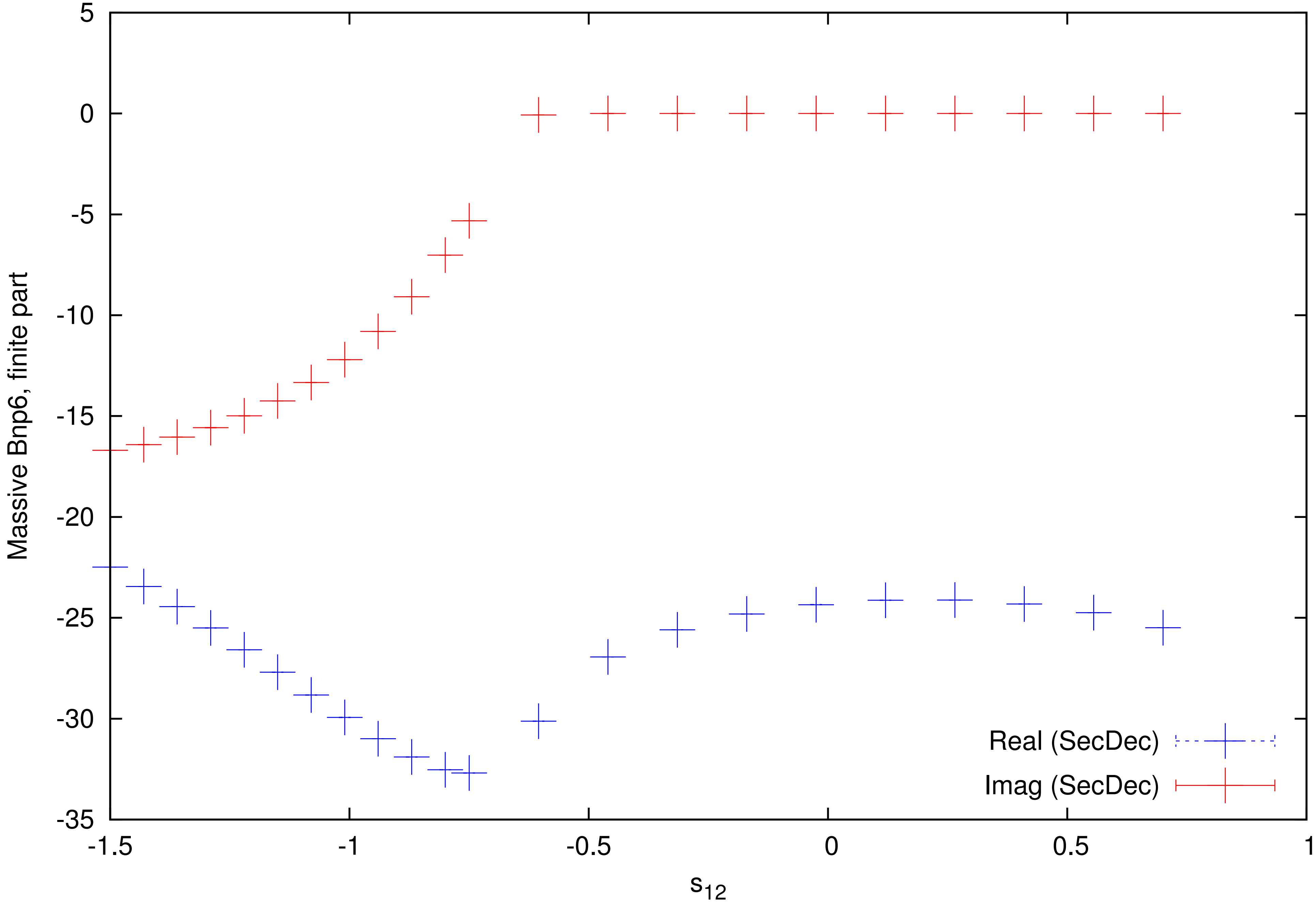
	Analytical	Numerical
Pro's	• get result for different kinematics in "no time"	• easier to automate • classes of diagrams can be computed similarly
Con's	• complicated integrands may need approximation • every integrand needs to be treated individually	• computation must be redone when changing kinematics • speed vs accuracy

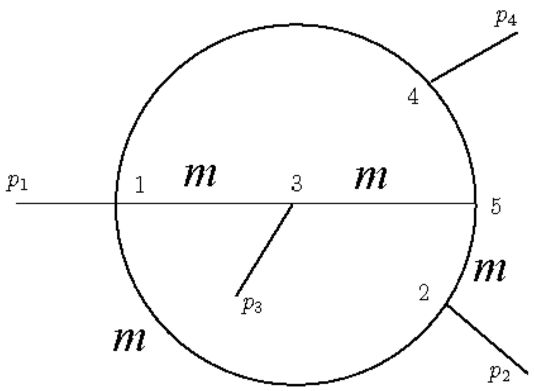
Result for the House

```
***********************************************
***OUTPUT: House h_ ***************************
 point: 3.0 5.0
 ext. legs: 0.0 0.0 0.0 0.0
 prop. mass: 0.0 0. 0. 0. 0.
Prefactor=-Exp[-2EulerGamma*eps]
***********************************************

********** eps^-3 coeff ***********************
result        =-0.2 + 0 I
error         =8.65745e-06 + 7.69808e-06 I

CPUtime (all eps^-3 subfunctions)  =0.00867425

********** eps^-2 coeff ***********************
result        =0.1416138 - 1.256629819413 I
error         =0.000112589057259132 + 0.000347590523927221 I

CPUtime (all eps^-2 subfunctions)  =0.03597475

********** eps^-1 coeff ***********************

result        =4.48469357326071 + 0.88977832278 I
error         =0.00197323555702034 + 0.000693096089967388 I

CPUtime (all eps^-1 subfunctions)  =0.30169275

********** eps^0 coeff ************************

result        =-0.955432257069887 + 10.9736953304604 I
error         =0.0110823059795104 + 0.0288369973195129 I

CPUtime (all eps^0 subfunctions)  =2.0828709

***********************************************
Time taken for decomposition = 1.332223

Total time for subtraction and eps expansion = 7.120933 secs
Time taken for longest subtraction and eps expansion = 2.63507 secs
```



More Results: Non-planar 4-Point Diagram

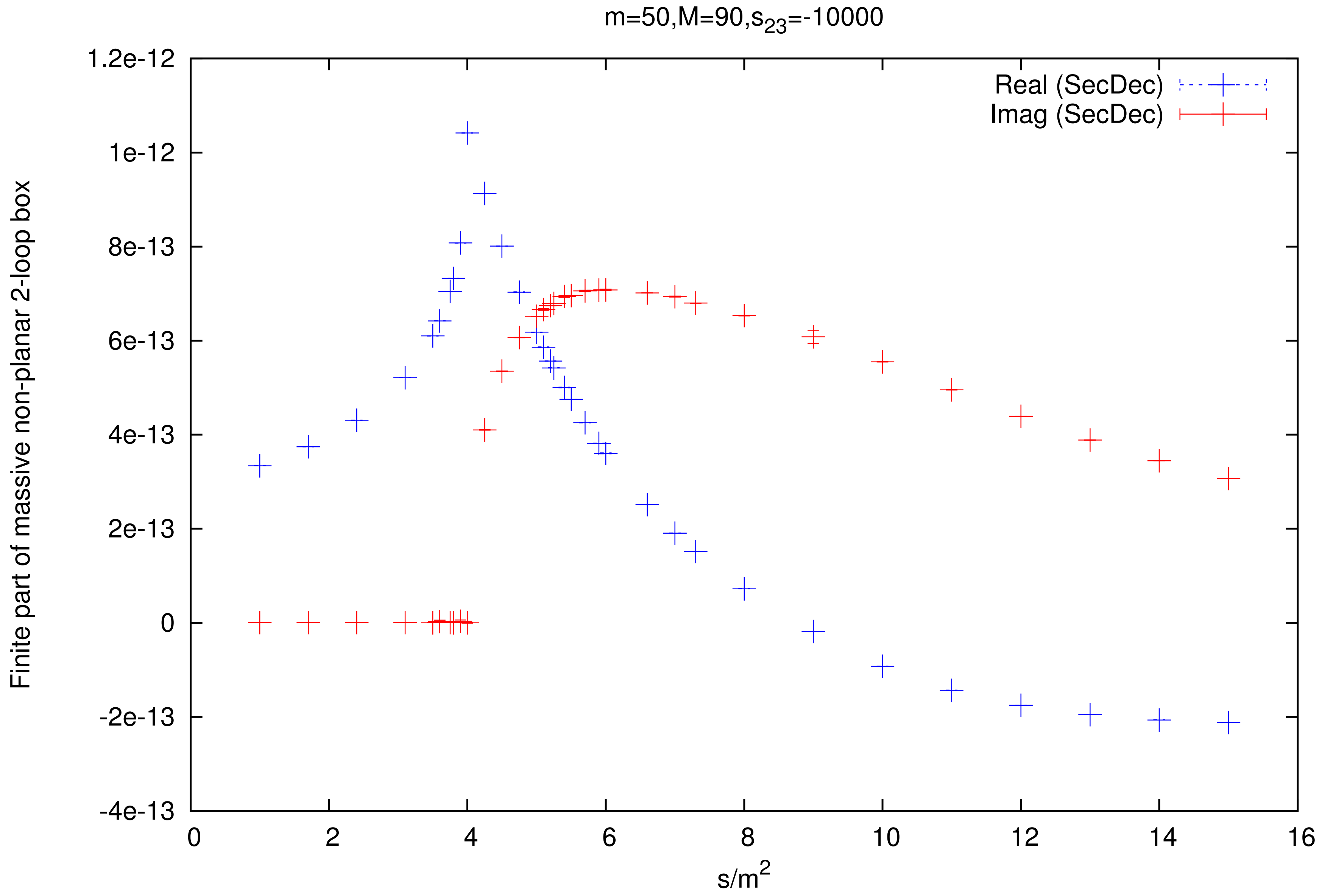


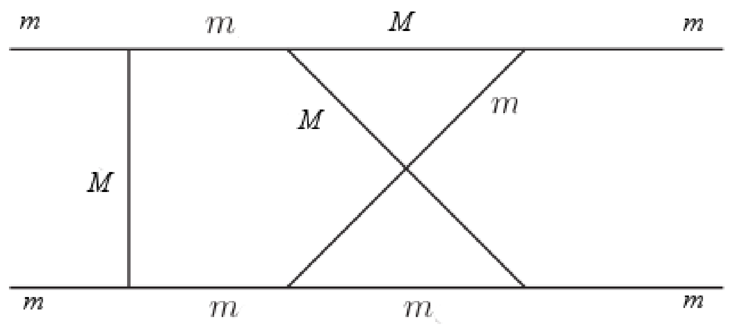


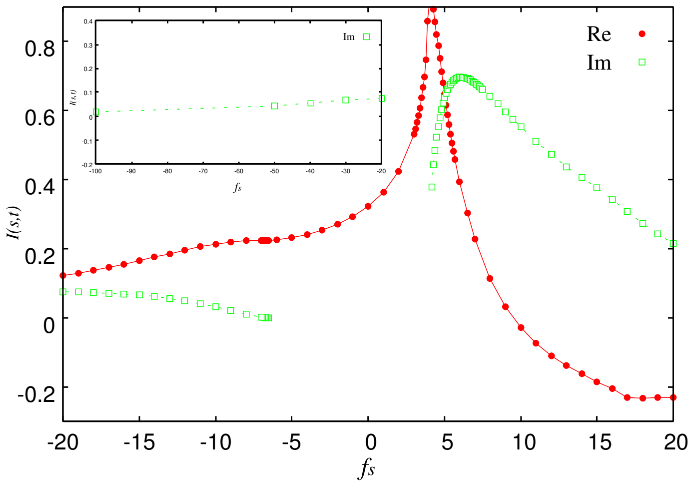
$m \neq 0$

massless case: Tausk '99

More Results: Non-planar 2-Loop Box Diagram







Fujimoto et al. '11

Summary

- Higher order computations can lead to large corrections
- Integrands can be constructed via topological cuts
- Overlapping divergences can be factorized with the help of sector decomposition
- Dealing with multiple scales, an additional deformation of the integration contour becomes necessary
- SecDec 2.0 is a tool to numerically compute (master) diagrams with arbitrary kinematics

What wasn't mentioned:

- SecDec 2.0 can compute much more (also tensor integrals, infrared divergent subtraction terms for real radiation or other more general functions)

Outlook

- Apply SecDec 2.0 to 2-loop corrections involving several mass scales, e.g. QCD/EW/MSSM corrections
- Improve detection and treatment of problematic kinematic regions, e.g. close to a (leading Landau) singularity
- Improve speed of computation of diagrams

Thank you for your attention.