(Quasi-) Poisson-geometry in string theory

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Reminder I: Poisson geometry and magnetic fields

$\rightarrow$ Think about phase spaces in classical mechanics

- coordinates: $\{x^i\} = \{x^1, \ldots, x^n, p_1, \ldots, p_n\}$
- observables: $f(x,p)$, e.g. Hamiltonian $H(x,p) = \frac{p^2}{2m} + V(x)$
  $\rightarrow$ Poisson bracket: $\{f,g\} = \beta^{ij}\partial_i f \partial_j g$
- Poisson-bivector $\beta = \frac{1}{2}\beta^{ij}\partial_i \wedge \partial_j$
- symplectic 2-form $\omega = \frac{1}{2}\omega_{ij}\, dx^i \wedge dx^j$
- canonical variables: $\{x^i, p_j\} = \delta^i{}_j$ $\rightarrow$ simplest (standard) symplectic structure: $\omega = dx^i \wedge dp_i$

$\rightarrow$ Math structures: Manifold M, together with a bi-vector β^{ij} with the condition $\beta^{[in}\partial_n\beta^{\underline{jk}]} = 0$ is called *Poisson*. If M has a two-form $\omega = \omega_{ij}dx^i \wedge dx^j$ which has $d\omega = 0$ then it is called *symplectic*.

Example: Particle in magnetic field

- Particle in 1-2-plane, with magnetic field in 3-direction:

$$\mathcal{L} = \frac{1}{2m}\left(p_1^2 + (p_2 + eA_2)^2 + p_3^2\right), \quad A = (0, Bx_1, 0)$$

Poisson-bracket: $\{f, g\} = \frac{\partial f}{\partial x^i}\frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial g}{\partial x^i}$
$\rightarrow$ standard symplectic structure: $\omega_{\mathcal{L}} = dx^i \wedge dp_i$

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- Describe it again by a free particle

$$\mathcal{L}' = \frac{1}{2m}\left(P_1^2 + P_2^2 + P_3^2\right)$$

Poisson bracket of the new momenta:

$$\{P_1, P_2\} = eB$$

$\rightarrow$ new symplectic structure

$$\omega_{\mathcal{L}'} = dx^i \wedge dP_i - eBdx^1 \wedge dx^2$$

Poisson structure in quantization

- Standard Poisson structure is used in *canonical quantization*:

  $\{x^i, p_j\} = \delta^i_j \to [\hat{x}^i, \hat{p}_j] = \frac{\hbar}{i}\delta^i_j \mathbf{1}$

  Associative algebra of operators in a Hilbert space

  $\Delta x^i \Delta p_i \geq \hbar$

- For general Poisson structures: *Deformation quantization*

  $\{f, g\} = \beta^{ij}\partial_i f \partial_j g$

  $f \cdot g \to f \star g = f \cdot g + i\hbar\beta^{ij}\partial_i f \partial_j g + \dots$

  Restriction for associativity: $\beta^{[\underline{i}n}\partial_n \beta^{\underline{jk}]} = 0$

  Remember: Poisson-condition

- Message: Poisson structure is needed in quantization

Reminder II: Gravity, bosonic string theory

- Manifold $\rightarrow$ spacetime $\{x^1, \dots x^n\}$
- Metric $\rightarrow$ measure distances $(\int_a^b \sqrt{g_{ij}\dot{x}^i\dot{x}^j}\,dt)$
- Connection $\nabla_i v^j = \partial_i v^j + \Gamma^j{}_{in} v^n$
  $\rightarrow$ define force-free motion (geodesic)
  - Define curvature: $R(X,Y)Z = [\nabla_X, \nabla_Y]\,Z - \nabla_{[X,Y]}Z$
  - in coordinates $R^i{}_{jkl} = \partial_l \Gamma^i{}_{kj} + \Gamma^i{}_{lm}\Gamma^m{}_{kj} - k \leftrightarrow l$
  - contract to Ricci scalar R
- Dynamics: Einstein-Hilbert action

$$\int d^n x \sqrt{-\det g}\, R$$

- Bosonic low energy string theory

$$\int d^n x \sqrt{-\mathrm{det} g}\, e^{-2\phi}\left(R - \frac{1}{12}H^2 + 4(\partial\phi)^2\right)$$

- ϕ: Dilaton, $H_{ijk} = \partial_{[i}B_{jk]}$: Field strength of string B-field

Problems...

- ...like in every classical field theory: Curvature singularities and all that...
  $\rightarrow$ Hope to resolve it in a theory of quantum gravity, e.g. by uncertainty relations, like

$$\Delta x^i \Delta x^j \geq l_s^2$$

- A Poisson structure on (classical) spacetime is needed for quantization!

  A. Connes, '94: "*A Poisson structure on a manifold is none other than the indication of the existence of a deformation parameter $\hbar$ of the algebra of functions*"

- Way to introduce Poisson structure only at quantum level: Noncommutative gravity (J.Wess et al.)

$$\Gamma^{\star\,i}{}_{jk} = \frac{1}{2} g^{im} \star \left(\partial^\star_j g_{mk} + \partial^\star_k g_{mj} - \partial^\star g_{jk}\right)$$
$$\int d^n x \, e^\star \star R$$

- BUT: Poisson structure arises just as a set of parameters, no clear physical interpretation (as for example in noncommutative gauge theories)

An Idea

T-duality in string theory relates large scales to small scales and in addition the string B-field to a bi-vector $\beta^{ij}\partial_i \wedge \partial_j$.

$\rightarrow$ Interprete it as a (Quasi-)Poisson structure on a manifold

Fact from math: A Poisson structure relates tangent and cotangent bundles of a manifold. Roughly: Makes a vector out of a form:

$$\beta(dx^i) = \beta^{ij}\partial_j$$

$\rightarrow$ Try to do differential geometry with forms instead of vectors in order to include the Poisson structure.

A glance at our work

R.Blumenhagen, A.Deser, E.Plauschinn, F.Rennecke, arXiv: 1210.1591, 1211.0030

- Ingredients: Extend differential geometry from the tangent bundle to a *Lie algebroid*

$\partial_a f$	$D^a f = \beta^{an}\partial_n f$
$g_{ab} = g(\partial_a, \partial_b)$	$\hat{g}^{ab} = \hat{g}(dx^a, dx^b)$
$[X, Y]_{\mathrm{Lie}}$	$[\xi, \eta]_{KS}$
$(X^i\partial_i Y^j - Y^i\partial_i X^j)\partial_j$	$(\xi_m D^m \eta_p - \eta_m D^m \xi_p - \xi_m\eta_n Q_p{}^{mn})\, dx^p$
$\nabla_m X^n = \partial_m X^n + \Gamma^n{}_{mp}X^p$	$\hat{\nabla}^m X^n = D^m X^n - \hat{\Gamma}_p{}^{mn}X^p$
$\Gamma^c{}_{ab} = \frac{1}{2}g^{cn}\left(\partial_a g_{bn} + \partial_b g_{an} - \partial_n g_{ab}\right)$	$\hat{\Gamma}_c{}^{ab} = \tilde{\Gamma}_c{}^{ab} - \hat{g}_{cq}\hat{g}^{p(a\vert}\mathcal{Q}_p{}^{\vert b)q} + \frac{1}{2}\mathcal{Q}_c{}^{ab}$
$R^m{}_{cab} = \partial_a\Gamma^m{}_{bc} - \partial_b\Gamma^m{}_{ac} + \Gamma^n{}_{bc}\Gamma^m{}_{an} - \Gamma^n{}_{ac}\Gamma^m{}_{bn}$	$\hat{R}_m{}^{cab} = D^a\hat{\Gamma}_m{}^{bc} - D^b\hat{\Gamma}_m{}^{ac} + \hat{\Gamma}_n{}^{bc}\hat{\Gamma}_m{}^{an} - \hat{\Gamma}_n{}^{ac}\hat{\Gamma}_m{}^{bn} - \mathcal{Q}_n{}^{ab}\hat{\Gamma}_m{}^{nc}$

- We can write down an action similar to the ordinary Einstein-Hilbert action!

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- Gravity with Poisson structure

$$\int d^n x \sqrt{-\det \hat{g}}\,(\det \beta^{-1})\;\hat{R}$$

- Same procedure with bosonic string theory

$$S = \int d^n x\,\sqrt{-\det g}\,e^{-2\phi}\Big(R - \tfrac{1}{12}H^2 + 4(\partial\phi)^2\Big)$$

$$\hat{S} = \int d^n x\,\sqrt{-\hat{g}}\,(\det \beta^{-1})\;e^{-2\phi}\Big(\hat{R} - \frac{1}{12}\Theta^2 + 4\,(D\phi)^2\Big)$$

- It turns out, that both actions are related by a field redefinition of the metric and B-field

$$\beta^{ij} = (B^{-1})^{ij}$$

$$\hat{g}^{ij} = \beta^{im}\beta^{jn}g_{mn}$$

Conclusion/discussion

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We constructed a gravity/low energy string theory on a Poisson manifold. The Poisson structure is connected to the stringy B-field and has non-trivial dynamics.

Nice features of this procedure

- Diffeomorphism invariance (as it should be for gravity)
- Invariance under an additional symmetry coming from gauge symmetries of stringy B-field
- Poisson structure appears at the classical level ($\rightarrow$ Deformation quantization?)
- Extendible to full superstring theory

Strange (interesting) feature

- Poisson manifolds are always even dimensional $\rightarrow$ what is the odd dimensional analogue of our procedure?

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Thank you