Ultrasoft Renormalization of the Potentials in vNRQCD

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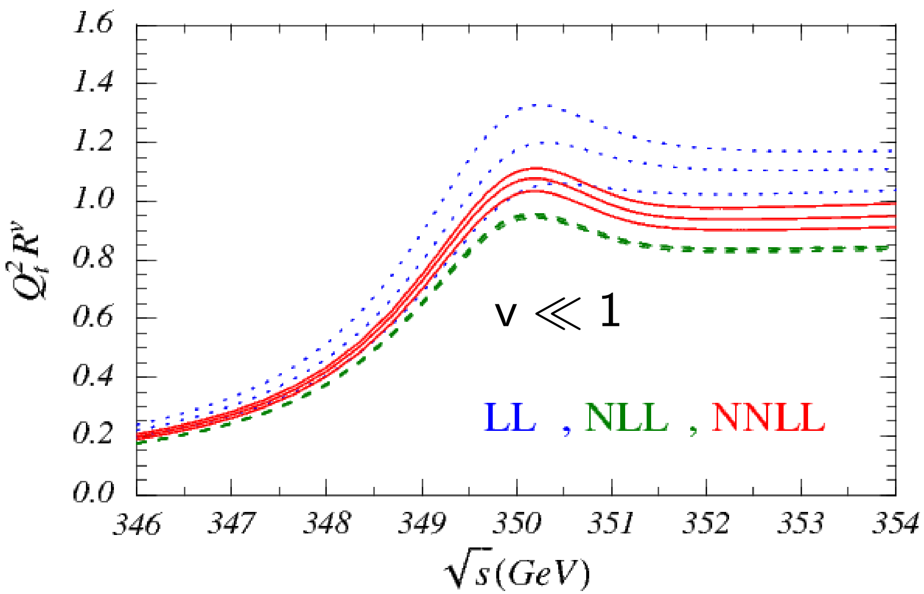
Outline

- Top Quark Threshold Physics
- vNRQCD
- Renormalization of the Potentials (at 2 loop level)
  - $\frac{1}{m^2}$ - Potentials
  - $\frac{1}{m\mathbf{k}}$ - Potentials
- Status of Calculations / Summary

Top Quark Threshold Physics

Top Physics at the ILC:

Focus: $t\bar{t}$ - production at threshold $(e^+ e^- \to t\bar{t})$



$\Gamma_t \approx 1.5\,\mathrm{GeV} \gg \Lambda_{QCD} \Rightarrow$

- nonpert. effects suppressed [Fadin, Khoze]
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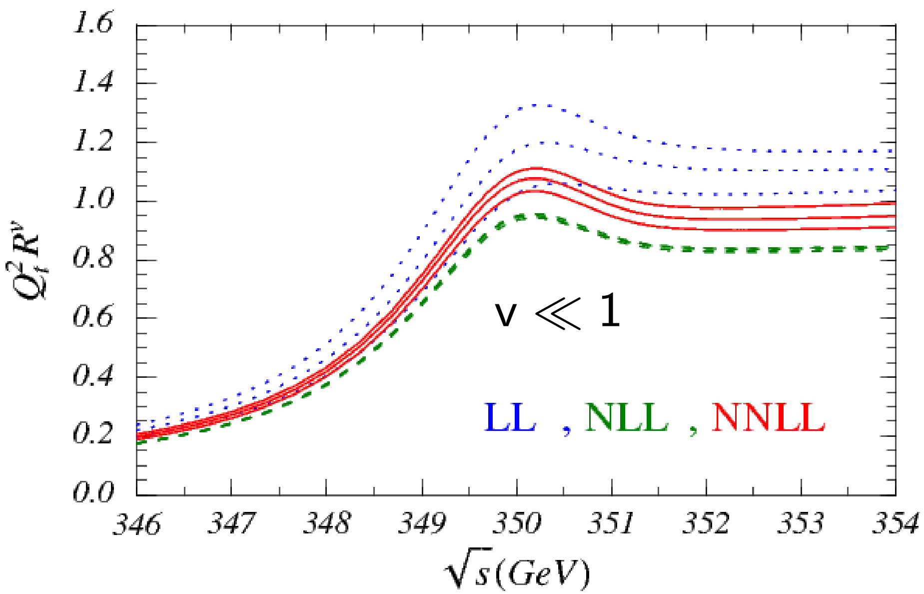

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Aim: precise determination of

m_t	status: $\delta m_t \sim 100\,\text{MeV}$ ✓
$y_t,\ \alpha_s,\ \Gamma_t$	status: $\delta\sigma_{tot}^{theo}/\sigma_{tot} \sim 6\,\%$ (NNLL incomplete)

needed: $< 3\,\%$

Top Quark Threshold Physics

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3 scales: m_t (hard) $\gg$ $\vec{p} \sim m_t v$ (soft) $\gg$ $E_{kin} \sim m_t v^2$ (ultrasoft) $(\sim \Gamma_t \gg \Lambda_{QCD})$

$\Rightarrow$ log's at the dyn. scales:

$$\ln\left(\frac{m^2}{E^2}\right),\; \ln\left(\frac{m^2}{\mathbf{p}^2}\right),\; \ln\left(\frac{\mathbf{p}^2}{E^2}\right) \qquad \text{e.g.: } \alpha_s \ln\left(\frac{m^2}{E^2}\right) \sim -\alpha_s \ln(v^4) \sim 1$$

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Solution:

two renormalization scales: $\mu_s \sim mv, \ \mu_u \sim mv^2$

correlation: $E \sim \frac{\mathbf{p}^2}{m} \rightarrow \mu_u \sim \frac{\mu_s^2}{m} \quad \longrightarrow \quad \mu_s = m\nu, \ \mu_u = m\nu^2 \quad \Rightarrow$ 'v'NRQCD

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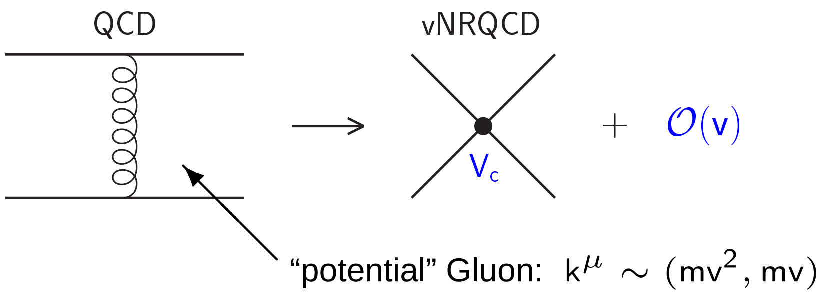
$\Rightarrow$ RGE's resum $\quad [\alpha_s \ln v]^n, \quad \alpha_s [\alpha_s \ln v]^n, \quad \alpha_s^2 [\alpha_s \ln v]^n, \; \ldots$ terms

LL NLL NNLL

vNRQCD

- Nonresonant dof's integrated out, e.g.:



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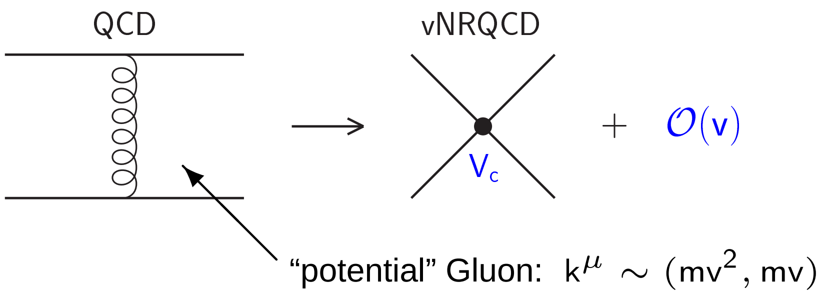

- Resonant dof's $\rightarrow$ fields in the vNRQCD Lagrangian:

nonrel. quark:	$(E, \mathbf{p}) \sim (mv^2, mv)$	$\psi_{\mathbf{p}}(x)$	
soft gluon:	$(q_0, \mathbf{q}) \sim (mv, mv)$	$A_q(x)$	
ultrasoft gluon:	$(q_0, \mathbf{q}) \sim (mv^2, mv^2)$	$A(x)$	

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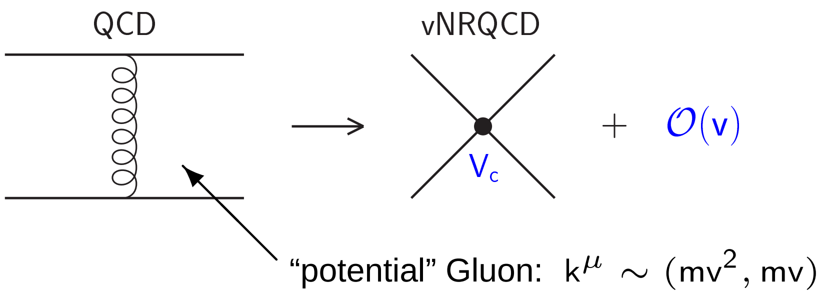

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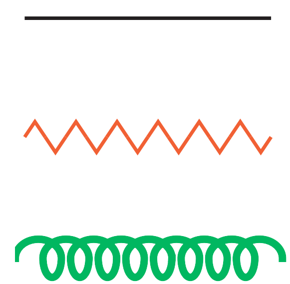

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- Systematic expansion in $v \Rightarrow$ consistent power counting in $v \sim \alpha_s$

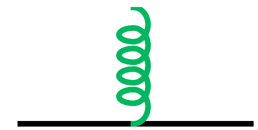
vNRQCD

[Luke, Manohar, Rothstein]

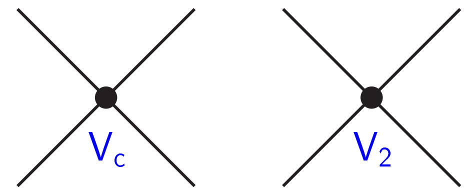
$$\boxed{\mathcal{L}_{\mathrm{vNRQCD}} = \mathcal{L}_{\mathrm{usoft}} + \mathcal{L}_{\mathrm{pot}} + \mathcal{L}_{\mathrm{soft}}}$$

$$D^\mu = \partial^\mu + igA^\mu(x)$$

$$\mathcal{L}_{\mathrm{usoft}}: \quad \psi^\dagger_{\mathbf{p}}(x)\left[iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m} + \ldots \right]\psi_{\mathbf{p}}(x) + \ldots$$



$$\mathcal{L}_{\mathrm{pot}}: \quad - V \;\psi^\dagger_{\mathbf{p}'}\psi_{\mathbf{p}}\chi^\dagger_{-\mathbf{p}'}\chi_{-\mathbf{p}} + \ldots$$



$$V \sim \frac{\mathcal{V}_c}{\mathbf{k}^2} + \frac{\mathcal{V}_k \pi^2}{m\mathbf{k}} + \frac{\mathcal{V}_r(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2}{m^2} + \frac{\mathcal{V}_s}{m^2}\mathbf{S}^2 + \ldots \qquad (\mathbf{k} = \mathbf{p}' - \mathbf{p})$$

vNRQCD

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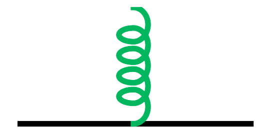
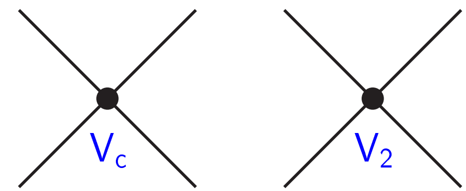


$$\mathcal{L}_{\rm pot}: \quad - V \;\; \psi^\dagger_{\mathbf{p}'}\psi_{\mathbf{p}}\chi^\dagger_{-\mathbf{p}'}\chi_{-\mathbf{p}} + \ldots$$



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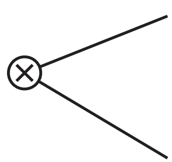
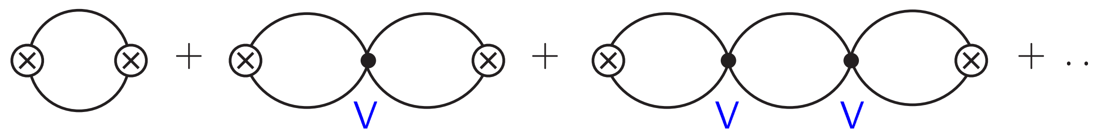
external production/annihilation current ($^3S_1$):



$$\sim \quad c_1(\nu)\cdot \underbrace{\vec{j}_1^{\,\rm eff}(x)}_{\psi^\dagger_{\mathbf{p}}\vec{\sigma}(i\sigma_2)\chi^*_{-\mathbf{p}}} + \ldots \qquad \text{(CMS)}$$

Renormalization of the Potentials

Anomalous dimension of V contributes to $\sigma(e^+ e^- \to t\bar{t})$:

$\sigma_{\rm tot} \sim {\rm Im}\Big[$  $+ \dots \Big]$

$$\sim |c_1(\nu)|^2 \cdot {\rm Im}\left[-i\int d^4x\ e^{i\hat{q}x}\,\langle 0|\,T\,\vec{j}_1^{\,\rm eff\,*}(x)\,\vec{j}_1^{\,\rm eff}(0)\,|0\rangle\right] + \dots$$

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$G^{\rm NNLL}$ known ✓

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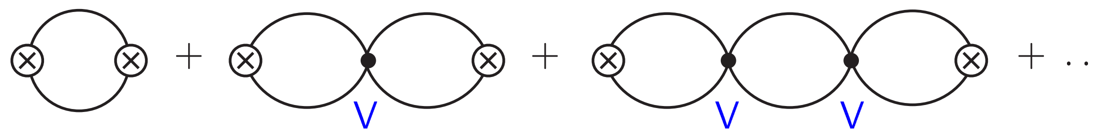
$\sigma_{\rm tot} \sim \mathrm{Im}\Big[$  $+ \dots \Big]$

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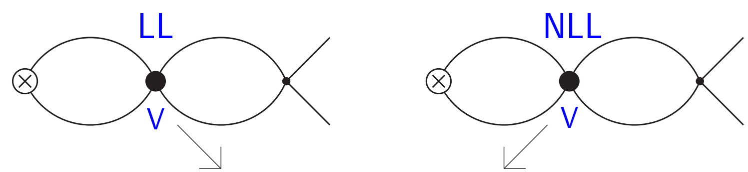
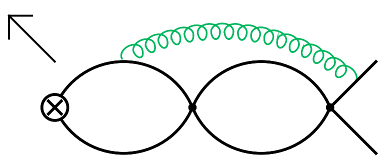


$$\ln\left[\frac{c_1(\nu)}{c_1(1)}\right] = \underbrace{\xi^{\rm LL}}_{0} + \xi^{\rm NLL} + \xi^{\rm NNLL}_{\rm mix} + \xi^{\rm NNLL}_{\rm nonmix}$$

[Luke, Manohar, Rothstein, Pineda, Hoang, Stewart]



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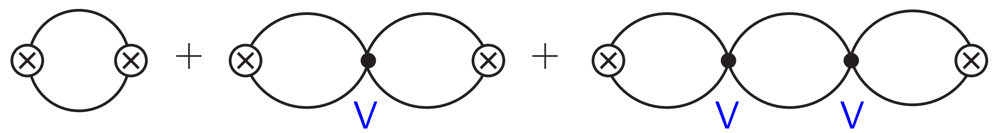

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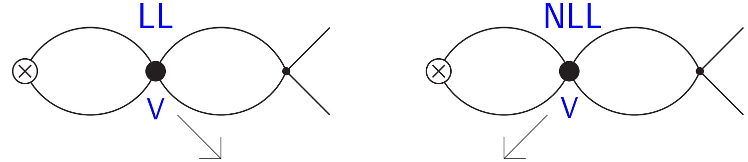

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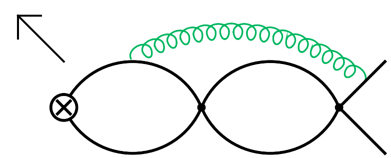

[Hoang]

missing $\Rightarrow$ $\boxed{V^{\rm NLL}(\nu)}$ needed!

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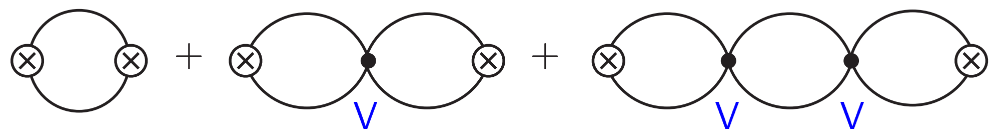
$\sigma_{\rm tot} \sim \mathrm{Im}\Big[$  $+ \dots \Big]$

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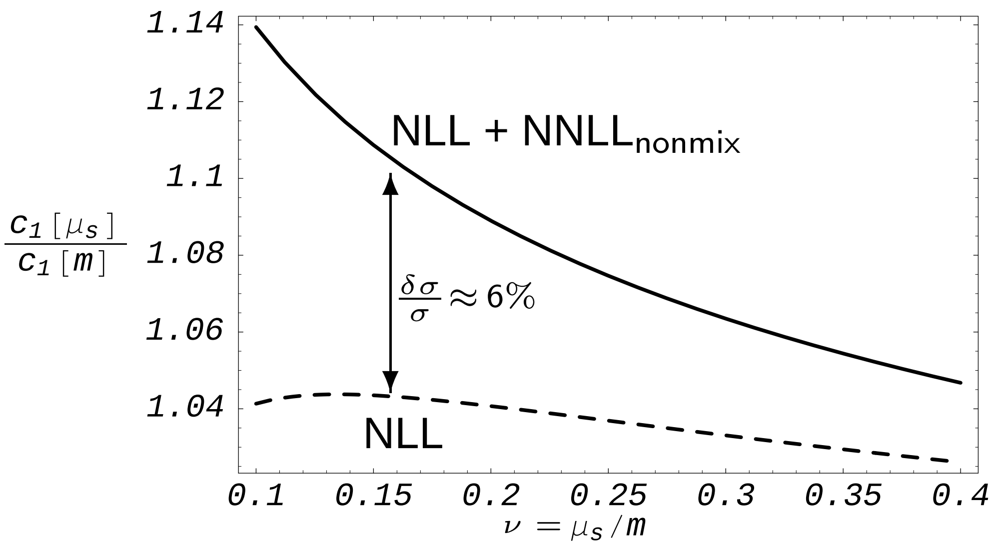

missing NNLL$_{\rm mix}$ contribution

may reduce theortical error of $\sigma_{\rm tot}$

$\Rightarrow$ $\boxed{V^{\rm NLL}(\nu)}$ needed!

Ultrasoft contributions dominant!

$[\alpha_s(mv^2) > \alpha_s(mv)]$

Renormalization of the $1/m^2$ - Potentials

$$\text{NLL}: \quad \nu \frac{\partial}{\partial \nu} \ln[c_1(\nu)] = -\frac{\mathcal{V}_c(\nu)}{16\pi^2}\left[\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu) + \mathbf{S}^2\,\mathcal{V}_s(\nu)\right] + \frac{1}{2}\mathcal{V}_k(\nu)$$

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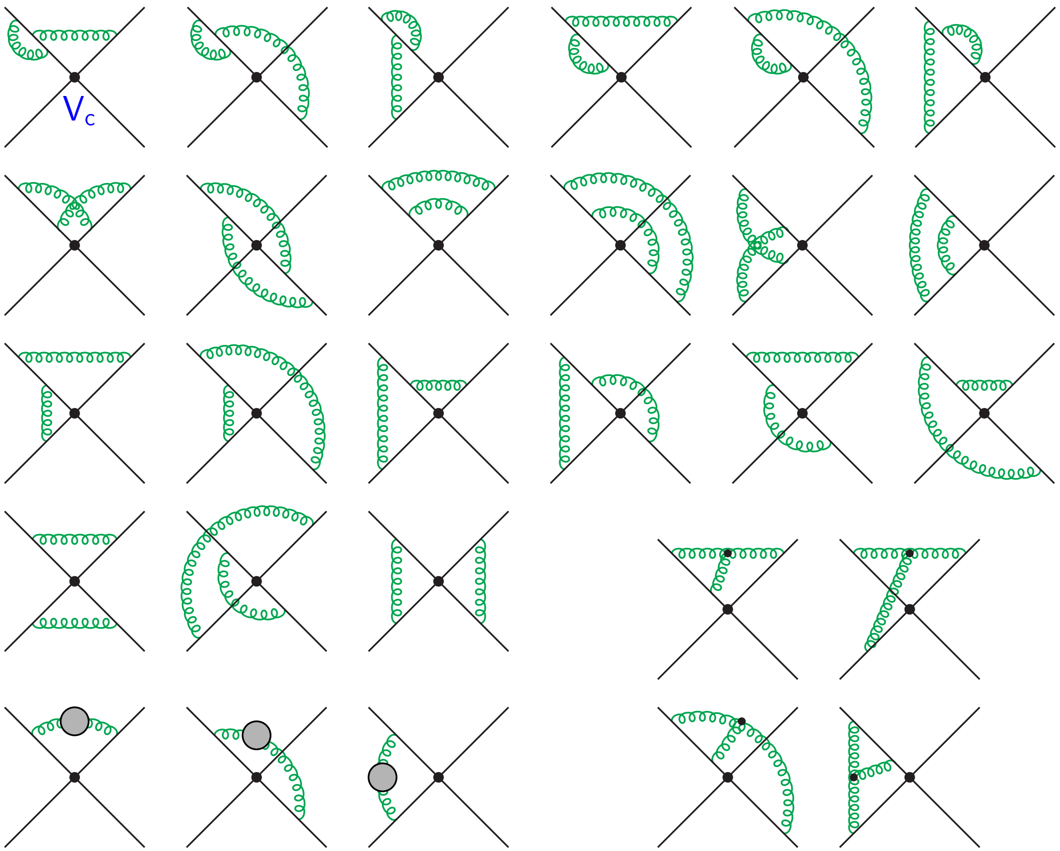
renormalize directly

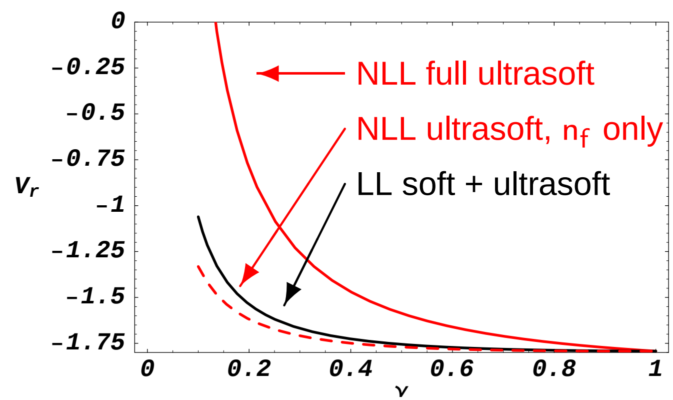
$\mathcal{O}(10^3)$ diagrams (Feynman gauge)



$\Rightarrow$

e.g.: [$\overline{\text{MS}}$ & Dim. Reg.]

$$\Rightarrow \delta V_r^{2\,\text{loop}} \xrightarrow{\text{RGE}} V_r^{\text{NLL}}(\nu)$$



V_2^{NLL} and V_r^{NLL} complete ✓

[MS, Hoang]

Renormalization of the $1/\mathrm{mk}$ - Potentials

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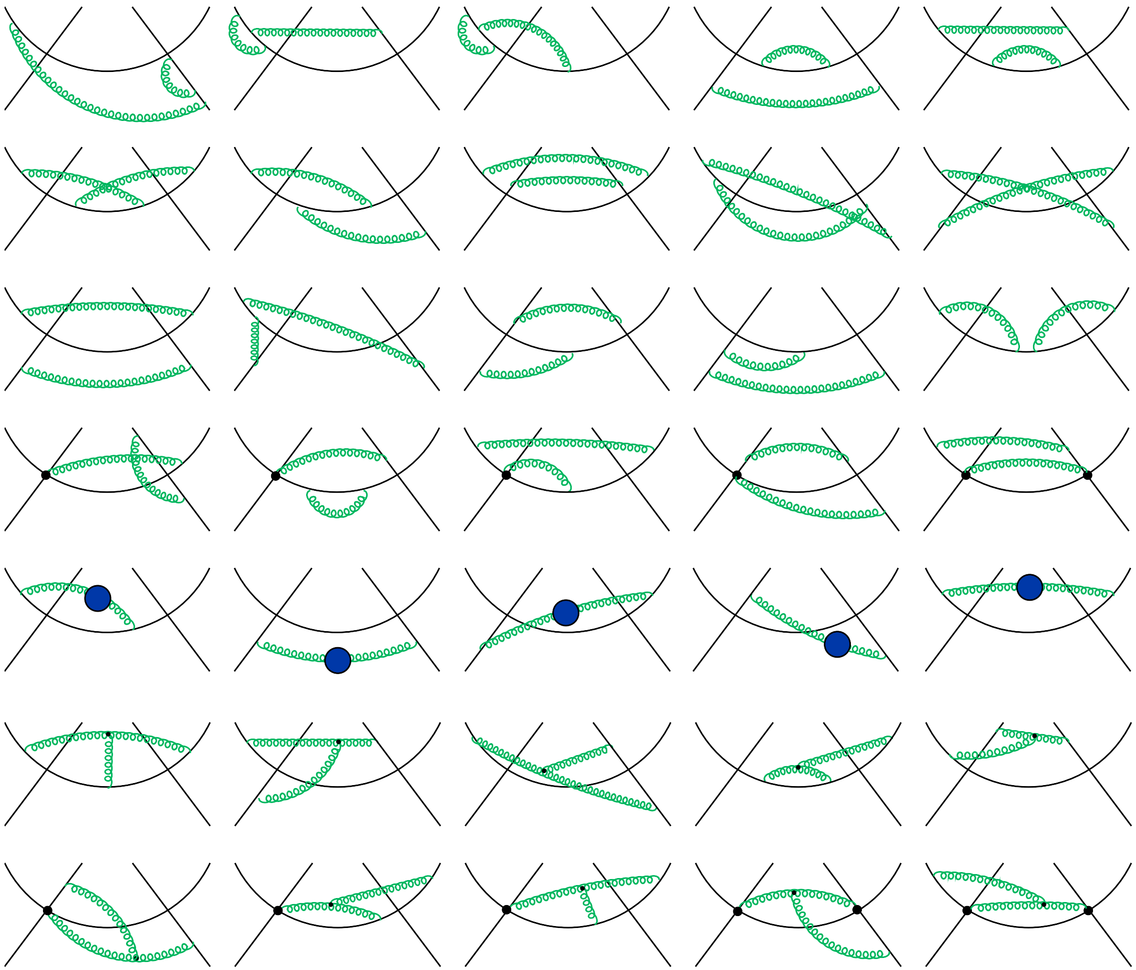
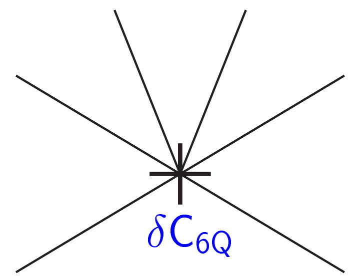
6 ext. HQ legs, $\mathcal{O}(10^4)$ diagrams (Feynman gauge)



$\Rightarrow$

Absorb divergence by 6Q-Op.:



$$\delta C_{6Q}^{2\text{loop}} \xrightarrow{\text{RGE}} C_{6Q}^{\text{NLL}}(\nu)$$

[$\overline{\mathrm{MS}}$ & Dim. Reg.]

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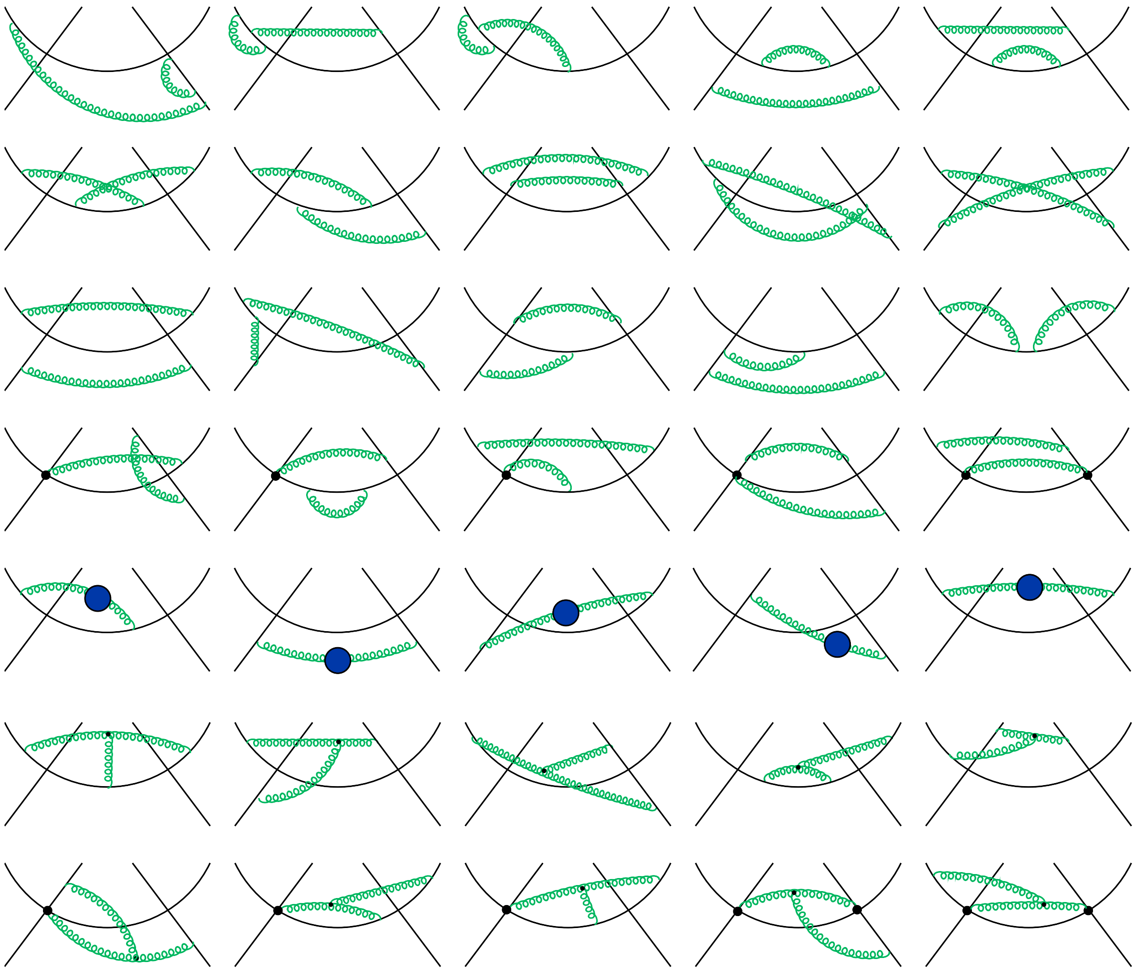
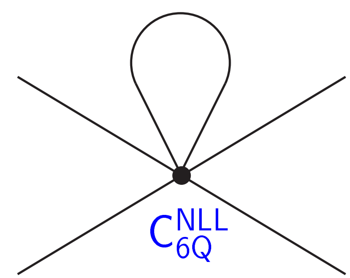


$\Rightarrow$

Close finite HQ - Loop:



$$= C_{6Q}^{NLL}(\nu)\cdot\frac{\#}{\mathbf{mk}} \equiv \boxed{(V_k^{eff})^{NLL}}$$

“effective” potential

Renormalization of the $1/mk$ - Potentials

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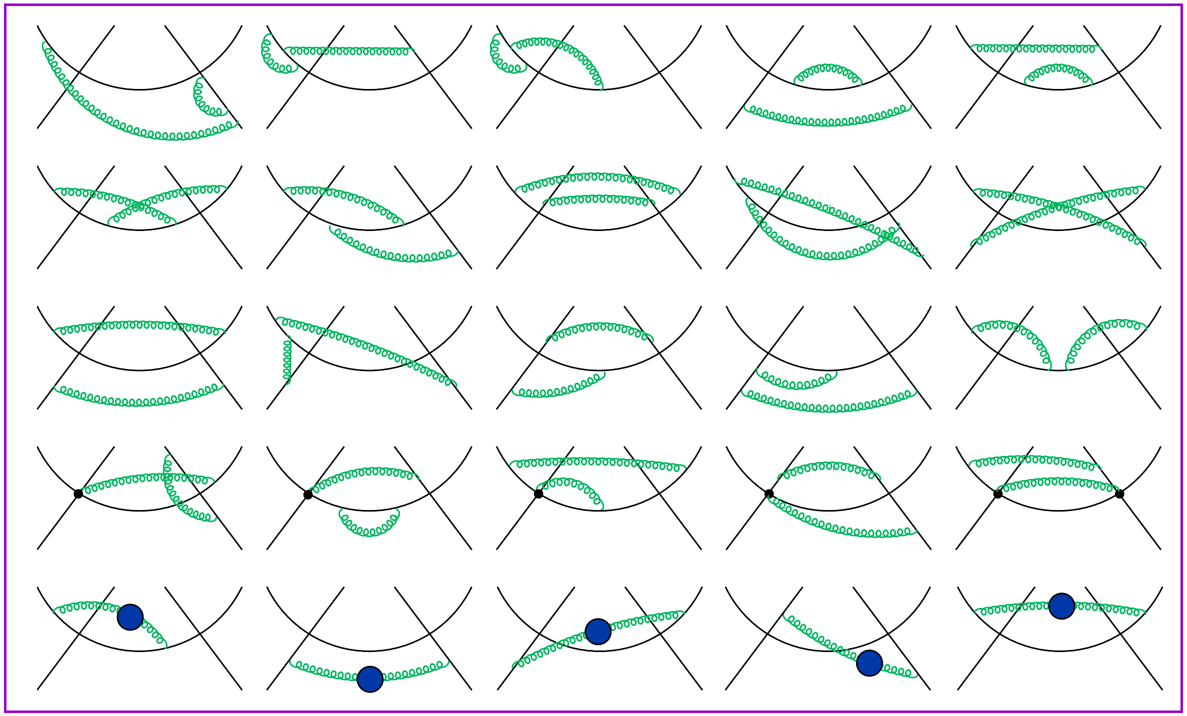
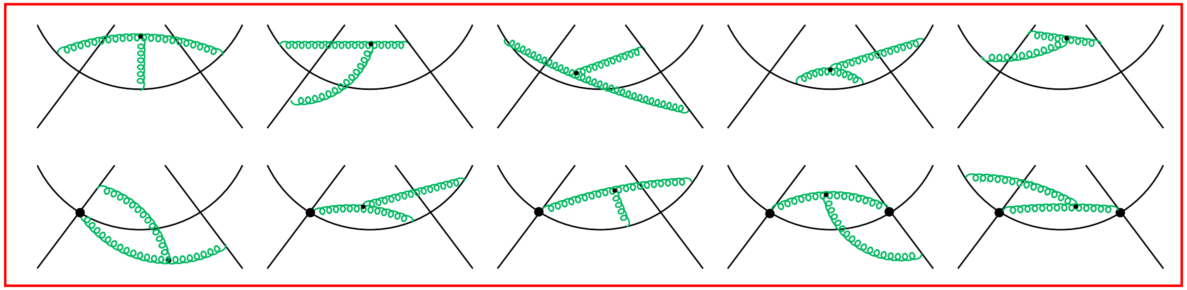


"Abelian" topologies:
Done ✓



"NonAbelian" topologies:
Work in progress

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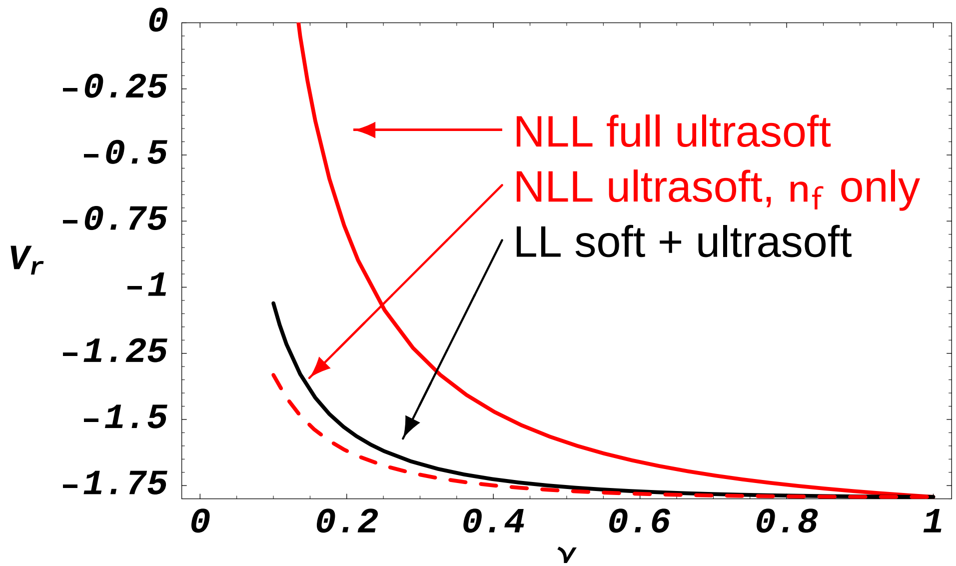
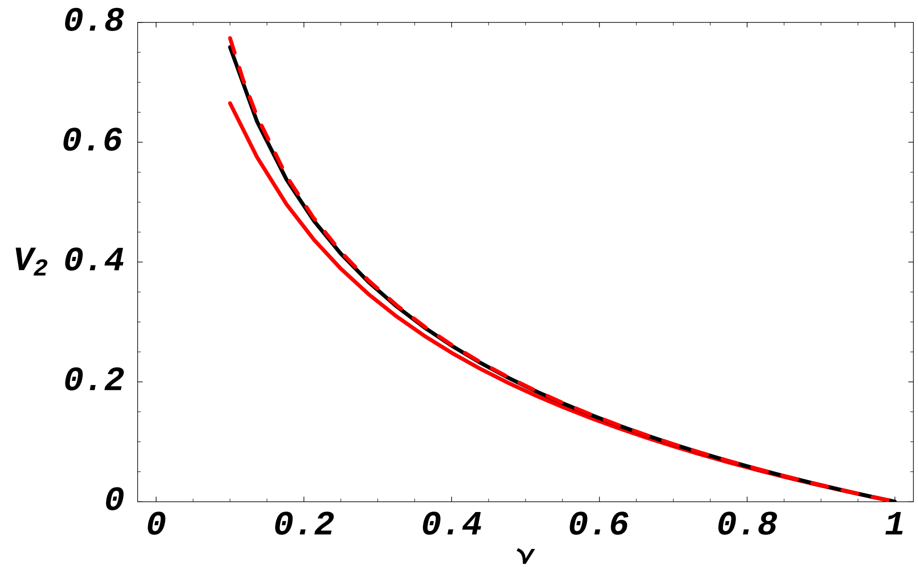
- Current status of the calculation: $[\alpha_S = \alpha_s(mv),\ \alpha_U = \alpha_s(mv^2)]$

Contribution	order / α_S	V_k	V_r	V_2	V_s
soft + usoft LL	$(\alpha_S \ln v)^n$, $(\alpha_U \ln v)^n$	✓	✓	✓	✓
usoft NLL n_f	$n_f\, \alpha_U (\alpha_U \ln v)^n$	✓	✓	✓	0
full usoft NLL	$\alpha_U (\alpha_U \ln v)^n$	w.i.p.	✓	✓	0
soft NLL	$\alpha_S (\alpha_S \ln v)^n$	–	–	–	✓

Back Up: Results for the $1/m^2$ - Potentials

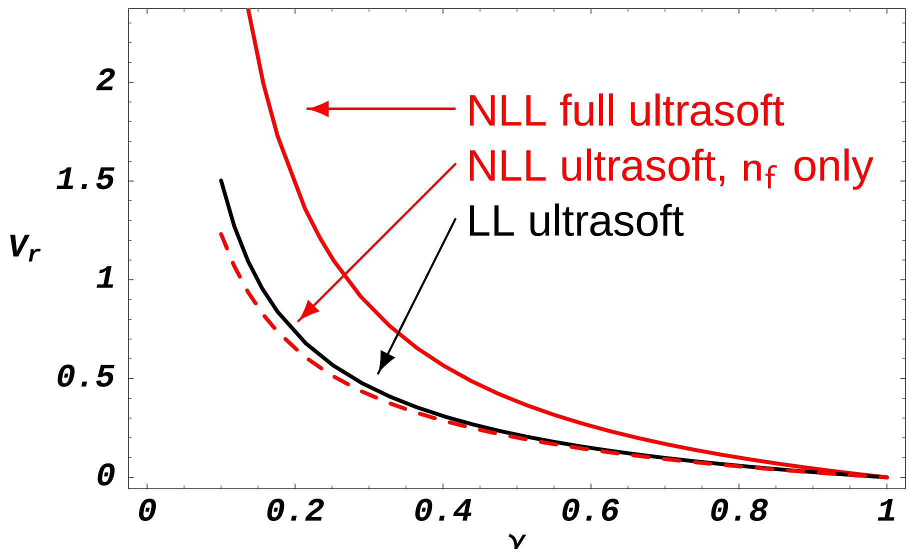
Results for $\frac{1}{m^2}$ potentials:

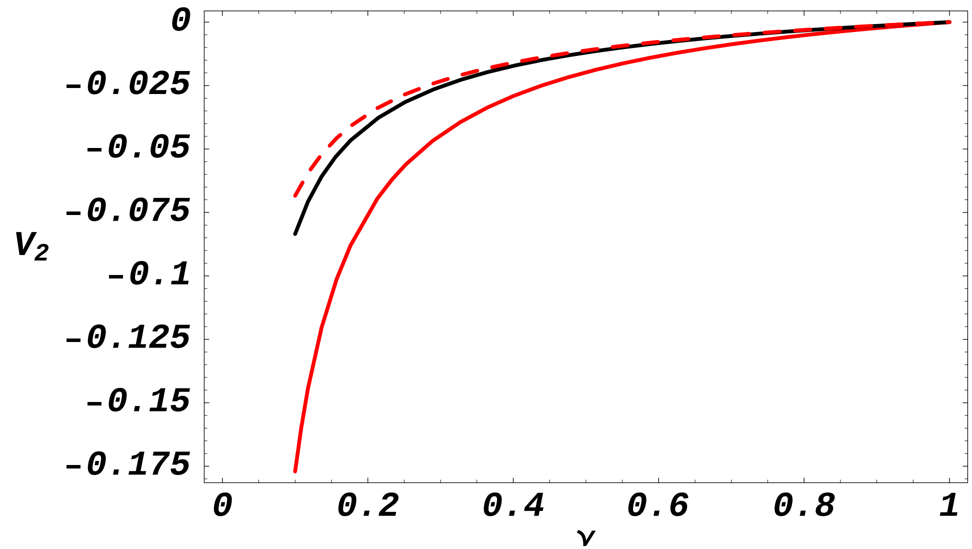




Back Up: Results for the $1/m^2$ - Potentials

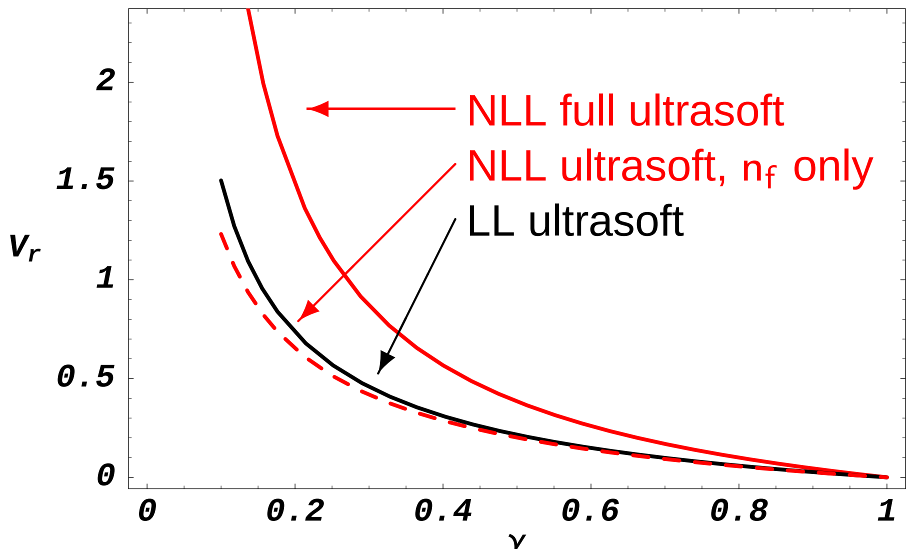
Results for $\frac{1}{m^2}$ potentials:

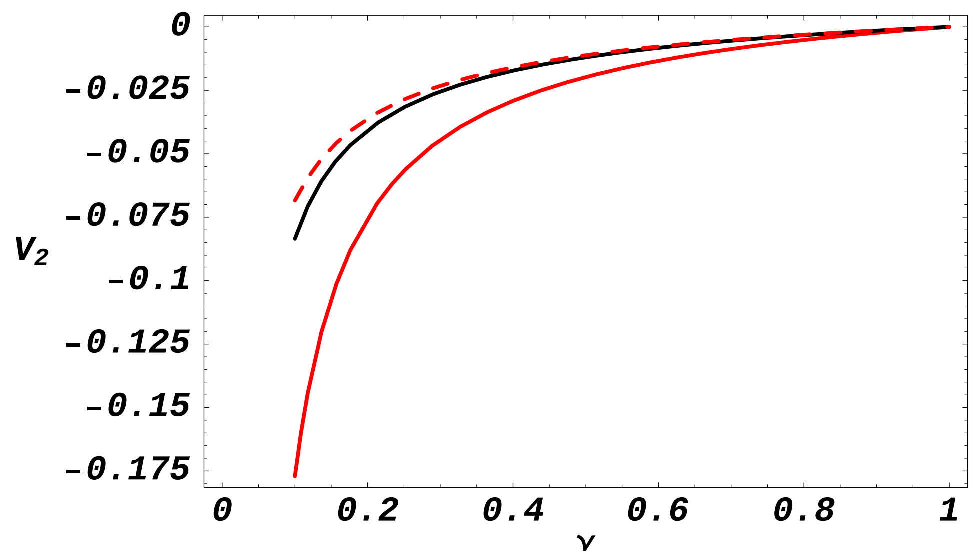




Back Up: Results for the $1/m^2$ - Potentials

Results for $\frac{1}{m^2}$ potentials:



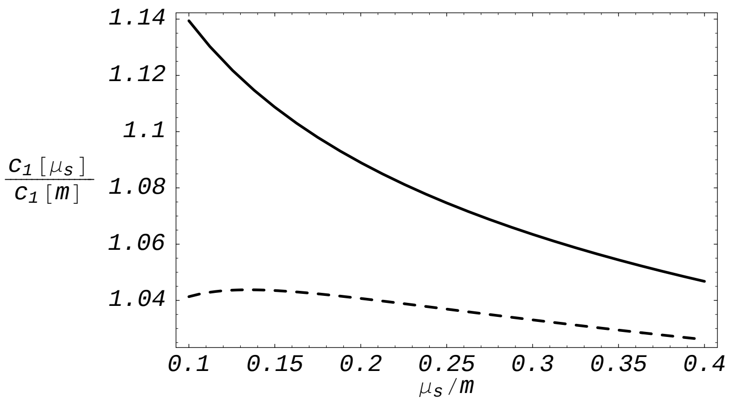


- Analysis shows that usoft LL $\sim$ usoft NLL

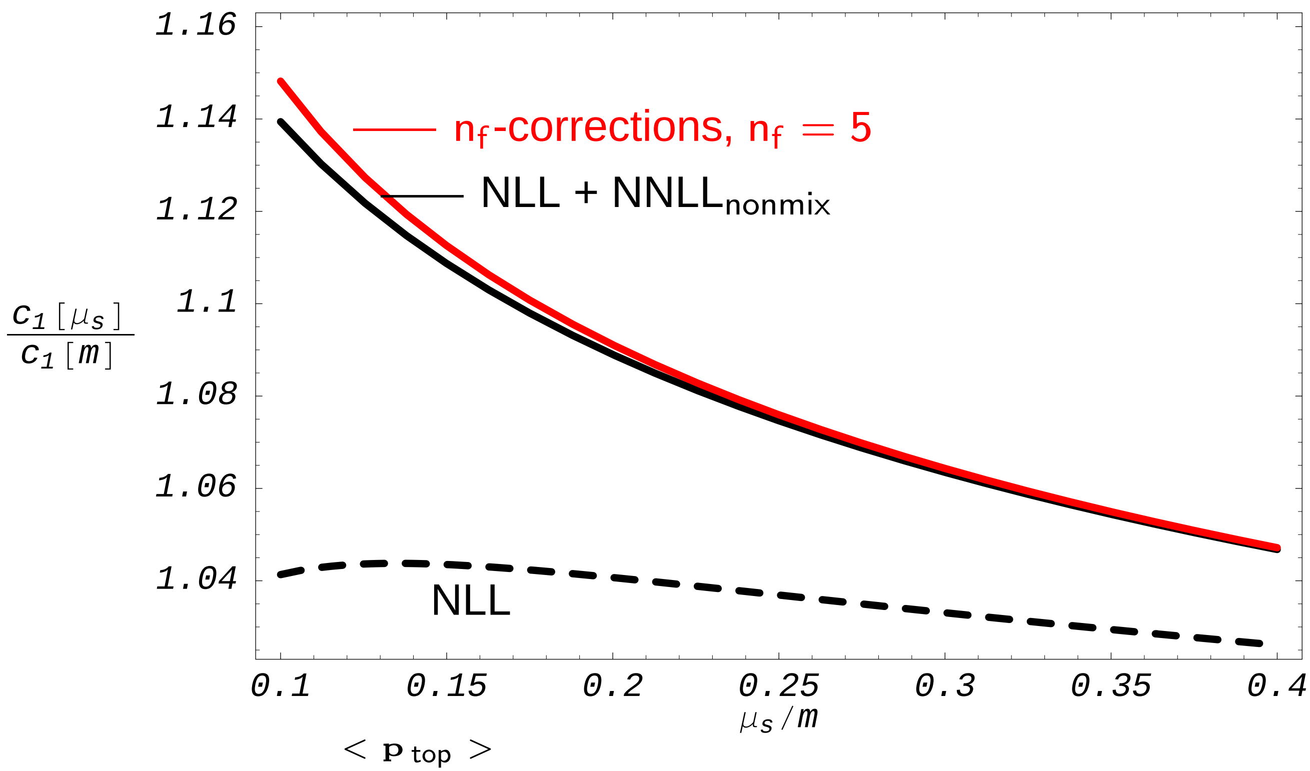
- $\delta c_1 = (-1.9\%, -0.5\%)$ for $\nu = 0.1, 0.2$

$\Rightarrow$ Big NNLL_{mix} contributions to c_1 expected

$\Rightarrow$ may compensate $\text{NNLL}_{\text{nonmix}}$ and reduce ν dependence of c_1!



Back Up: Old n_f Result



Back Up: Extra Formulae

$$\nu \frac{\partial}{\partial \nu} \ln[\mathsf{c}_1(\nu)] = -\frac{\mathcal{V}_c^{(0)}(\nu)}{16\pi^2}\left[\frac{\mathcal{V}_c^{(0)}(\nu)}{4} + \mathcal{V}_2^{(2)}(\nu) + \mathcal{V}_r^{(2)}(\nu) + \mathbf{S}^2\, \mathcal{V}_s^{(2)}(\nu)\right] + \frac{1}{2}\mathcal{V}_k^{(1)}(\nu) + \alpha_s^2(m\nu)\left[3\mathcal{V}_{k1}^{(1)}(\nu) + 2\mathcal{V}_{k2}^{(1)}(\nu)\right]$$

$$v \cong \alpha_s(mv) = \frac{4\pi}{\beta_0 \ln(m^2 v^2/\Lambda_{QCD}^2)} \quad \Rightarrow \quad v \cong \alpha_s \cong 0.14$$

$$v \equiv \sqrt{\frac{\sqrt{s}-2m_t}{m_t}} \longrightarrow \sqrt{\frac{\sqrt{s}-2m_t+i\Gamma_t}{m_t}}$$ [Fadin, Khoze]