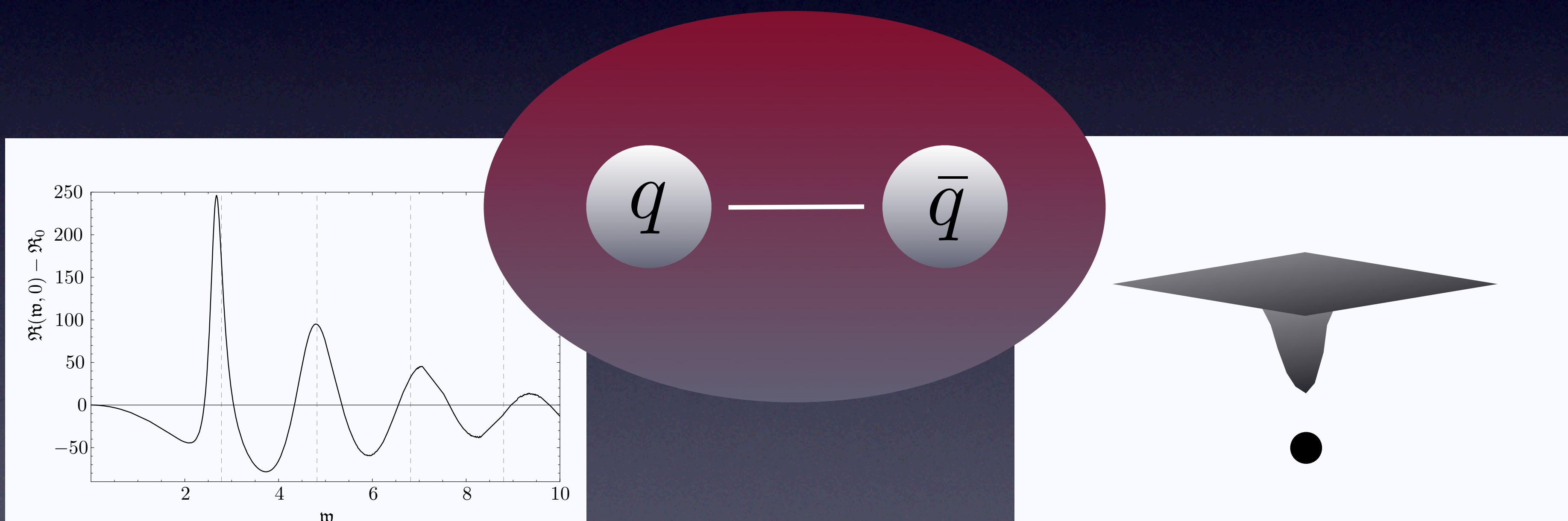
Thermal spectral functions and diffusion from AdS/CFT

by Matthias Kaminski

Max-Planck-Institut für Physik, München



[Erdmenger, M.K., Rust 0710.0334]

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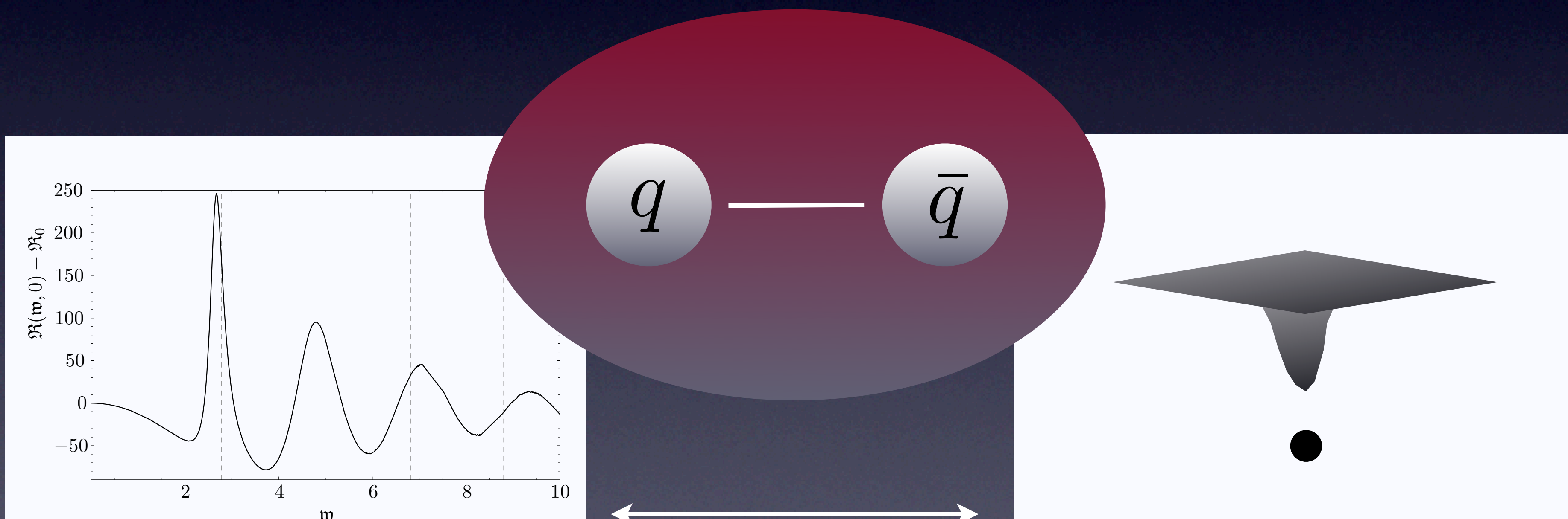

Thermal gauge theory ("CFT")

AdS/CFT duality

Supergravity in AdS

[Erdmenger, M.K., Rust 0710.0334]

Motivation & Introduction

Thermal spectral function $\mathfrak{R}$ contains all information about diffusion and quasiparticle resonances in QG-plasma.

$$\mathfrak{R}(\omega,\, \mathbf{q}) = -2\,\mathrm{Im}G^{\mathrm{ret}}(\omega,\, \mathbf{q})$$

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Find retarded two-point function of vector current $\hat{J}_\mu$ in QG-plasma.

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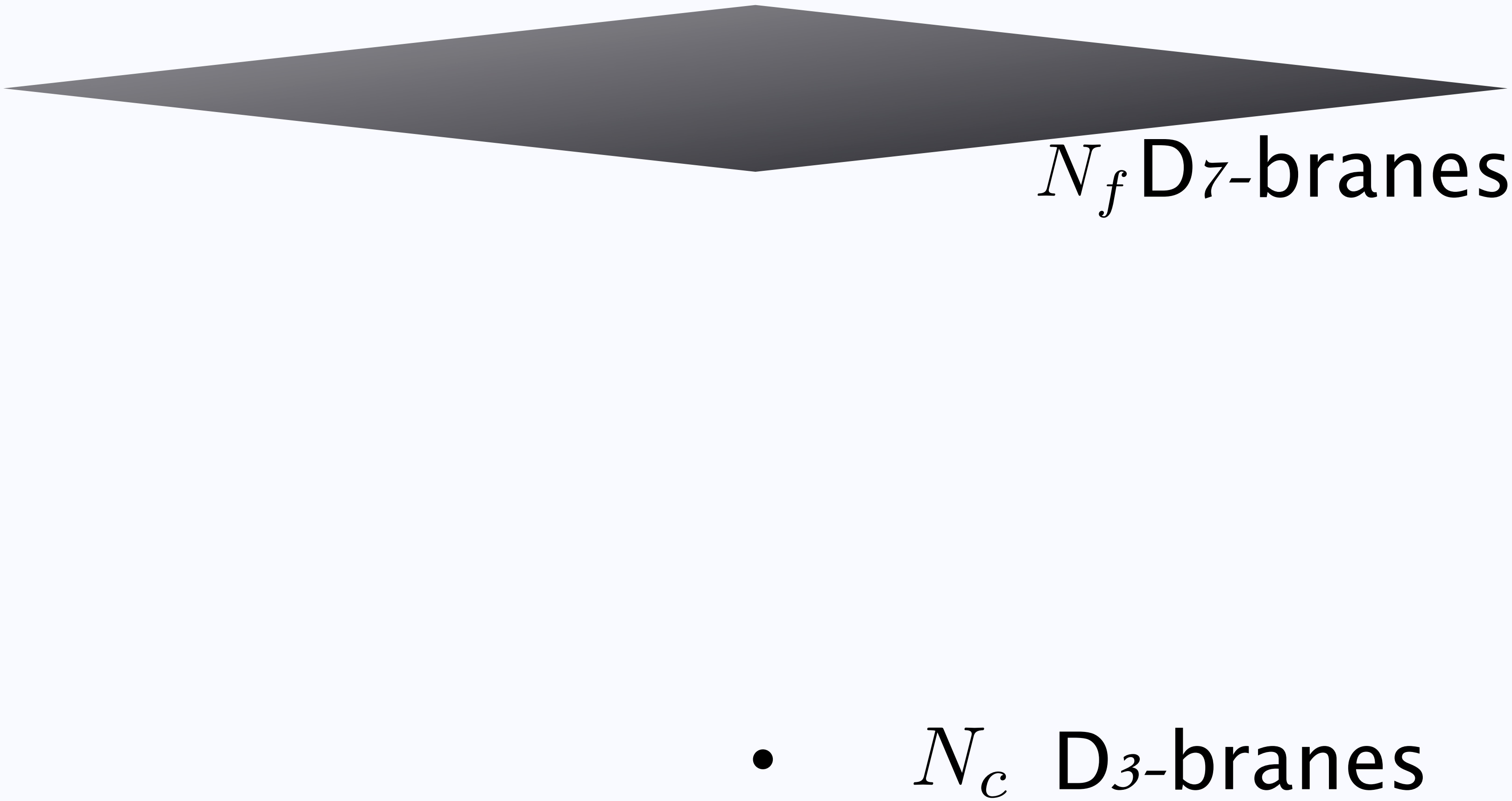
$\updownarrow$ AdS/CFT

Gravity problem (weak):
Find retarded two-point function of vector field $\hat{A}_\mu$ in SUGRA.

Gravity Setup & Problem

- N_c D3-branes

Gravity Setup & Problem



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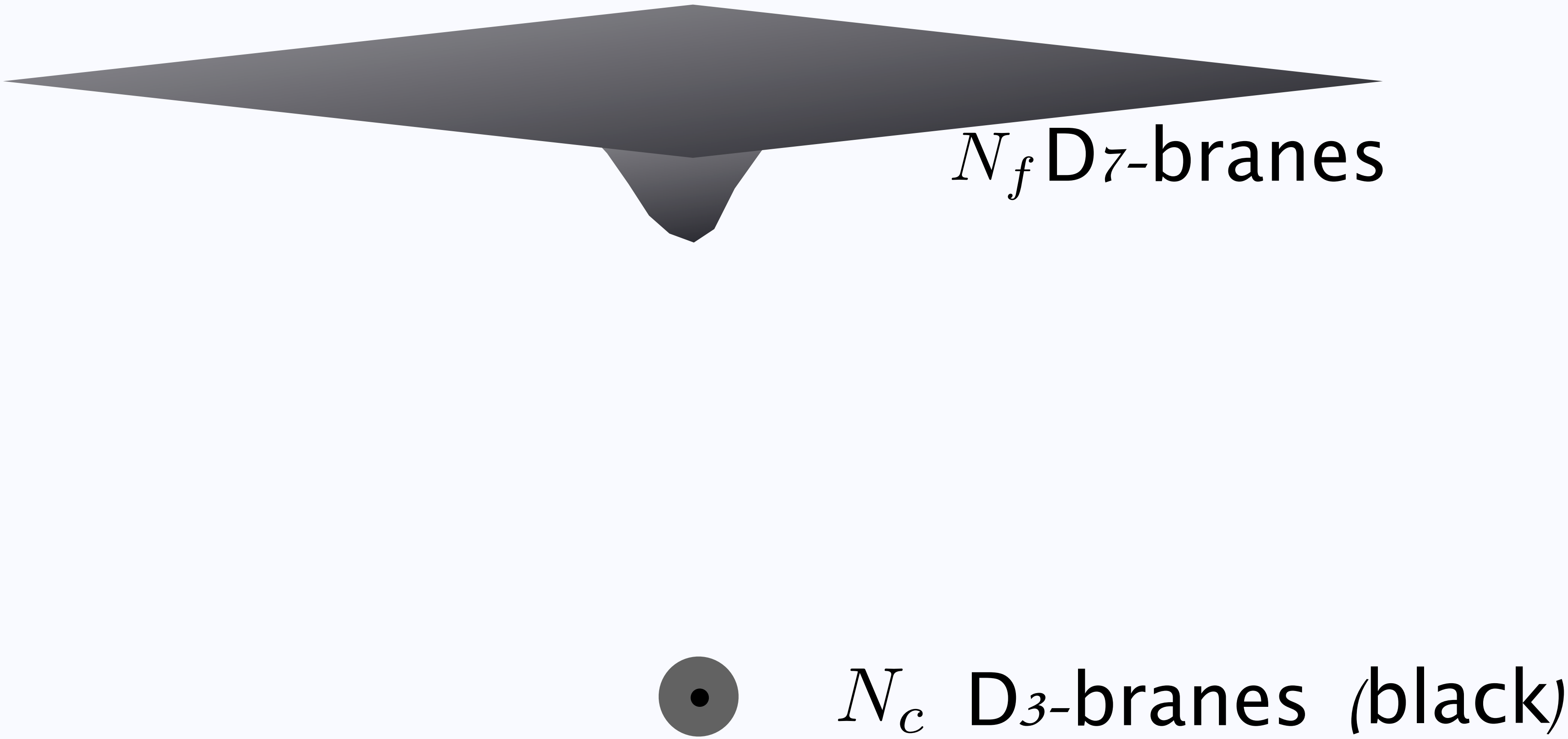

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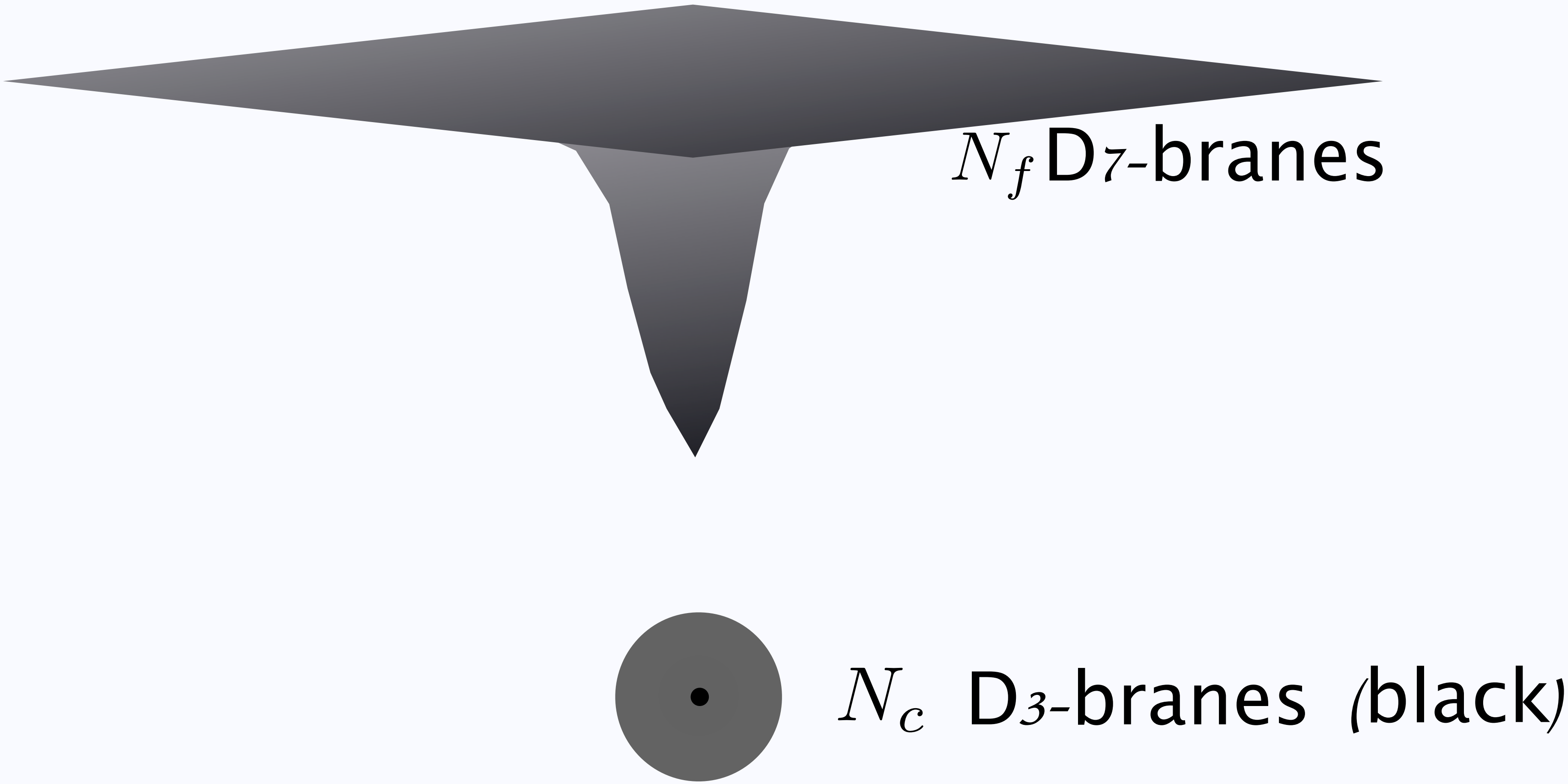

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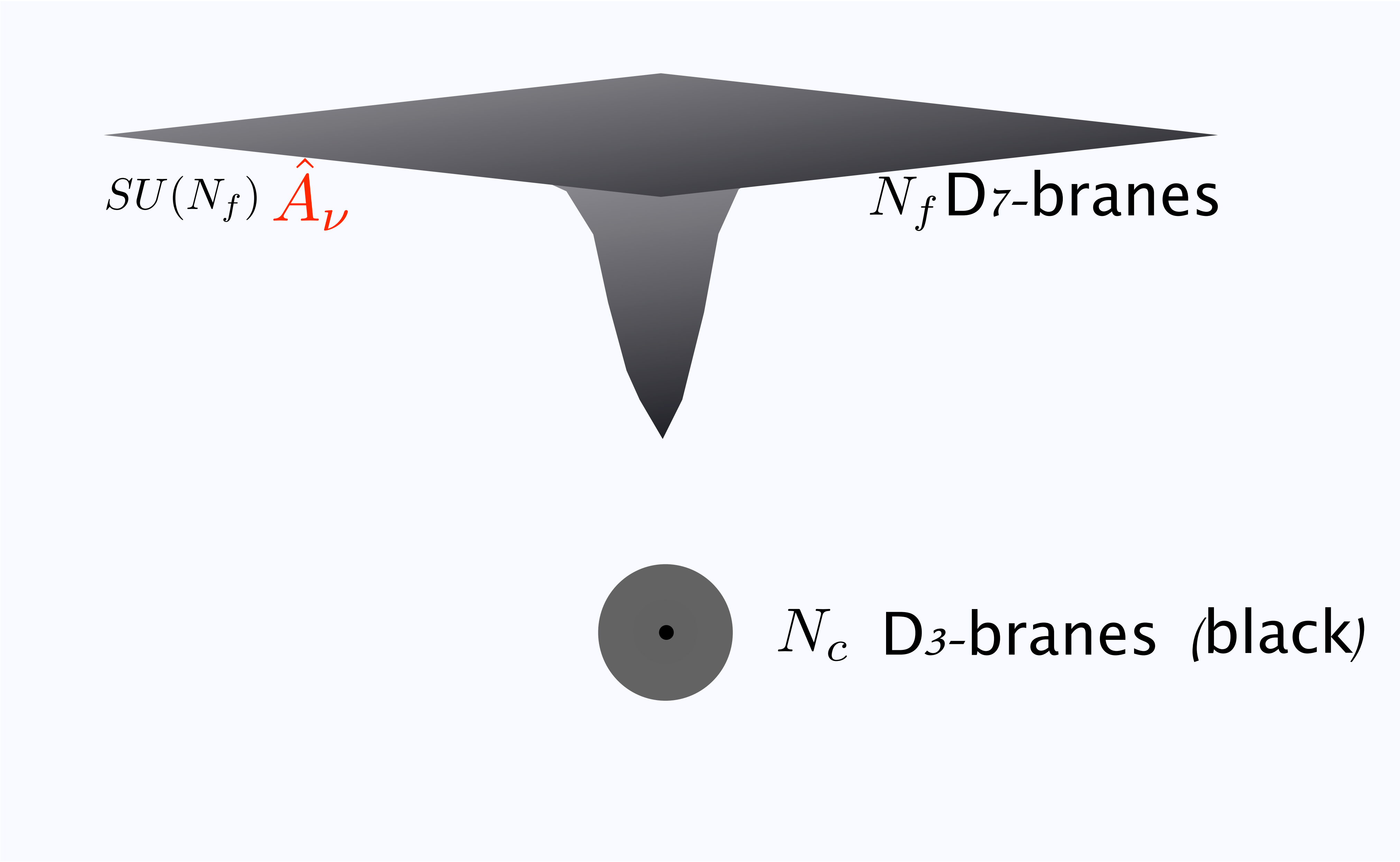

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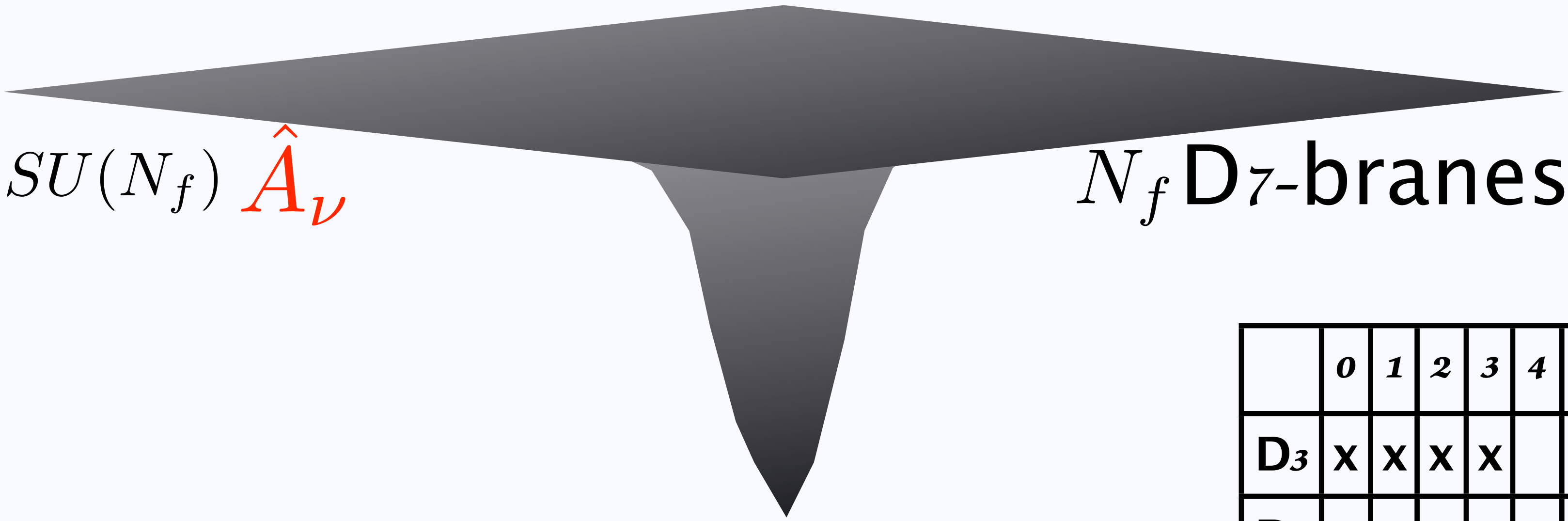
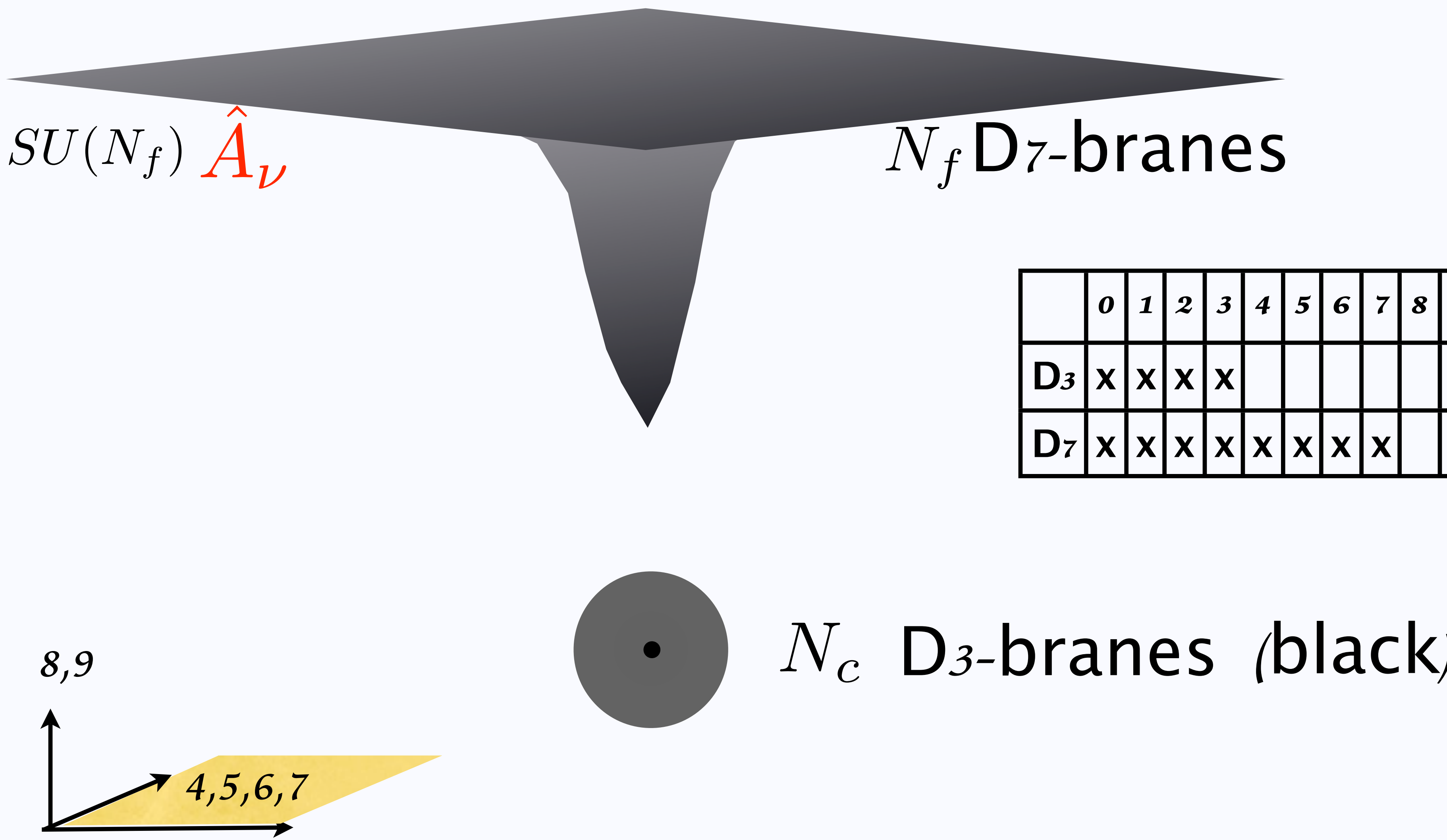


	0	1	2	3	4	5	6	7	8	9
D3	x	x	x	x						
D7	x	x	x	x	x	x	x	x		

Gravity Setup & Problem



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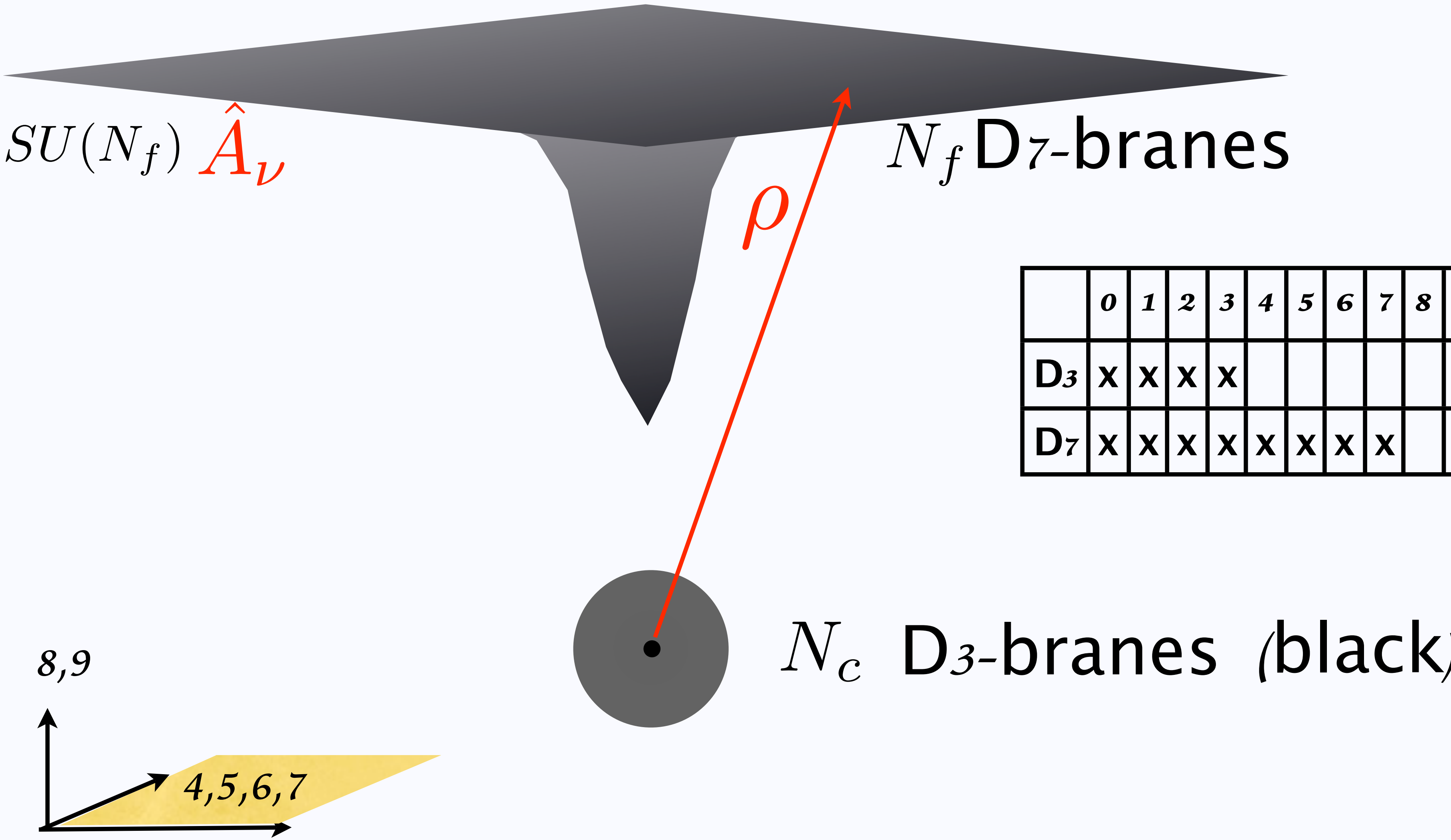

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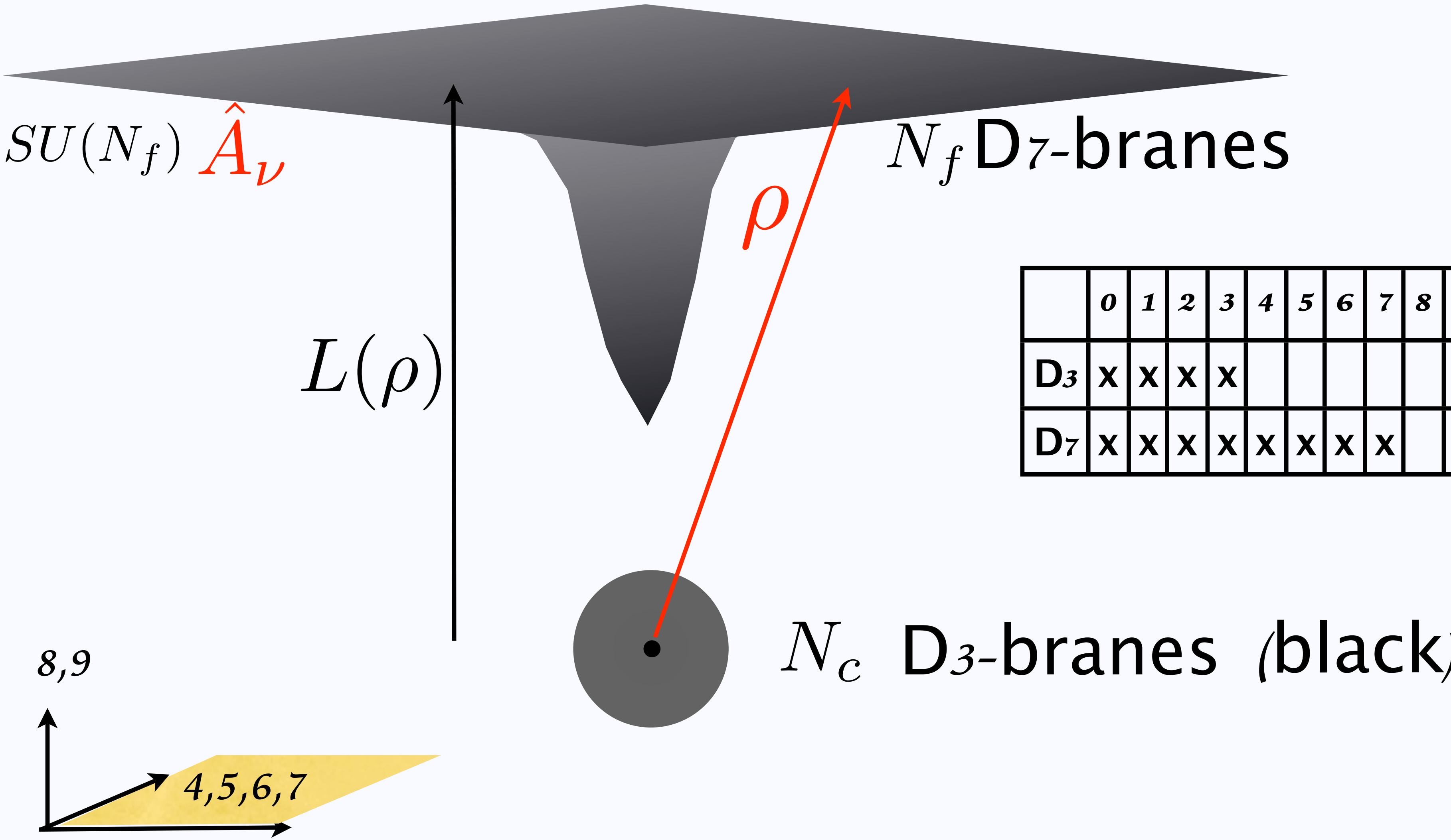

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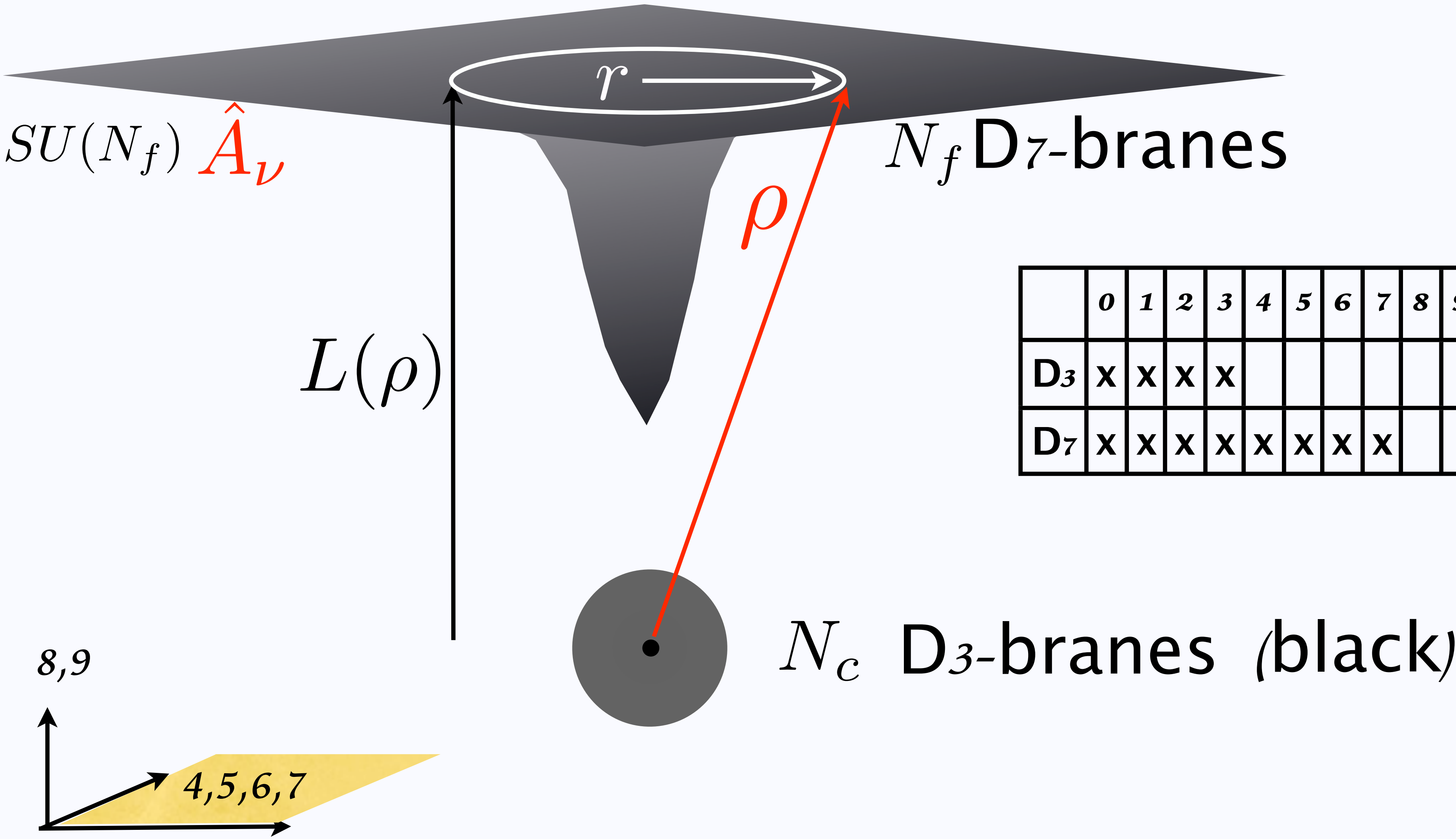

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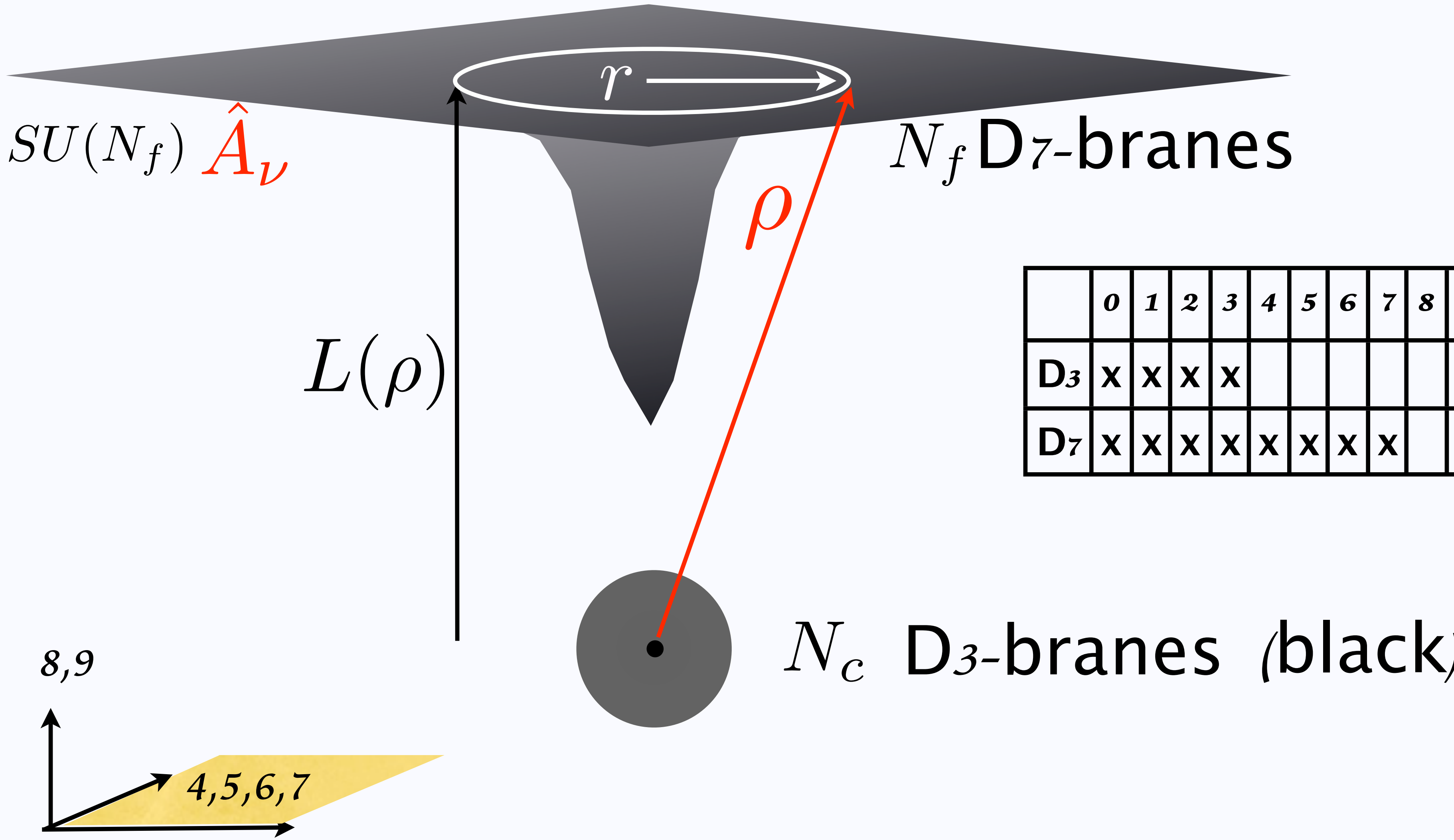

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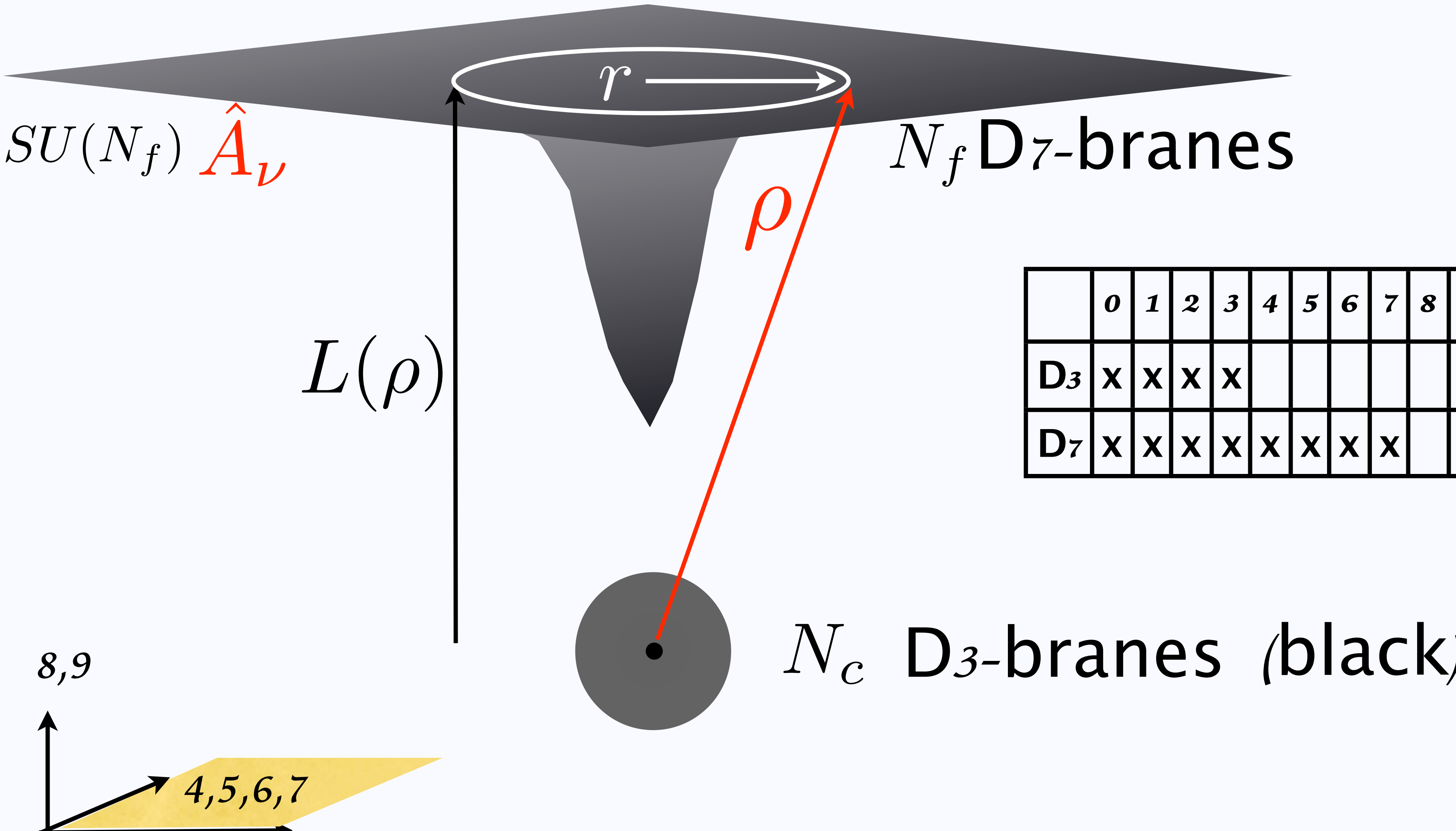
Chemical potential:

Gravity Setup & Problem



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Chemical potential: $\hat{A}_\mu = \delta_{\mu 0} A_0 + \tilde{A}_\mu$

Gravity Solution & Translation

Effective action: $$S_{\rm D7} = \int \mathrm{d}^8x \sqrt{\left|\det\{[g+F]+\tilde{F}\}\right|}\,, \quad F_{\mu\nu} = \partial_{[\mu}A_{\nu]}$$

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$$0 = \tilde{A}'' + \partial_\rho \ln\left(\frac{1}{8}\tilde{f}^2 f \rho^3 (1-\chi^2+\rho^2{\chi'}^2)^{3/2} \times \sqrt{1 - \frac{2\tilde{f}(1-\chi^2)(\partial_\rho A_0)^2}{f^2(1-\chi^2+\rho^2{\chi'}^2)}}\right)\tilde{A}' + 8\mathfrak{w}^2 \frac{\tilde{f}}{f^2}\frac{1-\chi^2+\rho^2{\chi'}^2}{\rho^4(1-\chi^2)}\tilde{A}$$

$$\rho = \frac{\varrho}{\varrho_H}\,, \quad \tilde{f}(\varrho) = 1 + \frac{{\varrho_H}^4}{\varrho^4}\,, \; f(\varrho) = 1 - \frac{{\varrho_H}^4}{\varrho^4}\,, \quad L(\varrho) = \varrho\,\chi(\varrho)\,,$$

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Translation to Gauge Theory by duality: $$A_\mu \overset{\text{AdS/CFT}}{\longleftrightarrow} J^\mu$$

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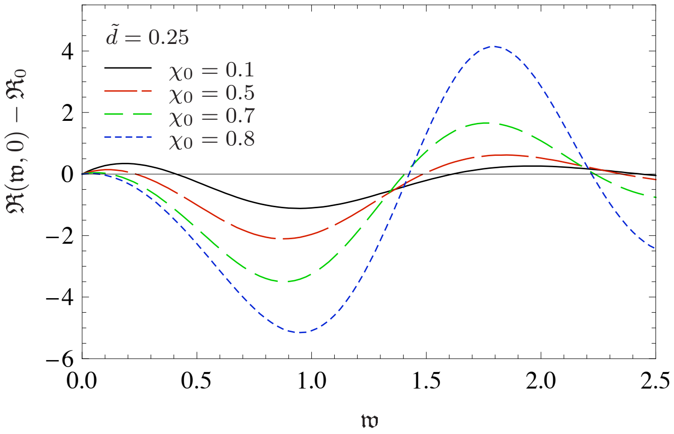
Translation to Gauge Theory by duality: $A_\mu \overset{\rm AdS/CFT}{\longleftrightarrow} J^\mu$

Gauge Correlator: $G^{\rm ret} = \frac{N_f N_c T^2}{8} \lim_{\rho\to\rho_{\rm bdy}} \left(\rho^3 \frac{\partial_\rho \tilde{A}(\rho)}{\tilde{A}(\rho)}\right)$

Gauge Results: Spectral Functions

Finite baryon density: $\chi_0 = \chi(\rho)\big|_{\rho\to\rho_H} \sim \dfrac{m_{\mathrm{quark}}}{T}$



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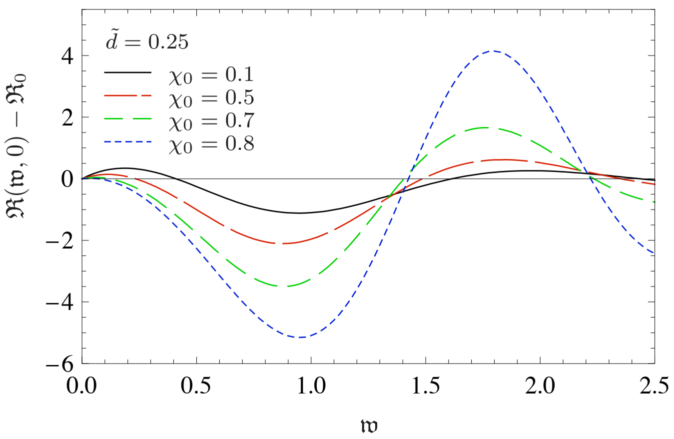

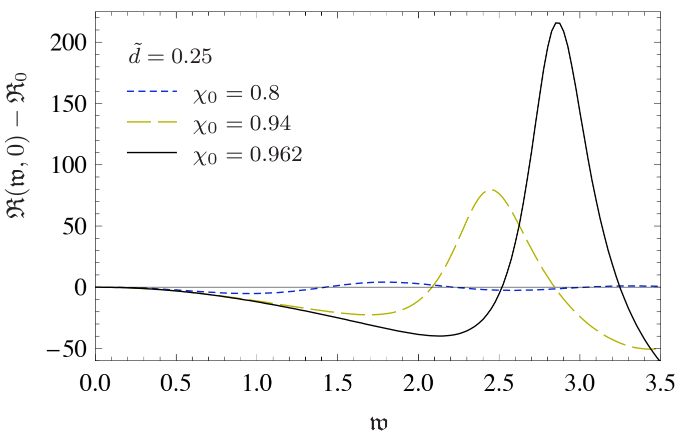

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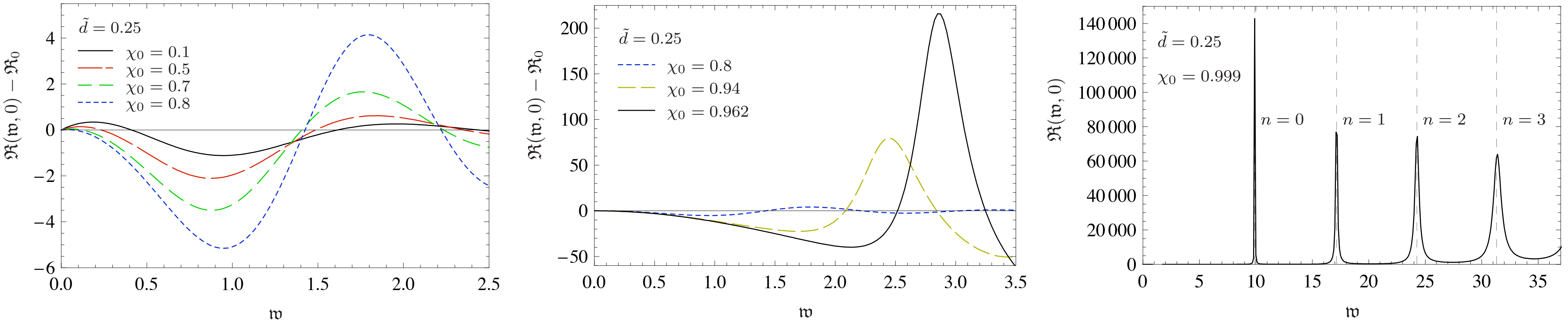

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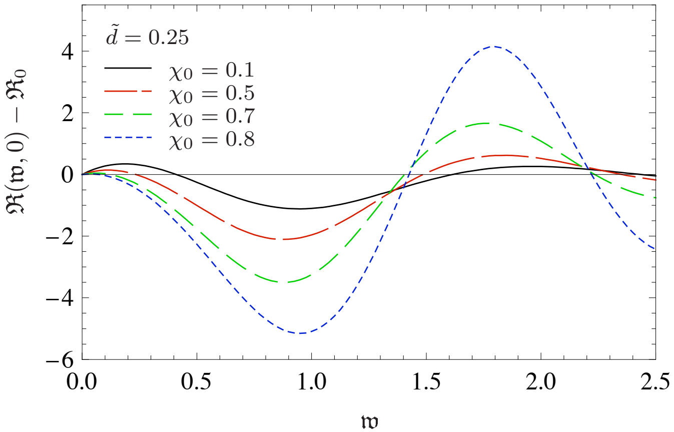

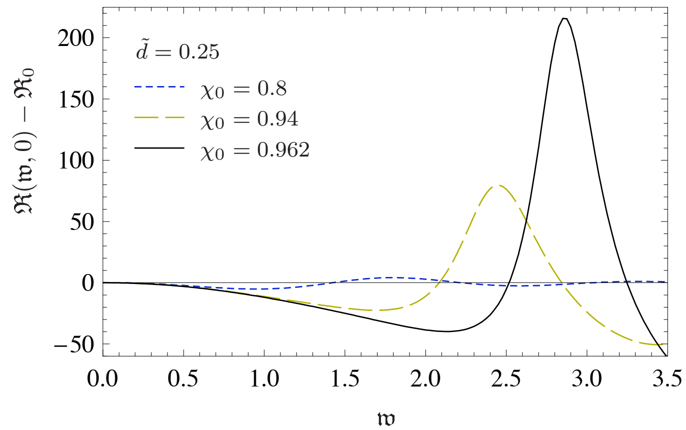

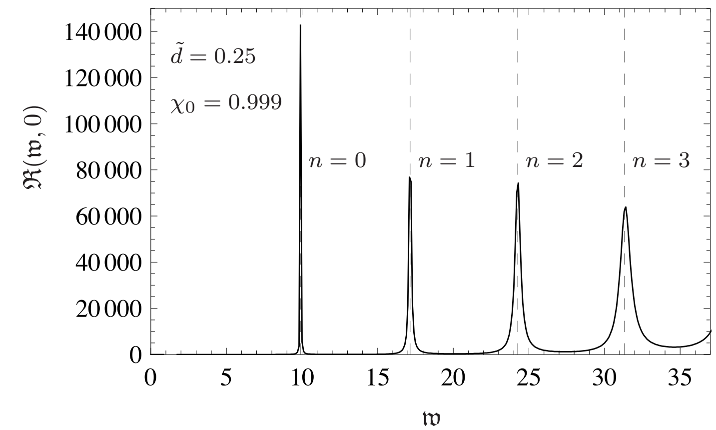

Phase diagram:

$$L(\varrho) = \varrho\,\chi(\varrho)$$

$$\chi = \chi(\tilde{d}\,,\rho)$$

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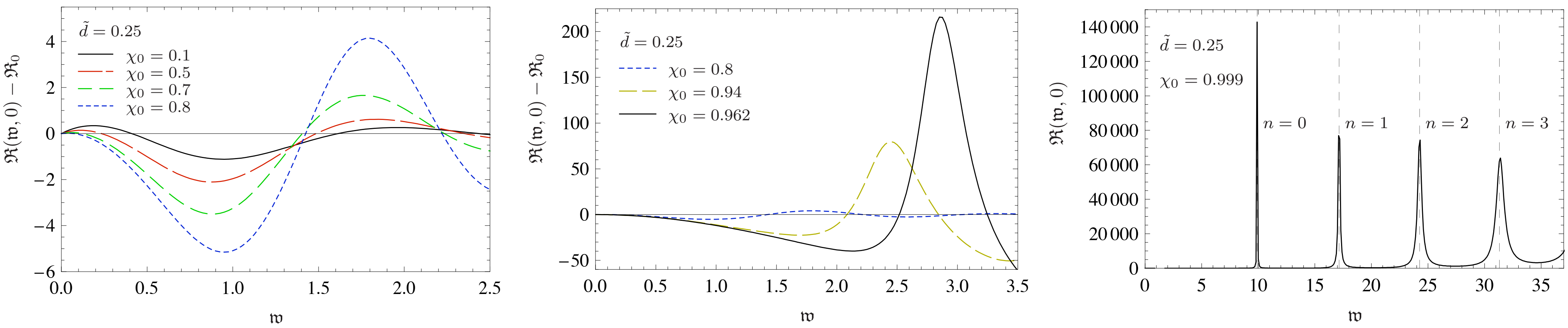

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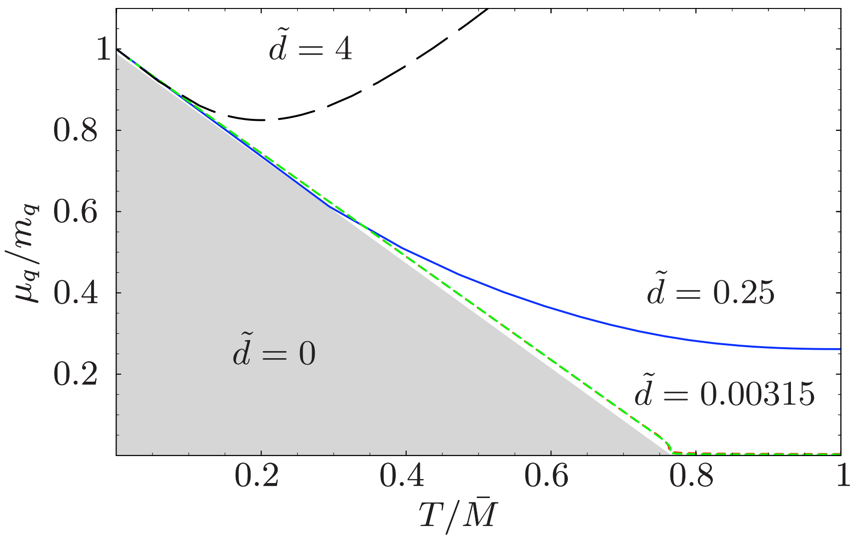

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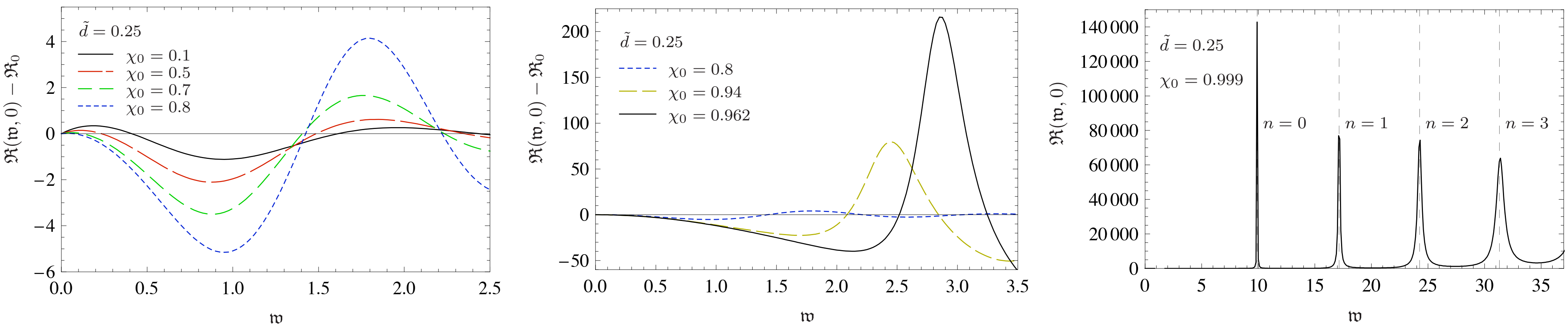

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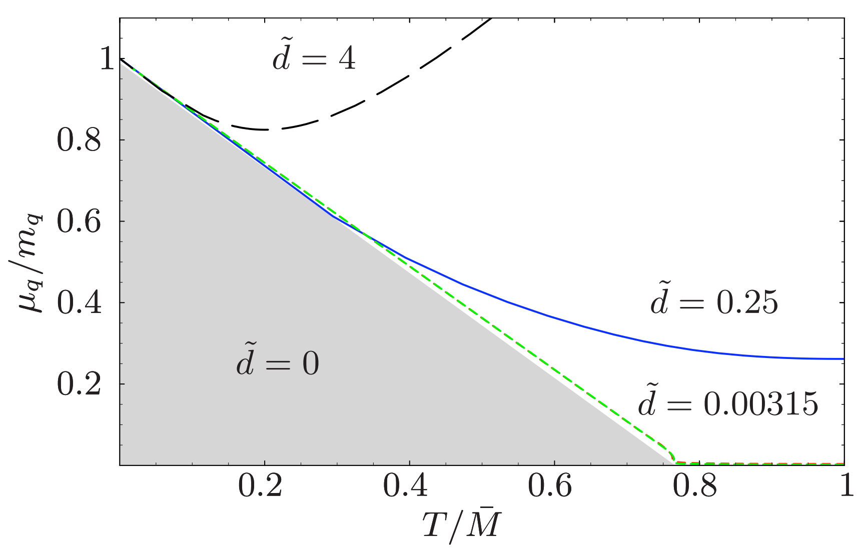
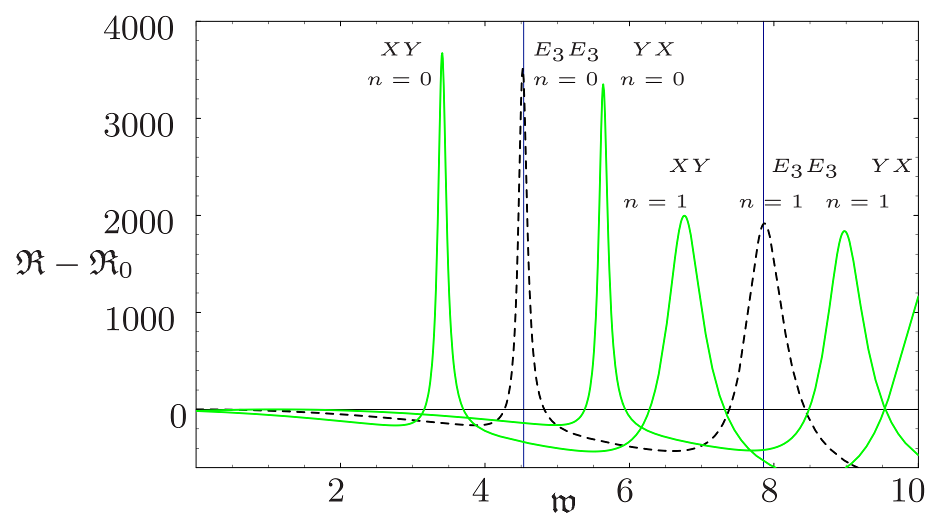


Finite isospin density:



Gauge Results: Quark diffusion

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Diffusion: $$\Xi D = \lim_{\omega \to 0} \frac{1}{2\omega} \Re(\omega, \mathbf{q} \to 0)$$

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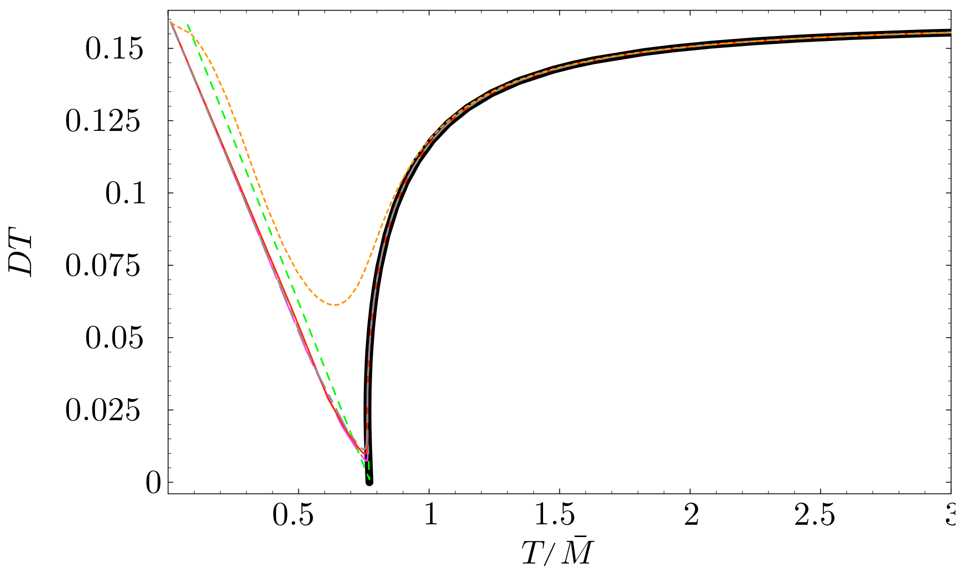

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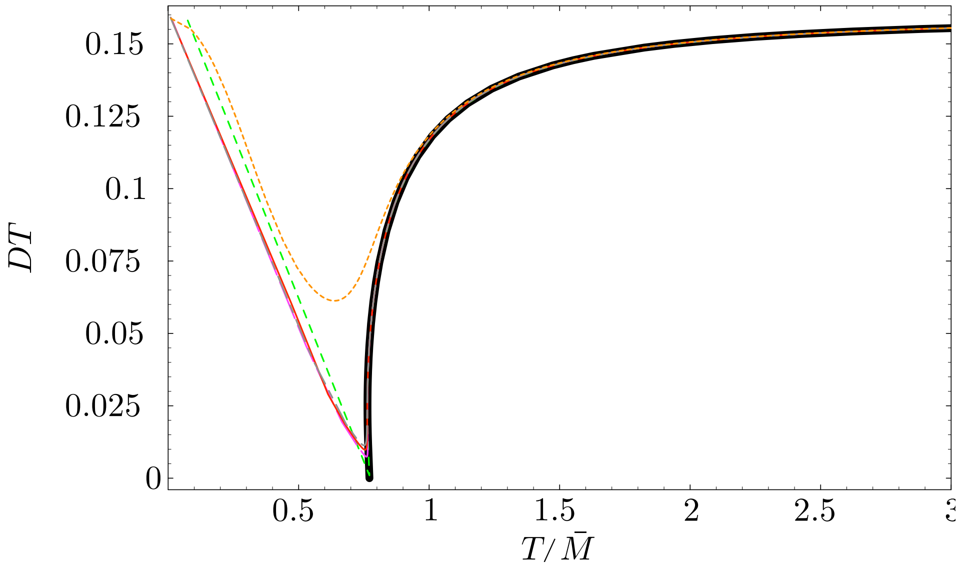
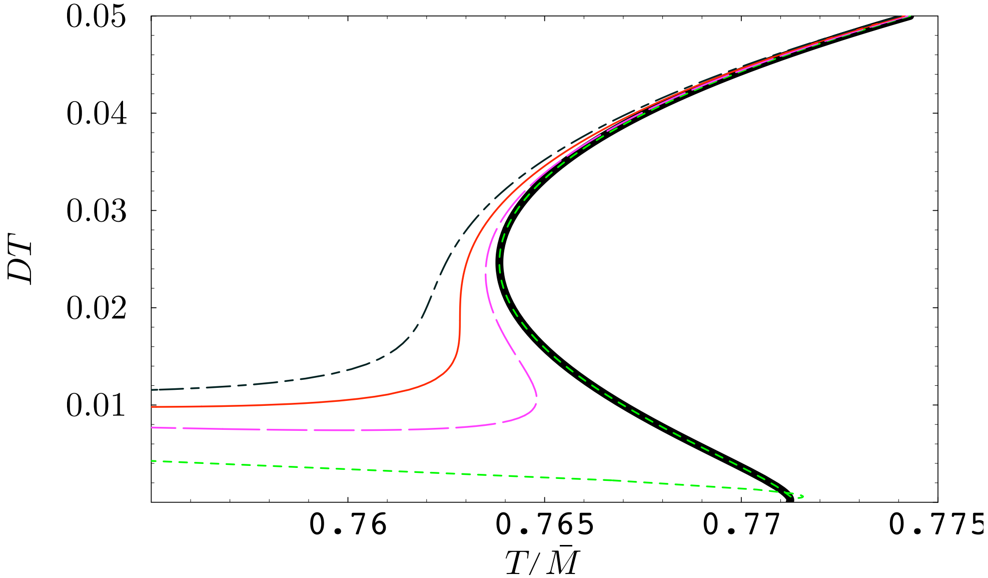


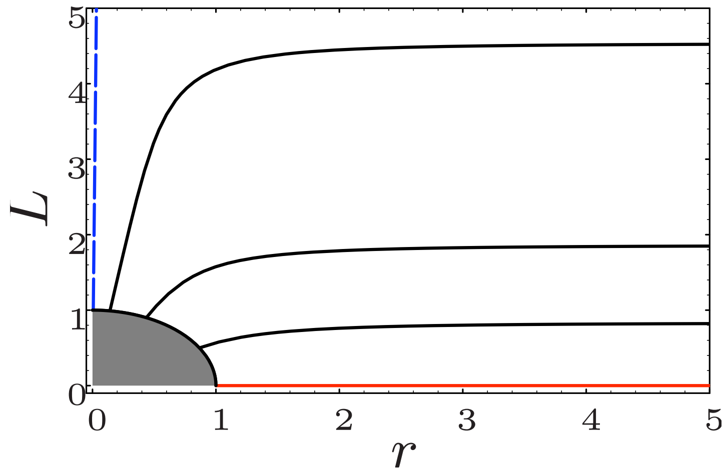


Conclusion

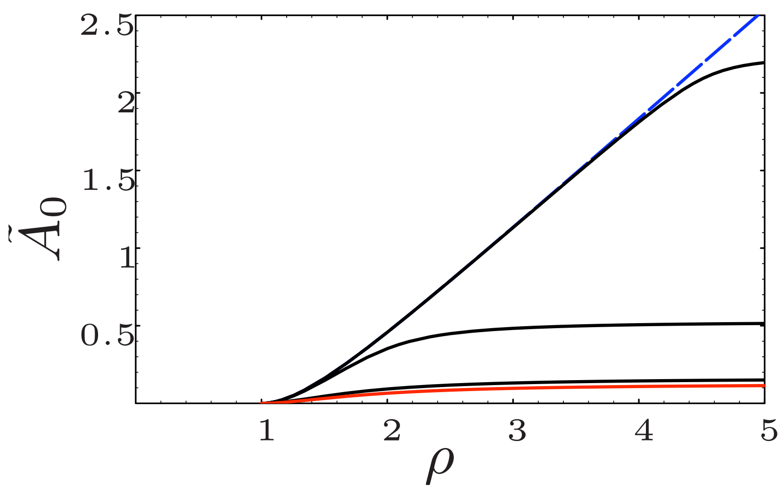
- [x] spectral functions in strongly coupled YM-plasma
- [x] sharp resonances signal stable vector mesons (a.d.)
- [x] resonances follow SUSY mass formula for small T
- [x] diffusion coefficient shows “softened” phase transition

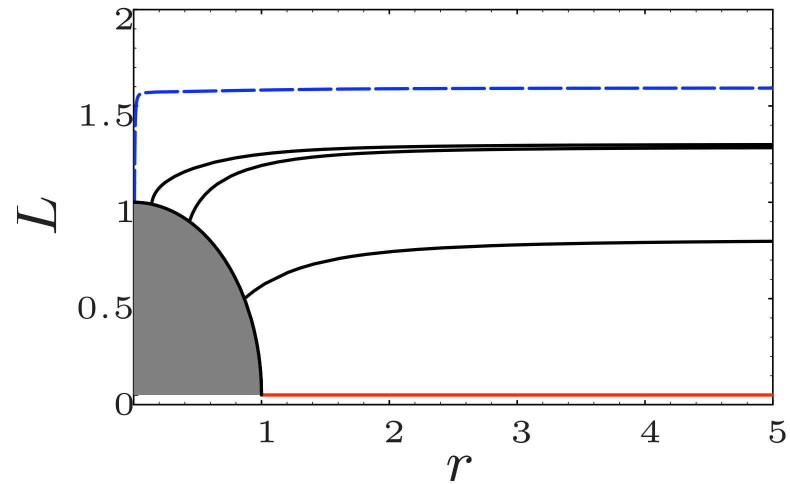
- [ ] explain resonance peak turning
- [ ] spectral functions at finite baryon and isospin density
- [ ] compute quasi-normal modes directly (resonances)

APPENDIX

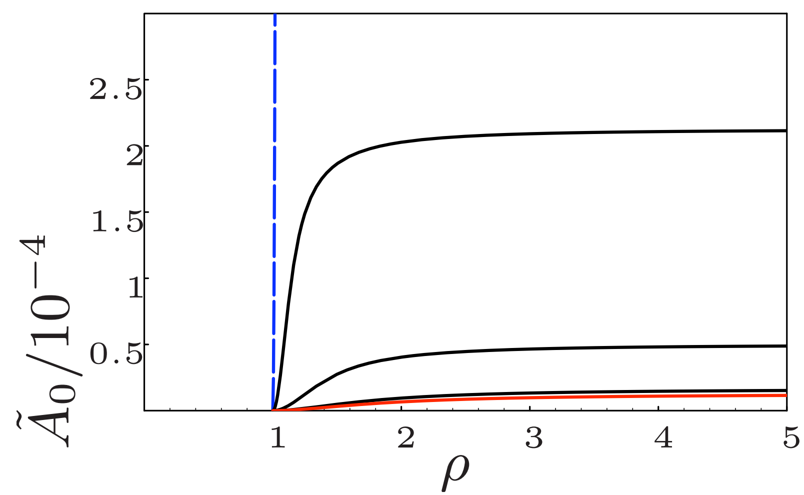


$$\tilde{d} = 0.25$$



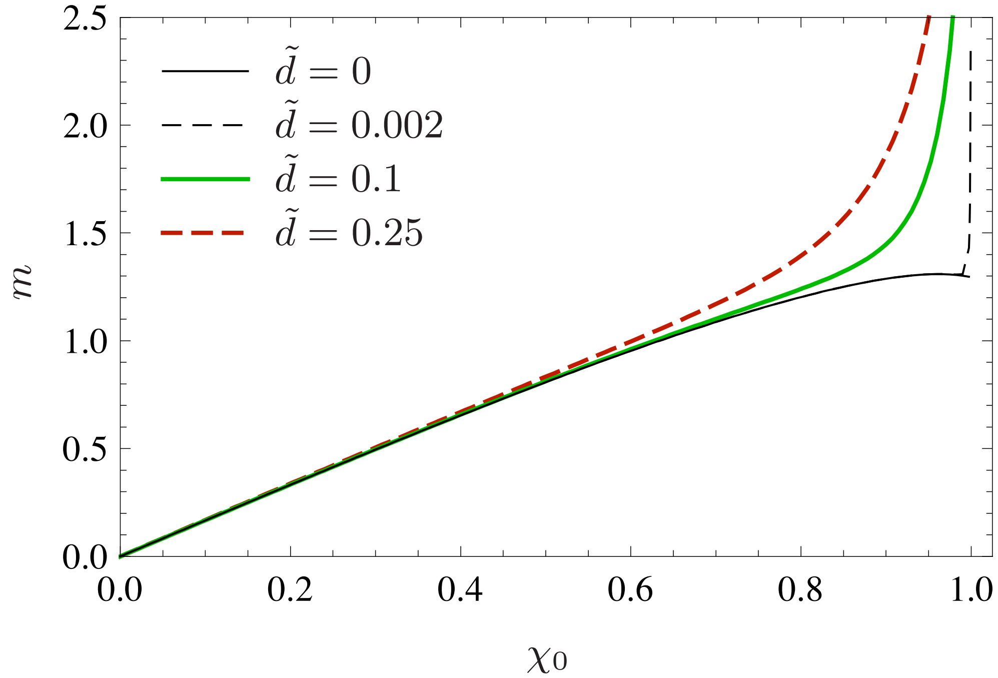


$$\tilde{d} = \frac{10^{-4}}{4}$$



APPENDIX

The mass parameter m depending on the parameter χ_0.



$$\chi_0 = \chi(\rho)\Big|_{\rho\to\rho_H}$$

$$m = \lim_{\rho\to\rho_{\text{bdy}}} \rho\,\chi(\rho) = \frac{2m_{\text{quark}}}{\sqrt{\lambda}T}$$

Near-boundary expansions:

$$\chi(\rho) = \frac{m}{\rho} + \frac{c}{\rho^3} + \dots$$

$$A_0 = \mu - \frac{1}{\rho^2}\frac{\tilde{d}}{2\pi\alpha'} + \dots$$

Other relations:

$$L(\varrho) = \varrho\,\chi(\varrho)\,, \quad \rho = \frac{\varrho}{\varrho_H}$$

Experiment
RHIC: Au-Au collisions
QuarkGluonPlasma

- strong QCD
- relativistic liquid

String theory

Experiment

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short string limit

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SUGRA in AdS (gravity)

- weak coupling

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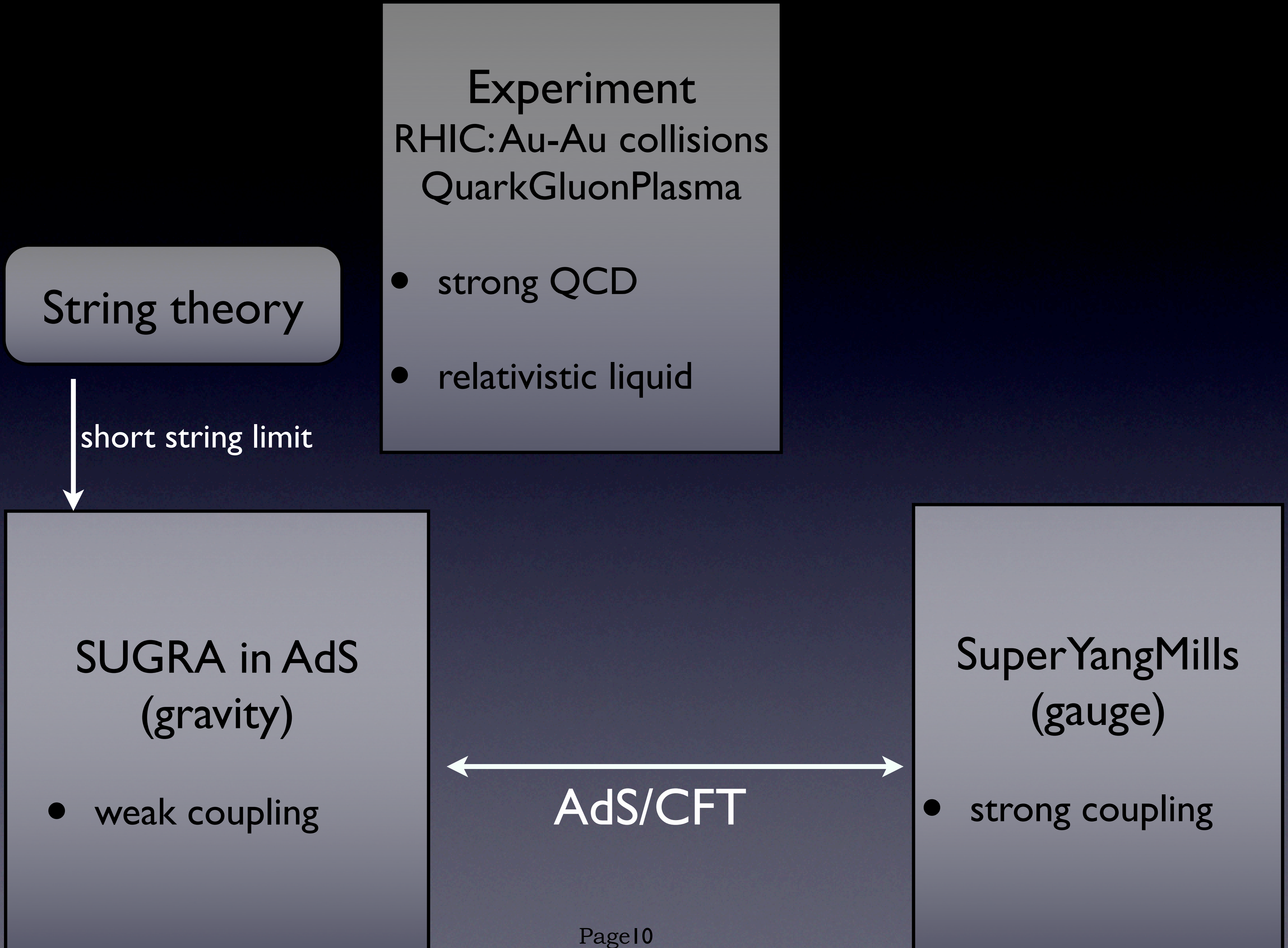
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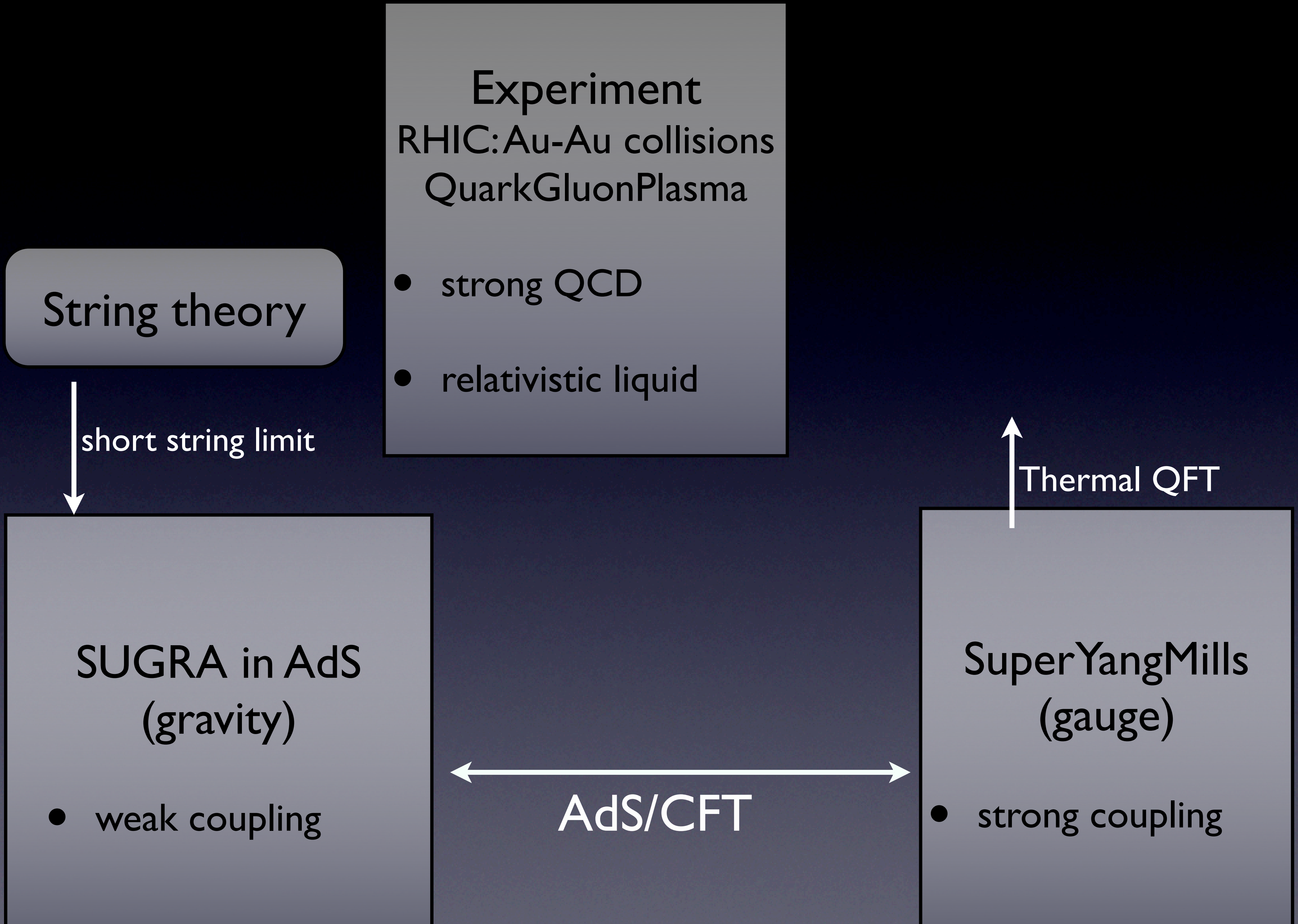
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⟷ AdS/CFT

SuperYangMills (gauge)

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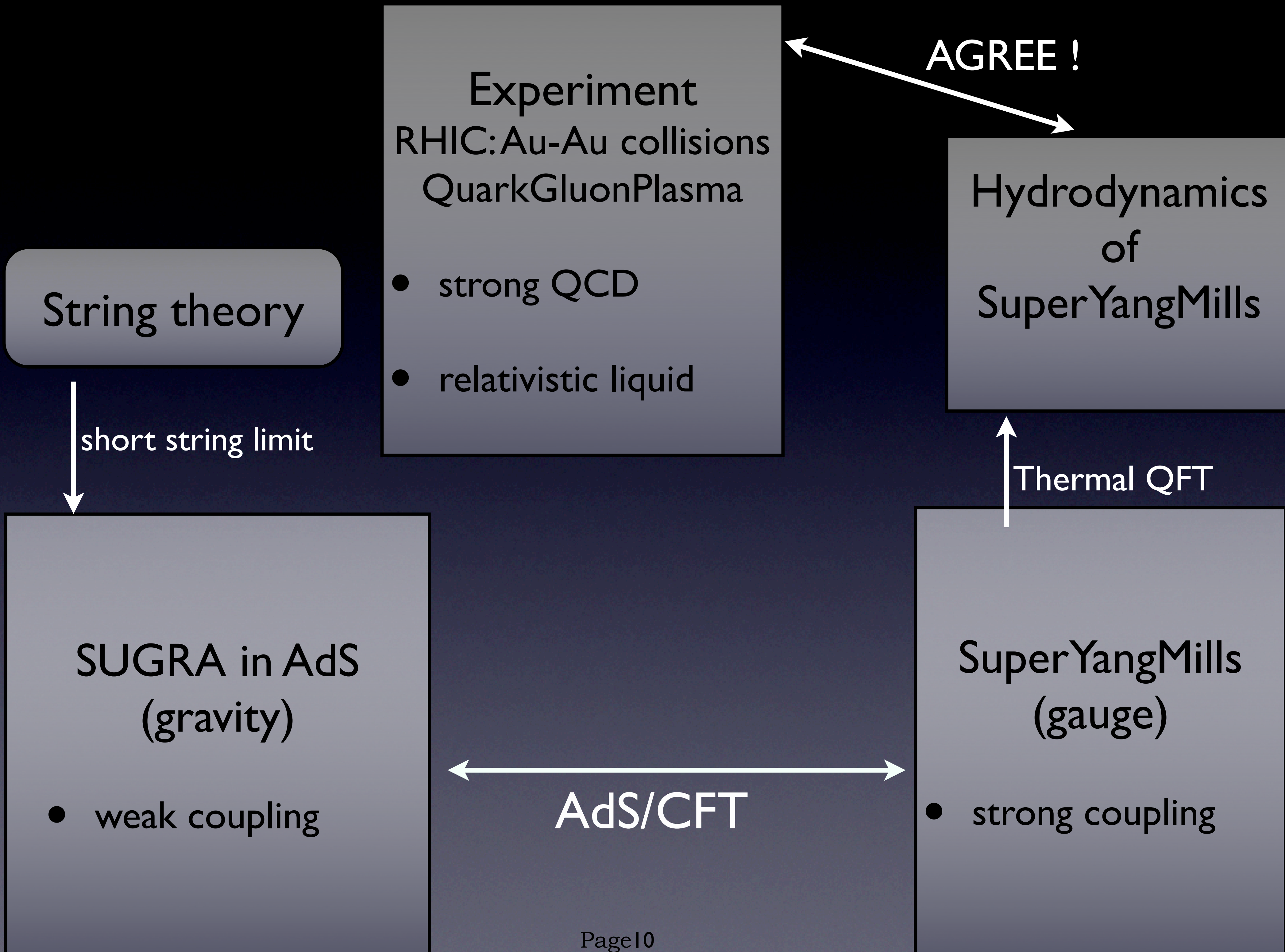
↑ Thermal QFT

Hydrodynamics of SuperYangMills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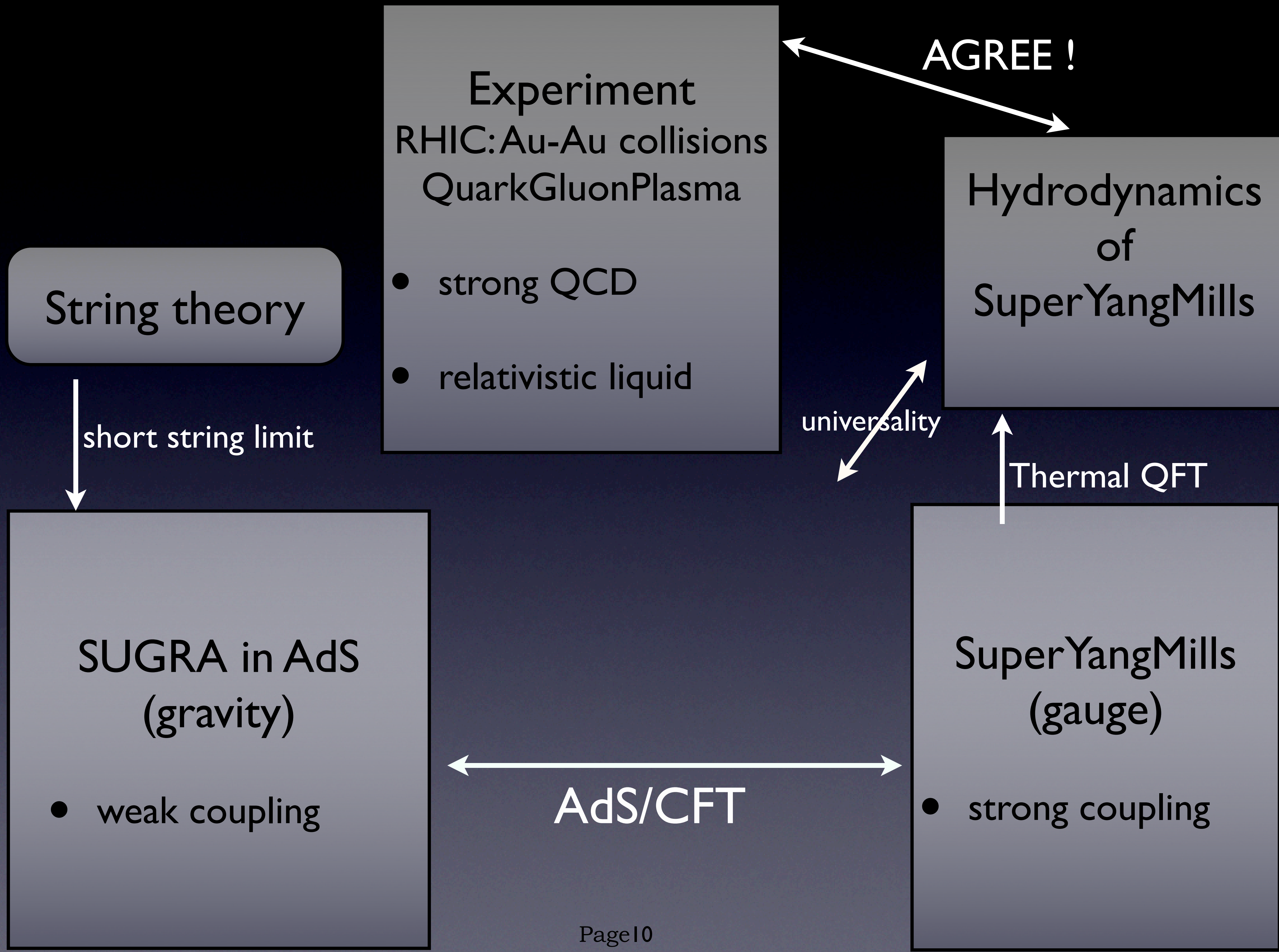
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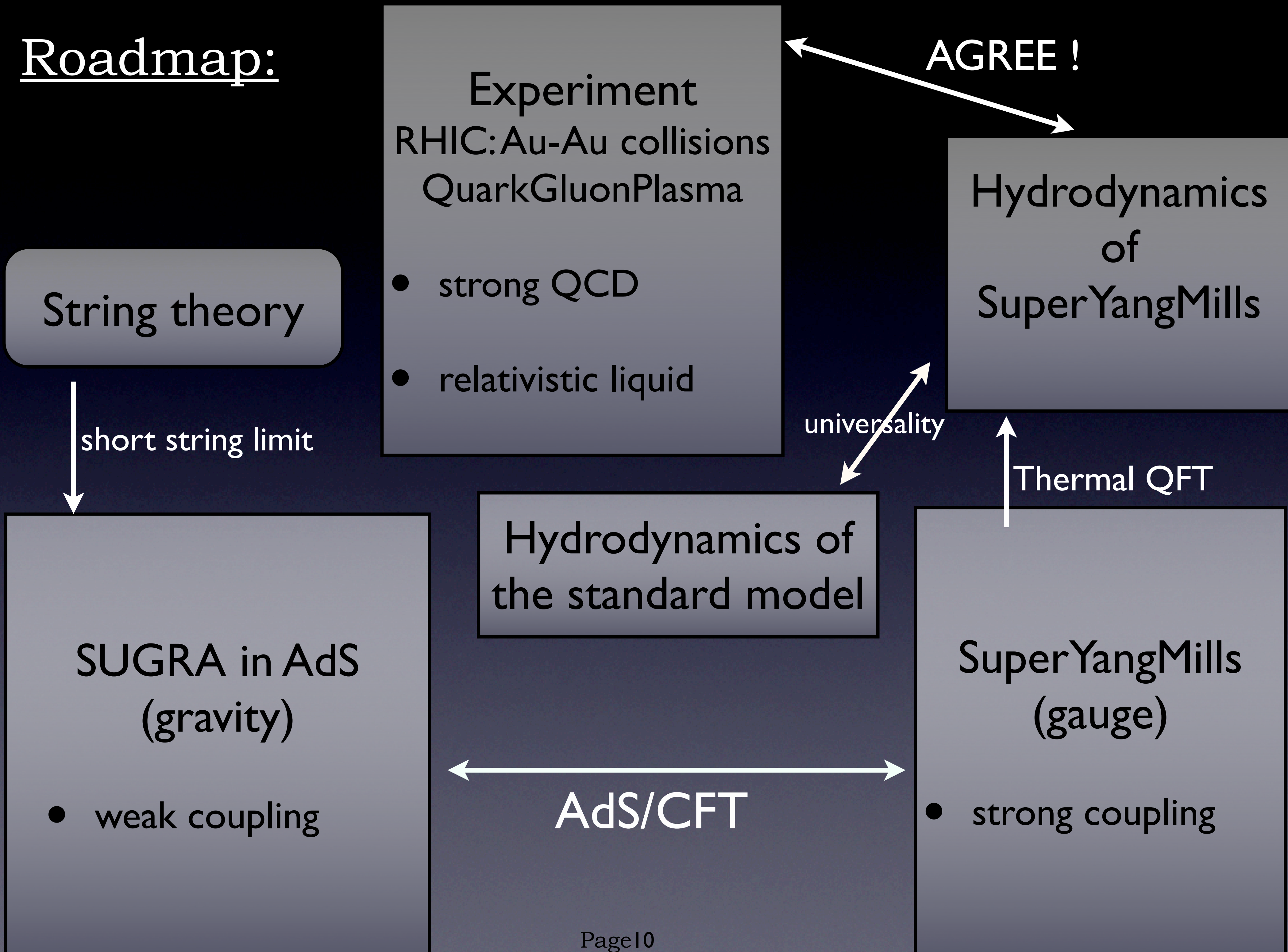
• strong coupling

Roadmap:



Experiment
RHIC: Au-Au collisions
QuarkGluonPlasma
• strong QCD
• relativistic liquid
AGREE !
Hydrodynamics of SuperYangMills
String theory
short string limit
universality
Thermal QFT
Hydrodynamics of the standard model
SUGRA in AdS (gravity)
• weak coupling
AdS/CFT
SuperYangMills (gauge)
• strong coupling