Universal results from gauge / gravity duality

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arXiv: (well, still not yet) M. Ammon, J. Erdmenger, St. St.

Outline

- Generalities about gauge / gravity duality
- Universality of $\eta/s = 1/4\pi$
- What the rough observation of $\eta/s \sim 1/4\pi$ in the QGP means for string theory and what it does not.
- My stuff: A new fermionic universal result?!

Large N limit

- Take a theory with gluons.
- As always, these carry both colour and anti-colour.
- Now take N colours instead of the usual 3 in QCD.
- SU(N) gauge theory
- In the limit $N \to \infty$ this looks like a string theory (´t Hooft '74):

Image from Maldacena's talk at X'th Quark Confinement and the Hadron Spectrum, Munich 2012

- Closed strings: glueballs; open strings: mesons

Gauge / string duality

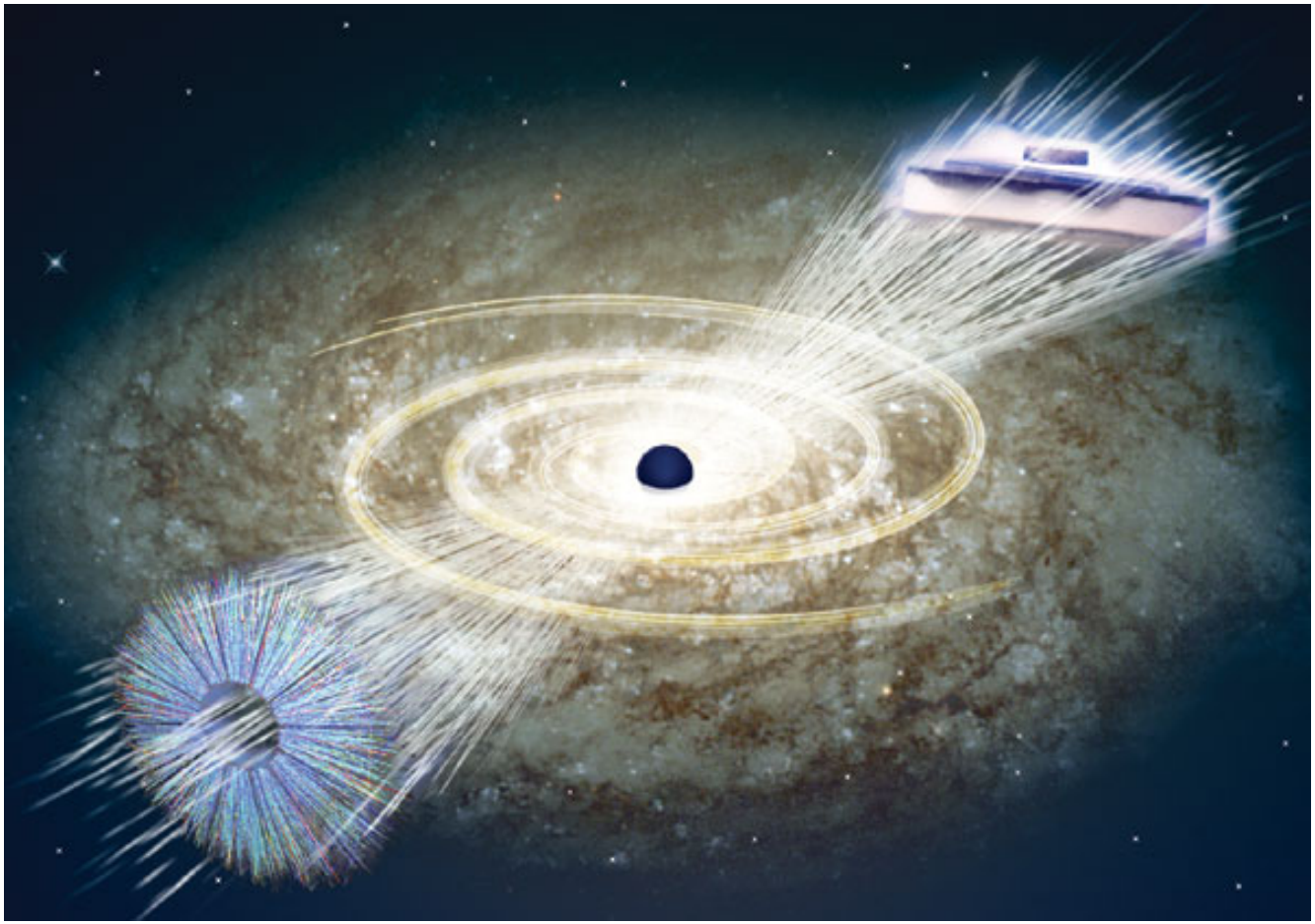
Take away message 1:

Any large N gauge theory is basically a string theory!

- But which string theory?
- In particular: conformal field theory in d dimensions
  $\rightarrow$ string theory on Anti de Sitter space in $d+1$ dimensions
- Field theory strongly coupled and string / gravity theory weakly coupled
- What can one do with it?
- Try to find results which apply to a very general class of theories, maybe even related to QCD or a condensed matter system!

Universality

Heavy-Ions and high T_C superconductor



Nature 448, 1000-1001 (30 August 2007)

Shear viscosity η and universality

- Hydrodynamics: long-distance, low-energy effective description of any interacting theory at finite temperature
- $T_{\mu\nu} = (\epsilon + p)u_\mu u_\nu + p\eta_{\mu\nu} - \sigma_{\mu\nu}$
- $\sigma_{ij} = \eta(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial_k u^k) + \zeta\delta_{ij}\partial_k u^k$
- Universal result: $\boxed{\frac{\eta}{s} = \frac{1}{4\pi}}$
- regardless of field theory being
  - conformal or not
  - confining or not
  - supersymmetric or not
  - with / without chemical potential

Universality proof (Kovtun, Son, Starinets '04)

- Absorption cross section of graviton h_{xy} in AdS black brane background (Klebanov '97; Gubser, Klebanov, Tseytlin '97):

$$\sigma_{\text{abs}}(\omega) = -\frac{16\pi G}{\omega}\text{Im}\,G^R(\omega) = \frac{8\pi G}{\omega}\int d^4x\, e^{i\omega t}\,\langle[T_{xy}(x), T_{xy}(0)]\rangle \tag{1}$$

- Use Kubo formula for η in field theory: (Policastro, Son, Starinets '01)

$$\eta = \frac{\sigma_{\text{abs}}(0)}{16\pi G} = \frac{A}{16\pi G} \tag{2}$$

for s-wave minimally coupled massless scalar ($\Box h^y_x = 0$) in spherically symmetric black hole (Das, Gibbons, Mathur '96)

- Black hole entropy $S = \frac{A}{4G}$

$$\Rightarrow \boxed{\frac{\eta}{s} = \frac{1}{4\pi}} \tag{3}$$

Its relevance.

- For QCD, current estimates from RHIC data show

$$\frac{\eta}{s} \sim (1 - 2{,}5)\frac{1}{4\pi} \tag{4}$$

- Remarkable agreement with the universal holographic result!
- Would agreement prove string theory?
- No, but it already gives strong evidence in favour of the methods being used, which originate from string theory.
- Certainly it means, we can learn something even about QCD using gauge / gravity duality!

Another message.

Take away message 2:

The universal gauge / gravity duality result for $\eta/s = 1/4\pi$ agrees remarkably well with results from QCD experiments! The precise value will neither prove nor disprove string theory. It tells something about the relevance of gauge / gravity duality!

An aside: Nutella

- In other real world systems $\eta = \eta(T)$.
- Can also get this from gauge / gravity duality by breaking rotational symmetry. (Erdmenger, Kerner, Zeller '11)

"Der Brotaufstrich unserer Nationalmannschaft"

Our project - a brief summary.

- Goal: find a (fermionic) universality result similar to the famous $\eta/s = 1/4\pi$ (Kovtun, Son, Starinets '04)
- Candidate: supersound diffusion constant D_s (Policastro '08)
- Try to look at correlator of the supersymmetry current

$$\langle [T_{\mu\nu}(x), T_{\rho\sigma}(0)] \rangle \rightarrow \langle \left\{ S^{\alpha}_{\mu}(x), \bar{S}^{\beta}_{\nu}(0) \right\} \rangle \qquad (5)$$

- Result:
  - explicitly computed for various theories (different dimensions)
  - related it to a universal absorption cross section result analogously to KSS's η/s proof

Phenomenology - what is D_s?

- spontaneous SUSY breaking due to temperature

  (Lebedev, Smilga '89; Kratzert '03)

- can see this from SUSY algebra in thermal state $\langle T_{00} \rangle_T \neq 0$

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^\mu_{\alpha\dot{\alpha}} T_{0\mu} \tag{6}$$

- Ward-Takahashi identity

$$\partial_\mu \langle T \left\{ S^\mu_\alpha, \bar{S}^\nu_{\dot{\alpha}} \right\} \rangle = \delta^4(x-y) 2 \langle T^\nu{}_\rho \rangle \sigma^\rho_{\alpha\dot{\alpha}} \tag{7}$$

- Phonino mode with pole at $\omega = v_s k - i D_s k^2$ with $v_s = \frac{p}{\epsilon}$

Background

AdS_{d+1} black branes (e.g. from D3, M2, M5)

$$ds^2 = -f(r)dt^2 + \frac{r^2}{l^2}\sum_{i=1}^{d-1} dx_i^2 + \frac{dr^2}{f(r)}\,, \qquad f(r) = \frac{r^2}{l^2} - \frac{R^d}{l^2 r^{d-2}} \tag{8}$$

from near horizon + sphere reduction of (Gibbons, Horowitz, Townsend '95)

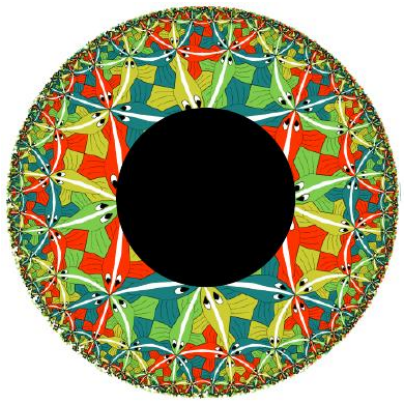


Image from Maldacena's talk at X'th Quark Confinement and the Hadron Spectrum, Munich 2012

Computation (1)

- Solve *massless* (Townsend '77) Rarita-Schwinger equation in AdS_{d+1} (for $ml = \frac{d-1}{2}$, Ψ_μ has d.o.f. of massless spin 3/2 field):

$$(\nabla\!\!\!/ + m)\Psi_\mu = 0\,, \text{ for gauge } \Gamma^\mu \Psi_\mu = 0 \tag{9}$$

- actually

$$\begin{aligned}
\gamma^d \psi_0' - \frac{i\omega}{f}\gamma^0\psi_0 - \frac{f'}{2f}\gamma^0\psi_d + \frac{f'}{4f}\gamma^d\psi_0 + \frac{ikl}{r\sqrt{f}}\gamma^1\psi_0 + \frac{d-1}{2r}\gamma^d\psi_0 + \frac{m}{\sqrt{f}}\psi_0 = 0\,,\\
\gamma^d \psi_d' - \frac{i\omega}{f}\gamma^0\psi_d - \frac{f'}{2f}\gamma^0\psi_0 + \frac{f'}{4f}\gamma^d\psi_d + \frac{ikl}{r\sqrt{f}}\gamma^1\psi_d + \frac{1}{r}\left(\frac{d+1}{2}\gamma^d\psi_d + \gamma^0\psi_0\right) + \frac{m}{\sqrt{f}}\psi_d = 0\,,\\
\gamma^d \psi_j' - \frac{i\omega}{f}\gamma^0\psi_j + \frac{f'}{4f}\gamma^d\psi_j + \frac{ikl}{r\sqrt{f}}\gamma^1\psi_j + \frac{1}{r}\gamma^j\psi_d + \frac{d-1}{2r}\gamma^d\psi_j + \frac{m}{\sqrt{f}}\psi_j = 0\,,
\end{aligned} \tag{10}$$

Computation (2)

- solve perturbatively up to first order in ω and k
- impose ingoing boundary conditions at the horizon (Son, Starinets '03)

$$\psi_d \sim (r-R)^{-3/4-\frac{i\omega}{4\pi T}}\psi_{d,0} \tag{11}$$

- identify source terms $\propto r^{\Delta-d} = r^{-1/2}$ for dimension $\Delta = \frac{1}{2}(d+2|m|)$ dual operators
- evaluate pole of retarded Green's function

Results

- Supersound velocity

$$v_s = \frac{1}{d-1} = \frac{p}{\epsilon} \quad \text{as expected for CFT}_d \tag{12}$$

- Main result: supersound diffusion constant

$$\boxed{2\pi T D_s = \frac{2^{2/d} d(d-2)}{2(d-1)^2}} \tag{13}$$

- in agreement with 4d $\mathcal{N} = 4$ SYM (Policastro '08) and (up to small numerical error) 3d results (Gauntlett, Sonner, Waldram '11)
- Transverse computation agrees precisely!

Relation to universal absorption cross sections (1)

- Can calculate closely related $D_{3/2}$ from Kubo formula

$$\bar{\epsilon}\, D_{3/2} = \frac{1}{(d-1)\mathrm{Tr}(-\gamma^0\gamma^0)} \mathrm{Tr} \left(\gamma^0 \, \mathrm{Im} \lim_{\omega,k\to 0} \langle S_T^i(\vec{k},\omega) \bar{S}_T^i(0) \rangle \right) \tag{14}$$

- Minimal coupling fermion absorption cross section:

$$\sigma_{1/2} = \frac{2\kappa^2}{\mathrm{Tr}(-\gamma^0\gamma^0)} \mathrm{Tr} \left(\gamma^0 \, \mathrm{Im} \int d^d x e^{ip.x} \langle \, S^x(x) \bar{S}^x(0) \, \rangle \right) \tag{15}$$

- Indeed these are look very similar, as for η and σ_0!
- $\exists$ similar universality result for $\sigma_{1/2}$ as $\sigma_0 = A$

Relation to universal absorption cross sections (2)

- Observation: for $\mu = 0$, the transverse gravitino is minimally coupled (!), however not massless
- Can look at *massless* gravitino in $\mathrm{AdS}_{d+1} \times \mathrm{S}^{D-d-1}$ along with *consistent truncation* condition
- Putting things together, e.g. $D_s = \left(\frac{d-2}{d-1}\right) D_{3/2}$, gives

$$\boxed{2\pi T D_s = \frac{2^{2/d} d(d-2)}{2(d-1)^2}} \tag{16}$$

from a universal absorption cross section result

Conclusion / Outlook

- We seem to have found a universality result similar to η/s, however fermionic.
- The interpretation is more difficult since the universal fermionic absorption cross section result is not as simple as the bosonic one.
- It is unclear, how general it is, e.g. if it persists for $\mu \neq 0$.

Take away messages

Take away message 1:

Any large N gauge theory is basically a string theory!

Take away message 2:

The universal gauge / gravity duality result for $\eta/s = 1/4\pi$ agrees remarkably well with results from QCD experiments! The precise value will neither prove nor disprove string theory. It tells something about the relevance of gauge / gravity duality!

Take away message 4:

Okay, whatever happened in the second part of the talk, it seems that they found a similar, however fermionic universal result!