

# Quantum Gravity from a Different Perspective

**Felix Rennecke**  
Max-Planck-Institut für Physik

In collaboration with Ralph Blumenhagen , Andreas Deser  
and Erik Plauschinn



---

Max-Planck-Institut für Physik  
(Werner-Heisenberg-Institut)

# Outline

---

- **A conception of Quantum Geometry**
- **Quantum Gravity from String Theory**
- **T-duality, fluxes and “non-geometry”**
- **Framework for fluxes**
- **Strings at low energies**

**Based on arXiv**    **1106.0316 (with Dieter Lüst)**  
**1202.4934**  
**1205.1522**  
**1210.1591**  
**1211.0030**

# A conception of Quantum Gravity

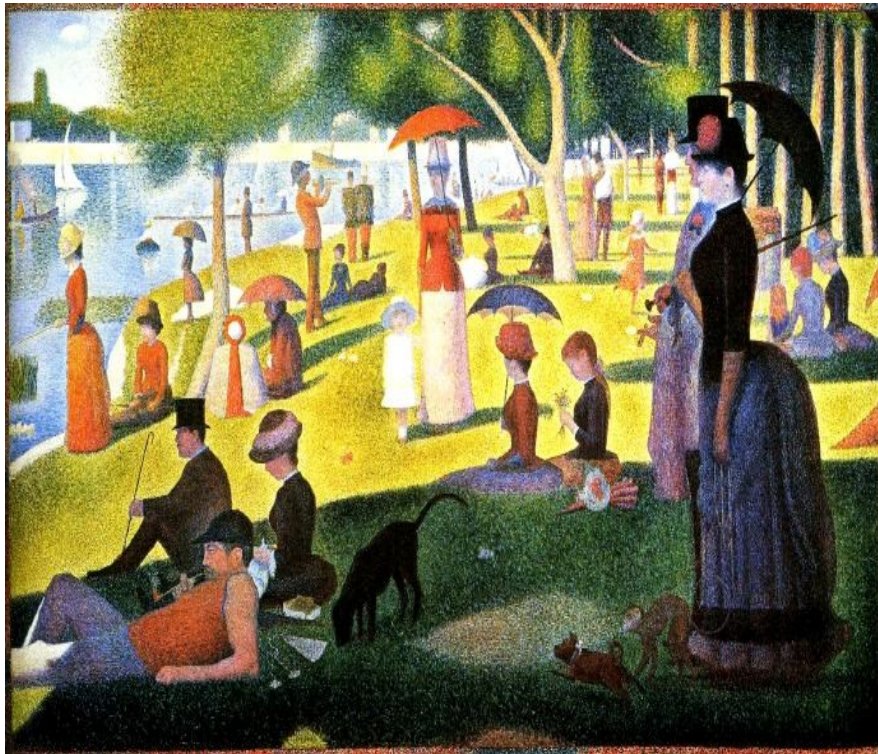
---

- general problem: **quantizing general relativity** gives a non-finite (**non-renormalizable**) theory
- at close range: **spacetime** is believed to be *fuzzy*

# A conception of Quantum Gravity

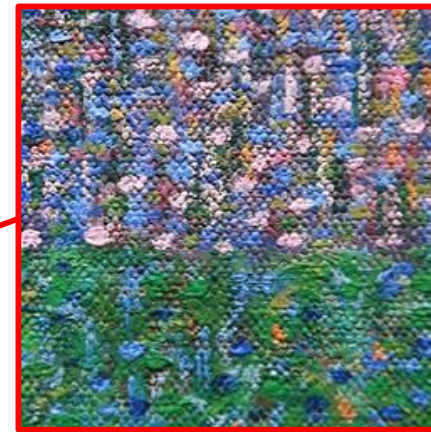
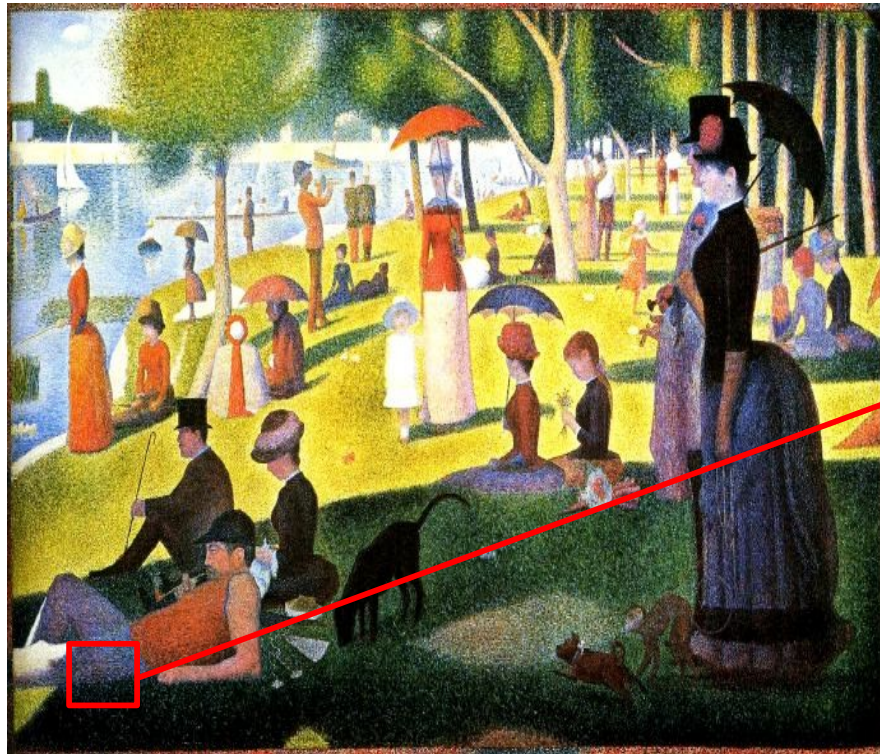
---

- general problem: **quantizing general relativity** gives a non-finite (**non-renormalizable**) theory
- at close range: **spacetime** is believed to be *fuzzy*



# A conception of Quantum Gravity

- general problem: **quantizing general relativity** gives a non-finite (**non-renormalizable**) theory
- at close range: **spacetime** is believed to be **fuzzy**



$$\Delta x \Delta x \geq \hbar$$

# A conception of Quantum Gravity

---

- general problem: **quantizing general relativity** gives a non-finite (**non-renormalizable**) theory
- at close range: **spacetime** is believed to be *fuzzy*



How to model this geometries?

$$\Delta x \Delta x \geq \hbar$$

# A conception of Quantum Gravity

---

- general problem: **quantizing general relativity** gives a non-finite (**non-renormalizable**) theory
- at close range: **spacetime** is believed to be *fuzzy*



How to model this geometries?

- **Non-commutative geometry** etc.

$$\Delta x \Delta x \geq \hbar$$

# A conception of Quantum Gravity

---

- general problem: **quantizing general relativity** gives a non-finite (**non-renormalizable**) theory
- at close range: **spacetime** is believed to be **fuzzy**



$$\Delta x \Delta x \geq \hbar$$

How to model this geometries?

- **Non-commutative geometry** etc.
- All direct approaches suffer from **inconsistencies** (symmetries, causality ...)



# A conception of Quantum Gravity

---

- general problem: **quantizing general relativity** gives a non-finite (**non-renormalizable**) theory
- at close range: **spacetime** is believed to be **fuzzy**



$$\Delta x \Delta x \geq \hbar$$

How to model this geometries?

- **Non-commutative geometry** etc.
- All direct approaches suffer from **inconsistencies** (symmetries, causality ...)
- and String Theory ?

# Quantum Gravity from String Theory

---

**Why String Theory?**

# Quantum Gravity from String Theory

---

## Why String Theory?

- gravity *naturally* included
- so far the only **consistent quantum gravity**

# Quantum Gravity from String Theory

---

## Why String Theory?

- gravity *naturally* included
- so far the only **consistent quantum gravity**

So how does the quantum geometry look like?

# Quantum Gravity from String Theory

---

## Why String Theory?

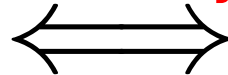
- gravity *naturally* included
- so far the only **consistent quantum gravity**

## So how does the quantum geometry look like?

- String Theory is *cheating*:



**T-Duality**



# T-duality, fluxes and “non-geometry”

---

- basic example: 3-Torus with  $H$ -flux

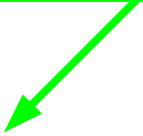
$$H \longrightarrow f \longrightarrow Q \longrightarrow R$$

# T-duality, fluxes and “non-geometry”

---

- basic example: 3-Torus with  $H$ -flux

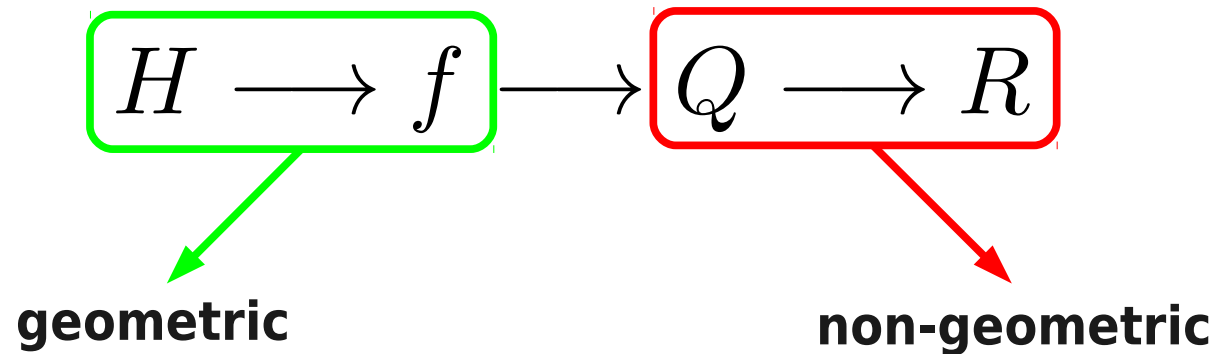
$$\boxed{H \longrightarrow f} \longrightarrow Q \longrightarrow R$$

  
**geometric**

# T-duality, fluxes and “non-geometry”

---

- basic example: 3-Torus with  $H$ -flux

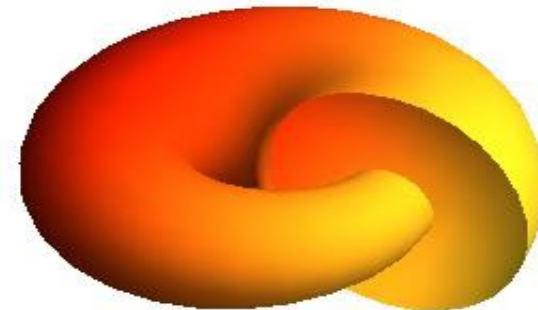
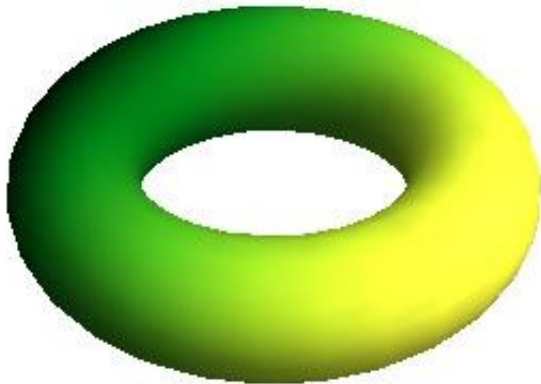
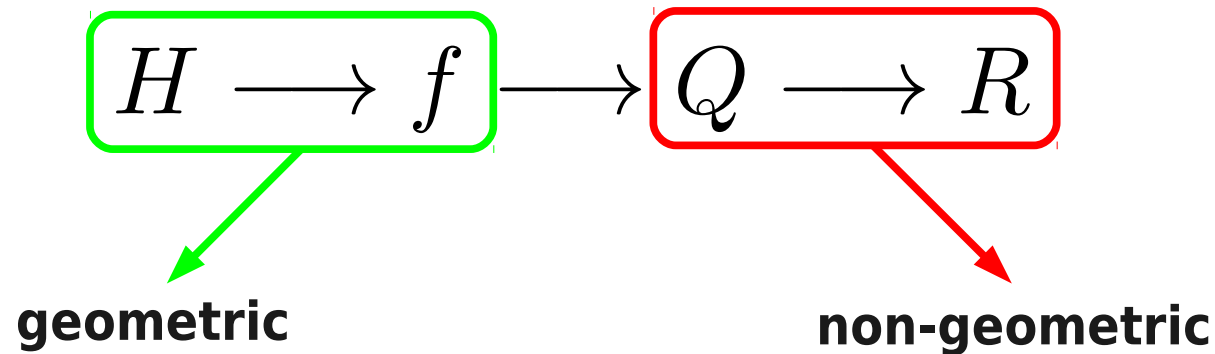




# T-duality, fluxes and “non-geometry”

---

- basic example: 3-Torus with  $H$ -flux



# T-duality, fluxes and “non-geometry”

---

- basic example: 3-Torus with  $H$ -flux

$$H \longrightarrow f \longrightarrow Q \xrightarrow{?} \boxed{R}$$

- cannot be obtained as others
- non-geometric locally
- **non-associative geometry**

# T-duality, fluxes and “non-geometry”

---

- **basic example: 3-Torus with  $H$ -flux**

$$H \longrightarrow f \longrightarrow Q \xrightarrow{?} R$$

- **$H$ - and  $R$ -flux *special***

$$H \longrightarrow \dots \longrightarrow R$$

# T-duality, fluxes and “non-geometry”

---

- basic example: 3-Torus with  $H$ -flux

$$H \longrightarrow f \longrightarrow Q \xrightarrow{?} R$$

- $H$ - and  $R$ -flux *special*

$$\boxed{H} \longrightarrow \dots \longrightarrow R$$

conventional  
geometry

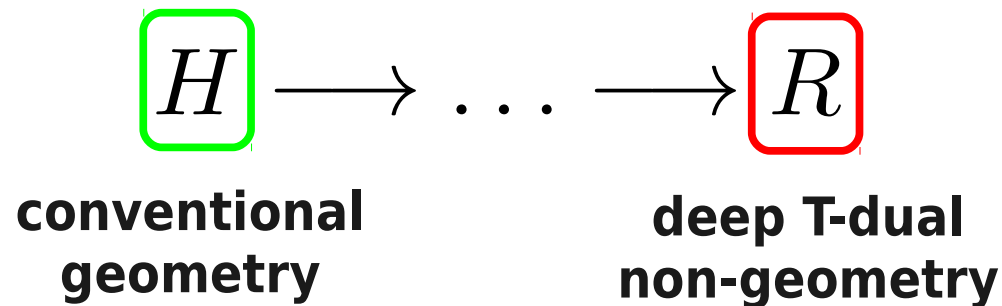
# T-duality, fluxes and “non-geometry”

---

- basic example: 3-Torus with  $H$ -flux

$$H \longrightarrow f \longrightarrow Q \xrightarrow{?} R$$

- $H$ - and  $R$ -flux *special*



# T-duality, fluxes and “non-geometry”

---

- basic example: 3-Torus with  $H$ -flux

$$H \longrightarrow f \longrightarrow Q \xrightarrow{?} R$$

- $H$ - and  $R$ -flux *special*

$$\boxed{H} \longrightarrow \dots \longrightarrow \boxed{R}$$

conventional geometry                      deep T-dual non-geometry

- $R$ -flux might describe small-scale quantum geometry

# Framework for fluxes

---

# Framework for fluxes

---

- ingredients for conventional geometry (**H-flux**)

space for points

$M$

space for *evolution*

$(TM, L)$



# Framework for fluxes

---

- ingredients for conventional geometry (**H-flux**)

space for points

$M$



manifold

space for *evolution*

$(TM, L)$

# Framework for fluxes

---

- ingredients for conventional geometry (***H-flux***)

space for points

$M$



manifold

space for *evolution*

$(TM, L)$



tangent bundle

# Framework for fluxes

---

- ingredients for conventional geometry (***H-flux***)

space for points

$M$

manifold

space for *evolution*

$(TM, L)$

tangent bundle

Lie derivative

# Framework for fluxes

- ingredients for conventional geometry (**H-flux**)

space for points

$M$

manifold

space for *evolution*

$(TM, L)$

tangent bundle

Lie derivative

$$L_X Y = [X, Y]_L$$

$$L_X \xi = d(\iota_X \xi) + \iota_X (d\xi)$$

describes the evolution of tensors  
along a path

# Framework for fluxes

---

- ingredients for conventional geometry (**H-flux**)

$$(TM, [\cdot, \cdot]_L, \partial_a)$$

# Framework for fluxes

---

- ingredients for conventional geometry (**H-flux**)

$$(TM, [\cdot, \cdot]_L, \partial_a) \implies \nabla_a, \mathcal{R}^a{}_{bcd} \dots$$

- covariant derivative
- curvature
- ...

# Framework for fluxes

---

- ingredients for conventional geometry (**H-flux**)

$$(TM, [\cdot, \cdot]_L, \partial_a) \implies \nabla_a, \mathcal{R}^a{}_{bcd} \dots$$

- ingredients for deep non-geometry (**R-flux**)

$$(T^*M, [\cdot, \cdot]_{\frac{H}{K}}, D^a)$$

# Framework for fluxes

---

- ingredients for conventional geometry (**H-flux**)

$$(TM, [\cdot, \cdot]_L, \partial_a) \implies \nabla_a, \mathcal{R}^a{}_{bcd} \dots$$

- ingredients for deep non-geometry (**R-flux**)

$$(T^*M, [\cdot, \cdot]_{\frac{H}{K}}, D^a)$$

co-tangent  
bundle





# Framework for fluxes

---

- ingredients for conventional geometry (**H-flux**)

$$(TM, [\cdot, \cdot]_L, \partial_a) \implies \nabla_a, \mathcal{R}^a{}_{bcd} \dots$$

- ingredients for deep non-geometry (**R-flux**)

$$(T^*M, [\cdot, \cdot]_{\frac{H}{K}}, D^a)$$

co-tangent  
bundle

*H*-twisted  
Koszul bracket

# Framework for fluxes

- ingredients for conventional geometry (**H-flux**)

$$(TM, [\cdot, \cdot]_L, \partial_a) \implies \nabla_a, \mathcal{R}^a{}_{bcd} \dots$$

- ingredients for deep non-geometry (**R-flux**)

$$(T^*M, [\cdot, \cdot]_K^H, D^a)$$

co-tangent  
bundle

*H*-twisted  
Koszul bracket

$$[\xi, \eta]_K^H = L_{\beta\#\xi}\eta - \iota_{\beta\#\eta}d\xi - \iota_{\beta\#\eta}\iota_{\beta\#\xi}H$$

# Framework for fluxes

- ingredients for conventional geometry (**H-flux**)

$$(TM, [\cdot, \cdot]_L, \partial_a) \implies \nabla_a, \mathcal{R}^a{}_{bcd} \dots$$

- ingredients for deep non-geometry (**R-flux**)

$$(T^*M, [\cdot, \cdot]_K^H, D^a)$$

co-tangent  
bundle

*H*-twisted  
Koszul bracket

$$[\xi, \eta]_K^H = L_{\beta^\# \xi} \eta - \iota_{\beta^\# \eta} d\xi - \iota_{\beta^\# \eta} \iota_{\beta^\# \xi} H$$

**anchor:** relates  $TM$  and  $T^*M$

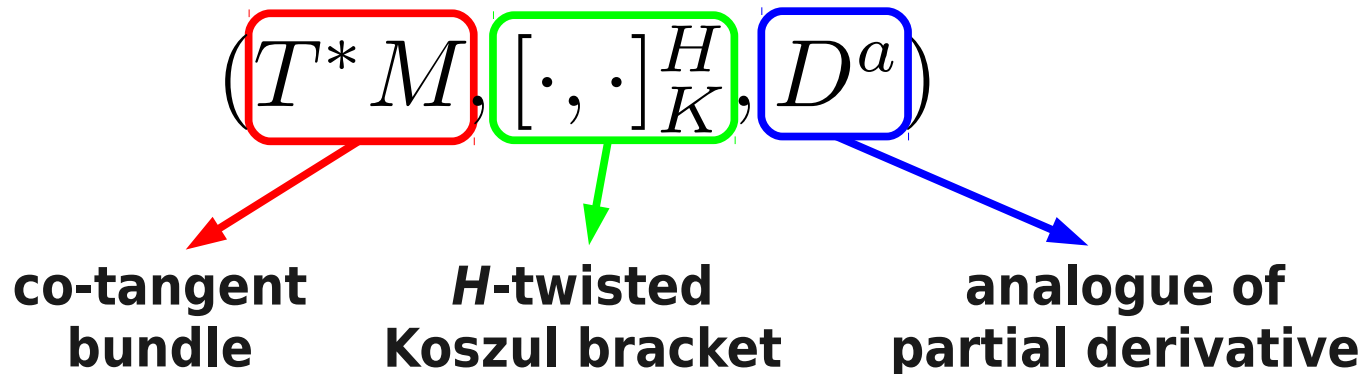
# Framework for fluxes

---

- ingredients for conventional geometry (**H-flux**)

$$(TM, [\cdot, \cdot]_L, \partial_a) \implies \nabla_a, \mathcal{R}^a{}_{bcd} \dots$$

- ingredients for deep non-geometry (**R-flux**)

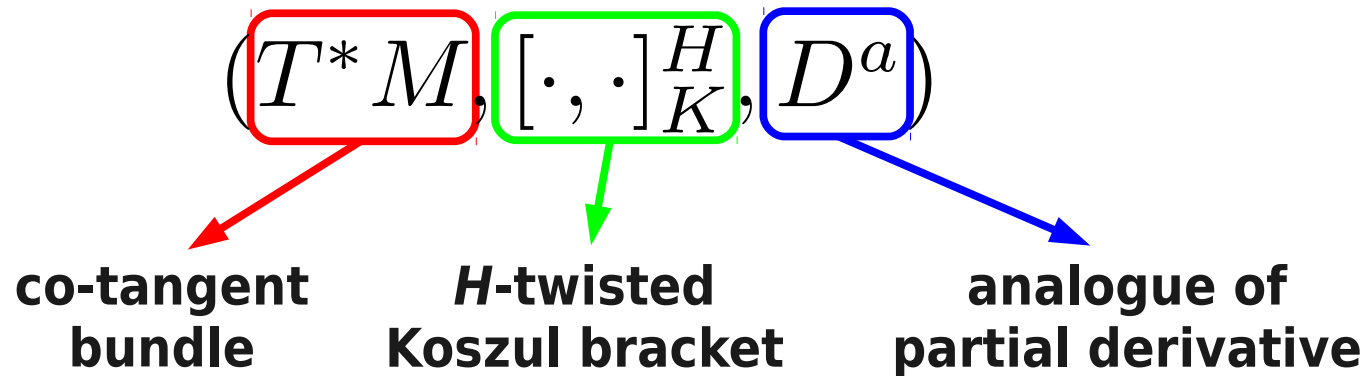


# Framework for fluxes

- ingredients for conventional geometry (**H-flux**)

$$(TM, [\cdot, \cdot]_L, \partial_a) \implies \nabla_a, \mathcal{R}^a{}_{bcd} \dots$$

- ingredients for deep non-geometry (**R-flux**)



$$D^a f = \beta^{am} \partial_m f$$

# Framework for fluxes

---

- ingredients for conventional geometry (**H-flux**)

$$(TM, [\cdot, \cdot]_L, \partial_a) \implies \nabla_a, \mathcal{R}^a{}_{bcd} \dots$$

- ingredients for deep non-geometry (**R-flux**)

$$(T^*M, [\cdot, \cdot]_{\frac{H}{K}}, D^a) \implies \hat{\nabla}^a, \hat{\mathcal{R}}_a{}^{bcd} \dots$$

# Framework for fluxes

---

- ingredients for conventional geometry (**H-flux**)

$$(TM, [\cdot, \cdot]_L, \partial_a) \implies \nabla_a, \mathcal{R}^a{}_{bcd} \dots$$

- ingredients for deep non-geometry (**R-flux**)

$$(T^*M, [\cdot, \cdot]_{\frac{H}{K}}, D^a) \implies \hat{\nabla}^a, \hat{\mathcal{R}}_a{}^{bcd} \dots$$

- both structures are instances of **Lie algebroids**

# Framework for fluxes

---

- ingredients for conventional geometry (**H-flux**)

$$(TM, [\cdot, \cdot]_L, \partial_a) \implies \nabla_a, \mathcal{R}^a{}_{bcd} \dots$$

- ingredients for deep non-geometry (**R-flux**)

$$(T^*M, [\cdot, \cdot]_{\frac{H}{K}}, D^a) \implies \hat{\nabla}^a, \hat{\mathcal{R}}_a{}^{bcd} \dots$$

- both structures are instances of **Lie algebroids**
- **non-associative geometry** apparent



# Framework for fluxes

---

- ingredients for conventional geometry (**H-flux**)

$$(TM, [\cdot, \cdot]_L, \partial_a) \implies \nabla_a, \mathcal{R}^a{}_{bcd} \dots$$

- ingredients for deep non-geometry (**R-flux**)

$$(T^*M, [\cdot, \cdot]_K^H, D^a) \implies \hat{\nabla}^a, \hat{\mathcal{R}}_a{}^{bcd} \dots$$

- both structures are instances of **Lie algebroids**
- **non-associative geometry** apparent

$$\{f, g\} = \beta(df, dg)$$

$$\{f, \{g, h\}\} + \{h, \{f, g\}\} + \{g, \{h, f\}\} = R(df, dg, dh)$$

# Strings at low energies

---

# Strings at low energies

---

- **usual fields;**  $(TM, [\cdot, \cdot]_L, \partial_a)$

$$S(G, B, \phi) = \int d^n x \sqrt{-|G|} e^{-2\phi} \left( \mathcal{R} - \frac{1}{12} H_{abc} H^{abc} + 4\partial_a \phi \partial^a \phi \right) + \dots$$

# Strings at low energies

---

- **usual fields;**  $(TM, [\cdot, \cdot]_L, \partial_a)$

$$S(G, B, \phi) = \int d^n x \sqrt{-|G|} e^{-2\phi} \left( \mathcal{R} - \frac{1}{12} H_{abc} H^{abc} + 4\partial_a \phi \partial^a \phi \right) + \dots$$

**metric**

# Strings at low energies

---

- **usual fields;**  $(TM, [\cdot, \cdot]_L, \partial_a)$

$$S(G, B, \phi) = \int d^n x \sqrt{-|G|} e^{-2\phi} \left( \mathcal{R} - \frac{1}{12} H_{abc} H^{abc} + 4\partial_a \phi \partial^a \phi \right) + \dots$$

**metric**      **Ricci scalar**

# Strings at low energies

---

- **usual fields;**  $(TM, [\cdot, \cdot]_L, \partial_a)$

$$S(G, B, \phi) = \int d^n x \sqrt{-|G|} e^{-2\phi} \left( \mathcal{R} - \frac{1}{12} H_{abc} H^{abc} + 4\partial_a \phi \partial^a \phi \right) + \dots$$

**metric**

**Ricci scalar**

**H-flux**  $H=dB$

# Strings at low energies

---

- **usual fields;**  $(TM, [\cdot, \cdot]_L, \partial_a)$

$$S(G, B, \phi) = \int d^n x \sqrt{-|G|} e^{-2\phi} \left( \mathcal{R} - \frac{1}{12} H_{abc} H^{abc} + 4\partial_a \phi \partial^a \phi \right) + \dots$$

**metric**

**Ricci scalar**

**H-flux** H=dB

**dilaton**

# Strings at low energies

---

- **usual fields;**  $(TM, [\cdot, \cdot]_L, \partial_a)$

$$S(G, B, \phi) = \int d^n x \sqrt{-|G|} e^{-2\phi} \left( \mathcal{R} - \frac{1}{12} H_{abc} H^{abc} + 4\partial_a \phi \partial^a \phi \right) + \dots$$

**symmetries: coordinate change**

**gauge transformation**  $B \rightarrow B + d\xi$



# Strings at low energies

---

- **usual fields;**  $(TM, [\cdot, \cdot]_L, \partial_a)$

$$S(G, B, \phi) = \int d^n x \sqrt{-|G|} e^{-2\phi} \left( \mathcal{R} - \frac{1}{12} H_{abc} H^{abc} + 4\partial_a \phi \partial^a \phi \right) + \dots$$

**symmetries: coordinate change**

**gauge transformation**  $B \rightarrow B + d\xi$

- **deep non-geometry;**  $(T^*M, [\cdot, \cdot]_K^H, D^a)$

$$\hat{S}(\hat{g}, \beta, \phi) = \int d^n x \sqrt{-|\hat{g}|} \det(\beta)^{-1} e^{-2\phi} \left( \hat{\mathcal{R}} - \frac{1}{12} R^{abc} R_{abc} + 4D^a \phi D_a \phi \right) + \dots$$

**metric**

**Ricci scalar**

**R-flux**  $R = d_\beta \beta$

**dilaton**

# Strings at low energies

---

- **usual fields;**  $(TM, [\cdot, \cdot]_L, \partial_a)$

$$S(G, B, \phi) = \int d^n x \sqrt{-|G|} e^{-2\phi} \left( \mathcal{R} - \frac{1}{12} H_{abc} H^{abc} + 4\partial_a \phi \partial^a \phi \right) + \dots$$

**symmetries: coordinate change**

**gauge transformation**  $B \rightarrow B + d\xi$

- **deep non-geometry;**  $(T^*M, [\cdot, \cdot]_K^H, D^a)$

$$\hat{S}(\hat{g}, \beta, \phi) = \int d^n x \sqrt{-|\hat{g}|} \det(\beta)^{-1} e^{-2\phi} \left( \hat{\mathcal{R}} - \frac{1}{12} R^{abc} R_{abc} + 4D^a \phi D_a \phi \right) + \dots$$

**symmetries: coordinate change**

**$\beta$ -diffeomorphisms**

# Strings at low energies

---

- **usual fields;**  $(TM, [\cdot, \cdot]_L, \partial_a)$

$$S(G, B, \phi) = \int d^n x \sqrt{-|G|} e^{-2\phi} \left( \mathcal{R} - \frac{1}{12} H_{abc} H^{abc} + 4\partial_a \phi \partial^a \phi \right) + \dots$$

**symmetries: coordinate change**

**gauge transformation**  $B \rightarrow B + d\xi$

- **deep non-geometry;**  $(T^*M, [\cdot, \cdot]_K^H, D^a)$

$$\hat{S}(\hat{g}, \beta, \phi) = \int d^n x \sqrt{-|\hat{g}|} \det(\beta)^{-1} e^{-2\phi} \left( \hat{\mathcal{R}} - \frac{1}{12} R^{abc} R_{abc} + 4D^a \phi D_a \phi \right) + \dots$$

**symmetries: coordinate change**

**$\beta$ -diffeomorphisms**

**a new kind of tensor**

# Strings at low energies

---

$$S(G, B, \phi) = \int d^n x \sqrt{-|G|} e^{-2\phi} \left( \mathcal{R} - \frac{1}{12} H_{abc} H^{abc} + 4\partial_a \phi \partial^a \phi \right) + \dots$$

$$\hat{S}(\hat{g}, \beta, \phi) = \int d^n x \sqrt{-|\hat{g}|} \det(\beta)^{-1} e^{-2\phi} \left( \hat{\mathcal{R}} - \frac{1}{12} R^{abc} R_{abc} + 4D^a \phi D_a \phi \right) + \dots$$

# Strings at low energies

---

$$S(G, B, \phi) = \int d^n x \sqrt{-|G|} e^{-2\phi} \left( \mathcal{R} - \frac{1}{12} H_{abc} H^{abc} + 4\partial_a \phi \partial^a \phi \right) + \dots$$



$$\hat{S}(\hat{g}, \beta, \phi) = \int d^n x \sqrt{-|\hat{g}|} \det(\beta)^{-1} e^{-2\phi} \left( \hat{\mathcal{R}} - \frac{1}{12} R^{abc} R_{abc} + 4D^a \phi D_a \phi \right) + \dots$$

# Strings at low energies

---

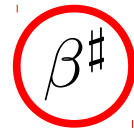
$$S(G, B, \phi) = \int d^n x \sqrt{-|G|} e^{-2\phi} \left( \mathcal{R} - \frac{1}{12} H_{abc} H^{abc} + 4\partial_a \phi \partial^a \phi \right) + \dots$$

$$\begin{array}{c} \beta = B^{-1} \\ \hat{g} = \beta^\#(G) \end{array} \quad \begin{array}{c} \uparrow \\ \beta^\# \\ \downarrow \end{array}$$

$$\hat{S}(\hat{g}, \beta, \phi) = \int d^n x \sqrt{-|\hat{g}|} \det(\beta)^{-1} e^{-2\phi} \left( \hat{\mathcal{R}} - \frac{1}{12} R^{abc} R_{abc} + 4D^a \phi D_a \phi \right) + \dots$$

# Strings at low energies

---



# Strings at low energies

---

- $\beta^\#$  translates everything!





# Strings at low energies

---

- $\beta^\#$  translates everything!

$$\widehat{\mathcal{R}}_a{}^{bcd} = (\beta^\# \mathcal{R})_a{}^{bcd}$$
$$R^{abc} = (\beta^\# H)^{abc}$$



$\beta^\#$

# Strings at low energies

---

- $\beta^\#$  translates everything!

$$\widehat{\mathcal{R}}_a{}^{bcd} = (\beta^\# \mathcal{R})_a{}^{bcd}$$
$$R^{abc} = (\beta^\# H)^{abc}$$

**full deep non-geometric  
quantum gravity**

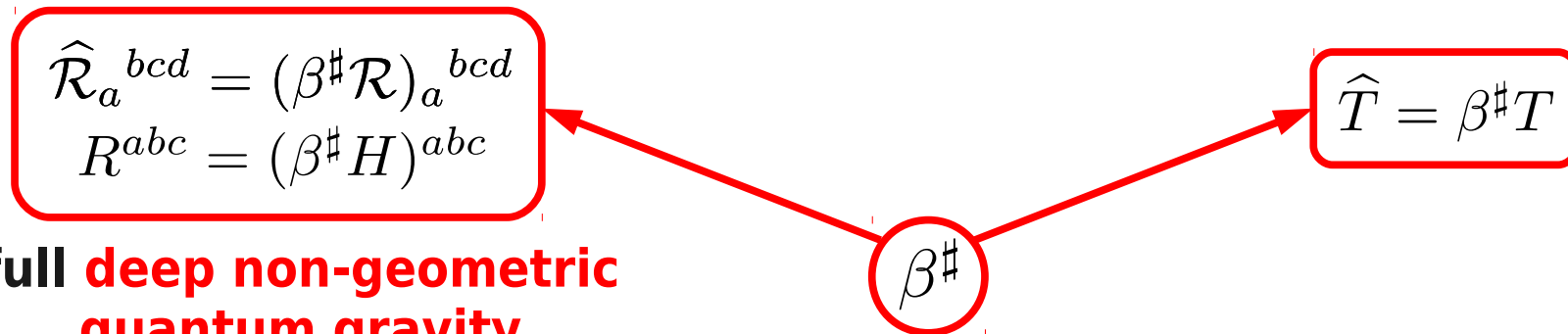


$\beta^\#$

# Strings at low energies

---

- $\beta^\#$  translates everything!



**full deep non-geometric  
quantum gravity**

# Strings at low energies

- $\beta^\#$  translates everything!

$$\begin{aligned}\widehat{\mathcal{R}}_a{}^{bcd} &= (\beta^\# \mathcal{R})_a{}^{bcd} \\ R^{abc} &= (\beta^\# H)^{abc}\end{aligned}$$

**full deep non-geometric  
quantum gravity**

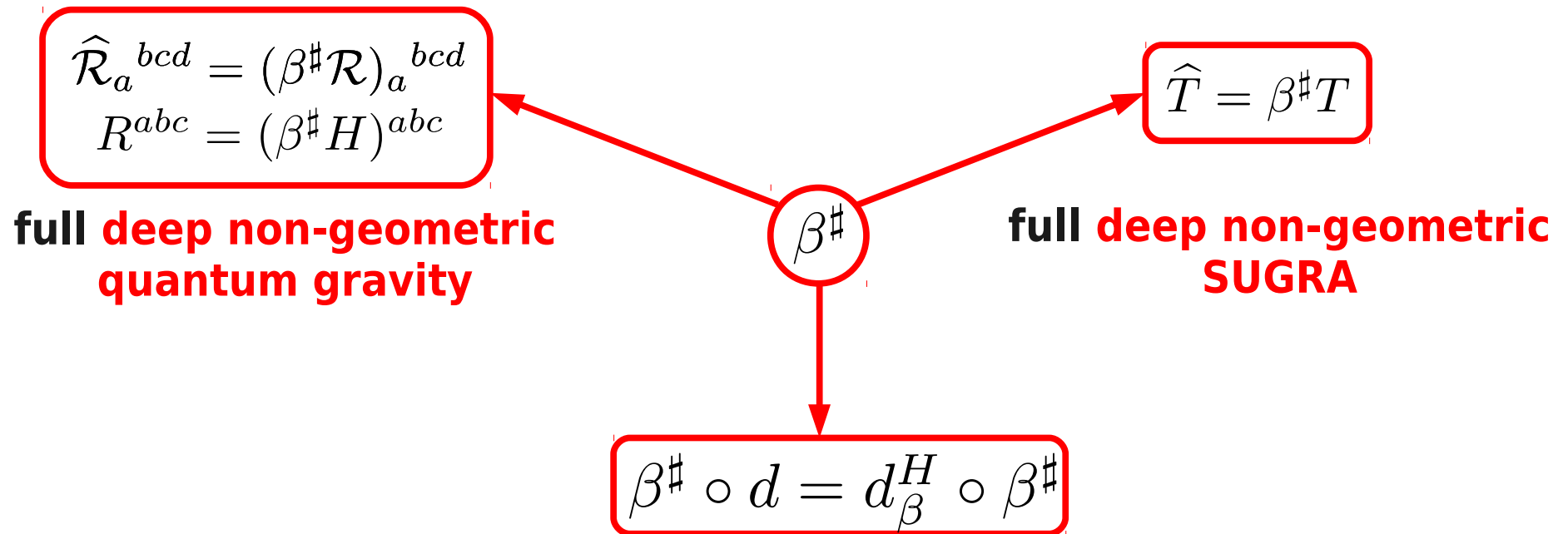
$\beta^\#$

$$\widehat{T} = \beta^\# T$$

**full deep non-geometric  
SUGRA**

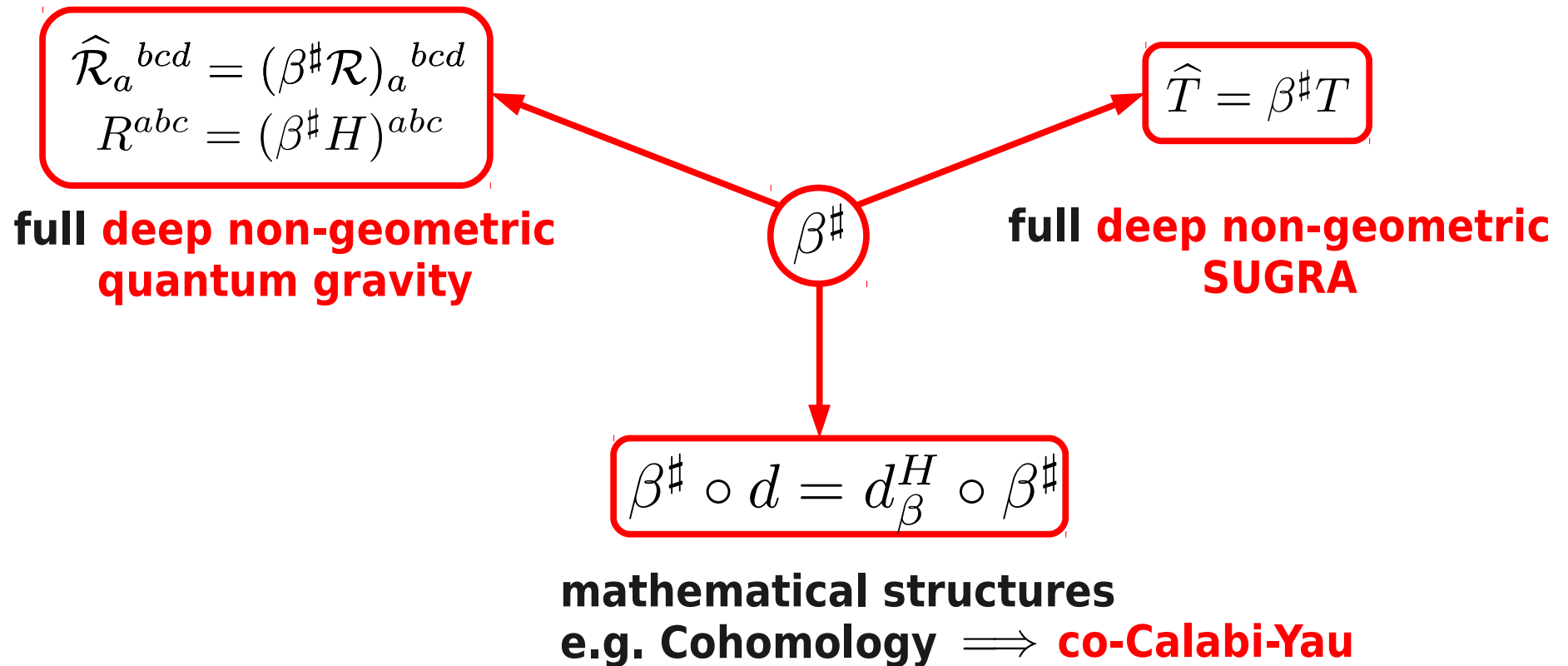
# Strings at low energies

- $\beta^\#$  translates everything!



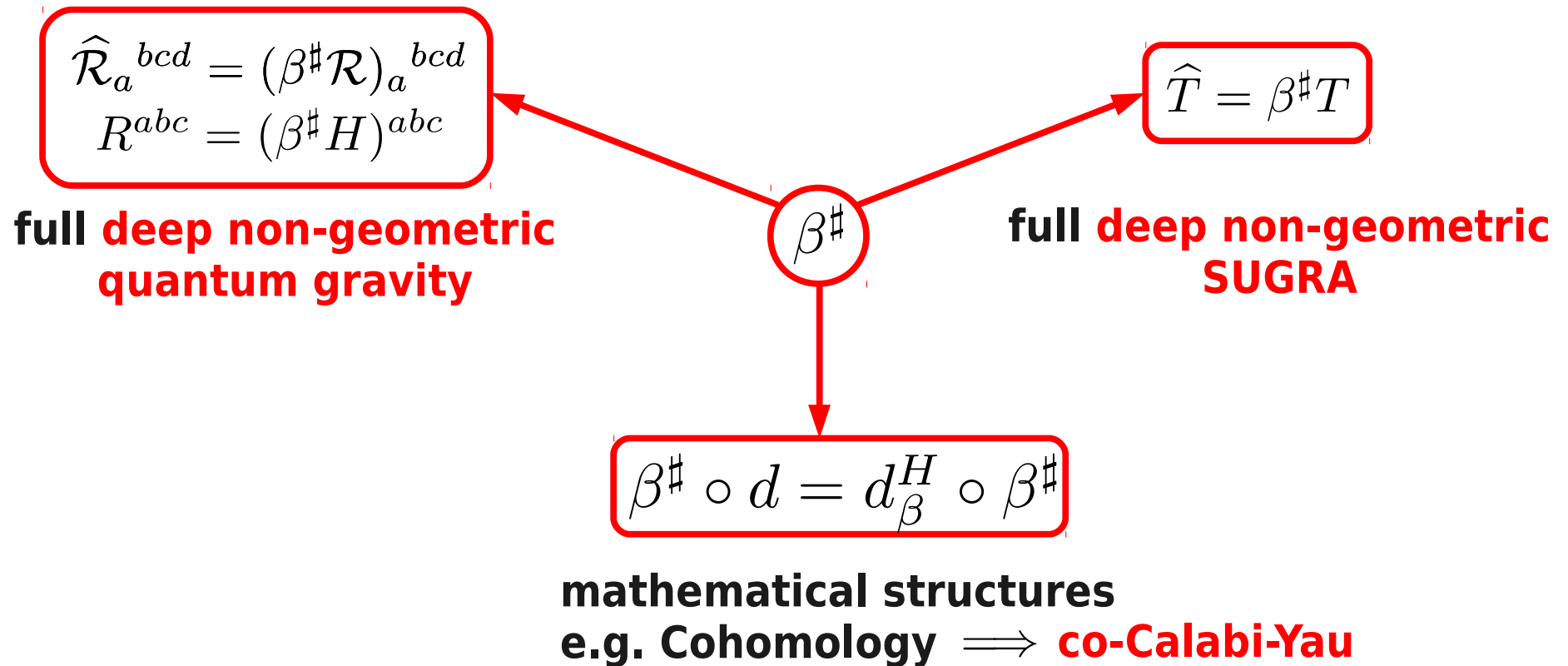
# Strings at low energies

- $\beta^\#$  translates everything!



# Strings at low energies

- $\beta^\#$  translates everything!



- EOM's are of the same form  $\implies$  non-geometric analogues

# Summary

---



# Summary

---

## ***H*-flux**

$$(TM, [\cdot, \cdot]_L, \partial_a)$$

## ***R*-flux**

$$(T^*M, [\cdot, \cdot]_K^H, D^a)$$

# Summary

---

**H-flux**

$$(TM, [\cdot, \cdot]_L, \partial_a)$$



**conventional geometry**

**R-flux**

$$(T^*M, [\cdot, \cdot]_K^H, D^a)$$



**deep non-geometry**

# Summary

---

## **H-flux**

$$(TM, [\cdot, \cdot]_L, \partial_a)$$



**conventional geometry**

$$S(G, B, \phi)$$

## **R-flux**

$$(T^*M, [\cdot, \cdot]_K^H, D^a)$$



**deep non-geometry**

$$\hat{S}(\hat{g}, \beta, \phi)$$

# Summary

**H-flux**

$$(TM, [\cdot, \cdot]_L, \partial_a)$$



**conventional geometry**

$$S(G, B, \phi)$$

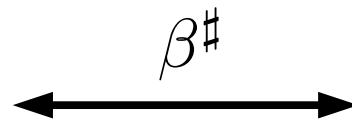
**R-flux**

$$(T^*M, [\cdot, \cdot]_K^H, D^a)$$



**deep non-geometry**

$$\hat{S}(\hat{g}, \beta, \phi)$$



# Summary

**H-flux**

$$(TM, [\cdot, \cdot]_L, \partial_a)$$



**conventional geometry**

$$S(G, B, \phi)$$

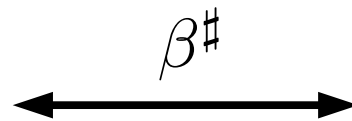
**R-flux**

$$(T^*M, [\cdot, \cdot]_K^H, D^a)$$



**deep non-geometry**

$$\hat{S}(\hat{g}, \beta, \phi)$$



# THANK YOU