Structure of Non-geometric Strings

Ralph Blumenhagen

Max-Planck-Institut für Physik, München

Activities

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String/M Theory

Basic structures

- Non-geometric/ asymmetric strings
- Non-associativity
- double field theory
- Lie algebroids

String Pheno

- compactifications
- F-theory GUTs
- LEEA=SUGRA
- moduli stabilization
- susy breaking
  soft terms

ST as a Tool

- AdS-CFT
  finite T/black hole
  AdS-CMP
- Scattering ampl.
  unitarity methods
  N=8 SUGRA finite

String Theory Research at MPP

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Johanna Erdmenger: AdS-CFT correspondence and applications to finite temperature effects

Stephan Stieberger: String scattering amplitudes with applications to field theory perturbation theory

Thomas Grimm(MPG Independent Research group): String compactifications, 6D superconformal field theories, mathematical structures of F-theory

Dieter Lüst, Ralph Blumenhagen: String Phenomenology, Non-geometric strings

Non-geometric Strings

Non-geometric Strings

String theory is described by 2D non-linear sigma model

$$\mathcal{S} = \frac{1}{2\pi\alpha'} \int_\Sigma d^2 z \left(G_{ab} + B_{ab} \right) \partial X^a \, \overline{\partial} X^b + \dots \; ,$$

where conformal invariance provides the string equations of motion, which are captured by the effective action

$$S = \frac{1}{2\kappa^2} \int d^n x \sqrt{-|G|} \, e^{-2\phi} \Big(R - \tfrac{1}{12} H_{abc} \, H^{abc} + 4 \partial_a \phi \, \partial^a \phi \Big) \, .$$

Leading order: Einstein gravity with a two-form B_{ab} and a scalar Φ.
There exist conformal field theories which cannot be identified with such simple large radius geometries.

Non-geometric Strings

Non-geometric Strings

These are left-right asymmetric like asymmetric orbifolds.
Applying T-duality leads to the chain of fluxes (Shelton, Taylor, Wecht, hep-th/0508133)

$$H_{abc} \leftrightarrow f_{ab}{}^{c} \leftrightarrow Q_a{}^{bc} \overset{?}{\leftrightarrow} R^{abc} ,$$

Q and R are non-geometric fluxes. What is the nature of R-flux?
Does there exist an effective field theory describing this non-geometric regime of string theory?

Classical non-associativity

Classical non-associativity

Given a VEV to the fluxes, a CFT analysis showed (Bhg, Plauschinn, arXiv:1010.1263), (Lüst, arXiv:1010.1361), (Bhg, Deser, Lüst, Plauschinn, Rennecke, arXiv:1106.0316)

$$[x^a, x^b] = \oint_{C_x} Q_c{}^{ab} dy^c \,, \qquad [x^a, x^b, x^c] = R^{abc} \,.$$

Non-commutativity: Wilson line of Q-flux

Non-associativity: local R-flux

This would result from the quantization of a classical quasi-Poisson structure

$$\{f, g\} = \beta^{ab} \, (\partial_a f) \, (\partial_b g) \,,$$

where $f, g \in \mathcal{C}^\infty(M)$.

Non-geometric Strings

Non-geometric Strings

In general, this bracket does not satisfy the Jacobi identity but one finds

$$\{\{f,g\},h\} + \text{cycl.} = R^{abc}\,(\partial_a f)\,(\partial_b g)\,(\partial_c h)\;.$$

with non-geometric R-flux is given by

$$R^{abc} = 3\,\beta^{[\underline{a}m}\,\partial_m \beta^{\underline{bc}]}$$

The non-geometric flux Q-flux is defined as $Q_c{}^{ab} = \partial_c \beta^{ab}$. The bi-vector β defines a new derivative operator as

$$D^a = \beta^\sharp(e^a) = \beta^{ab}\partial_b\;.$$

Field theory description

Field theory description

- Formulate a differential geometry for such a structure
- Identify symmetry principles
- Einstein-Hilbert type action
- Relation to the string effective action

Two approaches have been carried out:

- Double field theory (Hull, Zwiebach, arXiv:0904.4664) (Andriot, Hohm, Larfors, Lüst, Patalong, arXiv:1202.3060+1204.1979).
- Differential geometry of the Lie-algebroid $E^* = (T^*M, [\cdot,\cdot]^H_K, \rho = \beta^\sharp)$ (Bhg, Deser, Plauschinn, Rennecke, arXiv:1210.1591+1211.0030)

Symmetry: β-tensors

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A usual tensor transforms under an infinitesimal diffeomorphism as

$$\delta_X T^{a_1\ldots a_r}{}_{b_1\ldots b_s} = (L_X T)^{a_1\ldots a_r}{}_{b_1\ldots b_s} \ ,$$

with L_X the usual Lie derivative.

Definition: A tensor $T \in \Gamma\big((\otimes^r TM) \otimes (\otimes^s T^*M)\big)$ is called a β-tensor if for a one-form ξ it behaves as

$$\hat{\delta}_\xi T^{a_1\ldots a_r}{}_{b_1\ldots b_s} = \big(\hat{\mathcal{L}}_\xi T\big)^{a_1\ldots a_r}{}_{b_1\ldots b_s} \ ,$$

Symmetry: β-tensors

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where

$$
\begin{aligned}
\big(\hat{\mathcal{L}}_\xi T\big)^{a_1\ldots a_r}{}_{b_1\ldots b_s} = \quad & \xi_m\, D^m T^{a_1\ldots a_r}{}_{b_1\ldots b_s} \\
& - \sum_{i=1}^{s} \big(D^m \xi_{b_i} + \xi_n\, Q_{b_i}{}^{mn}\big)\, T^{a_1\ldots a_r}{}_{b_1\ldots b_{i-1}\, m\, b_{i+1}\ldots b_s} \\
& + \sum_{i=1}^{r} \big(D^{a_i} \xi_m + \xi_n\, Q_m{}^{a_i n}\big)\, T^{a_1\ldots a_{i-1}\, m\, a_{i+2}\ldots a_r}{}_{b_1\ldots b_s}\,.
\end{aligned}
$$

Require:

- The metric on T^*M, $\hat{g}^{ab}$, should be a β-tensor
- $D^a\phi$ should be a β-tensor

Covariant derivative

Covariant derivative

Introduce a covariant derivative so that

$$\hat{\nabla}^a \eta_b = D^a \eta_b + \hat{\Gamma}_b{}^{am}\, \eta_m$$

is a β-tensor.
The torsion $T \in \Gamma(\wedge^2 TM \otimes T^*M)$ is now defined as

$$\hat{T}(\xi, \eta) = \hat{\nabla}_\xi\, \eta - \hat{\nabla}_\eta\, \xi - [\xi, \eta]_K^H \ ,$$

with the twisted Koszul bracket

$$[\xi, \eta]_K = L_{\beta^\sharp(\xi)} \eta - \iota_{\beta^\sharp(\eta)}\, d\xi - \iota_{\beta^\sharp \eta} \iota_{\beta^\sharp \xi} H \ ,$$

where the Lie derivative on forms is given by
$L_X = \iota_X \circ d + d \circ \iota_X$.

Curvature

Curvature

The Levi-Civita connection is

$$\hat{\Gamma}_c{}^{ab} = \frac{1}{2}\,\hat{g}_{cm}\big(D^a\hat{g}^{bm} + D^b\hat{g}^{am} - D^m\hat{g}^{ab}\big) - \hat{g}_{cm}\,\hat{g}^{(a|n}\,\mathcal{Q}_n{}^{|b)m} + \frac{1}{2}\,\mathcal{Q}_c{}^{ab}\,,$$

The curvature is defined as

$$\hat{R}(\eta,\chi)\xi = \big[\hat{\nabla}_\eta, \hat{\nabla}_\chi\big]\xi - \hat{\nabla}_{[\eta,\chi]^H_K}\,\xi\ ,$$

The Ricci tensor is defined by

$$\hat{R}^{ab} = \hat{R}_m{}^{amb}\,,$$

and the Ricci scalar as $\hat{R} = \hat{g}_{ab}\hat{R}^{ab}$.

Symplectic gravity action

Symplectic gravity action

We propose the following bi-invariant Einstein-Hilbert action coupled to a dilaton ϕ and R-flux Θ^{abc}

$$\hat{S} = \frac{1}{2\kappa^2} \int d^n x \sqrt{-|\hat{g}|} \left|\beta^{-1}\right| e^{-2\phi} \Big(\hat{R} - \frac{1}{12} \Theta^{abc} \Theta_{abc} + 4 \hat{g}_{ab} D^a \phi D^b \phi \Big) .$$

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For the field redefinition

$$B = \hat{\beta}^{-1} \, , \qquad G = -\hat{\beta}^{-1} \, \hat{g} \, \hat{\beta}^{-1} \, ,$$

one finds the relation

$$S\big(\, G(\hat{g}, \hat{\beta}), \, B(\hat{g}, \hat{\beta}), \, \phi \, \big) = \hat{S}\big(\hat{g}, \hat{\beta}, \phi\big) \, .$$

Conclusions

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Research program on non-geometric string theory, motivated by

- Asymmetric CFTs
- T-duality of geometric configurations

Taken some initial steps

- CFT analysis of R-flux backgrounds $\rightarrow$ NCA structure
- Differential Geometry of Lie-algebroids

Future

- Solutions to clarify relation to DFT
- Supersymmetrization
- Deformation quantization of quasi-Poisson structures.

MPP people involved

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DFT: Dieter Lüst, David Andriot, Andre Betz, Peter Patalong

Lie-algebroids: Ralph Blumenhagen, Andreas Deser, Felix Rennecke, Christian Schmid

String Phenomenology: Xin Gao, Daniela Herschmann, Pramod Shukla, Zhang Xu (not covered in this talk),

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Scattering amplitudes: Stephan Stieberger, Georg Puhlfuerst

Group of Thomas Grimm

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Raffaele Savelli, Noppadol Mekareeya, Ioannis Florakis Thomas George Pugh, Federico Bonetti, Matthias Weissenbacher , Jan Keitel, Andreas Kapfer