Holographic Superfluidity from a Magnetic Field

Migael Strydom

Max-Planck-Institut für Physik, München

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In collaboration with Martin Ammon, Yan-Yan Bu,
Johanna Erdmenger, Patrick Kerner, Jonathan Shock

Gauge/Gravity Duality

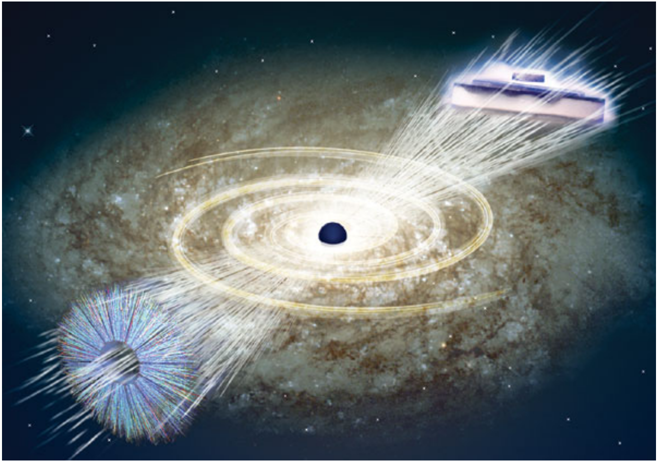
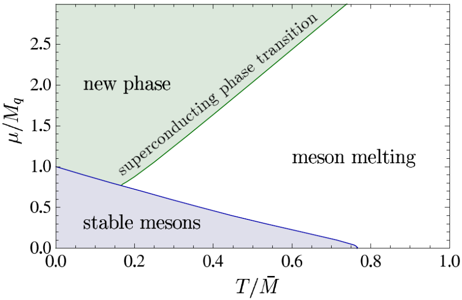
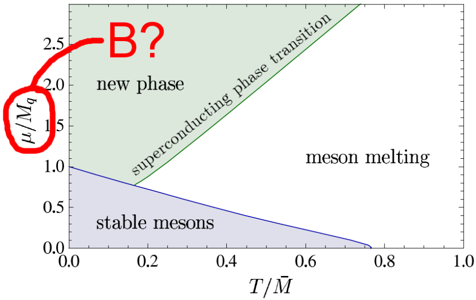


Image Source: Nature 448, 1000-1001(30 August 2007)



Gauge/gravity duality has been successful in exploring the phase diagram of large N gauge theories.
Is there more that we can learn?

Erdmenger, Kaminski, Kerner, Rust: arXiv:0807.2663



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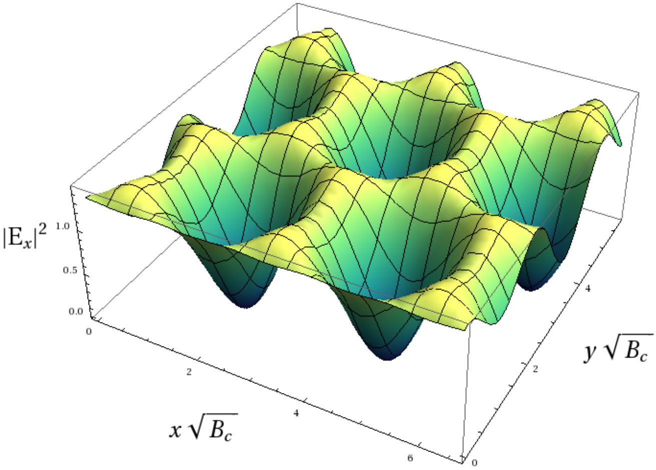
B field instead of μ

We set up a very simple holographic model representing a gauge theory at finite temperature and a magnetic field B. From this model we find that not only does a superconducting phase transition occur when B increases beyond a critical value B_c, but...

...the ground state forms a triangular Abrikosov lattice



Why is this interesting?

Holography gives us a handle on strongly coupled phenomena.

- QCD phenomena, like heavy ion collisions, ρ-meson physics.
- Condensed matter phenomena, like non-Fermi liquids, superconducting phase transitions.

We don't always know the underlying field theory — top-down vs bottom-up approach — but we get an idea of what strongly coupled matter can do.

Why is this interesting?

- Holographic superconductivity is an active field of research, with applications to both QCD phenomena and condensed matter physics.

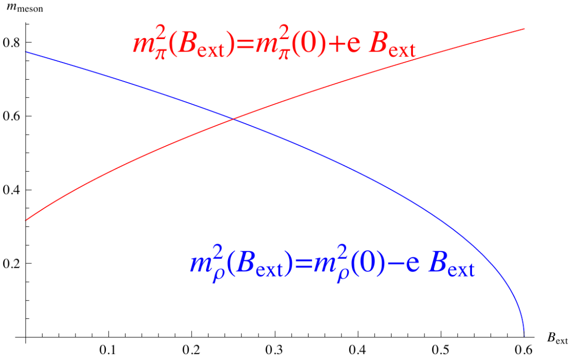
Where I'm coming from:

- Superconductors: Gubser (2008), Hartnoll, Herzog, Horowitz (2008)
- Theories on a lattice: Horowitz, Santos, Tong, 2012.
- Spontaneously broken translational symmetry: Domokos, Harvey, 2007.
- Spontaneous breaking with magnetic field: Donos, Gauntlett, Pantelidou, 2011.

- M. Chernodub proposed a mechanism by which the QCD vacuum can undergo a superconducting phase transition in 2010 (arXiv:1008.1055).

Strong magnetic field $\Rightarrow$ gluon-mediated attraction between quark-antiquark pairs leads to new, superconducting ground state.

Chernodub (2010): DSGS model and extended NJL model, with a $\sim F^{\mu\nu}\rho_{\mu\nu}$ interaction term.

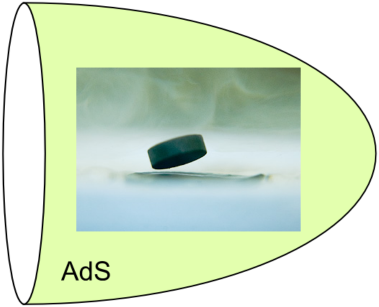


DSGS = Djukanovic, Schindler, Gegelia, Scherer
NJL = Nambu-Jona-Lasinio

How do we build our holographic superconductor?

To see if gauge/gravity duality can model this behaviour, and as a bonus yield a lattice ground state spontaneously, we need:

- Fundamental matter
- Magnetic field
- Finite temperature
- Order parameter



Holographic setup

The model:

$$S = \int d^5x\sqrt{-g}\ \left\{\frac{1}{16\pi G_N}\left(R+\frac{12}{L^2}\right)-\frac{1}{4\hat{g}^2}\mathrm{tr}\left(F_{\mu\nu}F^{\mu\nu}\right)\right\}+S_{\mathrm{bdy}}$$

Assume the probe limit. The metric in 5 dimensions, working in Poincaré coordinates with the boundary at $u=0$:

$$ds^2 = \frac{L^2}{u^2}\left(-f(u)dt^2+dx^2+dy^2+dz^2+\frac{du^2}{f(u)}\right)$$

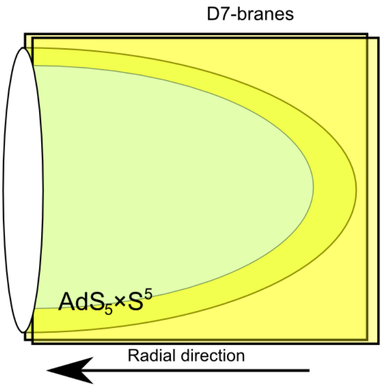
AdS-Schwarzschild: $f(u)=1-u^4/u_H^4$.

Fundamental matter $\Leftrightarrow$ D7-branes

- D7-branes $(\times 2)$

Strings with one endpoint on the D7-branes (and the other on the D3-branes) transform in the fundamental representation of $SU(2)$. In the small α' limit, we have just an $SU(2)$ gauge field.

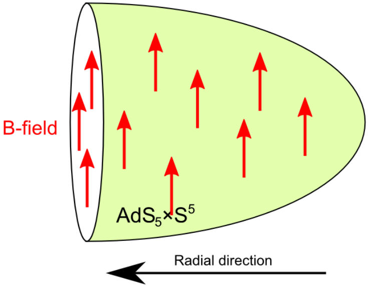
A. Karch, E. Katz



Magnetic field ⇔ Magnetic field

- $SU(2)$ flavour field

Choose $F^3_{xy} = B$, $F^a_{\mu\nu} = 0$ otherwise.
Also fix $\mathcal{A}^3_y = xB$ and other components so that only $U(1)$ gauge symmetry remains.



Magnetic field $\Leftrightarrow$ Magnetic field

Compare this to the case of a finite isospin chemical potential μ:

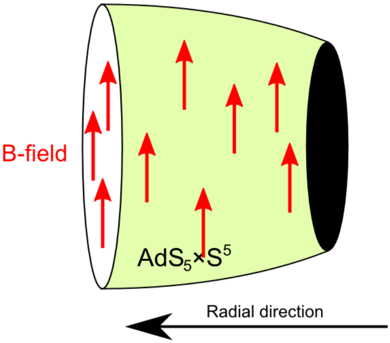
Isospin chemical potential	$\Leftrightarrow$	$\mathcal{A}_t^3$ non-zero at boundary.
Magnetic field	$\Leftrightarrow$	$\mathcal{A}_y^3$ non-zero at boundary.

Finite temperature ⇔ Black hole

- put a black hole with planar event horizon at $u = u_H$ $(f(u) = 1 - u^4/u_H^4)$.

This is needed to fix a scale.

E. Witten (1998)



Vector current ⇔ Gauge field

- The remaining components of A_μ^a act as an order parameter. Boundary expansion:

$$\mathcal{A}_\mu^a \approx 0 + u^2 \langle J_\mu^a \rangle + \mathcal{O}(u^4)$$

- It is consistent to switch on only $\mathcal{A}_{x,y}^{1,2}(x, y, u)$.
- When $B < B_c$, these components are zero.
- When $B > B_c$, some of these components become nonzero.

Gauge fixing

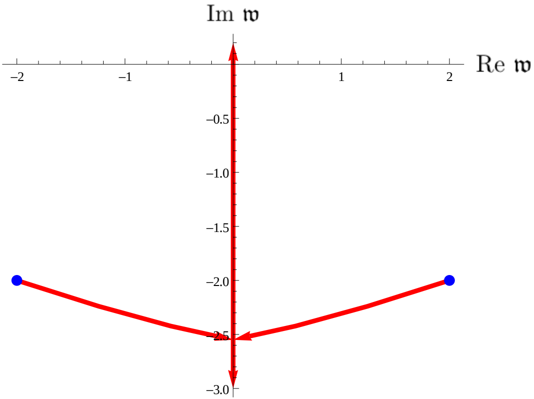
We look at $B \approx B_c$. Then we can focus on a small condensate and look at fluctuations of $\mathcal{A} \approx \varepsilon A + \varepsilon^3 a + \dots$.
Defining $E_{x,y} = A^1_{x,y} + iA^2_{x,y}$, we can focus on fields charged under the $U(1)$:

$$\begin{aligned} E_L &= x^2 B E_x - i(x\partial_x E_y - E_y) \\ &\to e^{-i\Lambda^3} E_L \end{aligned}$$

These source a vector condensate when they condense.

But do they condense?

Quasinormal mode analysis: $E_L \sim e^{-i\omega t} E_L$.



Ammon, Erdmenger, Kerner, MS (2011)

So what is the true ground state?

- The quasinormal mode analysis shows that the ground state with no condensate is unstable.
- Can we show that the ground state is an Abrikosov lattice?
- There are many different lattice configurations. We study a few of them.
- Maeda, Natsuume, Okamura: 2+1 dimensional YM with a scalar.

arXiv:0910.4475

Perturbative strategy

$$\nabla^\mu F^a_{\mu\nu} + \epsilon^{abc}\mathcal{A}^{b\mu}F^c_{\mu\nu} = 0.$$

$$\mathcal{E}_{x,y} = \mathcal{A}^1_{x,y} + i\mathcal{A}^2_{x,y}$$

$$\begin{aligned}
\mathcal{E}_{x,y} &= \quad \boxed{\varepsilon E_{x,y}} \quad + \quad \boxed{\varepsilon^3 e_{x,y}} \quad + \mathcal{O}(\varepsilon^5) \\
\mathcal{A}^3_y &= xB_c \quad + \quad \boxed{\varepsilon^2 a^3_y} \quad + \mathcal{O}(\varepsilon^4) \\
\mathcal{A}^3_x &= \quad \boxed{\varepsilon^2 a^3_x} \quad + \mathcal{O}(\varepsilon^4)
\end{aligned}$$

9 coupled equations for 3 gauge components in x, y and radial directions. $\varepsilon \sim \langle J \rangle$. We need to go to 3^{rd} order.

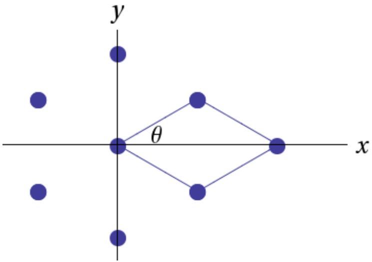
Linear order solution

$$E_x = \sum_{n=-\infty}^{\infty} C_n e^{-inky - \frac{1}{2}B_c\left(x - \frac{nk}{B_c}\right)^2} U(u)$$

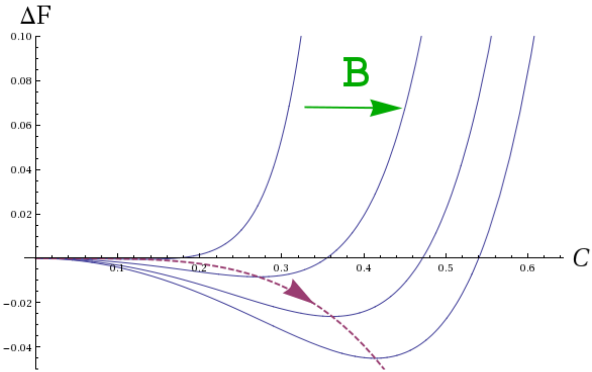
- This is the Abrikosov solution.
- $U(u)$ is the radial factor.
- Free parameters: k, C_n. We need to go to higher order to fix these.
- For $B \approx B_c$, C_n is small.

Fixing C_n

- We expect a lattice. This requires $C_n = C_{n+N}$ for some N.
- Choosing $N = 1$, $C_n = C$, and $k = \sqrt{2\pi B_c}$ gives a square lattice.
- Choosing $N = 2$, $C_1 = iC_0$, and $k = 3^{\frac{1}{4}}\sqrt{\pi B_c}$ gives a triangular lattice.
- Choosing $N = 2$, $C_1 = iC_0$, and varying k can give a rhombic lattice.

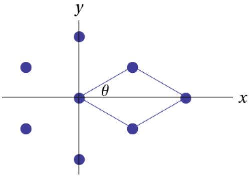


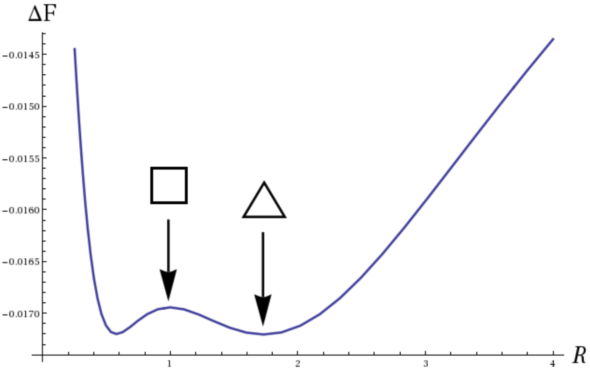
$$F \sim \int dudxdy\ f(u)\mathcal{A}^2 + g(u)\mathcal{A}^4$$
$$\sim \int du\tilde{f}(u)C^2 + \tilde{g}(u)C^4$$



As B increases, so does C.

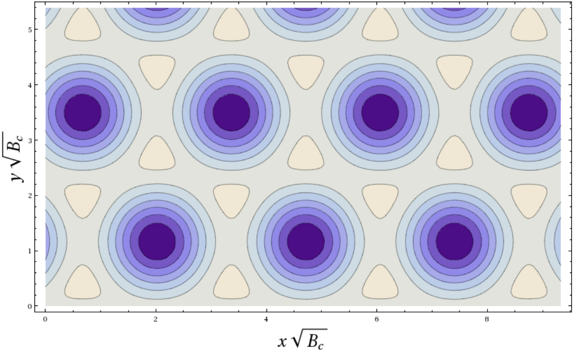
We can plot the free energy as a function of $R = \frac{L_x}{L_y}$.





Ground state lattice

We find a triangular lattice ground state dynamically appearing.



This agrees with the field theory calculations in a DSGS model, and Abrikosov lattices in type II superconductors.

What comes next?

- Calculate transport coefficients in this background.
- How generic are these results? We can try in different backgrounds/dimensions. (We got very similar results for AdS-Schwarzschild and hard wall models).
- Look for heavy ion and condensed matter applications.

Thank you!

M. Ammon, J. Erdmenger, P. Kerner and M. Strydom
arXiv:1106.4551

Y. Bu, J. Erdmenger, J. Shock and M. Strydom
(To appear)