Transport in Anisotropic Superfluids: A Holographic Description

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Motivation

- holographic p-wave superfluid
- possible interpretation: ρ meson superfluid generated by spontaneous symmetry breaking at finite isospin chemical potential
- here focus on transport phenomena
- possibly relevant for physics of Quark-Gluon-Plasma (RHIC, LHC)

- transport coefficients in strongly coupled theories are computable using gauge/gravity duality

- famous universal result by Kovtun, Son and Starinets (KSS):

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

conjectured to be lower bound for substances found in nature

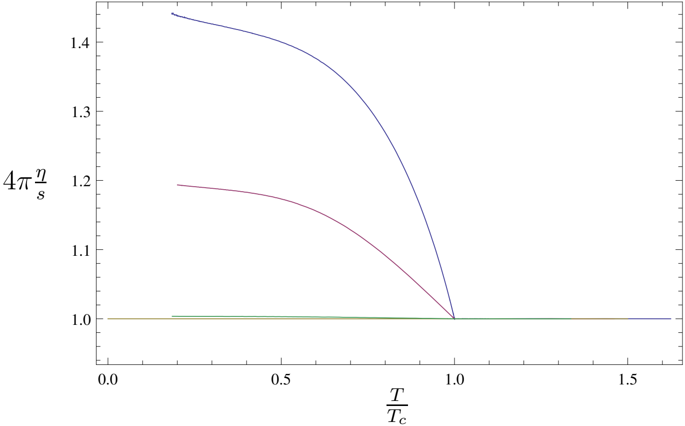
hep-th/0405231

- proven to be valid in all field theories dual to isotropic Einstein gravity, i.e. field theories at large N_c and infinite 't Hooft coupling λ

e.g.: 0809.3808

- η/s in Quark-Gluon-Plasma close to universal value novel feature of our holographic p-wave superfluid setup:

- non-universal result for η/s to leading order in $1/N_c$ and $1/\lambda$ (i.e. field theory dual to Einstein gravity)



- deviation due to broken symmetries
- KSS bound unbroken

Gauge/Gravity Duality – brief reminder I

- Maldacena in 1997:

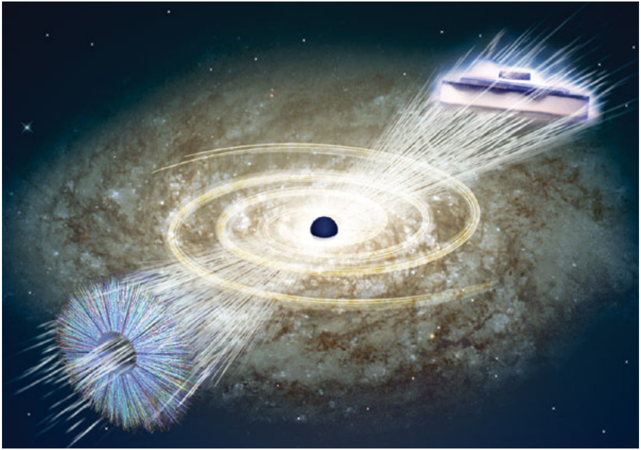
$\mathcal{N} = 4$ $SU(N_c)$ Super-Yang-Mills theory in D=4	$\Leftrightarrow$	type IIB Superstring theory in $\mathrm{AdS}_5 \times \mathrm{S}^5$

- Generalisation:

Large N_c gauge theory in D spacetime dimensions	$\Leftrightarrow$	Classical gravitational theory in asymptotically $\mathrm{AdS}_{\mathrm{D}+1}$ spacetime

$$\langle \mathrm{e}^{\int \mathrm{d}^{\mathrm{D}} x \mathcal{O} \phi_{\mathrm{bdy}}} \rangle_{\mathrm{FT}} = \mathcal{Z}_{\mathrm{GRAV}}[\Phi \to \phi_{\mathrm{bdy}}]$$

Gauge/Gravity Duality



Nature 448, 1000-1001 (30 August 2007)

Gauge/Gravity Duality – extensions

- finite temperature ⇔ black hole solution

hep-th/9803131

- chemical potential ⇔ Einstein-Maxwell theory with $A_t \neq 0$

- holographic superfluid:

s-wave: SSB global $U(1)$ ⇔ SSB gauged $U(1)$

0801.2977

p-wave: SSB global $U(1)$ and spatial $SO(3)$
⇔ SSB gauged $U(1)$ and spatial $SO(3)$

0805.2960

Hydrodynamics – brief reminder II

- effective theory: describes the macroscopic behaviour of a many body system
- long wavelength, small frequency fluctuations about thermal equilibrium
- response of system: transport coefficients
  e.g. shear viscosity, bulk viscosity, diffusion coefficient, conductivity
- constitutive equations

$$T^{\mu\nu} = T^{\mu\nu}_{\text{eq.}} + \Pi^{\mu\nu} \text{ and } J^{\mu} = J^{\mu}_{\text{eq.}} + \Upsilon^{\mu}$$

with $\Pi_{ij} \sim \eta\left(\partial_i u_j + \partial_j u_i\right)$

Gravitational Setup for holographic p-wave Superfluid

- $SU(2)$ Einstein-Yang-Mills theory in (4+1)-dimensional asymptotically AdS Space

$$S = \frac{1}{2{\kappa_5}^2} \int \mathrm{d}^5x \sqrt{-g} \left[R - \Lambda - \frac{\alpha^2}{2} F^a_{MN} F^{aMN} \right] + S_{\mathrm{bdy}}$$

- with

$$\alpha \equiv \frac{\kappa_5}{g_{\mathrm{YM}}}$$

- α measures the backreaction

Looking for solutions with...

Field Theory	$\Leftrightarrow$	Gravity
finite temperature T		black hole solutions
isospin chemical potential μ		$A_t^3 = \phi(r) \neq 0$
breaks $SU(2) \to U(1)_3$		breaks $SU(2) \to U(1)_3$
$\langle \mathcal{J}_1^x \rangle \neq 0$		$A_x^1 = w(r) \neq 0$
$U(1)_3 \to \mathbb{Z}_2$, $SO(3) \to SO(2)$		$U(1)_3 \to \mathbb{Z}_2$, $SO(3) \to SO(2)$

- $w(r_{\text{bdy}}) = 0 \Rightarrow$ SSB $U(1)_3 \to \mathbb{Z}_2$ & $SO(3) \to SO(2)$

$\Rightarrow$ holographic p-wave superfluid with backreaction

Ammon, Erdmenger, Grass, Kerner, O'Bannon: 0912.3515

Hairy Black Hole Solution

Depending on the temperature T, we find the following thermodynamically preferred solutions:

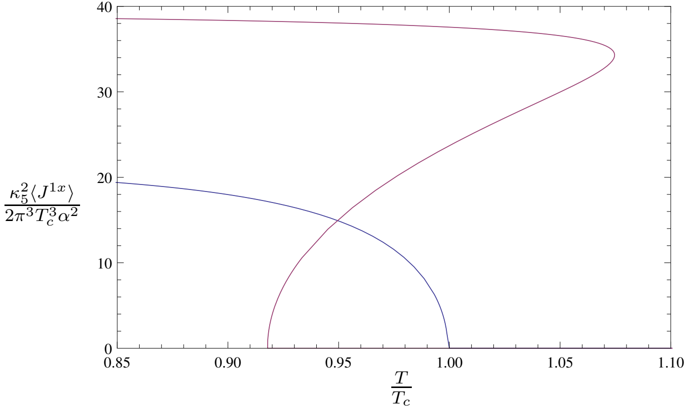
- $w(r) = 0$, AdS Reissner-Nordström black hole solution ($T > T_c$), or
- $w(r) \neq 0$, numerical solution ($T < T_c$).

Depending on the backreaction parameter α we find a

- 2nd order phase transition ($\alpha < \alpha_{\text{crit}}$), or
- 1st order phase transition ($\alpha > \alpha_{\text{crit}}$)

between the solutions.

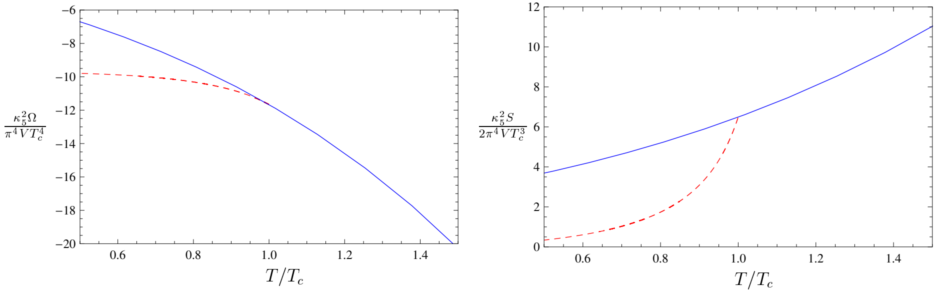
Condensate $\langle \mathcal{J}_1^x \rangle$



colour coding: $\alpha = 0.316 < \alpha_c$ and $\alpha = 0.447 > \alpha_c$
$\langle \mathcal{J}_1^x \rangle \propto (1 - T/T_c)^{1/2}$ for $\alpha = 0.316$

0912.3515

Potential Ω and Entropy density s for $\alpha = 0.316 < \alpha_c$



colour coding: Reissner-Nordström solution and $w(r) \neq 0$ solution

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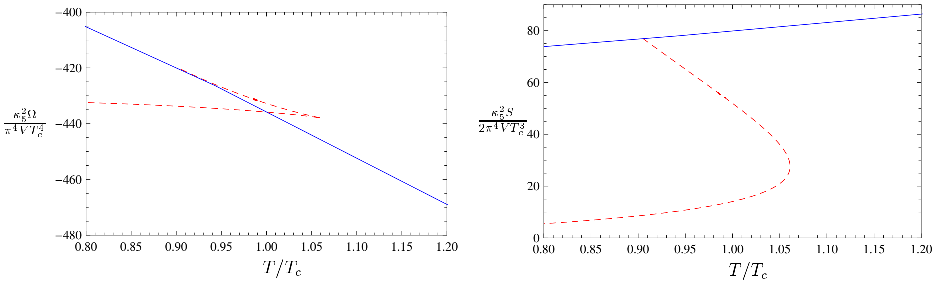
Potential Ω and Entropy density s for $\alpha = 0.447 > \alpha_c$



colour coding: Reissner-Nordström solution and $w(r) \neq 0$ solution

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Perturbations about the Thermodynamical Equillibrium

small perturbations:

- metric $\hat{g}_{MN} = g_{MN}(r) + h_{MN}(x^\mu, r)$
- gauge field $\hat{A}^a_M = A^a_M(r) + a^a_M(x^\mu, r)$
- x^μ-spacetime translational invariance still unbroken

$\Rightarrow$ Fourier decomposition of fluctuations possible:

$$h_{MN}(x^\mu, r) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{e}^{\mathrm{i} k_\mu x^\mu} h_{MN}(k^\mu, r)$$
$$a^a_M(x^\mu, r) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{e}^{\mathrm{i} k_\mu x^\mu} a^a_M(k^\mu, r)$$

- in the following set $\vec{k} = 0$

Classification of Perturbations

$\Rightarrow$ classification under $SO(2)$ rotational symmetry around x-axis possible:

	dynamical fields	constraints	# physical modes
helicity 2	$h_{yz}, h_{yy} - h_{zz}$	none	2
helicity 1	$h_{ty}, h_{xy}; a^a_y$	h_{yr}	4
	$h_{tz}, h_{xz}; a^a_z$	h_{zr}	4
helicity 0	$h_{tt}, h_{xx}, h_{yy} + h_{zz}, h_{xt}; a^a_t, a^a_x$	$h_{tr}, h_{xr}, h_{rr}; a^a_r$	4

gauge choice $h_{Mr} = 0$ and $a^a_r = 0 \Rightarrow$ 14 physical modes

Green's function

Using Son's and Starinets'recipe we compute the following Green's functions holographically:

hep-th/0205051

$$G^{\mu\nu,\rho\sigma}(\omega,\vec{k}) = \int \mathrm{d}^4x \; \mathrm{e}^{-ik_\mu x^\mu}\theta(t) \; \langle [T^{\mu\nu}(t,\vec{x}), \, T^{\rho\sigma}(0)] \rangle \, ,$$

$$G^{\mu\nu,\rho}_{a}(\omega,\vec{k}) = \int \mathrm{d}^4x \; \mathrm{e}^{-ik_\mu x^\mu}\theta(t) \; \langle [T^{\mu\nu}(t,\vec{x}), J^\rho_a(0)] \rangle \, ,$$

$$G^{\rho,\mu\nu}_a(\omega,\vec{k}) = \int \mathrm{d}^4x \; \mathrm{e}^{-ik_\mu x^\mu}\theta(t) \; \langle [J^\rho_a(t,\vec{x}), \, T^{\mu\nu}(0)] \rangle \, ,$$

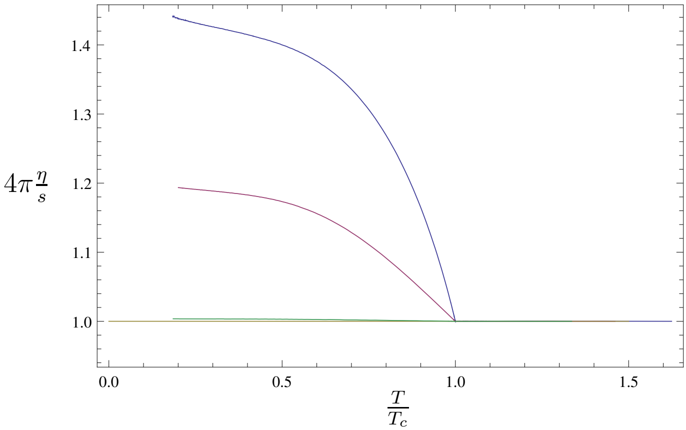
$$G^{\mu,\nu}_{a,b}(\omega,\vec{k}) = \int \mathrm{d}^4x \; \mathrm{e}^{-ik_\mu x^\mu}\theta(t) \; \langle [J^\mu_a(t,\vec{x}), J^\nu_b(0)] \rangle \, .$$

Relation between Green's function and transport

independent Green's functions	related transport property
$G^{yz,yz}$	shear viscosity η_{yz}
$G^{x\perp,x\perp}$	shear viscosity $\eta_{x\perp}$
$G^{x\perp,\perp}_{\ \ \ \ \ \ i}$	flexoelectric effect
$G^{\perp,\perp}_{i,j}$	?
$G^{\perp,\perp}_{3,3}$	conductivity $\sigma^{\perp\perp}$ thermoelectric effect perpendicular to condensate
$G^{x,x}_{3,3}$	conductivity σ^{xx} thermoelectric effect parallel to condensate
$G^{x,x}_{i,j}$, $G^{xx,x}_{\ \ \ \ i}$ and $G^{xx,xx}$	?

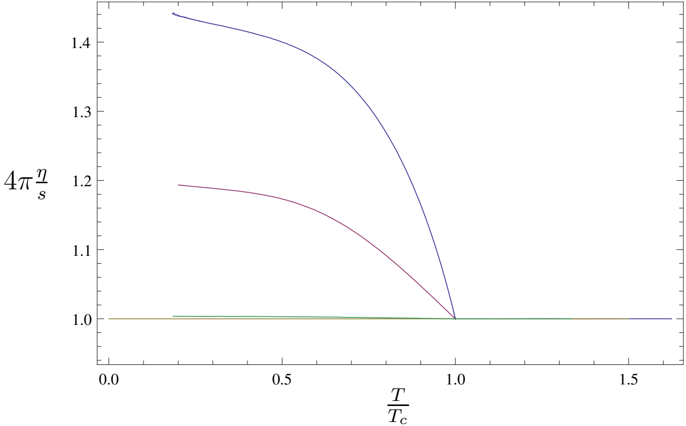
with $i, j \in \{1, 2\}$

Non-universal $\frac{\eta_{x\perp}}{s}$ for $\alpha < \alpha_c$



colour coding: $\frac{\eta_{yz}}{s} = \frac{1}{4\pi}$ and $\frac{\eta_{x\perp}}{s}$: $\alpha_c > \alpha_1 > \alpha_2 > \alpha_3$

Non-universal $\frac{\eta_{x\perp}}{s}$ for $\alpha < \alpha_c$



$$1 - 4\pi\frac{\eta_{x\perp}}{s} \propto \left(1 - \frac{T}{T_c}\right)^{\beta} \quad \text{with} \quad \beta = 1.00 \pm 3\%$$

confirmed analytically by Basu and Oh for small α and small condensate

1109.4592

Thermoelectric Effect

- Gauge/Gravity dictionary:
  $E_\perp = \mathrm{i}\omega \left((a^3_\perp)^b_0 + \mu (\Psi_t)^b_0 \right)$ and $-\frac{\nabla_\perp T}{T} = \mathrm{i}\omega (\Psi_t)^b_0$ with $\Psi_t = g^{\perp\perp} h_{t\perp}$

- field theory (Thermoelectric effect):

$$\begin{pmatrix} \langle J^\perp \rangle \\ \langle Q^\perp \rangle \end{pmatrix} = \begin{pmatrix} \sigma^{\perp\perp} & T\alpha^{\perp\perp} \\ T\alpha^{\perp\perp} & T\bar{\kappa}^{\perp\perp} \end{pmatrix} \begin{pmatrix} E_\perp \\ -(\nabla_\perp T)/T \end{pmatrix}$$

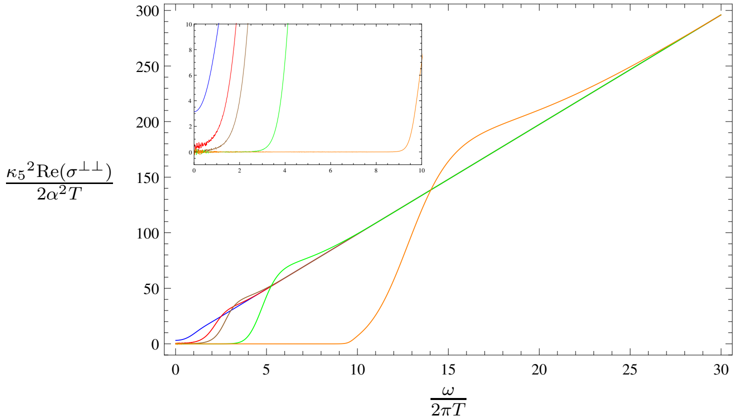
  with $\langle Q^\perp \rangle = \langle T^{t\perp} \rangle - \mu \langle J^\perp \rangle$

- identification with Green's functions:

$$\sigma^{\perp\perp} = -\frac{\mathrm{i} G^{\perp,\perp}_{3,3}}{\omega}$$

$$T\alpha^{\perp\perp} = \frac{\mathrm{i}}{\omega} \langle \mathcal{J}^t_3 \rangle - \mu \sigma^{\perp\perp} \quad , \quad T\bar{\kappa}^{\perp\perp} = \frac{\mathrm{i}}{\omega} \langle \mathcal{T}_{tt} \rangle + \mu^2 \sigma^{\perp\perp}$$

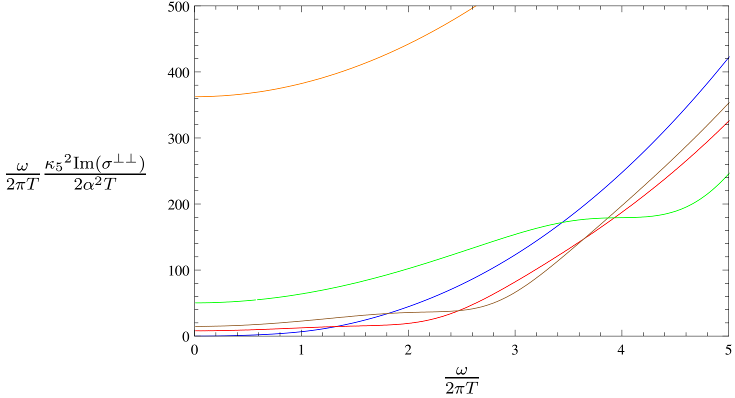
Electrical conductivity $\sigma^{\perp\perp}$ at $\alpha < \alpha_c$



colour coding:

$T = \infty > T = 1.00\,T_c > T = 0.88\,T_c > T = 0.50\,T_c > T = 0.19\,T_c$

$\omega \mathrm{Im}(\sigma^{\perp\perp})$ at $\alpha < \alpha_c$



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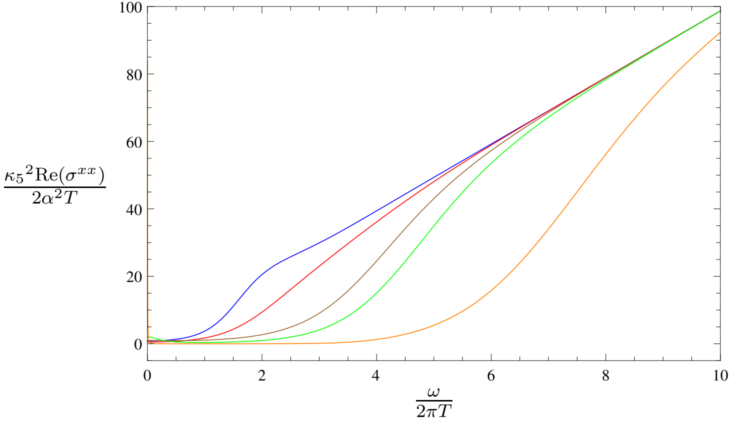
Electrical conductivity σ^{xx} at $\alpha < \alpha_c$



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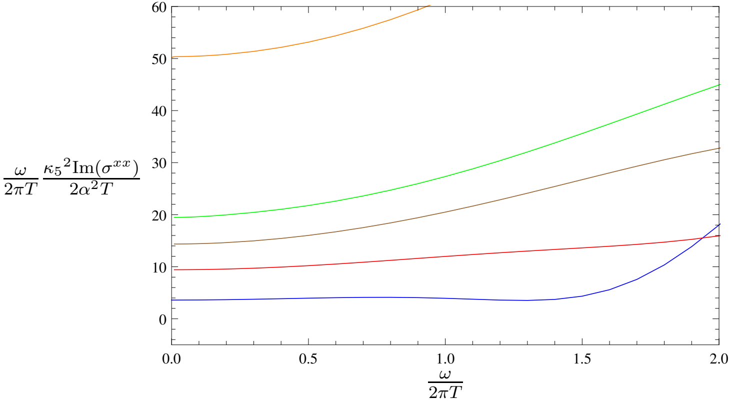
$T = 1.63\,T_c > T = 0.98\,T_c > T = 0.88\,T_c > T = 0.78\,T_c > T = 0.50\,T_c$

$\omega \mathrm{Im}(\sigma^{xx})$ at $\alpha < \alpha_c$



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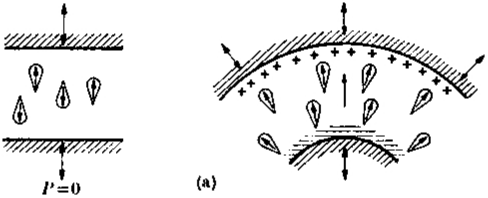
$T = 1.63\,T_c > T = 0.98\,T_c > T = 0.88\,T_c > T = 0.78\,T_c > T = 0.50\,T_c$

Flexoelectric effect

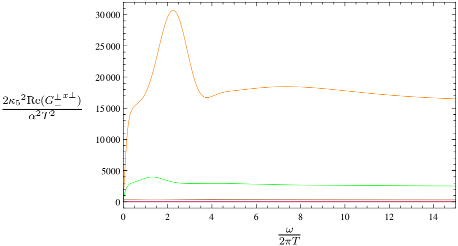
- strain leads to effective polarization
- electric field leads to stress

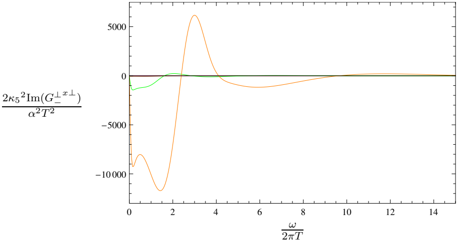


de Gennes

- our system: coupling between flavour fields and strain

$G_{-}^{\perp,x\perp}$ for $\alpha = 0.316$





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$T = \infty > T = 3.02\,T_c > T = 1.00\,T_c > T = 0.88\,T_c > T = 0.50\,T_c$

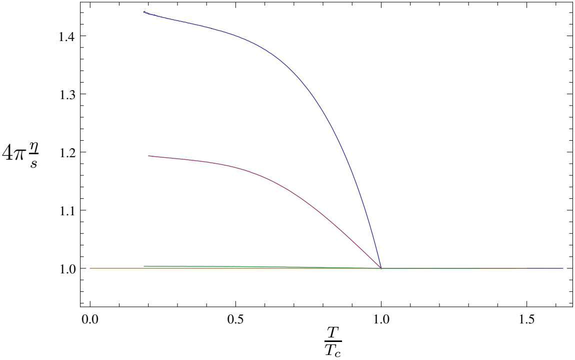
Conclusion

- holographic hydrodynamics: useful tool to compute transport coefficients in strongly coupled theories

- p-wave superfluid with backreaction $\Rightarrow$ two shear modes:

$$\frac{\eta_{yz}}{s} = \frac{1}{4\pi}$$

$$\frac{\eta_{x\perp}}{s}(T) \geq \frac{1}{4\pi}$$



- flexoelectric effect, thermoelectric effect

Open questions...

- Can we map holographic p-wave superfluids to systems found in nature?
- How can we change our setup to make it more realistic?

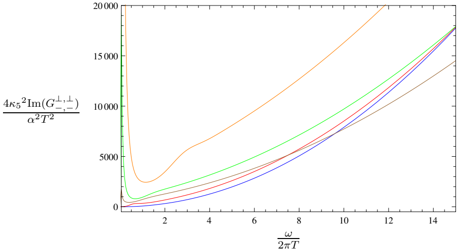
Thank you!

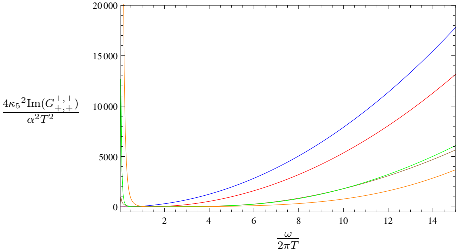
References: 1011.5912 and 1110.0007

$\mathrm{Im}(G^{\perp,\perp}_{\mp,\mp})$ for $\alpha = 0.316$

$$\begin{pmatrix} G^{\perp,\perp}_{+,+} & G^{\perp,\perp}_{+,-} & G_{+}^{\perp\ x\perp} \\ G^{\perp,\perp}_{-,+} & G^{\perp,\perp}_{-,-} & G_{-}^{\perp\ x\perp} \\ G^{x\perp\ \perp}_{\ \ +} & G^{x\perp\ \perp}_{\ \ -} & -\langle T_{xx}\rangle - \mathrm{i}\omega\eta_{x\perp} \end{pmatrix}$$

$\mathrm{Im}(G^{\perp,\perp}_{\mp,\mp})$ for $\alpha = 0.316$





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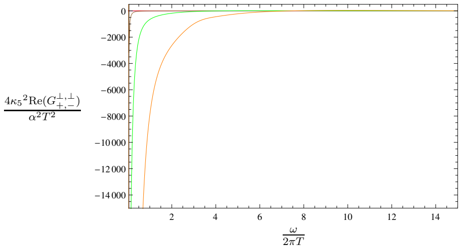
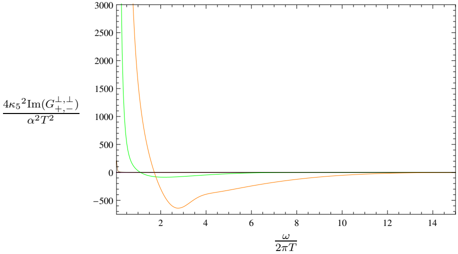
$T = \infty > T = 3.02\,T_c > T = 1.00\,T_c > T = 0.88\,T_c > T = 0.50\,T_c$

$G_{+,-}^{\perp,\perp}$ for $\alpha = 0.316$

$$\begin{pmatrix} G_{+,+}^{\perp,\perp} & G_{+,-}^{\perp,\perp} & G_{+}^{\perp\,x\perp} \\ G_{-,+}^{\perp,\perp} & G_{-,-}^{\perp,\perp} & G_{-}^{\perp\,x\perp} \\ G^{x\perp}{}_{+}^{\perp} & G^{x\perp}{}_{-}^{\perp} & -\langle T_{xx}\rangle - \mathrm{i}\omega\eta_{x\perp} \end{pmatrix}$$

$G_{+,-}^{\perp,\perp}$ for $\alpha = 0.316$





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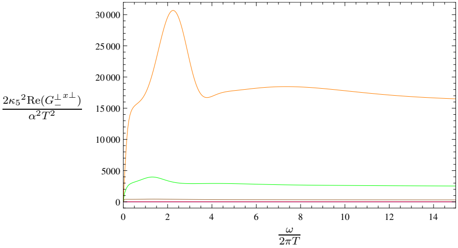
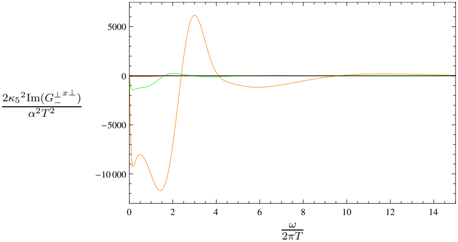
$G_{-}^{\perp\,x\perp}$ for $\alpha = 0.316$

$$\begin{pmatrix} G_{+,+}^{\perp,\perp} & G_{+,-}^{\perp,\perp} & G_{+}^{\perp\,x\perp} \\ G_{-,+}^{\perp,\perp} & G_{-,-}^{\perp,\perp} & G_{-}^{\perp\,x\perp} \\ G^{x\perp}{}_{+}^{\perp} & G^{x\perp}{}_{-}^{\perp} & -\langle T_{xx}\rangle - \mathrm{i}\omega\eta_{x\perp} \end{pmatrix}$$

$G_{-}^{\perp x \perp}$ for $\alpha = 0.316$





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