On predictions from spontaneously broken flavor symmetries

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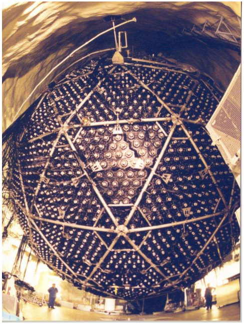
Based on:
M.-C. Chen, M. Fallbacher, M. Ratz, C. S.,
Phys. Lett. **B** (2012), [arXiv: 1208.2947]

Neutrinos have mass

Missing solar neutrinos at the Subury Neutrino detector in the 1960s.



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Neutrinos oscillate, therefore, they have mass.

Open questions:

- **origin of neutrino masses,**
- **mixing pattern**
  i.e. flavor structure.

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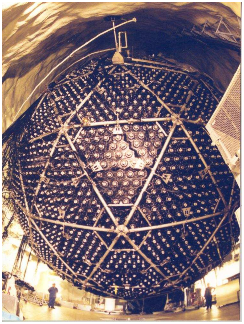

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Neutrinos oscillate, therefore, they have mass.

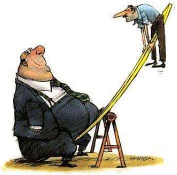
Flavor symmetries

Explain mixing patterns of neutrinos and might be window to new physics.

Outline

Neutrino oscillations

- Neutrinos get mass through higher dimensional operators, seesaw–mechanism . . .



therookiecynic.wordpress.com

- Cannot diagonalize neutrino mass matrix and charged lepton Yukawa matrix simultaneously:

$$V_{\nu,L}^T \, m_\nu \, V_{\nu,L} = \mathrm{diag}(m_1, m_2, m_3)$$
$$V_{e,R}^\dagger \, m_e \, V_{e,L} = \mathrm{diag}(m_e, m_\mu, m_\tau)$$

$$\boxed{U_{\mathrm{MNS}} = V_{e,L}^\dagger V_{\nu,L}}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} .$$

Parametrization of the mixing matrix

The mixing matrix can be described in standard parametrization, where $c_{xy} \equiv \cos(\theta_{xy})$ and $s_{xy} \equiv \sin(\theta_{xy})$.

$$
\begin{aligned}
U &= \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-\mathrm{i}\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{\mathrm{i}\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{\mathrm{i}\alpha_1/2} & 0 & 0 \\ 0 & e^{\mathrm{i}\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix} .
\end{aligned}
$$

- Three mixing angles θ_{23}, θ_{13} and θ_{12}.
- One CP–violating Dirac phase δ and two Majorana phases $\alpha_{1,2}$.

Known experimentally:

- existence (size) of mixing angles.
- two mass differences $\Delta m^2_{12}, \Delta m^2_{23}$.

Experimental results for the mixing angles

For early results, tri–bi–maximal mixing (TBM) seemed to be a good fit.
Harrison et al. [2002]

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \Rightarrow \begin{array}{l} \theta_{12} = 35.3^\circ \\ \theta_{23} = 45^\circ \\ \theta_{13} = 0^\circ \end{array}$$

	θ_{12}	θ_{23}	θ_{13}
Data 2005	$\left(34.0^{+1.3}_{-1.6}\right)^\circ$	$> 35.8^\circ$	$\left(6.5^{+2.7}_{-6.5}\right)^\circ$

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	2012	$\left(33.6^{+1.0}_{-1.0}\right)^\circ$	$\left(38.4^{+1.4}_{-1.2}\right)^\circ$	$\mathbf{\left(8.93^{+1.1}_{-1.0}\right)^\circ \neq 0}$

Beringer et al. [2012] , Fogli et al. [2012]

Supersymmetric flavor model building

The general idea:

- Horizontal symmetries relating different families with each other.
- Lepton and quark mixing patterns arise from flavor symmetry G_F. G_F can be continuous $(U(1), SU(2) \dots)$ or discrete $(A_4,\ T', S_3, S_4 \dots)$.
- Assign MSSM fields irreducible representations under G_F and introduce SM singlets in non–trivial G_F representations $\Rightarrow$ **'flavons'**.

Mixing pattern

Spontaneously break G_F by assigning VEVs to flavon fields:

"Correct" VEV alignment $\Rightarrow$ desired mixing pattern.

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In the following:

TBM mixing in the lepton sector based on a SUSY A_4 model.

Ingredients for an A_4 example

A_4 has ...

- four irreducible representations: $\mathbf{1}, \mathbf{1}', \mathbf{1}''$ and $\mathbf{3}$,
- multiplication law: $\mathbf{3} \otimes \mathbf{3} \;=\; \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'' \oplus \mathbf{3}_\mathrm{s} \oplus \mathbf{3}_\mathrm{a}$, e.g.

$$(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{3}_\mathrm{a}} \;=\; \mathrm{i}\sqrt{\frac{3}{2}}\begin{pmatrix} a_2\, b_3 - a_3\, b_2 \\ a_1\, b_2 - a_2\, b_1 \\ a_3\, b_1 - a_1\, b_3 \end{pmatrix} .$$

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For an A_4 model giving TBM one needs Altarelli and Feruglio [2005]

- three flavon fields: two A_4 triplets Φ_ν, Φ_e and the singlet ξ.
- The **left–handed lepton doublets** in an A_4 **triplet** $L = (L_e,\, L_\mu,\, L_\tau)^T$.
- The **right–handed charged leptons**, e_R, μ_R and τ_R, transforming as **singlets** $\mathbf{1}, \mathbf{1}''$, and $\mathbf{1}'$, respectively.

The superpotential

In the superpotential the flavons couple to the MSSM fields

$$\begin{aligned} \mathscr{W}_\nu &= \frac{\lambda_1}{\Lambda\,\Lambda_\nu}\left\{[(L\,H_u)\otimes(L\,H_u)]_{\mathbf{3}_s}\otimes\Phi_\nu\right\}_{\mathbf{1}} + \frac{\lambda_2}{\Lambda\,\Lambda_\nu}\,[(L\,H_u)\otimes(L\,H_u)]_{\mathbf{1}}\,\xi\,, \\ \mathscr{W}_e &= \frac{h_e}{\Lambda}\,(\Phi_e\otimes L)_{\mathbf{1}}\,H_d\,e_\mathrm{R} + \frac{h_\mu}{\Lambda}\,(\Phi_e\otimes L)_{\mathbf{1}'}\,H_d\,\mu_\mathrm{R} + \frac{h_\tau}{\Lambda}\,(\Phi_e\otimes L)_{\mathbf{1}''}\,H_d\,\tau_\mathrm{R}\,, \end{aligned}$$

with

- Λ being the cut–off scale,
- Λ_ν being the seesaw–scale,
- $h_{e,\mu,\tau}$ and $\lambda_{1,2}$ being constants.

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Spontaneous symmetry breaking

A_4 broken by flavon VEVs:

$$\begin{aligned}\langle\Phi_\nu\rangle &= (v,v,v)^T\\ \langle\Phi_e\rangle &= (v',0,0)^T\\ \langle\xi\rangle &= w\,.\end{aligned}$$

Electroweak symmetry broken by inserting Higgs VEVs

$$\begin{aligned}\langle H_u\rangle &= (0,v_u)^T\\ \langle H_d\rangle &= (v_d,0)^T\,.\end{aligned}$$

Lepton masses

After inserting all VEVs

$$\mathscr{W}_\nu = \frac{1}{2} L^T \overbrace{\begin{pmatrix} a+2d & -d & -d \\ -d & 2d & a-d \\ -d & a-d & 2d \end{pmatrix}}^{\boldsymbol{m_\nu}} L \quad \text{with} \quad \begin{array}{l} a = 2\lambda_2 \frac{v_u^2}{\Lambda_\nu} \frac{w}{\Lambda} \\ d = \sqrt{2}\lambda_1 \frac{v_u^2}{\Lambda_\nu} \frac{v}{\Lambda} \end{array}$$

$$\mathscr{W}_e = (e_R, \mu_R, \tau_R) \underbrace{\begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}}_{\boldsymbol{m_e}} L \quad \text{with} \quad y_{e,\mu,\tau} = h_{e,\mu,\tau} \frac{v'}{\Lambda} .$$

Tri–bi–maximal mixing

m_e is already diagonal, so we need to diagonalize m_ν:

$$V_{\nu,L}^T\, m_\nu\, V_{\nu,L} \;=\; \mathrm{diag}(m_1, m_2, m_3) \quad \text{with} \quad V_{\nu,L} \;=\; \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Since m_e diagonal $\Rightarrow$ $\boldsymbol{U_{\mathrm{MNS}} = V_{e,L}^\dagger V_{\nu,L} = V_{\nu,L} = U_{\mathrm{TBM}}}$

$$\Rightarrow \theta_{12} = 35.3^\circ, \qquad \theta_{23} = 45^\circ, \qquad \theta_{13} = 0^\circ\ .$$

Good until measurement of non–vanishing θ_{13} by MINOS, Super-K, T2K, Double Chooz, Reno and Daya Bay.

Adamson et al. [2011] ,Abe et al. [2011a] ,Abe et al. [2011b] ,Abe et al. [2012] ,An et al. [2012] ,Ahn et al. [2012]

Can one fix TBM?

Measured mixing angles do not fit TBM

$$\theta_{13}^{\mathrm{exp}} - \theta_{13}^{\mathrm{TBM}} \approx 9^\circ\ .$$

Fix TBM or start fresh?

- Include higher order corrections from the superpotential?
- Different symmetry group/representations?
- Other effects?
- How sensible is flavor model building?

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- **Other effects?**
- **How sensible is flavor model building?**

Address last two points with corrections from the **Kähler potential**.

Kähler potential

The supersymmetric Lagrangian is given by

$$\mathscr{L} \supset \int \mathrm{d}^2\theta\, \mathrm{d}^2\theta^\dagger\, K[\Psi, \Psi^*] + \left(\int \mathrm{d}^2\theta\, \mathscr{W}(\Psi) + \text{c.c.} \right) + \ldots$$

The **Kähler potential K** is a real non–holomorphic function and at tree level it is **canonical** $\boldsymbol{K = \Psi^i \Psi^*_i}$, i.e. in the leptonic sector:

$$K_{\text{canonical}} = \left(L^f\right)^\dagger \delta_{fg}\, L^g + \left(R^f\right)^\dagger \delta_{fg}\, R^g \; .$$

After integrating over superspace coordinates, proper kinetic terms i.e.

$$\mathscr{L} \supset -\partial^\mu \phi^* \, \partial_\mu \phi + \mathrm{i}\, \psi^\dagger \, \overline{\sigma}^\mu \, \partial_\mu \psi + \ldots$$

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Higher order corrections

What if one has **off–diagonal terms**, i.e. $K_{\text{canonical}} + \Delta K$ with

$$\Delta K = \left(L^f\right)^\dagger (\Delta K_L)_{fg}\, L^g + \left(R^f\right)^\dagger (\Delta K_R)_{fg}\, R^g\ ?$$

Kähler corrections for left–handed leptons

- Assume the corrections to be $\Delta K_L = -2\,x\,P$, with **infinitesimal *x*** and **Hermitian matrix *P***. Then,

$$K_L \;=\; L^\dagger\,(\mathbb{1} - 2\,x\,P)\;L\;.$$

- Rotate to get canonically normalized fields

$$L' \to L = (1 - x\,P)\;L'\;.$$

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- Field redefinition induces **change in the superpotential**

$$\begin{aligned}
\mathscr{W}_\nu \;&=\; \frac{1}{2}\,(L')^T\;m_\nu\,L' \\
&=\; \frac{1}{2}\,L^T\,\left(1 + x\,P^T\right)\,m_\nu\,(1 + x\,P)\;L \\
&=\; \frac{1}{2}\,L^T\,\left[m_\nu + x\,\left(P^T\,m_\nu + m_\nu\,P\right)\right]\;L\,.
\end{aligned}$$

Change in mixing parameters

- Now we have a x–dependent neutrino mass matrix

$$m_\nu(x) \;=\; m_\nu \;+ x \left(P^T\, m_\nu + m_\nu\, P\right) \;.$$

- Differential equation:

$$\frac{\mathrm{d}m_\nu(x)}{\mathrm{d}x} \;=\; P^T\, m_\nu + m_\nu\, P \;.$$

$\Rightarrow$ Differential equation for mixing parameters, i.e. angles, phases . . .

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$\Rightarrow$ Differential equation for mixing parameters, i.e. angles, phases ...

- This has same structure as RGE for neutrino mass operator.
  Antusch et al. [2003]
- Analytically solvable for all parameters.

Analytic computation

Formulae for change in mixing angles and phases available.

Where do these corrections come from?

Higher order Kähler terms with flavons

$$\Delta K \supset \Delta K_{\mathbf{X}}(\Phi) = \frac{1}{\Lambda^2} (L\,\Phi)^\dagger_{\mathbf{X}} \, (L\,\Phi)_{\mathbf{X}} \quad \xrightarrow{\langle\Phi\rangle} \quad L^\dagger \, \Delta K_L \, L$$

X is a representation of the flavor group G_F allowed by multiplication rules.

- These terms cannot be forbidden by any (conventional) symmetry.
- **All discrete flavor symmetries have them.**
- Structure of ΔK_L dependend on group G_F, flavon VEV $\langle\Phi\rangle$ and representation **X**.
- **Contributions can be sizable** $\Rightarrow$ see with help of A_4 example.

Back to our A_4 example

Returning to previous example:

- Based on A_4, four irreducible representations: $\mathbf{1}, \mathbf{1'}, \mathbf{1''}$ and $\mathbf{3}$, leading to six possible contractions of the form $(L \otimes \Phi)^\dagger_{\boldsymbol{X}} (L \otimes \Phi)_{\boldsymbol{X}}$,
- two flavon triplets $\boldsymbol{\Phi_\nu}$ and $\boldsymbol{\Phi_e}$.

$\Rightarrow$ **12 possible corrections.**

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Five independent corrections

$\langle \boldsymbol{\Phi_e} \rangle = (\boldsymbol{v}', \mathbf{0}, \mathbf{0})$ leads to:

$$P_{\text{I}} = \text{diag}(1,0,0) \ , \quad P_{\text{II}} = \text{diag}(0,1,0) \quad \text{and} \quad P_{\text{III}} = \text{diag}(0,0,1) \ ,$$

$\langle \boldsymbol{\Phi_\nu} \rangle = (\boldsymbol{v}, \boldsymbol{v}, \boldsymbol{v})$ leads to:

$$P_{\text{IV}} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad \text{and} \quad P_{\text{V}} = \begin{pmatrix} 0 & \mathrm{i} & -\mathrm{i} \\ -\mathrm{i} & 0 & \mathrm{i} \\ \mathrm{i} & -\mathrm{i} & 0 \end{pmatrix} .$$

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Corrections from P_V

The correction from

$$P_\mathsf{V} = \begin{pmatrix} 0 & \mathrm{i} & -\mathrm{i} \\ -\mathrm{i} & 0 & \mathrm{i} \\ \mathrm{i} & -\mathrm{i} & 0 \end{pmatrix},$$

due to the term

$$\begin{aligned} \Delta K &= \frac{\kappa_\mathsf{V}}{\Lambda^2}\left[(L\,\Phi_\nu)^\dagger_{\mathbf{3}_\mathrm{a}}\,(L\,\Phi_\nu)_{\mathbf{3}_\mathrm{s}} + \mathrm{h.c.}\right] \\ &= \kappa_\mathsf{V}\cdot\frac{v^2}{\Lambda^2}\cdot 3\sqrt{3}\cdot(L^f)^\dagger\,(P_\mathsf{V})_{fg}\,(L^g)\,. \end{aligned}$$

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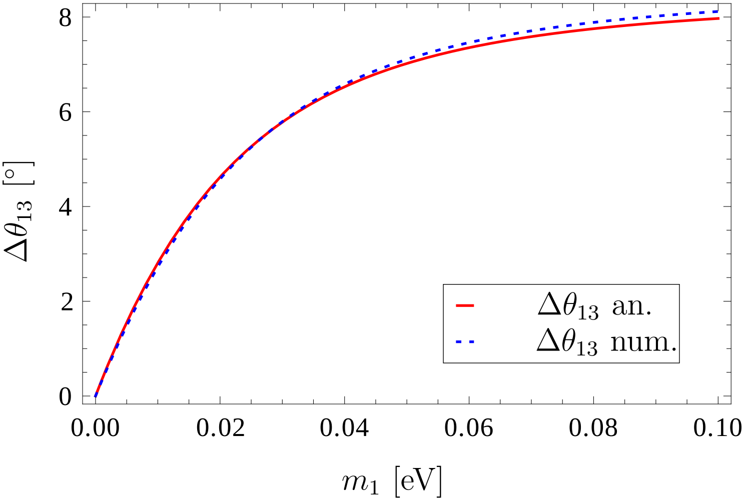
Analytic formulae for change in θ_{13}

Starting from TBM, we get

$$\Delta\theta_{13} \simeq \kappa_{\mathrm{V}} \cdot \frac{v^2}{\Lambda^2} \cdot 3\sqrt{6}\,\frac{m_1}{m_1 + m_3}$$ with m_i being neutrino masses .

Change in θ_{13}



Interpretation

P_V creates substantial deviation from TBM angles:

$\Rightarrow$ For large m_1 one gets $\boldsymbol{\Delta\theta_{13} \approx 8.42^\circ}$.

$\Rightarrow$ Other angles do not change much.

Problem with mixing angles solved?

Interpretation

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Other contributions from $P_{\mathrm{I},\ldots,\mathrm{IV}}$ incompatible with experiments, e.g. P_{IV} shifts θ_{12} and θ_{23} away from their best fit value.

How good are discrete flavor models?

Such corrections are always there and have rarely been considered.

- Several flavor groups with different VEV alignment have been used.
- Very fine–tuned models which try to describe mixing angles have not considered these effects.

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Implications for flavor model building

Positive view	**Negative view**
Simple models might work after all.	Successful models get spoiled.

Way out:

- Build models which include all effects and still work.
- Understand Kähler terms better. Possibility to control effects?

Conclusions

- **Neutrinos have mass** and there is a mismatch between mass– and weak–eigenstate $\Rightarrow$ **neutrinos oscillate**.
- Mixing patterns can be described by discrete **flavor symmetries** and clever breaking schemes lead to correct mixing pattern.
- Contributions to the mixing from higher order terms in the **Kähler potential**. They are always there.
- The mixing angles experience **substantial changes** and one can describe these changes **analytically**.

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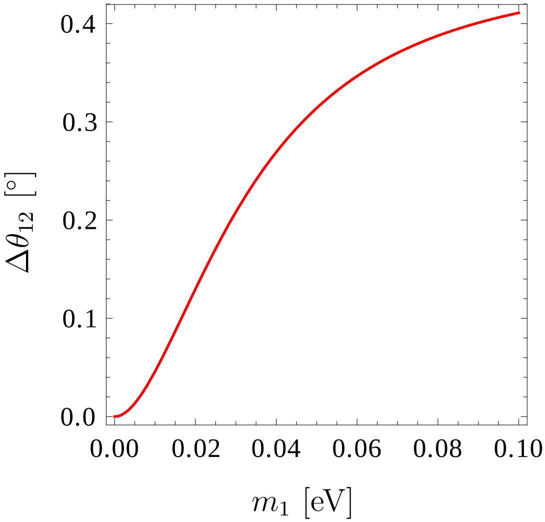
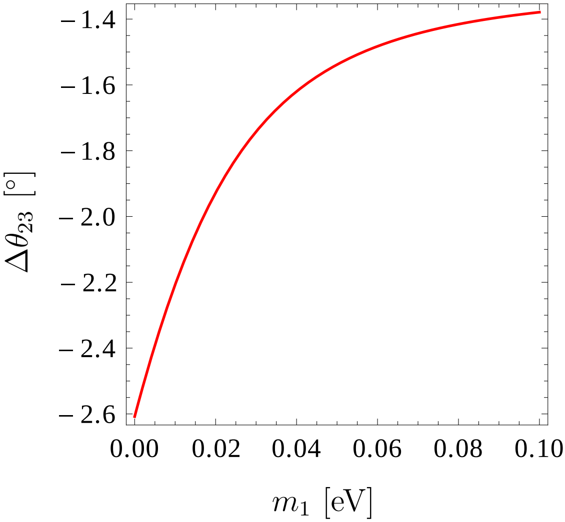
Save or destroy many models?

Need to understand/control terms from the Kähler potential in order to make predictions from flavor symmetries.

Change in θ_{12} from P_V



Change in θ_{23} from P_V



A_4 triplet multiplication:

$\mathbf{3} \otimes \mathbf{3} \;=\; \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'' \oplus \mathbf{3}_\mathrm{s} \oplus \mathbf{3}_\mathrm{a}$:

$$
\begin{aligned}
(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{1}} &= a_1\, b_1 + a_2\, b_3 + a_3\, b_2 \;, \\
(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{1}'} &= a_3\, b_3 + a_1\, b_2 + a_2\, b_1 \;, \\
(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{1}''} &= a_2\, b_2 + a_1\, b_3 + a_3\, b_1 \;, \\
(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{3}_\mathrm{s}} &= \frac{1}{\sqrt{2}} \begin{pmatrix} 2a_1\, b_1 - a_2\, b_3 - a_3\, b_2 \\ 2a_3\, b_3 - a_1\, b_2 - a_2\, b_1 \\ 2a_2\, b_2 - a_1\, b_3 - a_3\, b_1 \end{pmatrix} \;, \\
(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{3}_\mathrm{a}} &= \mathrm{i} \sqrt{\frac{3}{2}} \begin{pmatrix} a_2\, b_3 - a_3\, b_2 \\ a_1\, b_2 - a_2\, b_1 \\ a_3\, b_1 - a_1\, b_3 \end{pmatrix} \;.
\end{aligned}
$$

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